Studies on
Verification of Wireless Sensor Networks
and
Abstraction Learning for System Inference

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Chapter 1

Introduction

In daily life, computers are omnipresent. As a matter of fact, we are surrounded by computers, which we use to search for or share information on the Internet, to communicate via email and social networks, to keep track of financial data, or to express ourselves by writing blogs or by podcasting our voices. The vast majority of computers in use, however, are much less visible: They run the engine, brakes, seat belts, airbags, and audio system in our cars and control aircrafts and trains. They digitally encode our voices and construct radio signals to send from our cell phones to a base station. They control our microwave oven, refrigerator, and dishwasher. They command robots on a factory floor, power generation in a power plant, processes in a chemical plant, and traffic lights in a city. They search for microbes in biological samples, construct images of the inside of a human body, and measure vital signs. These less visible computers are called embedded systems.

Embedded systems are redefining how we perceive and interact with the physical world. Since the beginning of the 21st century, our society has witnessed a great increase in technological advances, and this has affected our lifestyle in various ways. On the one hand the young toddlers learn to deal with a wide variety of tools and devices, from digital toys to electronic games, as a part of the world in which they are supposed to live; on the other hand, the elderly adapt themselves appropriately to interact with every-day-coming new machines which are to make life easier. We go jogging with an iPod in our pocket which not only plays music for us but also computes our speed.

Strong demand for adding more features to software applications has led to much larger and more complex embedded systems. While mission-critical embedded applications raise obvious reliability concerns, unexpected or premature failures in even noncritical applications such as game boxes and portable video players can erode a manufacturer’s reputation and greatly diminish acceptance of new devices. The advent of more sophisticated embedded systems that support more powerful functions have brought reliability concerns to the forefront. The design of reliable systems requires assuring that the system never moves through
a dangerous state and verification and validation is the key. Indeed, verification and validation (V & V) is the process of checking that a software system meets its specifications and that it fulfills its intended purpose. Testing is used in association with verification and validation to disclose possible errors of the system, whereas model checking and theorem proving are using mathematical methods to prove the correctness of the model of the system.

In almost every engineering discipline, models have been used to manage system complexity. Developers have employed them as reusable and analyzable artifacts to bridge the conceptual gap between requirements and target-system implementations. Model-based development (MBD) relies on the use of explicit models to describe development activities and products. Applying MBD in embedded-system applications development encourages practitioners to use testing techniques that take another track than that of traditional techniques. The construction of models typically requires significant manual effort, implying that in practice often models are not available, or become outdated as the system evolves. Automated support for constructing behavioral models of implemented components would therefore be extremely useful.

This dissertation approaches V & V of embedded systems with two different analogies: in the first part, modeling and verification of a real-world case-study provided by the Chess eT International B.V. is described, whereas the second part investigates automata learning (automatic modeling of systems) using abstraction refinement.

This introduction is organized, as follows. In section 1.1, embedded systems are briefly introduced and a short account of their V & V challenges is presented, then in section 1.2 Chess case is informally described. Afterwards, section 1.3 presents automata learning and describes the problem which is explained in the second part of the thesis. Finally, the structure of this thesis is outlined in section 1.4.

1.1 V & V of Embedded Systems

Embedded systems are information processing systems embedded into enclosing products [60]. In other words, embedded systems are integrated hardware/software systems built into devices that are not necessarily recognized as computerized devices or computers. However, these embedded processing units control and actually define the functionality and quality of these devices. Embedded systems are typically not monolithic, but consist of multiple processing units, connected through wired or wireless networks. The size of the system components ranges from tiny battery-powered intelligent sensors and actuators, to large multiple-rack computing devices. These distributed or networked systems come with a large number of common characteristics, including real-time constraints, and dependability as well as efficiency requirements and strict resource constraints, and ranging from limited energy supply, memory and processing power to space and weight constraints. The
design of embedded systems is therefore intrinsically a multi-disciplinary activity, requiring skills from computer science, electronics and mechatronics and control, along with a thorough understanding and interaction with the application field. See [44].

For embedded systems, the link to physics and physical systems is rather important. Accordingly, the term cyber-physical systems (CPS) was coined by Lee [61] to refer to the integration of computation with physical processes. Cyber-physical systems are embedded computers and networks monitoring and controlling the physical processes, usually with feedback loops where physical processes affect computations and vice versa. Unlike more traditional embedded systems, a full-fledged CPS is typically designed as a network of interacting elements with physical input and output instead of as standalone devices [62]. This notion is closely tied to concepts of robotics and sensor networks.

In connection with the concept of cyber-physical systems, embedded systems are defined as integrations of software and hardware where the software reacts to sensory data and/or issues commands to actuators. The physical system is an integral part of the design and the software must be conceptualized to act jointly with that physical system. Physical systems are intrinsically concurrent and temporal. Actions and reactions happen simultaneously and over time, and the metric properties of time are an essential part of the behavior of the system [59]. In embedded software, therefore, time matters and concurrency and interaction with hardware are intrinsic, since embedded software engages the physical world in non-trivial ways (more than keyboards and screens).

Our reliance on embedded systems makes their reliable operation of large social importance. Besides, offering a good performance in terms of response times, processing capacity and the absence of annoying errors is one of the major quality indications. Consequently, there is a crucial need for formalisms, techniques, and tools that enable the efficient design of correct and well-functioning systems despite their complexity, and make it possible to guarantee their correctness, during maintenance. The availability of such tools could contribute to significant savings in the time and the cost of developing and certifying embedded systems. For safety-critical systems, these tools could save lives (e.g., in avionics and military systems).

Prevailing industrial practice in embedded software relies on bench testing for concurrency and timing properties. This has worked reasonably well, because programs are small, and because the software gets encased in a box with no outside connectivity that can alter the behavior of the software. However, applications today demand that embedded systems be feature-rich and networked, so bench testing become inadequate. In a networked environment, it becomes impossible to test the software under all possible conditions, because the environment is not known. Moreover, general-purpose networking techniques themselves make program behavior much more unpredictable.

*Formal methods* are a particular kind of mathematically-based techniques for
the specification, development and verification of software and hardware systems, where models are created that can be rigorously analyzed using techniques from mathematics and logic that provide objective evidence of well-formedness, soundness and completeness of the model. Formal methods can help detect flaws, and in the process can improve the understanding that a designer has of the behavior of a complex system.

In the coming subsections, a brief description of modeling and verification of embedded systems is presented.

1.1.1 Modeling

*Specification* is a description of how a system is supposed to behave. Specifications for embedded systems provide models of the system under design (SUD). A *model* is a simplification of another entity, which can be a physical thing or another model [50]. The model contains those characteristics and properties of the modeled entity that are relevant for a given task.

Working with models has a major advantage. Models can have formal properties. We can say definitive things about models. For example, we can assert that a model is deterministic, meaning that given the same inputs it will always produce the same outputs. If our model is a good abstraction of the physical system, then the definitive assertion about the model gives us confidence in the physical realization of the system. Such confidence is hugely valuable, particularly for embedded systems where malfunctions can threaten human lives. Studying models of systems gives us insight into how those systems will behave in the physical world.

Real-life systems are distributed, concurrent systems composed of components. It is therefore necessary to be able to specify concurrency conveniently. Components must be able to communicate and to synchronize. Furthermore, many embedded systems are real-time systems. Therefore, explicit timing requirements are one of the characteristics of embedded systems. The behavior of time-critical systems is typically subject to rather stringent timing constraints. For a train crossing it is essential that on detecting the approach of a train, the gate is closed within a certain time bound in order to halt car and pedestrian traffic before the train reaches the crossing. For a radiation machine the time period during which a cancer patient is subjected to a high dose of radiation is extremely important; a small extension of this period is dangerous and can cause the patients death. See [65].

Classical finite state machines do not provide information about time. In order to model time, classical automata have been extended to also include timing information. *Timed automata* [4] are essentially automata extended with real-valued variables. The variables model the logical clocks in the system, that are initialized with zero when the system is started, and then increase synchronously with the same rate. Clock constraints, i.e. guards on edges, are used to restrict the behavior of the automaton. *UPPAAL* [17] is an integrated tool environment for
modeling, validation and verification of real-time systems modeled as networks of
timed automata, extended with data types (bounded integers, arrays etc.).

1.1.2 Verification

Verification is the process of proving or demonstrating that the program correctly
complies to its specification. In a more formal way, verification shows that the
program satisfies a given specification by a (mathematical) proof. Briefly, system
verification is used to establish that the design or product under consideration
possesses certain properties. The properties to be validated are mostly obtained
from the system’s specification. A defect is found once the system does not fulfill
one of the specification’s properties. The system is considered to be “correct”
whenever it satisfies all properties obtained from its specification. Correctness is
always relative to a specification, and is not an absolute property of a system.
Model-based verification techniques are based on models describing the possible
system behavior in a mathematically precise and unambiguous manner. The sys-
tem models are accompanied by algorithms that systematically explore all states
of the system model. This provides the basis for a whole range of verification tech-
niques ranging from an exhaustive exploration (model checking) to experiments
with a restrictive set of scenarios in the model (simulation), or in reality (testing).

Model checking is a formal verification technique which allows for desired be-
havioral properties of a given system to be verified on the basis of a suitable model
of the system through systematic inspection of all states of the model. The attrac-
tiveness of model checking comes from the fact that it is completely automatic -i.e.
the learning curve for a user is very gentle- and that it offers counterexamples in
case a model fails to satisfy a property serving as indispensable debugging inform-
ation. Model checking requires a model of the system under consideration and
a desired property and systematically checks whether or not the given model sat-
sifies the property. Typical properties that can be checked are deadlock freedom,
invariants, and request-response properties [10]. Model checking is an automated
technique to check the absence of errors (i.e. property violation) and alternatively
can be considered as an intelligent and effective debugging technique.

With simulation, a model of the system at hand is constructed and simulated.
Based on inputs, execution paths of the system are examined using a simulator. A
mismatch between the simulators output and the output described in the specification
determines the presence of errors. Simulation is like testing, but is applied
to models. It suffers from the same limitations, though: the number of scenarios
to be checked in a model to get full confidence goes beyond any reasonable subset
of scenarios that can be examined in practice. In the other words, the main lim-
itation of simulation is that there is no guarantee that the corner cases are ever
reached (would require possibly infinite number of simulation runs, i.e. when real
values, concurrency or non-determinism are in the model).
1.1.3 Testing

Testing is the process of exercising a product to verify that it satisfies specified requirements or to identify differences between expected and actual results. In testing the implementation of the system is taken as already realized and is stimulated with certain (hopefully well-chosen) inputs and the reaction of the system is observed. Whereas verification proves conformance with a given specification, testing finds cases where a program does not meet its specification. It is important to note, that testing can never be complete, since it is built up solely of observations. Hence, only a small subset of all possible instances of a systems behavior can be taken into consideration. The purpose of testing is to make sure that a manufactured embedded system behaves as intended. Testing can be done during or after the fabrication (fabrication testing) and also after the system has been delivered to the customer (field testing).

Software testing is a dynamic technique that actually runs the system. Software testing constitutes a significant part of any software engineering project. Usually, between 30% and 50% of the total software project costs are devoted to testing [10]. Testing takes the piece of software under consideration and provides its compiled code with input, called tests. Correctness is thus determined by forcing the software to traverse a set of execution paths, sequences of code statements representing a run of the software. Based on the observations during the test execution, the actual output of the software is compared to the output as documented in the system specification. Although test generation and test execution can partly be automated, the comparison is usually performed by human-beings. The main advantage of testing is that it can be applied to all sorts of software, ranging from application software (e.g., e-business software) to compilers and operating systems. As exhaustive testing of all execution paths is practically infeasible; in practice only a small subset of these paths is treated. Testing can thus never be complete. That is to say, testing can only show the presence of errors, not their absence.

Software testing methods are traditionally divided into white- and black-box testing. White-box testing is when the tester has access to the internal data structures and algorithms including the code that implement these. Black-box testing treats the software as a “black-box” i.e. without any knowledge of internal implementation. These two approaches are used to describe the point of view that a test engineer takes when designing test cases.

Hardware testing is achievable through hardware description languages (HDL). One of the first steps in hardware design is to write down the logical structure and behavior of the circuit using an HDL which is a software-like language. This logical description is later compiled into circuit elements. Functional design verification aims to find problems at this early stage of design by analyzing the description written in HDL. The behavior and the structure of designs are usually so complex that, in many cases, the design is not entirely correct and will behave differently
than expected once implemented as a circuit. Since it is prohibitively expensive to fix problems after the design is fabricated, functional design verification proves to be indispensable by finding problems early on, before additional work is done on the design. With automatic test vector generation (see for example \cite{26,8,55}), it is possible to run more tests more often and earlier in the development process which probably results in improved quality of the system. The test vector generation \cite{25} produces a set of test vectors that include the inputs, expected outputs, and requirement traceability link. Boundary-scan, as defined by the IEEE Std.-1149.1 \cite{94} standard, is an integrated method for testing interconnects on printed circuit boards (PCBs) that are implemented at the integrated circuit (IC) level.

Testing embedded/cyber-physical systems in their real environment may be dangerous. For example, testing control software in a nuclear power plant can be a source of serious, far-reaching problems. Model-based testing is the application of model-based design for designing and optionally executing the necessary artifacts to perform software testing. Models can be used to represent the desired behavior of the System Under Test (SUT), or to represent the desired testing strategies and testing environment. Innovative work is needed to make effective connections to the design environments and tools that can produce formal models automatically for embedded systems.

### 1.2 The Chess Wireless Sensor Network

The next evolutionary development step in building, utilities, industrial, home, shipboard, and transportation systems automation is represented by smart environments. Like any reactive system, the smart environment relies first and foremost on sensory data from the real world. Sensory data comes from multiple sensors of different modalities in distributed locations. The smart environment needs information about its surroundings as well as about its internal workings. Most information needed by smart environments is provided by sensor networks, which are responsible for sensing as well as for the first stage of the processing hierarchy. The challenges in monitoring the environment, detecting the relevant events, collecting the data, assessing and evaluating the information, formulating meaningful reports for the users, and performing decision-making and alarm functions are enormous.

A wireless sensor network (WSN) is a collection of nodes organized into a cooperative network. Each node has a processing capability (one or more microcontrollers, CPUs or DSP chips), may contain multiple types of memory (program, data and flash memories), has a RF transceiver (usually with a single omnidirectional antenna), has a power source (e.g., batteries and solar cells), and accommodate various sensors and actuators. The nodes communicate wirelessly and often self-organize after being deployed in an ad hoc fashion. Systems of 1000s or even 10,000 nodes are anticipated. Wireless sensor networks are currently beginning to
be deployed at an accelerated pace. Such systems can revolutionize the way we live and work. It is not unreasonable to expect that in 10-15 years that the world will be covered with wireless sensor networks which are connected to the Internet.

This new technology is exciting with unlimited potential for numerous application areas including environmental, medical, military, transportation, entertainment, crisis management, homeland defense, and smart spaces. The proper design of wireless sensor networks is challenging as it requires an broad breadth of knowledge from a wide variety of disciplines, such as communications, wireless technologies, smart sensors, self-organization and signal processing.

An effective protocol for wireless sensor networks must consume little power, avoid collisions, be implemented with a small code size and memory requirements, be efficient for a single application, and be tolerant to changing radio frequency and networking conditions. One of the greatest challenges in the design of wireless sensor networks is to find suitable mechanisms for clock synchronization.

The Chess eT International B.V. has developed a WSN platform using a gossip (epidemic) communication model. Gossiping in distributed systems refers to the repeated probabilistic exchange of information between two members. In gossip networks, information can spread within a group just as it would in real life. The main advantage of gossiping is that the absence of explicit routing provides a potentially scale free network, whereby message flooding provides robustness. In order to meet strict energy constraints, Chess used a Time Division Multiple Access (TDMA) protocol where the period in which nodes are active is limited and for the remainder of the time, nodes switch to an energy saving mode. One of the greatest challenges in the design of communication protocols is to find suitable mechanisms for clock synchronization: we must ensure that whenever some node is sending all its neighbors are listening. Each wireless sensor node has a low-cost 32 KHz crystal oscillator driving an internal clock used to determine the start and end of each slot. The TDMA time slot boundaries might drift (i.e. oscillators are sensitive to temperature changes) that makes the nodes go out of sync. Many clock synchronization protocols have been proposed for WSNs. In most of these protocols, clocks are synchronized to an accurate real-time standard like Universal Coordinated Time (UTC). However Chess has employed a different approach in which a node only needs to be synchronized to its immediate neighbors, not to faraway nodes or to UTC. In the first part of this thesis, clock synchronization in the Chess WSN is studied.

1.3 Automata Learning

The construction of models typically requires specialized expertise. It is time consuming and involves significant manual effort, implying that in practice often models are not available, or become outdated as the system evolves. In practice, 80% of software development involves legacy code, for which only poor documentation is available. Manual construction of models of legacy components is typically
very labor intensive and often not cost effective. The possible solution that is investigated in this thesis is to infer models automatically through observations and test, that is, through black-box reverse engineering.

The problem of inducing, learning or inferring grammars and automata has been studied for decades, but only in recent years grammatical inference a.k.a. grammar induction has emerged as an independent field. Grammatical inference techniques aim at building a grammar or automaton for an unknown language, given some data about this language. Within the setting of active learning, it is assumed that a learner interacts with a teacher. Inspired by the work of Angluin [6] on the $L^*$ algorithm, Niese [76] developed an adaptation of the $L^*$ algorithm for active learning of deterministic Mealy machines. This algorithm has been further optimized in [83]. In the algorithm it is assumed that the teacher knows a deterministic Mealy machine $M$. Initially, the learner only knows the action signature (the sets of input and output symbols $I$ and $O$) and her task is to learn a Mealy machine that is equivalent to $M$. The teacher will answer two types of questions – output queries (“what is the output generated in response to input $i \in I^*$?”) and equivalence queries (“is a hypothesized machine $H$ correct, i.e., equivalent to the machine $M$?”). The learner always records the current state $q$ of Mealy machine $M$. In response to query $i$, the current state is updated to $q'$ and answer $o$ is returned to the learner. At any point the learner can “reset” the teacher, that is, change the current state back to the initial state of $M$. The answer to an equivalence query $H$ is either yes (in case $M \approx H$) or no (in case $M \not\approx H$). Furthermore, the teacher will give the learner a counterexample that proves that the learner’s hypothesis is wrong with every negative equivalence query response, that is, an input sequence $u \in I^*$ such that $obs_M(u) \neq obs_H(u)$. This algorithm has been implemented in the LearnLib tool [83].

State-of-the-art tools for active learning of state machines are able to learn state machines with at most in the order of 10,000 states. This is not enough for learning models of realistic software components which, due to the presence of program variables and data parameters in events, typically have much larger state spaces.

Abstraction is the key when learning behavioral models of realistic systems. Hence, in most practical applications where automata learning is used to construct models of software components, researchers manually define abstractions which, depending on the history, map a large set of concrete events to a small set of abstract events that can be handled by automata learning tools. Recently, Aarts, Jonsson & Uijen have proposed a framework for regular inference with abstraction in which, depending on the history, a large set of concrete events is mapped to a small set of abstract events [1]. Using this framework they succeeded to automatically infer models of several realistic software components with large state spaces, including fragments of the TCP and SIP protocols.

In the second part of this thesis, it is shown how such abstractions can be constructed fully automatically for a class of extended finite state machines in
Introduction

which one can test for equality of data parameters, but no operations on data are allowed. This aim is reached through counterexample-guided abstraction refinement: whenever the current abstraction is too coarse and induces nondeterministic behavior, the abstraction is refined automatically. Using a prototype implementation of the algorithm, models of several realistic software components, including the biometric passport and the SIP protocol were learned fully automatically.

1.4 Thesis Statement

This thesis is funded by the NWO project ARTS, Abstraction Refinement for Timed Systems. In 2008, when I started as a PhD candidate, the european project Quasimodo was started which introduced several industrial challenges including Chess wireless sensor network case. In line with that project, we planned to use Counterexample-guided Abstraction Refinement (CEGAR) to verify the synchronization of an arbitrary size WSN. Although we succeeded to use invariant proof techniques to verify an arbitrary size WSN with clique topology, and despite the fact that we discovered a potential flaw in Chess implementation, after two years of working on the project, we did not succeed to use CEGAR method in order to conquer the state space explosion problem when verifying the Chess synchronization protocol. However, in 2010, a new research project started in MBSD in the field of automata learning, which seemed to be a good place for performing CEGAR method. Within that project, we decided to design and implement a CEGAR-based algorithm which is capable of fully automatically learning a class of parametric systems. Correspondingly, this dissertation is organized in two parts.

Part one is written based on the publications


In part one, the industrial case-study of Chess on wireless sensor networks is investigated. UPPAAL is used for modeling and verification of two synchronization algorithms for wireless sensor networks. Chapter 2 introduces Chess WSN, in detail. Afterwards, in chapter 3 a synchronization algorithm for wireless sensor
networks is fully described. Furthermore, the timed automata model of the synchronization protocol is depicted in detail and it is shown how UPPAAL is used to extract the error scenarios presenting the situations the network goes out of sync. Based on such error scenarios, three conditions are introduced for a fully connected network to work correctly. The conditions are proved to be necessary and sufficient using invariant proof techniques. Isabelle/HOL supports the proofs. In chapter 4 another synchronization protocol, named Median, is fully described, and a detailed timed automata model of the protocol is presented in UPPAAL. The model is checked for synchronization, and an error scenario showing how the network goes out of sync is presented. The error scenario is reproducible in reality. While in chapter 3 the focus of modeling is simplicity to make verification easier, in chapter 4 the model is attempted to be constructed as close to real implementation as possible. Chapter 5 concludes the research described in part one and compares the obtained results with the most related researches.

Part two is written based on the publications


Part two describes how counterexample-guided abstraction refinement can be employed to expand the learning ability of automata learning tools. Chapter 6 gives a short introduction of automata learning and the way abstraction refinement can be used to strengthen the abilities of the current tools. Chapter 7 provides a solid theoretical foundation for learning interface automata using a large class of abstractions. In chapter 8 it is explained how this theoretical results support building the tool Tomte to learn a limited class of interface automata, called scalarset symbolic interface automata. Chapter 9 concludes the investigation and represents the most related works. Finally, I end the thesis in chapter 10.
Part I

Formal Analysis of Synchronization Protocols for Wireless Sensor Networks
Chapter 2

Introduction to Part One

The research reported in the first part of this thesis was carried out within the context of the EU project Quasimodo. The main goal of Quasimodo was to develop new techniques and tools for model-driven design, analysis, testing and code-generation for advanced embedded systems where ensuring quantitative bounds on resource consumption is a central problem. Case studies have been the driving momentum behind the project. Quasimodo followed an iterative approach where fundamental research on theory and algorithms—challenged by real-life case studies—was developed and implemented in methods and tools, which were evaluated through case studies. The Chess eT International B.V. was an industrial party of Quasimodo project who provided several case studies including a wireless sensor network running an epidemic communication protocol.

The first part of this thesis is devoted to modeling and analysis of synchronization algorithms for Chess wireless sensor network case study.

A wireless sensor network (WSN) consists of spatially distributed autonomous devices that communicate via radio and use sensors to cooperatively monitor physical or environmental conditions, such as temperature, sound, vibration, pressure, motion or pollutants, at different locations. WSNs consist of potentially thousands of nodes where each node comes equipped with one (or sometimes several) sensors. Each such sensor node has typically several parts: a radio transceiver with an internal antenna or connection to an external antenna, a microcontroller, an electronic circuit for interfacing with the sensors and an energy source, usually a battery or an embedded form of energy harvesting. WSNs have numerous applications, ranging from monitoring of dikes to smart kindergartens, and from forest fire detection to monitoring of the Matterhorn.

The Chess eT International B.V. develops a WSN platform using an epidemic (gossip) communication model, in the context of the MyriaNed project [80]. Figure 2.1 displays a sensor node developed by Chess.

Gossiping in distributed systems refers to the repeated probabilistic exchange of information between two members [53, 23]. The effect is that information can
spread within a group just as it would in real life. Their simplicity, robustness and flexibility make gossip based algorithms attractive for data dissemination and aggregation in wireless sensor networks. However, formal analysis of gossip algorithms is a challenging research problem [11]. The Chess WSN distinguishes three protocol layers: the Medium Access Control (MAC) layer, which is responsible for regulating the access to the wireless shared channel, the intermediate Gossip layer, which is responsible for insertion of new messages, forwarding of current messages and deletion of old messages, and the Application layer, which has the business logic that interprets messages and may generate new messages. This research concentrates on the MAC layer of the Chess WSN.

The rest of this introduction is organized as follows. Section 2.1 gives a brief account of MyriaNed design for MAC layer. Section 2.2 introduces a proposed synchronization algorithm for Chess WSN and explains how the suggested protocol is formally analyzed in chapter 3 this thesis. Section 2.3 presents the basics of Median algorithm which is the focus of chapter ??.

2.1 Chess MAC model

The MAC layer uses specific protocols to ensure that signals sent from different stations across the same channel don’t collide, as RF broadcasting is used to transfer message. Characteristics of the other layers influence the design decisions for the MAC layer. For instance, the redundant nature of the Gossip layer justifies occasional message loss in the MAC layer.

Chess used a Time Division Multiple Access (TDMA) protocol for the MAC layer. In this approach, time is divided in fixed length frames, and each frame is subdivided into slots (see Figure 2.2). Slots can be either active or idle. During active slots, a node is either listening for incoming messages from neighboring nodes (“RX”) or it is sending a message itself (“TX”). During idle slots a node is switched to energy saving mode. In WSNs, nodes are usually battery operated devices with an expected uninterrupted field deployment of several years. Hence, energy efficiency is a major concern in the design of WSNs. For this reason, in
MyriaNed design the number of active slots is typically much smaller than the total number of slots (less than 1% in the current implementation \[80\]). The active slots are placed in one contiguous sequence which is placed at the beginning of the frame. A node can only transmit a message once per time frame in its TX slot. If two neighboring nodes choose the same send slot, a collision will occur in the intersection of their ranges preventing delivery of either node’s message in that intersection. Ideally, no neighboring pair would ever choose the same send slot. This has proven to be very hard to achieve, especially in settings with node mobility. In this thesis, the issue of slot allocation is not addressed and it is simply assumed that the TX slots of all nodes are fixed and have been chosen in such a way that no collisions occur. Actually, receiving is typically more expensive than sending, as radio needs to be turned on longer. Furthermore, receiving usually consumes more energy per bit.

One of the greatest challenges in the design of the MAC layer is to find suitable mechanisms for clock synchronization, that is a distributed algorithm to ensure that the start of all active periods of the nodes are synchronous. More precisely, it must be guaranteed that whenever some node is sending all its neighbors are listening.

In the setting of Chess, each wireless sensor node comes equipped with a low-cost 32 KHz crystal oscillator that drives an internal clock that is used to determine the start and end of each slot. This may cause the TDMA time slot boundaries to drift and thus lead to situations in which nodes get out of sync. To overcome this problem, the notion of guard time is introduced: at the beginning of its TX slot, before actually starting transmission, a sender waits a certain amount of time for the receiver to be ready to receive messages. Similarly, the sender also waits for some time period at the end of its TX slot (see Figure \[2.3\]).

In the implementation of Chess, each slot consisted of 29 clock cycles, out of which 18 cycles were used as guard time. Assegei \[7\] calculated how the battery life of a wireless sensor node is influenced by the guard time. Figure \[2.4\] taken from \[7\], summarizes these results. Clearly, it is of vital importance to reduce the guard time as much as possible, since this directly affects the battery life, which is a key characteristics of WSNs. Intuitively, larger guard-time leads to larger slots which leads to larger active time which implies more energy consumption. Reduction of
18 Introduction to Part One

Figure 2.4: Battery life as a function of guard time

the guard time is possible if the hardware clocks are properly synchronized.

To experiment with its designs, Chess builds prototypes and uses advanced simulation tools. However, due to the huge number of possible network topologies and clock speeds of nodes, although it is not difficult to discover flaws in the clock synchronization algorithm via these methods, it is difficult to understand the root cause of the flaws and to provide objective evidence that the algorithm is correct under all circumstances.

Timed automata model checking has been successfully used for the analysis of worst case scenarios for protocols that involve clock synchronization, see for instance [16, 41, 97]. To enable model checking, models need to be much more abstract than for simulation, and also the size of networks that can be tackled is much smaller, but the big advantage is that the full state space of the model can be explored.

The purpose of the research of this thesis was to use timed automata model checking to help Chess with designing synchronization protocols for their WSN.

2.2 MyriaNed Protocol

In chapter 3 based on the model of Chess for the MAC layer, a synchronization algorithm is proposed for WSN in which a node adjusts its clock whenever a message arrives, and a timed automata model is presented for the suggested algorithm. Then the use of timed automata model checker UPPAAAL [15] for analyzing WSNs with full connectivity is explained. Various instances are verified and three different scenarios are identified which may lead to situations where the network is out of sync. Besides, a full parametric analysis of the protocol for cliques (networks with a connection between every pair of nodes), is presented, that is, constraints on the parameters are given that are both necessary and sufficient for correctness. The results are checked using the proof assistant Isabelle [77]. In order to make verification feasible, the model of chapter 3 abstracts from several aspects in the implementation, including radio switching time: there is some time involved in
the transition from sending mode to receiving mode (and vice versa), which in some cases may affect the correctness of the algorithm. Finally, a result for the special case of line topologies is presented: for any instantiation of the parameters, the protocol will eventually fail if the network grows. Although this approach has advantages, the practical usefulness of this algorithm still needs to be explored further.

2.3 Median Protocol

Chess has implemented Median algorithm, an extension of an algorithm proposed by Tjoa et al [91], on WSN. The idea is that in every frame, each node computes its phase error to any of its direct neighbors. After the last active slot, each node adjusts its frame length by the median of the phase errors of its immediate neighbors. In chapter 4, a detailed model of the Chess Median algorithm is presented using the input language of UPPAAL. The aim is to construct a model that comes as close as possible to the specification of the clock synchronization algorithm presented in [80]. Nevertheless, the model still does not incorporate some features of the full algorithm and network, such as dynamic slot allocation, synchronization messages, uncertain communication delays, and unreliable radio communication. At places where the informal specification of [80] was incomplete or ambiguous, the engineers from Chess kindly provided additional information on the way these issues are resolved in the current implementation of the network [100]. The Median algorithm works reasonably well in practice, but by means of simulation experiments, Assegei [7] points out that the performance of the Median algorithm decreases if the network becomes more dynamic. In some test cases where new nodes join or networks merge, the algorithm fails to converge or nodes may stay out of sync for a certain period of time. Analysis with UPPAAL as presented in chapter 4 confirms these results. In fact, it is shown in chapter 4 that the situation is even worse: in certain cases a static, fully synchronized network may eventually become unsynchronized if the Median algorithm is used, even in a setting with infinitesimal clock drifts. This theoretical result has been later reproduced experimentally in a real network of Chess. Assegei [7] proposes a variation of the Median algorithm that uses Kalman filters, but as it is shown in [42], also this variation leads to serious synchronization problems.
Chapter 3

Modeling and Verification of MyriaNed Synchronization Protocol for Wireless Sensor Networks

In this chapter, the MyriaNed algorithm for the Chess WSN is analyzed. In this protocol a node adjusts its clock whenever a message arrives. This chapter is structured as follows. In Section 3.1 the synchronization algorithm is modeled using timed automata. Section 3.2 describes the use of the timed automata model checker UPPAAL to analyze WSNs with full connectivity. Various instances are verified and three different scenarios that may lead to situations where the network is out of sync, are identified. Section 3.3 presents a full parametric analysis of the protocol for cliques. In Section 3.4 an exhaustive analysis is reported using UPPAAL of all networks with 4 nodes. Section 3.5 presents a result for the special case of line topologies: for any instantiation of the parameters, the protocol will eventually fail if the network grows. Section 3.6 finally, discusses related work and draws conclusions.

UPPAAL models, Isabelle sources and invariant proofs for this study are available at http://www.mbsd.cs.ru.nl/publications/papers/fvaan/HSV09/.

3.1 Uppaal Model

In this section, the UPPAAL model of the Chess protocol is described. A detailed account of the timed automata model checking tool UPPAAL, is presented in [15] and the website http://www.uppaal.com.

A wireless sensor network is defined as a finite, fixed set of wireless nodes
Nodes $= \{0, \ldots, N - 1\}$. The behavior of each individual node $i \in \text{Nodes}$ is described by three timed automata: $\text{Clock}(i)$, $\text{WSN}(i)$ and $\text{Synchronizer}(i)$. Automaton $\text{Clock}(i)$ models the hardware clock of the node, automaton $\text{WSN}(i)$ takes care of sending messages, and the $\text{Synchronizer}(i)$ automaton resynchronizes the hardware clock upon receipt of a message. The complete model consists of the composition of timed automata $\text{Clock}(i)$, $\text{WSN}(i)$ and $\text{Synchronizer}(i)$, for each $i \in \text{Nodes}$.

Figure 3.1 illustrates the architecture of the model for a single node $i$. For each $i$, there is a state variable $\text{clk}[i]$ that records the (integer) value of $i$’s hardware clock (initially 0), and a variable $\text{csn}[i]$ that records the current slot number of node $i$ (also 0 initially). Variable $\text{clk}[i]$ is incremented cyclically whenever $\text{Clock}[i]$ ticks, but it can also be reset by $\text{Synchronizer}[i]$. Automaton $\text{WSN}[i]$ reads $\text{clk}[i]$ in order to determine when to transmit. Automaton $\text{WSN}[i]$ both reads and write variable $\text{csn}[i]$. The $\text{Synchronizer}[i]$ needs to read variable $\text{csn}[i]$ in order to determine whether node $i$ is active or idle. In UPPAAL, a broadcast channel can be used to synchronize transitions of multiple automata. If $a$ is a broadcast channel and one automaton in the network is in a state with an outgoing $a!$ transition, then this transition may always occur (provided the guard evaluates to true). In this case, the transition synchronizes with the $a?$ transitions of all automata that enable such a transition. Automata that do not enable an $a?$ transition remain in the same state. Within the model, broadcast channel $\text{tick}[i]$ is used to synchronize the activities within node $i$, and broadcast channel $\text{start\_message}[i]$ is used to inform all the nodes in the network that node $i$ has started transmission. More
specifically, automaton $\text{WSN}[i]$ performs a $\text{start\_message}[i]$! action to indicate that node $i$ starts transmission, and whenever some node $j$ starts transmission and node $i$ is in an active slot ($\text{csn}[i] < n$), automaton $\text{Synchronizer}[i]$ may perform a $\text{start\_message}[j]$? transition.

Table 3.1 lists the parameters (constants in UPPAAL terminology) that are used in the model, together with some basic constraints. The domain of all parameters is the set of natural numbers.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>number of nodes</td>
<td>$1 &lt; N$</td>
</tr>
<tr>
<td>$C$</td>
<td>number of slots in a time frame</td>
<td>$0 &lt; C$</td>
</tr>
<tr>
<td>$n$</td>
<td>number of active slots in a time frame</td>
<td>$0 &lt; n \leq C$</td>
</tr>
<tr>
<td>$\text{tsn}[i]$</td>
<td>TX slot number for node $i \in \text{Nodes}$</td>
<td>$0 \leq \text{tsn}[i] &lt; n$</td>
</tr>
<tr>
<td>$k_0$</td>
<td>number of clock ticks in a time slot</td>
<td>$0 &lt; k_0$</td>
</tr>
<tr>
<td>$g$</td>
<td>guard time</td>
<td>$0 &lt; g$</td>
</tr>
<tr>
<td>$t$</td>
<td>tail time</td>
<td>$0 &lt; t, g + t + 2 \leq k_0$</td>
</tr>
<tr>
<td>$\text{min}$</td>
<td>minimal time between two clock ticks</td>
<td>$0 &lt; \text{min}$</td>
</tr>
<tr>
<td>$\text{max}$</td>
<td>maximal time between two clock ticks</td>
<td>$\text{min} \leq \text{max}$</td>
</tr>
</tbody>
</table>

Table 3.1: Protocol parameters

3.1.1 Clock

Timed automaton $\text{Clock}(i)$, displayed in Figure 3.2, models the behavior of the hardware clock of node $i$. It has a single location and a single transition. It comes equipped with a local clock variable $x$, which is initially 0, that is used to measure the time in between clock ticks. Whenever $x$ reaches the value $\text{min}$, the automaton enables a $\text{tick}[i]$! action. The $\text{tick}[i]$! action must occur before $x$ has reached value $\text{max}$. Then $x$ is reset to 0 and the (integer) value of $i$’s hardware clock $\text{clk}[i]$ is incremented by 1. For convenience and in order to make model checking feasible, the hardware clock is reset after $k_0$ ticks, that is, the clock takes integer values modulo $k_0$ (UPPAAL’s modulo operator % is used). This is not an essential modeling assumption and it is easy to change it. In the implementation of the protocol, the clock domain is larger and additional program variables are used to record, for instance, the clock value at which a frame starts. However, in order to avoid state space explosion, it is tried to reduce the number of state variables and the domains of these variables.

3.1.2 Wireless Sensor Node

Automaton $\text{WSN}(i)$, displayed in Figure 3.3, is the most important component in the model. It has three locations and four transitions. The automaton stays
in initial location WAIT until the current slot number of $i$ equals the TX slot number of $i$ ($\text{csn}[i] = \text{tsn}[i]$) and the $g^{th}$ clock tick in this slot occurs. It then jumps to location GO_SEND. This is an urgent location that is left immediately via a start_message[$i$]!-transition to location SENDING. Broadcast channel start_message[$i$] is used to inform all neighboring nodes that a new message transmission has started. The automaton stays in location SENDING until the start of the tail interval, that is, until the $(k_0 - t)^{th}$ tick in the current slot (cf. Figure 2.3), and then returns to location WAIT. At the end of each slot, that is, when the $k_0^{th}$ tick occurs, the automaton increments its current slot number (modulo $C$).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig_3.2}
\caption{Timed automaton $\text{Clock}(i)$}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig_3.3}
\caption{Timed automaton $\text{WSN}(i)$}
\end{figure}

### 3.1.3 Synchronizer

Automaton Synchronizer($i$), displayed in Figure 3.4, is the last component of the model. It performs the role of the clock synchronizer in the TDMA protocol. The automaton has two locations and two transitions. The automaton waits in its initial location $S0$ until it detects the start of a new message, that is, until a start_message[$j$]? event occurs, for some $j$. The UPPAAL select statement is used
3.2. Uppaal Analysis Results for Cliques

A wireless sensor network is called *synchronized* if whenever a node is sending, all neighboring nodes have the same slot number as the sending node. For networks to nondeterministically select a $j \in \text{Nodes}$. The automaton then moves to location $S_1$, provided node $i$ is active ($\text{csn}[i] < n$). Remember that at the moment when the $\text{start\_message}[j]$? event occurs, the hardware clock of node $j$, $\text{clk}[j]$, has value $g$. Therefore, node $i$ resets its own hardware clock $\text{clk}[i]$ to $g + 1$ upon occurrence of the first clock tick following the $\text{start\_message}[j]$? event. The automaton then returns to its initial location $S_0$.

Figure 3.4: Timed automaton $\text{Synchronizer}(i)$

In the model there is no delay between sending and receipt of messages. Following Meier & Thiele [70], it is assumed delay uncertainties are negligible, and therefore the delays are eliminated from the analysis. When communication is infrequent, this is reasonable since the impact of clock drift dominates over the influence of delay uncertainties.

Automaton $\text{Synchronizer}(i)$ has no constraint on the value of $j$, that is, node $i$ is assumed to be able to receive messages from any node in the network. Hence the network has full connectivity. It is easy to generalize this model to a setting with arbitrary network topologies by adding a guard $\text{neighbor}(i, j)$ to the transition from $S_0$ to $S_1$ that indicates that $i$ is a direct neighbor of $j$. It is assumed that $\text{neighbor}(i, j) \Rightarrow i \neq j$. The $\text{neighbor}(i, j)$ predicate does not have to be symmetric since in a wireless sensor network it may occur that $i$ can receive messages from $j$, but not vice versa. For networks with full connectivity, it is assumed that all nodes have unique TX slot numbers:

$$i \neq j \Rightarrow \text{tsn}[i] \neq \text{tsn}[j] \quad (3.1)$$

For networks that are not fully connected, this assumption generalizes to the requirements that neighboring nodes have distinct TX slot numbers, and distinct nodes with the same TX slot number do not have a common neighbor:

$$\text{neighbor}(i, j) \Rightarrow \text{tsn}[i] \neq \text{tsn}[j] \quad (3.2)$$

$$\text{neighbor}(i, j) \land \text{neighbor}(i, k) \Rightarrow \text{tsn}[j] \neq \text{tsn}[k] \quad (3.3)$$

3.2 Uppaal Analysis Results for Cliques
with full connectivity this means that all nodes in the network agree on the current slot, which leads to the following formal definition of correctness.

**Definition** A network with full connectivity is synchronized if and only if for all reachable states \((\forall i, j \in \text{Nodes})(\text{SENDING}_i \Rightarrow \text{csn}[i] = \text{csn}[j])\).

The objective is to find necessary and sufficient constraints on the system parameters that ensure that a network with full connectivity is synchronized. To this end, different values are assigned to the parameters of the model and UPPAAL is used to verify the property of Definition 3.2. Based on the outcomes (and in particular the counterexamples generated by UPPAAL) general constraints are derived. For networks with up to 4 nodes, the UPPAAL model checker is able to explore the state space within a few seconds.

Table 3.2 shows some example values of the parameters for which the model is synchronized. In fact, via a series of model checking experiments, using binary search, \(\min\) and \(\max\) are found to be the smallest consecutive natural numbers for which the model with the values assigned to \(N, C, n, k_0\) and \(g\) is synchronized. Note that for correctness of the WSN algorithm the exact values of \(\min\) and \(\max\) are not important: what matters is their ratio. By setting \(\min = m, \max = m + 1\) and letting \(m\) grow, the hardware becomes more and more accurate, until (hopefully) we reach the point at which the algorithm becomes correct. Parameter \(t\) is chosen equal to \(g\) and \(\text{tsn}(i)\) is chosen equal to \(i\). \(n, k_0\) and \(g\) are kept constant while \(C\), the number of slots in a frame, is varying. Observe that if the value of \(C\) increases also the values of \(\min\) and \(\max\) increase, i.e., if the length of a frame increases then the hardware clocks must become more accurate to maintain synchronization.

<table>
<thead>
<tr>
<th>(N)</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C)</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>(n)</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>(k_0)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>(g)</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>(\min)</td>
<td>49</td>
<td>69</td>
<td>89</td>
</tr>
<tr>
<td>(\max)</td>
<td>50</td>
<td>70</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 3.2: Some UPPAAL verification results

Observe that these parameter values are not realistic: a realistic clock accuracy is around 30 ppm (parts-per-million), \(C\) is about 1000 (instead of 10), and \(g\) is 9 (instead of 2). UPPAAL cannot handle realistic values because of the state explosion problem. Nevertheless, as we will see, the counterexamples provided by UPPAAL do provide insight.

In Table 3.3 all the parameters are kept constant and then the values of \(\min\) and \(\max\) are considered for different numbers of nodes when \(n\) changes. Since, in
3.2. UPPAAL Analysis Results for Cliques  

accordance with the specification of the protocol, only slot allocations in which the sending slots are placed at the very beginning of a frame are considered, increasing \( n \) has no impact on network behavior: when no node is transmitting anyway, it makes no difference whether nodes are sleeping or listening. In Table 3.4 all the

<table>
<thead>
<tr>
<th>( N )</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>( n )</td>
<td>4</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>( k_0 )</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>( g )</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( \min )</td>
<td>189</td>
<td>189</td>
<td>189</td>
</tr>
<tr>
<td>( \max )</td>
<td>190</td>
<td>190</td>
<td>190</td>
</tr>
</tbody>
</table>

Table 3.3: Numerical results, changing \( n \)

parameters are kept constant and then the smallest values of \( \min \) and \( \max \) are considered for different number of nodes when \( k_0 \) changes. It turns out that increasing \( k_0 \) necessitates the increase of \( \min \) and \( \max \). In Table 3.5 all the parameters are kept constant and then the smallest values of \( \min \) and \( \max \) are considered for different number of nodes when \( g \) changes. Increasing \( g \), facilitates the decrease of \( \min \) and \( \max \).

The concrete counterexamples produced by the UPPAAL model checker can easily be transformed into parametric counterexamples: it is enough to replace the concrete values of the timing constants by parameters and collect the constraints on these parameters that are imposed by the guards and invariants that occur in the counterexample execution. Inspection of the counterexamples for the WSN protocol, which one can rerun step by step in the simulator, reveals that there are essentially three different scenarios that may lead to a state in which the network is not synchronized. In order to describe these scenarios parametrically at an abstract level, a bit of notation is needed. \( s \in \{0, \ldots, C - 1\} \) is said to be a
transmitting slot, notation TX\(s\), if there is some node \(i\) that is transmitting in \(s\), that is,
\[
\text{TX}(s) \iff (\exists i \in \text{Nodes}) (\text{tsn}[i] = s).
\]
PREV\(s\) denotes the nearest transmitting slot that precedes \(s\) (cyclically). Formally, function PREV : \([0, \ldots, C-1]\) \(\rightarrow\) \([0, \ldots, C-1]\) is defined by
\[
\text{PREV}((s + 1)\%C) = \begin{cases} 
  s & \text{if } \text{TX}(s) \\
  \text{PREV}(s) & \text{otherwise}
\end{cases}
\]
\[
(3.4)
\]
D\(s\) denotes the number of slots visited when going from PREV\(s\) to \(s\), that is, \(D(s) = (s - \text{PREV}(s))\%C\). \(M = \max_s D(s)\) is defined to be the maximal distance between transmitting slots. As we will see, \(M\) plays a key role in defining correctness.

### 3.2.1 Scenario 1: Fast Sender - Slow Receiver

In the first error scenario, a sending node is proceeding maximally fast whereas a receiving node runs maximally slow. The sender starts with the transmission of a message while the receiver is still in an earlier slot. The scenario is illustrated in Figure 3.5. It starts when the fast and the slow node receive a synchronization message. Immediately following receipt of this message (at the same point in time), the hardware clock of fast node ticks and the synchronizer resets this clock to \(g + 1\). Now, in the worst case, it may take \(M \cdot k_0 - 1\) ticks before the fast node is in its TX slot with its hardware clock equal to \(g\). Since the hardware clock of the fast node ticks maximally fast, the length of the corresponding time interval is \((M \cdot k_0 - 1) \cdot \min\). The slow node will reach the TX slot of the fast node after \(M \cdot k_0 - g\) ticks. With a clock that ticks maximally slow, this may take \((M \cdot k_0 - g) \cdot \max\) time. If \((M \cdot k_0 - g) \cdot \max\) is greater than or equal to \((M \cdot k_0 - 1) \cdot \min\) then we may end up in a state where the network is no longer synchronized since the fast node is sending before the slow node has moved to the same slot. Hence, in order to exclude this scenario, we must have:
\[
(M \cdot k_0 - g) \cdot \max < (M \cdot k_0 - 1) \cdot \min \quad (3.5)
\]

<table>
<thead>
<tr>
<th>N</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>n</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>(k_0)</td>
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<td>10</td>
</tr>
<tr>
<td>(g)</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>min</td>
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</tr>
<tr>
<td>max</td>
<td>50</td>
<td>25</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 3.5: Numerical results, changing \(g\)
This constraint is consistent with the results in Table \ref{tab:uppaal}. Consider, for instance the first column. According to UPPAAL the protocol is correct if \( N = 2, C = 6, n = 4, k_0 = 10, g = 2, \min = 49 \) and \( \max = 50 \). Since it is assumed that the two nodes are sending in the first two slots of a frame, it is easy to see that \( M = 5 \). Now we can verify that

\[(5 \cdot 10 - 2) \cdot 50 = 48 \cdot 50 = 2400 < 2401 = 49 \cdot 49 = (5 \cdot 10 - 1) \cdot 49\]

However, if we increase the clock drift slightly by setting \( \min \) and \( \max \) to 48 and 49, respectively, then the protocol fails according to UPPAAL. And indeed

\[(5 \cdot 10 - 2) \cdot 49 = 48 \cdot 49 = 2352 = 49 \cdot 48 = (5 \cdot 10 - 1) \cdot 48\]

Instead of the lower bound \( \min \) and the upper bound \( \max \) on the time between clock ticks, sometimes it is convenient to consider the ratio

\[\rho = \frac{\min}{\max}\]

Since \( 0 < \min \leq \max \), it follows that \( \rho \) is contained in the interval \((0, 1]\). The following elementary lemma turns out to be quite useful.

**Lemma 3.2.1** Constraint (3.5) is equivalent to \( g > (1 - \rho) \cdot M \cdot k_0 + \rho \).

This implies that the worst case scenario occurs when the distance between TX slots is maximal: if the constraint holds for \( M \) it also holds when we replace \( M \) by a smaller value.

**The Chess implementation** Constraint (3.5) allows us to infer a lower bound on the guard time \( g \). In the current implementation of the protocol by Chess \cite{Chess}, a quartz crystal oscillator is used with a clock drift rate \( \theta \) of at most 20 ppm. This means that

\[\rho = \frac{1 - \theta}{1 + \theta} = \frac{1 - 20 \cdot 10^{-6}}{1 + 20 \cdot 10^{-6}} \approx 0.99996\]
In the Chess implementation, one time frame lasts for about 1 second. It consists of \( C = 1129 \) slots and each slot consists of \( k_0 = 29 \) clock ticks. The number of active slots is small \((n = 10)\). A typical value for \( M \) is \( C - n = 1119 \). Hence

\[
g > (1 - \rho) \cdot M \cdot k_0 + \rho \approx 0.00004 \cdot 1119 \cdot 29 + 0.99996 = 2.298
\]

Thus, according to the theoretical model, a value of \( g = 3 \) should suffice. Chess actually uses a guard time of 9. Of course one should realize here that the model is overly simplified and, for instance, does not take into account (uncertainty in) message delays and partial connectivity. We will see that these restrictions greatly influence the minimal guard time.

### 3.2.2 Scenario 2: Fast Receiver - Slow Sender - before transmission

In the second error scenario, a receiving node runs maximally fast whereas a sending node proceeds maximally slow. The receiving node already leaves the slot in which it should receive a message from the sender before the sender has even started transmission. This scenario is illustrated in Figure 3.6. Again, the scenario starts when the fast and the slow node receive a synchronization message. But now the node that has to send the next message runs maximally slow. It sends this message after \( M \cdot k_0 \) ticks have occurred, which takes \( M \cdot k_0 \cdot \text{max time} \). Meanwhile, the fast node has made maximal progress: immediately after receipt of the first synchronization message (at the same point in time), the hardware clock of the fast node ticks and the synchronizer resets this clock to \( g + 1 \). Already after \( (k_0 - g - 1) \cdot \text{min time} \) the node proceeds to the next slot. Another \( (M \cdot k_0 - 1) \cdot \text{min time} \) units later the fast node sets its clock to \( k_0 - 1 \) and is about to leave the slot in which the slow node will send a message. If the slow node starts transmission after this point it is too late: after the next clock tick the fast node will increment its slot counter and the network is no longer synchronized. In order to exclude the
second scenario, the following constraint must hold:
\[ M \cdot k_0 \cdot \max < ((M + 1) \cdot k_0 - g - 2) \cdot \min \]  \( \text{(3.6)} \)
Also this constraint can be rewritten:

**Lemma 3.2.2** Constraint (3.6) is equivalent to \( g < (1 - \frac{1}{\rho}) \cdot M \cdot k_0 + k_0 - 2 \).

Thus constraint (3.6) imposes an upper bound on guard time \( g \). Since in practice one will always try to minimize the guard time in order to save energy, this constraint is only of theoretical interest. If we fill in the values of Example 3.2.1, we obtain \( g < 25.8 \), which is close to the slot length \( k_0 = 29 \).

### 3.2.3 Scenario 3: Fast Receiver - Slow Sender - during transmission

The third scenario involves a fast receiver and a slow sender. The receiver moves to a new slot while the sender is still transmitting a message. Figure 3.7 illustrates the scenario. As in the previous scenarios, the hardware clock of the fast node is set to \( g + 1 \) immediately after receipt of the synchronization message.

![Figure 3.7: Scenario 3: Fast Receiver - Slow Sender - during transmission](image)

Essentially, constraint (3.7) provides a lower bound on \( t \): to rule out the scenario in Figure 3.7, the sender should wait long enough before proceeding to the next slot.

**Lemma 3.2.3** Constraint (3.7) is equivalent to \( t > (1 - \rho)(k_0 - g) + \rho \).

If we fill in the values of Example 3.2.1 with \( g \) set to 3, we obtain \( t > 1.001 \). Hence a value of \( t = 2 \) should suffice. Hence, for the simple case of a static network with full connectivity and no uncertainty in message delays, we only need to reserve 5 clock cycles for guard and tail time together. In Section 3.5, we will see that for different network topologies indeed much larger values are required.
3.3 Proving Sufficiency of the Constraints

This section outlines the proof that the three constrains derived in Section 3.2 are sufficient to ensure synchronization in networks with full connectivity.

First the key invariants used in the proof are presented and then the formalization of the full proof using Isabelle/HOL is discussed.

3.3.1 Invariants

Let’s start the proof by stating some elementary invariants.

**Lemma 3.3.1** For any network with full connectivity the following invariant assertions hold, for all reachable states and for all \( i \in \text{Nodes} \):

\[
0 \leq x_i \leq \max
\]

\[
0 \leq \text{clk}[i] < k_0
\]

\[
0 \leq \text{csn}[i] < C
\]

\[
\text{GO}_\text{SEND}_i \Rightarrow x_i = 0
\]

\[
\text{GO}_\text{SEND}_i \Rightarrow \text{csn}[i] = \text{tsn}[i]
\]

\[
\text{GO}_\text{SEND}_i \Rightarrow \text{clk}[i] \in \{g, g + 1\}
\]

\[
\text{SENDING}_i \Rightarrow \text{csn}[i] = \text{tsn}[i]
\]

\[
\text{SENDING}_i \Rightarrow g \leq \text{clk}[i] < k_0 - t
\]

Invariants (3.8)-(3.10) assert that the state variables indeed take values in their intended domains: clock variables stay within the (real-valued) range \([0, \max]\), hardware clocks stay within the integer range \([0, k_0]\), and current slot numbers stay within the integer range \([0, C]\). Invariants (3.11)-(3.15) directly follow from the definitions of the individual automata in the network. For invariant (3.13), observe that since the tick?-transition from WAIT to GO_SEND may synchronize with the tick?-transition from S1 to S0, the value of clk\([i]\) in GO_SEND\(_i\) is potentially \(g + 1\).

In order to be able to state more interesting invariants, two auxiliary global history (or ghost) variables are introduced. Clock \( y \) records the time that has elapsed since the last synchronization message (or the beginning of the protocol). Variable last records the last slot in which a synchronization message has been sent (initially last = \(-1\)). Figure 3.8 shows the version of the WSN\(_i\) automaton obtained after adding these variables.

The only change is that upon occurrence of a synchronization start_message\([i]\)! clock \( y \) is reset to 0 and variable last is reset to csn\(_i\). First a few basic invariants are stated which restrict the values of the new variables.

**Lemma 3.3.2** For any network with full connectivity the following invariant as-
3.3. Proving Sufficiency of the Constraints

Assertions hold, for all reachable states and for all $i \in \text{Nodes}$:

\[
0 \leq y \quad \quad \quad \quad (3.16)
\]
\[
-1 \leq \text{last} < C \quad \quad (3.17)
\]
\[
S1_i \quad \Rightarrow \quad y \leq x_i \quad \quad (3.18)
\]
\[
\text{last} = -1 \quad \Rightarrow \quad S0_i \quad \quad (3.19)
\]

Invariant (3.16) says that $y$ is always nonnegative, and invariant (3.17) says that \text{last} takes values in the integer domain $[-1, C - 1]$. If the system is in $S1_i$ then a synchronization occurred after the last clock tick (invariant (3.18)), and if the system is in $S0_i$ then no synchronization occurred yet (invariant (3.19)).

The key idea behind the correctness proof is that, given the local state of some node $i$ and the value of \text{last}, we can compute the number $c(i)$ of ticks of $i$’s hardware clock that has occurred since the last synchronization. Since the minimal and maximal clock speeds are known, we can then derive an interval that contains the value of $y$, the amount of real-time that has elapsed since the last synchronization. Next, given the value of $y$, we can compute an interval that contains the value of $c(j)$, for arbitrary node $j$. Once we know the value of $c(j)$, this gives us some information about the local state of node $j$. Through these correspondences, we are able to infer that if node $i$ is sending the slot number of $i$ and $j$ must be equal.

Figure 3.8: $\text{WSN}(i)$ with history variables
Formally, for \( i \in \text{Nodes} \), the state function \( c(i) \) is defined by

\[
c(i) = \quad \text{if } \text{last} = -1 \text{ then } \text{clk}[i] \text{ else }
\]
\[
\quad \text{if } S1_i \text{ then } 0 \text{ else }
\]
\[
\quad ((\text{csn}[i] - \text{last})\%C) \cdot k_0 + \text{clk}[i] - g
\]
\[
\fi
\]

If there has been no synchronization yet (\( \text{last} = -1 \)) then \( c(i) \) is just equal to the hardware clock \( \text{clk}[i] \). If the synchronizer is in location \( S1_i \), then we know that there has been no tick since the last synchronization, so \( c(i) \) is set to 0. Otherwise, \( c(i) \) is \( k_0 \) times the number of slots since the last synchronization, incremented by the number of ticks in the current slot, minus \( g \) to take into account that the hardware clock has been reset to \( g + 1 \) after the last synchronization.

Now the main invariant result from this section can be stated.

**Theorem 3.3.3** Assume constraints (3.5), (3.6) and (3.7), \( 3 \leq N \) and assume that some node transmits in the initial slot. Then for any network with full connectivity the following invariant assertions hold, for all reachable states and for all \( i, j \in \text{Nodes} \):

\[
y \leq c(i) \cdot \max + x_i \quad (3.20)
\]
\[
c(i) > 0 \quad \Rightarrow \quad y \geq (c(i) - 1) \cdot \min + x_i \quad (3.21)
\]
\[
\text{csn}[i] = \text{tsn}[i] \land (\text{clk}[i] < g \lor \text{GO_SEND}_i) \quad \Rightarrow \quad \text{last} \neq \text{csn}[i] \quad (3.22)
\]
\[
(\text{csn}[i] = \text{tsn}[i] \land \text{clk}[i] = g) \quad \Rightarrow \quad (\text{GO_SEND}_i \lor \text{SENDING}_i) \quad (3.23)
\]
\[
(\text{csn}[i] = \text{tsn}[i] \land \text{clk}[i] > g) \quad \Rightarrow \quad \text{last} = \text{csn}[i] \quad (3.24)
\]
\[
\text{SENDING}_i \quad \Rightarrow \quad \text{csn}[i] = \text{csn}[j] = \text{last} \quad (3.25)
\]
\[
\text{GO_SEND}_i \quad \Rightarrow \quad \text{csn}[i] = \text{csn}[j] \land \text{clk}[i] = g \quad (3.26)
\]
\[
\text{last} \neq -1 \land \text{last} \neq \text{PREV}(\text{csn}[i]) \quad \Rightarrow \quad (\text{TX} (\text{csn}[i]) \land \text{last} = \text{csn}[i]) \quad (3.27)
\]
\[
\text{TX} (\text{csn}[i]) \land \text{clk}[i] = k_0 - 1 \quad \Rightarrow \quad \text{last} = \text{csn}[i] \quad (3.28)
\]
\[
S1_i \quad \Rightarrow \quad \text{clk}[i] < k_0 - 1 \land \text{last} = \text{csn}[i] \quad (3.29)
\]
\[
c(i) \geq 0 \quad (3.30)
\]
\[
\text{last} = -1 \quad \Rightarrow \quad \text{csn}[i] = 0 \quad (3.31)
\]

**Proof** By induction, using the invariants from Lemma’s 3.3.1 and 3.3.2.


\(^{1}\text{The last two assumptions have been made for convenience. They are not needed, even though they are used in the proof.}\)
3.3. Proving Sufficiency of the Constraints

Invariants (3.20) and (3.21) are the key invariants that relate the values of \( c(i) \) and \( y \). Invariant (3.26) implies that the network is synchronized. This is the main correctness property we are interested in. All the other invariants in Theorem 3.3.3 are auxiliary assertions, needed to make the invariant inductive.

3.3.2 On the formal proof

The manual proof of the invariants from the previous subsection has been fully checked using the proof assistant Isabelle [77]. Below some general remarks about the formalization are made and some of the subtleties encountered are discussed. The main motivation for discussing some of the proof details is that this sheds light on the type of reasoning that will be necessary in order to completely automate the verification.

The length of the Isabelle/HOL proof is about 5300 lines, whereas the manual proof is around 1000 lines. Formal proofs are usually longer than their manual counterpart. Wiedijk [101, 21] proposes the De Bruijn factor as a way to quantify this difference. This factor basically compares the size of two proof files, compressed using the Unix utility gzip. Wiedijk [101] observes that the average De Bruijn factor is about 4. In this case, 4.58 is obtained. This is a bit larger than usual, since the formal proof includes the definition of the UPPAAL model and its semantics, which are not included in the manual proof. All the invariants are also needed to be defined (about 500 lines). In the manual proof, the 12 basic invariants defined in Lemmas 3.3.1 and 3.3.2 are all disposed of by the word “trivial”. The formal proof is indeed straightforward but still occupies about 440 lines.

Key aspects of the Isabelle formalization are (1) an alternative definition of function \( \text{PREV} \) and a proof of lemmas showing particular properties of it, and (2) a formalization of the claim that there are at least three transmitting slots per frame. Common to these two issues is the introduction of the largest slot number in which a message is transmitted. This is the maximum of function \( \text{tsn} \) and is obtained for node \( i_{\text{max}} \). The properties needed are basic facts like \( \text{PREV}(s) \) cannot be \( s \) or that in the idle period of a frame \( \text{PREV}(s) \) equals the transmitting slot of \( i_{\text{max}} \), i.e., \( \text{tsn}[i_{\text{max}}] \). Altogether, the definition of \( \text{PREV} \), the introduction of \( i_{\text{max}} \), the formal proof that there are at least three transmitting slots, and the proof of basic properties about these notions occupy about 600 lines.

In the remainder of this section, first \( i_{\text{max}} \) is formally introduced. Then, the definition of function \( \text{PREV} \) is rephrased and a sequel of properties of that function is proved. After that, the claim that there are at least three transmitting slots is formalized. Finally, the formal proof is illustrated by two simple but representative examples.

Definition of \( i_{\text{max}} \) and \( \text{PREV} \)

As shown in Figure 2.2, a frame is composed of an active period and an idle period. In the active period, there are slots where a node is transmitting and the other
nodes are listening, and also slots where no node is sending and all nodes are
listening. Consequently, there is a last slot in which a message is emitted. Let $i_{\text{max}}$
be the node that is transmitting in this slot. This transmitting node maximizes function $\text{tsn}$:

$$TX_{\text{max}}(i_{\text{max}}) \equiv TX(\text{tsn}[i_{\text{max}}]) \land \forall i \neq i_{\text{max}}:\text{tsn}[i_{\text{max}}] > \text{tsn}[i]$$ (3.33)

The formal definition of function $\text{PREV}$ in Isabelle slightly differs from Equation 3.4. The combination of modulo and the incrementation in the argument does not translate to Isabelle, where functions must be total and proved to terminate. Basically, the modulo is removed and frames are considered to be unbounded. The assumption is still that function $\text{tsn}$ returns a natural number strictly less than $n$. The first basic invariants then prove that parameters take values in their intended domain. Function $\text{PREV}$ is the recursive function below:

**Definition**

$$\begin{align*}
\text{PREV}(0) &= \text{tsn}[i_{\text{max}}] \\
\text{PREV}(s + 1) &= \text{if } TX(s) \text{ then } s \text{ else } \text{PREV}(s)
\end{align*}$$

**Properties of $\text{PREV}$**

In the formal proof, a sequel of properties showing the structure of a frame is needed. The next lemma asserts that function $\text{PREV}$ is constant during the idle period, that is, if slot $s$ is transmitting and all slots from $s$ to $y$ are not transmitting, then $\text{PREV}(y)$ is slot $s$.

**Lemma 3.3.4** $(TX(s) \land y > s \geq 0 \land (\forall z.s < z < y \Rightarrow \neg TX(z))) \Rightarrow \text{PREV}(y) = s$

**Proof** By induction on $s$.

From this above lemma it directly follows that after the last transmitting slot, function $\text{PREV}$ equals this slot:

**Lemma 3.3.5** $\forall y > \text{tsn}[i_{\text{max}}].\text{PREV}(y) = \text{tsn}[i_{\text{max}}]$

**Proof** By definition $i_{\text{max}}$ is such that there is no transmitting slot after it. This fact is used for instantiating Lemma 3.3.4 above.

The aim is to prove that the previous slot of slot $s$ is strictly less than $s$. Because of the cyclic nature of a frame, this is only true if $s > 0$.

**Lemma 3.3.6** $s > 0 \implies \text{PREV}(s) < s$

**Proof** By induction on $s$.

Another useful lemma asserts that the “PREV” of a transmitting slot cannot be $\text{tsn}[i_{\text{max}}]$. 
Lemma 3.3.7 \((s > 0 \land TX(s)) \implies PREV(s) < tsn[i_{\text{max}}]\)

**Proof** From \(TX(s)\) we obtain \(j\) such that \(tsn[j] = s\). By definition of \(i_{\text{max}}\) we have \(tsn[j] \leq tsn[i_{\text{max}}]\). Using Lemma 3.3.6, we obtain \(PREV(s) < s\). Hence \(PREV(s) < tsn[i_{\text{max}}]\).

**At least three sending nodes**

In the informal case study description \([80]\), it is assumed that for each node there is a transmission slot. Translated to the setting of the model, this means that \(tsn\) is a total function from nodes to slots. Interestingly, the Isabelle formalization revealed that the assumption that \(tsn\) is total, is never used in the proof. \(^2\) The only assumption that is made is that there are at least three sending nodes.

In the formalization, a predicate \(TX_n(i)\) is introduced which states that for node \(i\) there exists a slot \(s\) that equals the transmitting slot of node \(i\), that is, node \(i\) is a transmitting node. Predicate \(TX_n(i)\) complements predicate \(TX(s)\) defined earlier. Predicate \(TX_n(i)\) is defined as follows:

**Definition** \(TX_n(i) = \exists s. tsn[i] = s\)

The assumption that there are at least three transmitting slots is formalized by assuming that predicate \(TX_n\) holds for nodes 0 to 2.

\[
\forall i \leq 2. \; TX_n(i) \quad (3.34)
\]

Two important facts are derived: \(tsn[i_{\text{max}}]\) is at least 2, and between slot number 0 and slot number \(n - 1\) there is at least one transmitting slot.

**Lemma 3.3.8** \(tsn[i_{\text{max}}] \geq 2 \land \exists s. 0 < s < n - 1 \land TX(s)\)

**Proof** The first part is trivial. Function \(tsn\) assigns different slots to different nodes. Together with Equation 3.34 we can derive three distinct nodes \(i, j, k\) with distinct transmitting slots \((tsn[i], tsn[j], tsn[k])\). We do a case analysis on their different possible orderings (e.g., \(tsn[i] < tsn[j] < tsn[k]\)). By definition a slot number is not greater than \(n - 1\) and positive. Consequently, the “\(tsn\)” in the middle of the ordering is strictly positive and strictly less than \(n - 1\). This shows the second term of the conclusion.

A consequence of Lemma 3.3.8 is that function \(PREV\) is at least one for all slots not smaller than \(n - 1\).

**Lemma 3.3.9** \(s \geq n - 1 \implies PREV(s) \geq 1\)

**Proof** We consider two cases. If \(s > n - 1\), then \(s > tsn[i_{\text{max}}]\). Moreover there is no transmitting slot between \(s\) and \(n - 1\). So, from Lemma 3.3.4 we obtain \(PREV(s) > tsn[i_{\text{max}}] > 1\). If \(s = n - 1\), then we know from Lemma 3.3.8 that there is at least one transmitting node between slot 0 and \(n - 1\) and \(PREV(s)\) is then at least equal to this slot.

\(^2\)This observation may have practical implications. The results suggest that, at least in certain situations, if nodes have nothing to say they may in fact remain silent.
Proof samples

In the remainder of this section, two examples are presented that show some of the subtleties in the proof. These example illustrates why some of the lemmas introduced earlier are needed, e.g., Lemma 3.3.7 and Lemma 3.3.8.

Example The situation of the first proof sample is pictured in Figure 3.9. This situation appeared in the proof of Invariant 21 and 23 of Theorem 3.3.3. It involves nodes $k$ and an arbitrary different node $j$. Node $k$ is sending in its current slot number, i.e. we have $\text{csn}[k] = \text{tsn}[k]$ and TX($\text{csn}[k]$). The last transmitting slot (depicted in the gray slot) is the previous transmitting slot of both nodes $j$ and $k$.

\[ \text{PREV}(\text{csn}[k]) = \text{PREV}(\text{csn}[j]) = \text{last}(3.35) \]

The goal is to prove that these two nodes agree on the current slot number, i.e., that $\text{csn}[k] = \text{csn}[j]$.

The formal proof needs a case analysis on the relative positions of $\text{csn}[j]$ and $\text{csn}[k]$. Assume $\text{csn}[j] < \text{csn}[k]$. Node $k$ is in a later slot. Because of the cyclic nature of frames, we must consider two cases: $\text{csn}[j] = 0$ and $\text{csn}[j] > 0$.

If $\text{csn}[j] = 0$, by definition of PREV we have $\text{PREV}(\text{csn}[j]) = \text{tsn}[i_{\text{max}}]$, and $\text{PREV}(\text{csn}[k]) = \text{tsn}[i_{\text{max}}]$ also. From Lemma 3.3.7 we have $\text{PREV}(\text{csn}[k]) < \text{csn}[k]$. So, we have $\text{tsn}[i_{\text{max}}] < \text{csn}[k] = \text{tsn}[k]$. By definition of $i_{\text{max}}$, this is clearly impossible.

The case when $\text{csn}[k] < \text{csn}[j]$ is similar. We know that TX($\text{csn}[k]$), so $k$ is transmitting and has to be in the active region, i.e., $\text{csn}[k] < n$. We do not have such information for $\text{csn}[j]$ and need to consider extra cases: $\text{csn}[j] < n$ and $\text{csn}[j] \geq n$.

Example Now the proof of Invariant 3.22 in Theorem 3.3.3 is illustrated. This invariant assumes that node $i$ is in a transmitting slot. We have $\text{csn}[i] = \text{tsn}[i]$, hence TX($\text{csn}[i]$). It also considers that node $i$ is either in state WAIT with $\text{clk}[i] < g$ or in state GO_SEND. (By basic Invariant 3.15 node $i$ cannot be in state SENDING.) In brief, node $i$ is about to send a message. The conclusion asserts that the last slot with a synchronization is not the current slot ($\text{last} \neq \text{csn}[i]$). Before
the first synchronization, \( \text{last} \) is negative \( (\text{last} = -1) \) and the conclusion holds as any \( \text{csn} \) is a nonnegative number (basic Invariant \[3.10\]). Before a \text{start\_message} action, node \( i \) is in state \text{GO\_SEND} with its clock equal to \( g \). Variable \( \text{clk}[i] \) is not modified by this action. Hence the invariant is trivially true in the target state because its premises are false. The case of a tick action is more complicated.

We only consider the end of the current slot \( (\text{csn}[i]) \). The situation is as follows. Node \( i \) is in state \text{WAIT} with its clock counting the last tick of a slot \( (\text{clk}[i] = k_0 - 1) \). After the tick action the clock is reset to 0, a new slot starts \( (\text{csn}[i] = (\text{csn}[i] + 1)\%C) \), and other variables are left unchanged, in particular \( \text{last}' = \text{last} \). The last transmitting slot is the previous transmitting slot of \( i \) \( (\text{last} = \text{PREV}(\text{csn}[i])) \). The case where \( \text{csn}[i] = 0 \) or \( \text{csn}[i] = n - 1 \) is illustrated.

3.4 Uppaal Analysis Results: Networks with 4 Nodes

In the two previous sections, the correctness of the clock synchronization protocol for networks with full connectivity was studied. In practice, however, wireless sensor networks are rarely fully connected. A fully parametric analysis of the protocol for arbitrary network topologies will be quite involved.

In order to illustrate some of the complications, a small script is written to explore all possible network topologies with 4 nodes. As explained in Section \[3.1\] it is easy to model arbitrary network topologies in \text{Uppaal} by appropriate instantiation of the \text{neighbor} function. For parameters \( k_0 = 15, C = 6 \) and \( n = 4 \), \text{Uppaal} was used to find out for each topology for which guard time and \( \text{min}/\text{max} \) ratio the synchronization property was satisfied. This took 30 hours for the \( 4^6 = 4096 \) possible topologies. The results of the model checking experiments for the connected networks of Figure \[3.10\] in which communication is symmetric, is summarized in Table \[3.6\]. As in Section \[3.2\] \( \text{min} \) and \( \text{max} \) in this table are the smallest consecutive natural numbers for which the model with the values assigned to \( C, n, k_0 \) and \( g \) is synchronized. As expected, topology number 6, requires the highest guard time of all networks with 4 nodes. We observe that the more connected the network is, the lower the guard time can be.

When communication is not symmetric, that is, it may occur that some node \( A \) receives messages from a node \( B \) but not vice versa, the clock synchronization behavior becomes highly unpredicatable and depends on time slot numbers assigned to each node. Table \[3.7\] summarizes the analysis results for the networks depicted in Figure \[3.11\]. Surprisingly, network 1 allows for smaller guard times.
Figure 3.10: Connected networks with 4 nodes

<table>
<thead>
<tr>
<th>ID</th>
<th>Diameter</th>
<th>Time Slot Numbers</th>
<th>Guard Time</th>
<th>min/max Ratio</th>
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<td>any</td>
<td>2</td>
<td>44/45</td>
</tr>
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<td>2</td>
<td>{0,1,2,3}</td>
<td>3</td>
<td>58/59</td>
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<td></td>
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<td>3</td>
<td>73/74</td>
</tr>
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<td>2</td>
<td>{0,1,2,3}</td>
<td>3</td>
<td>58/59</td>
</tr>
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<td></td>
<td></td>
<td>{0,2,1,3}</td>
<td>3</td>
<td>73/74</td>
</tr>
<tr>
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<td>2</td>
<td>{0,1,2,3}</td>
<td>3</td>
<td>58/59</td>
</tr>
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<td>3</td>
<td>{0,1,2,3}</td>
<td>4</td>
<td>88/89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>{0,3,1,2}</td>
<td>3</td>
<td>43/44</td>
</tr>
</tbody>
</table>

Table 3.6: Analysis results for the networks of Figure 3.10

than network 3, even though it has fewer links.

### 3.5 Uppaal Analysis Results: Line Topologies

Since the experiments indicate, among the symmetric topologies, line topologies have the worst clock synchronization behavior, UPPAAL was used for model checking of some further instances of the protocol that involve line topologies, that is, connected networks in which each node is connected to exactly two other nodes, except for two nodes that only have a single neighbor.
A 3-node network with line topology was defined in UPPAAL and the behavior of the system for different variable valuations was checked. It turns out that, unlike the fully connected network with three nodes (see Table 3.2), the network will not always remain synchronized for $g = 2$, even when the clocks are perfect. Table 3.8 lists some of the verification results. On the left the results are given for a line network of size 3 and on the right those for a clique network of size 3. If we compare these results then we see that, in order to keep the network synchronized, the hardware clocks in a line topology must be more accurate than the hardware clocks in a fully connected network (i.e., the min/max ratio must be closer to 1 if we want the network to be synchronized). Intuitively, the reason is that in a line topology the frequency of synchronization for each node is less than that in a fully connected network.

In order to maintain synchronization, a line topology requires more accurate hardware clocks and a larger guard time. The claim is that, for a fixed value of the guard time, the network may become unsynchronized if we keep increasing the number of nodes. In fact, the claim is that for a line topology of size $N$, the guard
Model checking of synchronization for line topology entails exploring a state space that grows exponentially with the number of nodes. In order to reduce the state space, only networks with perfect clocks are considered. However, even with perfect clocks, UPPAAL can only handle networks with at most 8 nodes. Table 3.9 shows the resource usage of UPPAAL required for model checking of networks with line topologies. A Sun Fire X4440 machine with 4 Opteron 8356 2.3 Ghz quad-core processors and 128 Gb DDR2-667 memory was used. One processor on this machine needs about half an hour to establish that a line network with 8 nodes is synchronized if the guard time is 8.

The reason why we run into state space explosions even in a setting with perfect clocks, is that race conditions are possible involving arrival of messages and ticking of hardware clocks. As a result even a network with perfect clocks will not necessarily remain synchronized for any parameter valuation. Figures 3.12 and 3.13 illustrate how race conditions may affect the time interval between two synchronization events in the model. (For simplicity, we assume \( g = t \).) We consider the case where a node is the receiver in one slot and the sender in the next slot. We know that the sender sends a message when the value of its clock equals \( g \), and that the receiver resets its clock counter to \( g + 1 \) at the first clock tick after receiving the message. Figure 3.12 shows that a synchronization signal is received immediately after a clock tick at the receiver. In this scenario, the receiver waits a full clock cycle before resetting its clock counter to \( g + 1 \). Figure 3.13 illustrates a

<table>
<thead>
<tr>
<th>C</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( k_0 )</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>( g )</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>min</td>
<td>58</td>
<td>78</td>
<td>98</td>
<td>118</td>
</tr>
<tr>
<td>max</td>
<td>59</td>
<td>79</td>
<td>99</td>
<td>119</td>
</tr>
</tbody>
</table>

Table 3.8: Results for line network of size 3 (left) and for clique network of size 3 (right)
Table 3.9: CPU time and memory usage of Uppaal for line networks of different sizes

<table>
<thead>
<tr>
<th>Nodes</th>
<th>g</th>
<th>Collision</th>
<th>Time</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>yes</td>
<td>0.008 s</td>
<td>240852 KB</td>
</tr>
<tr>
<td>2</td>
<td>no</td>
<td>0.039 s</td>
<td>240852 KB</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>yes</td>
<td>0.160 s</td>
<td>240852 KB</td>
</tr>
<tr>
<td>3</td>
<td>no</td>
<td>0.200 s</td>
<td>240852 KB</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>yes</td>
<td>1.007 s</td>
<td>240852 KB</td>
</tr>
<tr>
<td>4</td>
<td>no</td>
<td>1.012 s</td>
<td>240852 KB</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>yes</td>
<td>2.570 s</td>
<td>240852 KB</td>
</tr>
<tr>
<td>5</td>
<td>no</td>
<td>2.587 s</td>
<td>240852 KB</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>yes</td>
<td>17.000 s</td>
<td>240852 KB</td>
</tr>
<tr>
<td>6</td>
<td>no</td>
<td>18.006 s</td>
<td>240852 KB</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>yes</td>
<td>163.154 s</td>
<td>326892 KB</td>
</tr>
<tr>
<td>7</td>
<td>no</td>
<td>173.922 s</td>
<td>336672 KB</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>yes</td>
<td>1624.481 s</td>
<td>2328572 KB</td>
</tr>
<tr>
<td>8</td>
<td>no</td>
<td>1681.874 s</td>
<td>2451884 KB</td>
<td></td>
</tr>
</tbody>
</table>

different scenario in which a synchronization signal is received immediately before the receiver clock ticks and the receiver immediately resets its clock counter to \( g + 1 \). We see that the length of the time interval between two synchronization events in the first scenario is one clock cycle longer than that in the second scenario.

Now it will be shown that in a line network of size \( N \) and with guard time \( g = N - 1 \), there is a reachable state in which the network is no longer synchronized. Thus synchronization of line networks can only be ensured if \( g \geq N \). In the examples, \( \text{tsn}[i] = i \% 3 \), that is, the transmission slot number of node \( i \) equals \( i \) modulo 3. Note that, for line topologies, this allocation of transmission slot numbers satisfies the conditions (3.2) and (3.3) defined at the end of Section 3.1. Figures 3.14, 3.16 and 3.17 illustrate three abstract error scenarios, extracted from concrete counterexamples produced by Uppaal, resulting in a loss of synchronization. Figure 3.14 applies to the case in which \( N \) modulo 3 equals 0, Figure 3.16 to the case in which \( N \) modulo 3 equals 1, and Figure 3.17 to the case in which \( N \) modulo 3 equals 2. The example of Figure 3.14 is explained in detail. The other two scenarios are similar.

The scenario of Figure 3.14 consists of two "staircases". One "fast" staircase has steps with minimum time between synchronizations (using the mechanism of Figure 3.13), where a synchronization signal is received immediately before the receiver clock ticks and the receiver resets its clock counter to \( g + 1 \) immediately,
A message is received **immediately before** a clock tick.

Figure 3.13: Minimum distance between two consecutive clock synchronization events

Figure 3.14: Error scenario for line topologies when $N \mod 3 = 0$

while the other “slow” staircase has steps with the maximum time between synchronizations (using the mechanism of Figure 3.12), where a synchronization signal is received immediately after the receiver clock ticks, and it takes an additional clock tick before resetting the clock is reset to $g+1$. Both staircases start from the same point, viz. when node number 1, the second node in the line, sends messages to its neighboring nodes 0 and 2. After $N-1$ steps the two staircases join again when node $N-2$ tries to communicate with node $N-1$. At that point, node $N-2$ has gone through $g$ time units since its previous synchronization and is about to send a message to node $N-1$. However, node $N-1$ is about to make a clock tick and enter its new time slot, which is convenient for receiving the message from its neighbor. Synchronization is lost when node $N-2$ starts sending before node $N-1$ ticks.

This proof, in sum, shows that for each network of line topology in which guard
3.6 Conclusion

This chapter demonstrated the application of timed automata model checking and UPPAAL in modeling and analysis of a synchronization protocol for wireless sensor networks, provided by the Chess eT International B.V.

We have seen timed automata model checking led to discovery of some interesting error scenarios for line topologies: for any instantiation of the parameters, the protocol will eventually fail if the network grows.
Moreover, in this chapter, a parametric verification for the very restrictive case of cliques (network with full connectivity) was presented. Indeed, model checking was used to find the key error scenarios that underly the parameter constraints for correctness. The parameter constraints were then proved to be sufficient and necessary for the network to be synchronized. Afterwards, the correctness of the manual invariant proof was checked by automatic theorem proving.

Despite its limitations, Uppaal proved to be indispensable for the formal analysis of the Chess protocol. Modeling the protocol in terms of timed automata is natural, the graphical user interface helped to visualize the models, the simulator was of great help during the initial validation of the model, and the ability of Uppaal to generate counterexamples and to replay them in the simulator was helpful for finding the parameter constraints that are needed for correctness. Since Uppaal does not support parametric verification, the sufficiency of the constraints was proved manually. But Uppaal was also helpful in checking the validity of various invariants for instances of the model, and obtaining confidence in their correctness before embarking on the (long and tedious) invariant proofs.

Using Uppaal, we have only been able to analyze models of some really small networks, and in order to carry out the analysis making some drastic simplifying assumptions were inevitable. Clearly, there is a lot of room for improving timed automata model checking technology, enabling the analysis of larger and dynamic network topologies. Nevertheless, it is concluded that the ability of model checkers to find worst-case error scenarios appears to be quite useful in this application.
domain.

In practical applications of WSNs, cliques rarely occur and therefore the verification results should primarily be seen as a first step towards a correctness proof for arbitrary and dynamically changing network topologies. Nevertheless, these results give us an upper bound on allowable clock drift of a generic WSN.

Methodologically, the approach of this research is similar to the study of the Biphase Mark Protocol [97], which also uses UPPAAL to analyze instances of the protocol and a theorem prover for the full parametric analysis. Theorem provers have been frequently and successfully applied for the analysis of clock synchronization protocols, see for instance [86, 88]. An interesting research challenge is to synthesize (or prove the correctness of) the parameter constraints for the Chess protocol fully automatically. Recently, some approaches have been presented by which, for instance, the (parametric) Biphase Mark Protocol can be verified fully automatically [20, 96]. Very interesting also is the work of [22, 39] on parameterized verification (using the SMT based tool MCMT) of networks with an arbitrary number of identical timed automata. However, it seems that these approaches are not powerful enough (yet) to handle this WSN protocol in which the number N of sensor nodes is not fixed, and the parameter constraints and the length of the corresponding counterexamples depend on N. Moreover, in the case of this WSN algorithm the parameter constraints involve a product of three parameters, whereas the mentioned techniques can only handle linear constraints on the parameters. Finally, these new tools still lack the graphical user interface and expressive input language of UPPAAL, which are key features that enable the application of formal methods in practice.
Chapter 4

Modeling of Median Algorithm for Synchronization of Wireless Sensor Networks

In this chapter, a detailed model of the synchronization algorithm used in the implementation of the Chess wireless sensor network is presented. The algorithm, named Median, is an extension of an algorithm proposed by Tjoa et al [91], and the idea is that in every frame each node computes its phase error to any of its direct neighbors. After the last active slot, each node adjusts its clock by the median of the phase errors of its immediate neighbors. A detailed model of the Chess Median algorithm is presented in this chapter using the input language of Uppaal. The objective is to construct a model that comes as close as possible to the specification of the clock synchronization algorithm presented in [80]. This chapter is organized as follows. In Section 4.1 the Median algorithm is presented in detail. Section 4.2 describes the Uppaal model of Median algorithm. In Section 4.3 the analysis results are described. Finally, in Section 4.4 some conclusions are drawn.


4.1 The Median Protocol

In this section additional details are provided about the Median protocol as it has been implemented by Chess.
4.1.1 The Synchronization Algorithm

In each frame, each node broadcasts one message to its neighbors. The timing of this message is used for synchronization purposes: a receiver may estimate the clock value of a sender based on the time when the message is received. Thus there is no need to send around (logical) clock values. In the current implementation of Chess, clock synchronization is performed once per frame using the following algorithm [7, 100]:

1. In its sending slot, a node broadcasts a packet which contains its transmission slot number.

2. Whenever a node receives a message it computes the phase error, that is the difference (number of clock cycles) between the expected receiving time and the actual receiving time of the incoming message. Note that the difference between the sender’s sending slot number (which is also the current slot number of the sender) and the current slot number of the receiving node must also be taken into account when calculating the phase errors.

3. After the last active slot of each frame, a node calculates the offset from the phase errors of all incoming messages in this frame with the following algorithm:

   ```java
   if (number of received messages == 0)
       offset = 0;
   else if (number of received messages <= 2)
       offset = gain * the phase error of the first received message;
   else
       offset = gain * the median of all phase errors
   ```

   Here gain is a coefficient with value 0.5, used to prevent oscillation of the clock adjustment.

4. During the sleeping period, the frame length of each node is adjusted by the computed offset obtained from step 3.

In situations when two networks join, it is possible that the phases of these networks differ so much that the nodes in one network are in active slots whereas the nodes in the other network are in sleeping slots and vice versa. In this case, no messages can be exchanged between two networks. Therefore in the Chess design, a node will send an extra message in one (randomly selected) sleeping slot to increase the chance that networks can communicate and synchronize with each other. This slot is called the synchronization slot and the message is in the same format as in the transmission slot. The extreme value of offset can be obtained when two networks join: it may occur that the offset is larger than half the total
number of clock cycles of sleeping slots in a frame. Chess uses another algorithm called \textit{join} to handle this extreme case. For simplicity, the \textit{join} algorithm is not modeled, so joining of networks and synchronization messages are not covered in this research.

4.1.2 Guard Time

The desirable correctness condition for gMAC is that whenever a node is sending all its neighbors are in receiving mode. However, at the moment when a node enters its TX slot it cannot be guaranteed, due to the phase errors, that its neighbors have entered the corresponding RX slot. This problem is illustrated in Figure \ref{fig:guard_time}(a). Given two nodes 1 and 2, if a message is transmitted during the entire sending slot of node 1 then this message may not be successfully received by node 2 because of the imperfect slot alignment. Taking the clock of node 1 as a reference, the clock of node 2 may drift backwards or forwards. In this situation, node 1 and node 2 may have a different view of the current slot number within the time interval where node 1 is sending a message.

To cope with this problem, messages are not transmitted during the entire sending slot but only in the middle, as illustrated in Figure \ref{fig:guard_time}(b). Both at the beginning and at the end of its sending slot, node 1 does not transmit for a preset period of \(g\) clock ticks, in order to accommodate the forwards and backwards clock drift of node 2. Therefore, the time available for transmission equals the total length of the slot minus \(2g\) clock ticks.

4.1.3 Radio Switching Time

The radio of a wireless sensor node can either be in sending mode, or in receiving mode, or in idle mode. Switching from one mode to another takes time. In the current implementation of the Chess gMAC protocol, the radio switching time is around 130\(\mu\text{sec}\). The time between clock ticks is around 30\(\mu\text{sec}\) and the guard time \(g\) is 9 clock ticks. Hence, in the current implementation the radio switching time
is smaller than the guard time, but this may change in future implementations. If the first slot in a frame is an RX slot, then the radio is switched to receiving mode some time before the start of the frame to ensure that the radio will receive during the full first slot. However if there is an RX slot after the TX slot then, in order to keep the implementation simple, the radio is switched to the receiving mode only at the start of the RX slot. Therefore messages arriving in such receiving slots may not be fully received. This issue may also affect the performance of the synchronization algorithm.

### 4.2 Uppaal Model

In this section, the UPPAAL model of the gMAC protocol is described.

A finite, fixed set of wireless nodes \( \text{Nodes} = \{0, \ldots, N - 1\} \) is assumed. The behavior of an individual node \( \text{id} \in \text{Nodes} \) is described by five timed automata \( \text{Clock(id)} \), \( \text{Receiver(id)} \), \( \text{Sender(id)} \), \( \text{Synchronizer(id)} \) and \( \text{Controller(id)} \). Figure 4.2 shows how these automata are interrelated. All components interact with the clock, although this is not shown in Figure 4.2. Automaton \( \text{Clock(id)} \) models the hardware clock of node \( \text{id} \), automaton \( \text{Sender(id)} \) the sending of messages by the radio, automaton \( \text{Receiver(id)} \) the receiving part of the radio, automaton \( \text{Synchronizer(id)} \) the synchronization of the hardware clock, and automaton \( \text{Controller(id)} \) the control of the radio and the clock synchronization.

![Figure 4.2: Message flow in the model](image)

Table 4.1 lists the parameters that are used in the model (constants in UPPAAL terminology), together with some basic constraints. The domain of all parameters is the set of natural numbers. Now, the five automaton templates used in the model are described.

**Clock** Timed automaton \( \text{Clock(id)} \) models the behavior of the hardware clock of node \( \text{id} \). The automaton is shown in Figure 4.3. At the start of the system state variable \( \text{csn[id]} \), that records the current slot number, is initialized to \( C - 1 \), that is, to the last sleeping slot. Hardware clocks are not perfect and so a
### Table 4.1: Protocol parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>number of nodes</td>
<td>1 &lt; N</td>
</tr>
<tr>
<td>C</td>
<td>number of slots in a time frame</td>
<td>0 &lt; C</td>
</tr>
<tr>
<td>n</td>
<td>number of active slots in a time frame</td>
<td>0 &lt; n ≤ C</td>
</tr>
<tr>
<td>tsn[id]</td>
<td>TX slot number for node id</td>
<td>0 ≤ tsn[id] &lt; n</td>
</tr>
<tr>
<td>k₀</td>
<td>number of clock ticks in a time slot</td>
<td>0 &lt; k₀</td>
</tr>
<tr>
<td>g</td>
<td>guard time</td>
<td>0 &lt; g</td>
</tr>
<tr>
<td>r</td>
<td>radio switch time</td>
<td>0 ≤ r</td>
</tr>
<tr>
<td>min[id]</td>
<td>minimal time between two clock ticks of node id</td>
<td>0 &lt; min[id]</td>
</tr>
<tr>
<td>max[id]</td>
<td>maximal time between two clock ticks of node id</td>
<td>min[id] ≤ max[id]</td>
</tr>
</tbody>
</table>

minimal time min[id] and a maximal time max[id] between successive clock ticks are assumed. Integer variable clk[id] records the current value of the hardware clock. For convenience (and to reduce the size of the state space), the hardware clock is assumed to be reset at the end of each slot, that is after k₀ clock ticks. Also, a state variable csn[id], which records the current slot number of node id, is updated each time at the start of a new slot.

![Automaton Clock[id]](image)

**Sender** The sending behavior of the radio is described by the automaton Sender[id] shown in Figure 4.4. The behavior is rather simple. When the controller asks the sender to transmit a message (via a start_sending[id] signal), the radio first switches to sending mode (this takes r clock ticks) and then transmits the message (this takes k₀ − 2 · g ticks). Immediately after the message transmission has been completed, an end_sending[id] signal is sent to the controller to indicate that the message has been sent.

**Receiver** The automaton Receiver[id] models the receiving behavior of the radio. The automaton is shown in Figure 4.5. Again the behavior is rather simple. When the controller asks the receiver to start receiving, the receiver first switches to receiving mode (this takes r ticks). After that, the receiver may receive messages...
from all its neighbors. A function $\text{neighbor}$ is used to encode the topology of the network: $\text{neighbor}(j, \text{id})$ holds if messages sent by $j$ can be received by $\text{id}$. Whenever the receiver detects the end of a message transmission by one of its neighbors, it immediately informs the synchronizer via a $\text{message\_received}[\text{id}]$ signal. At any moment, the controller can switch off the receiver via an $\text{end\_receiving}[\text{id}]$ signal.

**Controller** The task of the $\text{Controller}[\text{id}]$ automaton, displayed in Figure 4.6, is to put the radio in sending and receiving mode at the appropriate moments. Figure 4.7 shows the definition of the predicates used in this automaton. The radio should be put in sending mode $r$ ticks before message transmission starts (at time $g$ in the transmission slot of $\text{id}$). If $r > g$ then the sender needs to be activated $r - g$ ticks before the end of the slot that precedes the transmission slot. Otherwise, the sender must be activated at tick $g - r$ of the transmission slot. If the first slot in
a frame is an RX slot, then the radio is switched to receiving mode $r$ time units before the start of the frame to ensure that the radio will receive during the full first slot. However if there is an RX slot after the TX slot then, as described in Section 4.1.3, the radio is switched to the receiving mode only at the start of the RX slot. The controller stops the radio receiver whenever either the last active slot has passed or the sender needs to be switched on.

![Automaton Controller](image)

```cpp
bool go_send(){return (r>g) ?((csn[id]+1)%C==tsn[id] & clk[id]==k0-(r-g)) :((csn[id]==tsn[id] & clk[id]==g-r));}

bool go_receive(){return (r>0 && 0!=tsn[id] && csn[id]==C-1 && clk[id]==k0-r) || (r==0 && 0!=tsn[id] && csn[id]==0) || (0<csn[id] && csn[id]<n && csn[id]-1==tsn[id]);}

bool go_sleep(){return csn[id]==n;}
```

![Predicates used in Controller](image)

All the channels used in the Controller[id] automaton (`start_sending`, `end_sending`, `start_receiving`, `end_receiving` and `synchronize`) are urgent, which means that these signals are sent at the moment when the transitions are enabled.

**Synchronizer** Finally, automaton **Synchronizer[id]** is shown in Figure 4.8. The automaton maintains a list of phase differences of all messages received in the current frame, using a local array `phase_errors`. Local variable `msg_counter` records the number of messages received. Whenever the receiver gets a message from a neighboring node (`message_received[id]`), the synchronizer computes and stores the phase difference using the function `store_phase_error` at the next clock tick. Here the phase difference is defined as the expected time at which the message transmission ends ($tsn[sender] * k0 + k0 - g$) minus the actual time at which the message transmission ends ($csn[id] * k0 + clk[id]$), counting from
void store_phase_error(int sender)
{
    phase_errors[msg_counter] =
        (tsn[sender] * k0 + k0 - g) - (csn[id] * k0 + clk[id]);
    msg_counter++
}

As explained in Section 4.1.1, the synchronizer computes the value of the phase correction (offset) and adjusts the clock during the sleeping period of a frame. Hence, in order to decide in which slot the synchronization should be performed, the maximal phase difference between two nodes should be known. In this model, it is assumed no joining of networks occurs. When a node receives a message from another node, the phase difference computed using this message will not exceed the length of an active period. Otherwise one of these two nodes will be in sleeping period while the other is sending, hence no message can be received at all. In practice, the number of sleeping slots is much larger than the number of active slots. Therefore it is safe to perform the adjustment in the middle of sleeping period because the desired property described above holds. When the value of gain is smaller than 1 the maximal phase difference will be even smaller.

The function of compute_phase_correction implements exactly the algorithm listed in Section 4.1.1.

---

Footnote:

1 Actually, in the implementation the offset is used to compute the corrected wakeup time, that is the moment when the next frame will start $t_{next}$. In this model the clock is reset, but this should be equivalent.
4.3 Analysis Results

In this section, the verification results for simple instances of the model described in Section 4.2 are presented. The following invariant properties were checked using the UPPAAL model checker:

INV1 : A[] forall (i: Nodes) forall (j : Nodes)
       SENDER(i).Sending & neighbor(i,j) imply RECEIVER(j).Receiving

INV2 : A[] forall (i:Nodes)forall (j:Nodes) forall (k:Nodes)
       SENDER(i).Sending & neighbor(i,k) &
       SENDER(j).Sending & neighbor(j,k) imply i == j

INV3 : A[] not deadlock

The first property states that always when some node is sending, all its neighbors are listening. The second property states that never two different neighbors of a given node are sending simultaneously. The third property states that the model contains no deadlock, in the sense that in each reachable state at least one component can make progress. The three invariants are basic sanity properties of the gMAC protocol, at least in a setting with a static topology and no transmission failures.

UPPAAL was used on a Sun Fire X4440 machine (with 4 Opteron 8356 2.3 Ghz quad-core processors and 128 Gb DDR2-667 memory) to verify instances of the model with different number of nodes, different network topologies and different parameter values. Table 4.2 lists some of the verification results, including the resources UPPAAL needed to verify if the network is synchronized or not. In all experiments, C = 10 and k₀ = 29.

Clearly, the values of network parameters, in particular clock parameters min and max, affect the result of the verification. Table 4.2 shows several instances where the protocol is correct for perfect clocks (min = max) but fails when the ratio min/max is decreased. It is easy to see that the protocol will always fail when r ≥ g. Consider any node i that is not the last one to transmit within a frame. Right after its sending slot, node i needs r ticks to get its radio into receiving mode. This means that — even with perfect clocks — after g ticks another node already has started sending even though the radio of node i is not yet receiving. Even when r < g, the radio switching time has a clear impact on correctness: the larger the radio switching time is, the larger the guard time has to be in order to ensure correctness. Using UPPAAL, it is possible to fully analyze line topologies with at most seven nodes if all clocks are perfect. For larger networks UPPAAL runs out of memory. A full parametric analysis of this protocol will be challenging, also due to the impact of the network topology and the selected slot allocation. Using UPPAAL, it is discovered that for certain topologies and slot allocations the Median algorithm may always violate the above correctness assertions, irrespective of the
<table>
<thead>
<tr>
<th>N/n</th>
<th>Topology</th>
<th>g</th>
<th>r</th>
<th>min(\text{max})</th>
<th>CPU Time</th>
<th>Peak Memory Usage</th>
<th>Sync</th>
</tr>
</thead>
<tbody>
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Table 4.2: Model checking experiments
choice of the guard time. For example, in a 4 node-network with clique topology and min and max of 100.000 and 100.001, respectively, if the median of the clock drifts of a node becomes −1, the median algorithm divides it by 2 and generates 0 for clock correction value and indeed no synchronization happens. If this scenario repeats in three consecutive time frames for the same node, that node runs \( g = 3 \) clock cycles behind and gets out of sync.

Another example in which the algorithm may fail is displayed in Figure 4.10. This network has 4 nodes, connected by a line topology, that send in slots 1, 2, 3, 1, respectively. Since all nodes have at most two neighbors, the Median algorithm prescribes that nodes will correct their clocks based on the first phase error that they see in each frame. For the specific topology and slot allocation of Figure 4.10, this means that node 0 adjusts its clock based on phase errors of messages it gets from node 1, node 1 adjusts its clock based on messages from node 0, node 2 adjusts its clock based on messages from node 3, and node 3 adjusts its clock based on messages from node 2. Hence, for the purpose of clock synchronization, we have two disconnected networks! Thus, if the clock rates of nodes 0 and 1 are lower than the clock rates of nodes 2 and 3 by just an arbitrary small margin, then two subnetworks will eventually get out of sync. These observations are consistent with results obtained using Uppaal. If, for instance, we set \( \min[id] = 99 \) and \( \max[id] = 100 \), for all nodes id then neither INV1 nor INV2 holds. In practice, it is unlikely that the above scenario will occur due to the fact that in the implementation slot allocation is random and dynamic. Due to regular changes of the slot allocation, with high probability node 1 and node 2 will now and then adjusts their clocks based on messages they receive from each other.

However, variations of the above scenario may occur in practice, even in a setting with dynamic slot allocation. In fact, the above synchronization problem is also not restricted to line topologies. A subset \( C \) of nodes in a network is called a community if each node in \( C \) has more neighbors within \( C \) than outside \( C \). For any network in which two disjoint communities can be identified, the Median algorithm allows for scenarios in which these two parts become unsynchronized. Due to the median voting mechanism, the phase errors of nodes outside a community will not affect the nodes within this community, independent of the slot allocation. Therefore, if nodes in one community \( A \) run slow and nodes in another community \( B \) run fast then the network will become unsynchronized eventually, even in a setting with infinitesimal clock drifts. Figure 4.11 gives an example of a network with two communities.
Figure 4.11: Another problematic network topology with two communities

Using UPPAAL, instances of the simple network with two communities displayed in Figure 4.12 were analyzed. The numbers on the vertices are the node identifiers and the transmission slot numbers, respectively. Table 4.3 summarizes the results of the model checking experiments.

It is still needed to explore how realistic these counterexamples are. Network topologies with multiple communities occur in many WSN applications. Nevertheless, in practice the gMAC protocol appears to perform quite well for static networks. It might be that problems do not occur so often in practice due to the probabilistic distributions of clock drift and jitter.

4.4 Conclusion

This chapter represented a detailed UPPAAL model of relevant parts of the clock synchronization algorithm implemented in a wireless sensor network developed by Chess [80, 100].

Using UPPAAL, it was established that in certain cases a static, fully synchr-
Table 4.3: Model checking experiments of a network with two communities

<table>
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<th>$r$</th>
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<th>Slow Clock Cycle Length</th>
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A synchronized network may eventually become unsynchronized if the Median algorithm is used, even in a setting with infinitesimal clock drifts.

In another research, presented in chapter 3, another synchronization algorithm for WSN was presented that does not have the correctness problems of the Median algorithm. However, that algorithm has never been tested in practice. Advantages of that approach are (a) unlike the Median approach and its variants we need almost no guard time at the end of a sending slot (2 clock ticks suffice instead of 9 ticks in the current implementation), and (b) the computational overhead becomes essentially zero.

Assegei proposed and simulated three alternative algorithms, to be used instead of the Median algorithm, in order to achieve decentralized, stable and energy-efficient synchronization of the Chess gMAC protocol. It should be easy to construct UPPAAL models for Assegei’s algorithms: basically, only the definition of the compute_phase_correction function should be modified.

Overall, starting from the UPPAAL model introduced in this chapter it should be relatively easy to construct models for alternative synchronization algorithms in order to explore their properties.
Chapter 5

Conclusion of Part One

Wireless sensor networks are beginning to be deployed at an accelerated pace due to their wide range of application potential in areas such as target detection and tracking, environmental monitoring, industrial process monitoring, and tactical systems. Low power capacities of sensor nodes makes the design of medium access protocols pretty challenging. One of the greatest challenges in the design of the MAC layer is to find suitable mechanisms for clock synchronization: we must ensure that whenever some node is sending all its neighbors are listening.

Many clock synchronization protocols have been proposed for WSNs, see e.g. [90, 35, 91, 70, 7, 63, 79]. However, these protocols (with the exception of [91, 7] and possibly [79]) involve a computation and/or communication overhead that is unacceptable given the extremely limited resources (energy, memory, clock cycles) available within the Chess nodes. In most of these protocols, clocks are synchronized to an accurate real-time standard like Coordinated Universal Time (UTC). In [90] an overview of this type of protocols is presented. However, these protocols are based on the exchange of time stamp messages, and for the Chess WSN this creates an unacceptable computation and communication overhead. It is possible to come up with more efficient algorithms, since for the MAC layer a weak form of clock synchronization suffices: a node only needs to be synchronized to its immediate neighbors, not to faraway nodes or to UTC. Fan & Lynch [35] study the gradient clock synchronization (GCS) problem, in which the difference between any two network nodes’ clocks must be bounded from above by a non-decreasing function. Thus nearby nodes must be closely synchronized but faraway nodes are allowed to be more loosely synchronized. In the approach of Fan & Lynch [35], nodes compute logical clock values based on their hardware clocks and message exchanges, and the goal is to synchronize the nodes’ logical clocks as closely as possible, while satisfying certain validity conditions. Logical clocks have been introduced by Lamport [56] to totally order the events in a distributed system. A key property of Lamport’s logical clocks is that they never run backwards: their value can only increase. In fact, Fan & Lynch [35] assume a constant lower bound
on clock speed. Also Meier & Thiele [70] and Pussente & Barbosa [79], who adapt the work of Fan & Lynch to the setting of wireless sensor networks, make a similar assumption (with minimal clock rates $\frac{1}{2}$ and $\frac{1}{D}$, respectively, where $D$ is the network diameter). For certain applications of WSNs it is important to have Lamport style logical clocks. For example, if two sensor nodes observe a moving object, then logical clocks allow one to establish the object's direction by determining which node observed the object first [70]. However, for the MAC layer there is no need to compute a total order on events: we only need to ensure that whenever one node is sending all neighbors are listening. Since it is allowed to set back clocks, the lower bounds of [35, 70] do not apply in this case.

Quasimodo research on the Chess WSN case study had a significant impact on the design of the network [82]. But equally important, it also provided major challenges for research on theory and algorithms for model checking. Most industrial applications of Uppaal thus far involve small networks with a fixed topology. The size and dynamic nature of the Chess WSN, and the resulting complexity of clock synchronization, provides a major challenge for model checking technology that goes beyond what we have seen in other Uppaal case studies. Another challenge raised by the present research is in the area of parametric model checking: the parameter constraints that were derived in chapter 3 for clique topologies are nonlinear, and appear to be beyond reach of existing algorithms for parametric model checking. We have seen, even the analysis of a basic clock synchronization algorithm for an industrial WSN platform turns out to be quite difficult.

Meier & Thiele [70] provide a lower bound for the achievable synchronization quality in sensor networks, but no algorithms that attain or come close to this bound. Pussente & Barbosa [79] also proposed a clock synchronization algorithm that achieves an $O(1)$ worst-case skew between the logical clocks of neighbors, but this cannot be applied in the TDMA based setting of the Chess algorithm. Basic assumptions of [70, 79] are that (a) messages sent between neighbors are always delivered instantaneously, and (b) consecutive communications between any two neighbors in the same direction are no farther apart in time than some given time $d$. Pussente & Barbosa [79] derive a strict upper bound of $c + 2(1 + 2\hat{\rho})d$ on the difference between the clocks of neighboring nodes, where $c > 0$ is a constant and $\hat{\rho} \in [0, 1)$ is the maximal clock drift. But since this bound exceeds $2d$ and in a TDMA setting $d$ basically equals the length of a frame, the algorithm of [79] is unable to guarantee that whenever some node is sending all its neighbors are listening.

All in all, a fundamental open question is to establish an impossibility result along the lines of Fan & Lynch [35] for the setting of the Chess MAC layer in which clocks can be set back. The MyriaNed algorithm analyzed in chapter 3 appears to perform well for networks with a small diameter, but the results of section 3.5 show that performance may degrade if the diameter increases. However, the counterexample scenarios described in this thesis require a very specific combination of events and clock drifts; hence it may not appear so often in practice. Chess has
already demonstrated twice that a network of the size in the order of 1000 nodes works without encountering these error scenarios.

Wireless sensor networks also constitute a potentially very important but also extremely challenging application area for formal methods. For quantitative formal methods, one challenge is to come up with the right abstractions that will facilitate verifying larger instances of the model.

Using state-of-the-art model checking technology, we have only been able to analyze models of some really small networks. In order to carry out the analysis making some drastic simplifying assumptions were inevitable. Clearly, there is a lot of room for improving timed automata model checking technology, enabling the analysis of larger and dynamic network topologies. Nevertheless, it is concluded that the ability of model checkers to find worst-case error scenarios appears to be quite useful in this application domain. In particular, error scenarios—found using Uppaal by exploring simple models of small networks—are reproducible in real implementations of larger networks [81].

The use of simulations is essential for providing insight into the robustness and usefulness of MAC layer protocols, also because occasional flaws of the MAC layer protocols may be resolved by the redundancy of the gossip layer. However, this research asserts that it is unlikely that simulation techniques will be able to produce worst case counterexamples, such as the example of Figure 3.14 that was produced by Uppaal. Work of [24] also shows that one has to be extremely careful in using the results of MANET simulators.

Wireless sensor networks algorithms pose many challenges for probabilistic model checkers and specification tools such as PRISM [54] and CaVi [36]. A first challenge is to make a more detailed, probabilistic model of radio communication that involves the possibility of message loss. Another challenge is to consider dynamic slot allocation. In the research of this thesis, a fixed slot allocation was assumed. However, in the actual implementation of Chess, a sophisticated probabilistic algorithm is used for dynamic slot allocation. Formal analysis of this algorithm would be very interesting. Another simplifying assumption made in this research is that the network topology is fixed. A probabilistic model in which nodes may join or leave will be more realistic. Finally, the gossiping algorithms used by the Chess network are intrinsically probabilistic in nature. Formal analysis of the gossip layer is a largely unexplored research field [11]. Practical obstacles for the application of probabilistic model checkers to the Chess case study are the limited expressivity of the input language of existing tool, and the small network sizes that can be handled. An interesting alternative approach for some of the problems in this area is the use of mean-field analysis, as proposed by Bakhshi et al [12].

Several other researches report on the application of Uppaal for the analysis of protocols for wireless sensor networks, see e.g. [37, 36, 95, 43]. In [103], Uppaal is also used to automatically test the power consumption of wireless sensor networks. This research confirms the conclusions of [37, 95]: despite the small
number of nodes that can be analyzed, model checking provides valuable insight in the behavior of protocols for wireless sensor networks, insight that is complementary to what can be learned through the application of simulation and testing or theorem proving.
Part II

Automata Learning through Counterexample-guided Abstraction Refinement
Chapter 6

Introduction to Part Two

Model-based system development is becoming an increasingly important driving force in the software and hardware industry. In this approach, models become the primary artifacts throughout the engineering lifecycle of computer-based systems. Requirements, behavior, functionality, construction and testing strategies of computer-based systems are all described in terms of models. Models are not only used to reason about a system, but also used to allow all stakeholders to participate in the development process and to communicate with each other, to generate implementations, and to facilitate reuse. The construction of models typically requires significant manual effort, implying that in practice often models are not available, or become outdated as the system evolves. Tools that are able to infer state machine models automatically by systematically “pushing buttons” and recording outputs have numerous applications in different domains. For instance, they support understanding and analyzing legacy software, regression testing of software components [48], protocol conformance testing based on reference implementations, reverse engineering of proprietary/classified protocols, fuzz testing of protocol implementations [29], and inference of botnet protocols [27]. Automated support for constructing behavioral models of implemented components would therefore be extremely useful.

The problem to build a state machine (automata) model of a system by providing inputs to it and observing the outputs resulting, which often referred to as black-box system identification, is fundamental and has been studied for decades. A major challenge is to let computers perform this task in a rigorous manner for systems with large numbers of states. Moore [74] first proposed the problem of learning automata, provided an exponential algorithm, and proved that this problem is inherently exponential. Many techniques for constructing models from observation of component behavior have been proposed, for instance in [6, 84, 83, 45].

The most efficient such techniques use the setup of active learning, where a model of a system is learned by actively performing experiments on that system.
In other words, within the setting of active learning, a learner interacts with a teacher. This problem was addressed by Angluin [6] in her $L^*$ algorithm for learning of finite state automata (FSA). Niese [76] adapted the $L^*$ algorithm for active learning of deterministic Mealy machines. This algorithm has been further optimized in [83]. The assumption, in the algorithm, is that the teacher knows a deterministic Mealy machine $M$, and the learner, initially, knows the action signature (the sets of input and output symbols $I$ and $O$). The learner’s task is to learn a Mealy machine that is equivalent to $M$. The teacher reacts to two types of queries: –output queries (“what is the output generated in response to input $i \in I^*$?”) and equivalence queries (“is a hypothesized machine $H$ correct, i.e., equivalent to the machine $M$?”). The learner always records the current state $q$ of Mealy machine $M$. In response to output query $i$, the current state is updated to $q'$ and answer $o$ is returned to the learner. At any point the learner can “reset” the teacher, that is, change the current state back to the initial state of $M$. The answer to an equivalence query $H$ is either yes (in case $M \approx H$) or no (in case $M \not\approx H$). Furthermore, with every negative equivalence query response, the teacher will provide the learner with a counterexample, that is an input sequence $u \in I^*$ such that $\text{obs}_M(u) \neq \text{obs}_H(u)$. This algorithm has been implemented in the LearnLib tool [83, 46, 71], the winner of the 2010 Zulu competition on regular inference. In practice, when a real implementation is used instead of an idealized teacher, the implementation cannot answer equivalence queries. Therefore, LearnLib “approximates” such queries by generating a long test sequence that is computed by standard methods such as state cover, transition cover, W-method, and the UIO method among which fully randomized testing turn out to be most effective. (see [58]).

LearnLib has been applied successfully to learn various kinds of systems, such as computer telephony integrated (CTI) systems [48]. Nevertheless, a lot of further research will be required to make automata based learning tools suitable for routine use on industrial case studies. An issue is the extension of automata learning techniques to nondeterministic systems (see e.g. [102]). Furthermore, in practice, the characteristic of Mealy machines that each input corresponds to exactly one output is overly restrictive. Sometimes several inputs are required before a single output occurs, sometimes a single input triggers multiple outputs, etc. In [3], Aarts & Vaandrager addressed this problem by developing a method for active learning of I/O automata. The I/O automata of Lynch & Tuttle [67, 68] and Jonsson [52] constitute a popular modeling framework which does not suffer from the restriction that inputs and outputs have to alternate. Aarts & Vaandrager used LearnLib, and their idea was to place a transducer in between the IOA teacher and the Mealy machine learner, which translates concepts from the world of I/O automata to the world of Mealy machines, and vice versa. The transducer and Mealy machine learner together then implement an IOA learner. Another important issue, clearly, is the development of abstraction techniques in order to be able to learn much larger state spaces (see [1], also for further references). State-
of-the-art methods for learning automata such as LearnLib are currently able to only learn automata with at most in the order of 10,000 states. Hence, powerful abstraction techniques are needed to apply these methods to practical systems. There are many reasons to expect that by combining ideas from verification, model-based testing and automata learning, it will become possible to learn models of realistic software components with state-spaces that are many orders of magnitude larger than what state-of-the-art tools can currently handle. See e.g. [18, 64, 83].

The second part of this dissertation presents a framework for automata learning. In chapter 7 this framework is described in full mathematical details. Thereafter, in chapter 8 the tool Tomte is introduced. Tomte enables learning of a restricted class of parametrized Mealy machines (i.e. Mealy machines, for which each action contains a few parameters), which are called scalarset Mealy machines. In scalarset Mealy machines, one can test for equality of data parameters, but no operations on data are allowed. In the rest of this introduction, a brief history of this research is presented together with a rough description of the idea proposed in this thesis. Furthermore, Tomte functionality is shortly explained.

6.1 History Dependent Abstraction

Applying existing automata learning methods on realistic applications is typically achievable only through abstraction. Dawn Song at al [27], for instance, succeeded to infer models of realistic botnet command and control protocols by placing an emulator between botnet servers and the learning software, which concretizes the alphabet symbols into valid network messages and sends them to botnet servers. When responses are received, the emulator does the opposite — it abstracts the response messages into the output alphabet and passes them on to the learning software. The idea of an intermediate component that takes care of abstraction is very natural and is used, implicitly or explicitly, in many case studies on automata learning.

Within process algebra [19], the most prominent abstraction operator is the $\tau_I$ operator from ACP, which renames actions from a set $I$ into the internal action $\tau$. In order to establish that an implementation $\text{Imp}$ satisfies a specification $\text{Spec}$, one typically proves $\tau_I(\text{Imp}) \approx \text{Spec}$, where $\approx$ is some behavioral equivalence or preorder that treats $\tau$ as invisible. In state based models of concurrency, such as TLA+ [57], the corresponding abstraction operator is existential quantification, which hides certain state variables. Both $\tau_I$ and $\exists$ abstract in a way that does not depend on the history of the computation. In practice, however, we frequently describe and reason about reactive systems in terms of history dependent abstractions. For instance, most of us have dealt with the following protocol: “If you forgot your password, enter your email and user name in the form below. You will then receive a new, temporary password. Use this temporary password to login and immediately select a new password.” Here, essentially, the huge name spaces for user names and passwords are abstracted into small sets with abstract values.
such as “temporary password” and “new password”. The choice which concrete password is mapped to which abstract value depends on the history, and may change whenever the user selects a new password.

History dependent abstractions are the key for scaling methods for active learning of automata to realistic applications. History dependent abstractions can be described formally using the state operator known from process algebra \([9]\), but this operator has been mostly used to model state bearing processes, rather than as an abstraction device. Implicitly, history dependent abstractions play an important role in the work of Pistore et al \([73, 38]\): whereas the standard automata-like models for name-passing process calculi are infinite-state and infinite-branching, they provide models using the notion of a history dependent automaton which, for a wide class of processes (e.g. finitary \(\pi\)-calculus agents), are finite-state and may be explored using model checking techniques. Aarts, Jonsson and Uijen \([1]\) formalized the concept of history dependent abstractions within the context of automata learning. Inspired by ideas from predicate abstraction \([66]\) and abstract interpretation \([30]\), they defined the notion of a mapper \(\mathcal{A}\), which is placed in between the teacher or system-under-test (SUT), described by a Mealy machine \(\mathcal{M}\), and the learner. The mapper transforms the concrete actions of \(\mathcal{M}\) (in a history dependent manner) into a small set of abstract actions. Each mapper \(\mathcal{A}\) induces an abstraction operator \(\alpha_{\mathcal{A}}\) that transforms a Mealy machine over the concrete signature into a Mealy machine over the abstract signature. A teacher for \(\mathcal{M}\) and a mapper for \(\mathcal{A}\) together behave like a teacher for \(\alpha_{\mathcal{A}}(\mathcal{M})\). Hence, by interacting with the mapper component, the learner may learn an abstract Mealy machine \(\mathcal{H}\) that is equivalent \((\approx)\) to \(\alpha_{\mathcal{A}}(\mathcal{M})\). Mapper \(\mathcal{A}\) also induces a concretization operator \(\gamma_{\mathcal{A}}\). The main technical result of \([1]\) is that, under certain assumptions, \(\alpha_{\mathcal{A}}(\mathcal{M}) \approx \mathcal{H}\) implies \(\mathcal{M} \approx \gamma_{\mathcal{A}}(\mathcal{H})\).

Aarts et al \([1]\) demonstrated the feasibility of their approach by learning models of fragments of realistic protocols such as SIP and TCP \([1]\), and the new biometric passport \([2]\). The learned SIP model, for instance, is an extended finite state machine with 29 states, 3741 transitions, and 17 state variables with various types (booleans, enumerated types, (long) integers, character strings,..). This corresponds to a state machine with an astronomical number of states and transitions, which is thus far, fully out of reach of current automata learning techniques.

![Figure 6.1: Active learning with an abstraction mapping.](image-url)
Despite its success, the theory of [1] has several limitations when facing real-world case studies. Here is an example: In a (deterministic) Mealy machine, each sequence of input actions uniquely determines a corresponding sequence of output actions. This means that the login protocol that was described above cannot be modeled in terms of a Mealy machine, since a single input (a request for a temporary password) may lead to many possible outputs (one for each possible password). The theory of chapter 7 applies to interface automata that are determinate in the sense of Milner [72]. In a determinate interface automaton multiple output actions may be enabled in a single state, which makes it straightforward to model the login protocol. In order to learn the resulting model, it is crucial to define an abstraction that merges all outputs that are enabled in a given state to a single abstract output.

In brief, the theory presented in chapter 7 improved the framework of [1] in four important directions: (a) interface automata instead of the more restricted Mealy machines, (b) the concept of a learning purpose, which allows one to restrict the learning process to relevant behaviors only, (c) a richer class of abstractions, which includes abstractions that over-approximate the behavior of the system-under-test, and (d) a conceptually superior approach for testing correctness of the hypotheses that are generated by the learner.

6.2 Tomte

Chapter 8 presents an algorithm to construct fully automatically mappers for a restricted class of extended finite state machines, called scalarset Mealy machines, in which one can test for equality of data parameters, but no operations on data are allowed. The notion of a scalarset data type originates from model checking, where it is been used by Ip & Dill for symmetry reduction [49]. The algorithm of chapter 8 is implemented in a tool named Tomte, after the creature that shrank Nils Holgersson into a gnome and (after numerous adventures) changed him back to his normal size again.

For the whole research, LearnLib is used as the basic learning tool and therefore the abstraction of the SUT may not exhibit any nondeterminism: if it does then LearnLib crashes. On the other hand, when abstraction is applied, nondeterminism arises naturally: it may occur that the behavior of a SUT is fully deterministic but that due to the mapper (which, for instance, abstracts from the precise value of certain input parameters), the system appears to behave nondeterministically from the perspective of the learner. In case of nondeterminism, abstraction has to be refined in order to dissolve it. This procedure is formalized and the construction of the mapper is described in terms of a counterexample guided abstraction refinement (CEGAR) procedure, similar to the approach developed by Clarke et al [28] in the context of model checking. This is exactly what has been done repeatedly during the manual construction of the abstraction mappings in the case studies of [1].
Using the prototype tool implementation Tomte, models of several realistic software components, including the biometric passport and the SIP protocol were learned *fully automatically*. The Tomte tool and all the models used in the experiments are available via [http://www.italia.cs.ru.nl/](http://www.italia.cs.ru.nl/).
Chapter 7

A Theory of History Dependent Abstractions for Learning Interface Automata

This chapter offers a general theory of history dependent abstractions for learning interface automata. In section 7.1 the primary prerequisites are presented. Section 7.2 gives a detailed account of automata learning. Next, mappers and their functionality are introduced in section 7.3. Thereafter, the theory of history dependent abstractions for learning interface automata is fully described in section 7.4. Finally, section 7.5 concludes the work.

7.1 Preliminaries

7.1.1 Interface automata

In this chapter reactive systems are modeled by a simplified notion of interface automata [32], essentially labeled transition systems with input and output actions.

Definition An interface automaton (IA) is a tuple \( \mathcal{I} = \langle I, O, Q, q^0, \rightarrow \rangle \) where

- \( I \) and \( O \) are disjoint sets of input and output actions, respectively,
- \( Q \) is a non-empty set of states,
- \( q^0 \in Q \) is the initial state, and
- \( \rightarrow \subseteq Q \times (I \cup O) \times Q \) is the transition relation.
We write \( q \xrightarrow{a} q' \) if \((q,a,q') \in \rightarrow \). An action \( a \) is *enabled* in state \( q \), denoted \( q \xrightarrow{a} \), if \( q \xrightarrow{a} q' \) for some state \( q' \). We extend the transition relation to sequences by defining, for \( \sigma \in (I \cup O)^* \), \( \rightarrow^* \) to be the least relation that satisfies, for \( q, q', q'' \in Q \) and \( a \in I \cup O \),

- \( q \xrightarrow{\epsilon} q \), and
- if \( q \xrightarrow{\sigma} q' \) and \( q' \xrightarrow{a} q'' \) then \( q \xrightarrow{\sigma a} q'' \).

Here \( \epsilon \) is used to denote the empty sequence. A state \( q \) is said to be *reachable* if \( q^0 \xrightarrow{\sigma} q \), for some \( \sigma \). We write \( q \xrightarrow{\sigma} q' \) and \( q' \xrightarrow{a} q'' \) if \( q \xrightarrow{\sigma a} q'' \). A state \( q \) is said to be a *trace* of \( I \) if \( q^0 \xrightarrow{\sigma} \), and write \( \text{Traces}(I) \) for the set of traces of \( I \).

A bisimulation on \( I \) is a symmetric relation \( R \subseteq Q \times Q \) such that \((q_0, q_0) \in R\) and

\[
(q_1, q_2) \in R \land q_1 \xrightarrow{a} q_1' \Rightarrow \exists q_2' : q_2 \xrightarrow{a} q_2' \land (q_1', q_2') \in R.
\]

Two states \( q, q' \in Q \) are said to be *bisimilar*, denoted \( q \sim q' \), if there exists a bisimulation on \( I \) that contains \((q, q')\). Recall that relation \( \sim \) is the largest bisimulation and that \( \sim \) is an equivalence relation \([72]\).

Interface automaton \( I \) is said to be:

- *finite* if \( Q, I, \) and \( O \) are finite sets.
- *finitary* if \( I \) and \( O \) are finite and there exists a bisimulation \( R \subseteq Q \times Q \) that is an equivalence relation with finitely many equivalence classes.
- *deterministic* if for each state \( q \in Q \) and for each action \( a \in I \cup O \), whenever \( q \xrightarrow{a} q' \) and \( q \xrightarrow{a} q'' \) then \( q' = q'' \).
- *determinate* \([72]\) if for each reachable state \( q \in Q \) and for each action \( a \in I \cup O \), whenever \( q \xrightarrow{a} q' \) and \( q \xrightarrow{a} q'' \) then \( q' \sim q'' \).
- *output-determined* if for each reachable state \( q \in Q \) and for all output actions \( o, o' \in O \), whenever \( q \xrightarrow{o} \) and \( q \xrightarrow{o'} \) then \( o = o' \).
- *behavior-deterministic* if \( I \) is both determinate and output-determined.
- *active* if each reachable state enables an output action.
- *output-enabled* if each state enables each output action.
- *input-enabled* if each state enables each input action.

An \( I/O \) automaton (IOA) is an input-enabled IA. Our notion of an \( I/O \) automaton is a simplified version of the notion of IOA of Lynch & Tuttle \([68]\) in which the set of internal actions is empty, the set of initial states has only one member, and the equivalence relation is trivial.
7.1.2 The ioco relation

A state \( q \) of \( I \) is quiescent if it enables no output actions. Let \( \delta \) be a special action symbol. In this article, we only consider IAs \( I \) in which \( \delta \) is not an input action. The \( \delta \)-extension of \( I \), denoted \( I^\delta \), is the IA obtained by adding \( \delta \) to the set of output actions, and \( \delta \)-loops to all the quiescent states of \( I \). Write \( O^\delta = O \cup \{ \delta \} \). The following lemma easily follows from the definitions.

Lemma 7.1.1 Let \( I \) be an IA with outputs \( O \). Then

1. \( I^\delta \) is active,
2. \( I^\delta \) is an IOA iff \( I \) is an IOA,
3. if \( I \) is determinate then \( I^\delta \) is determinate,
4. if \( \delta \not\in O \) and \( I^\delta \) is determinate then \( I \) is determinate,
5. \( I^\delta \) is output-determined iff \( I \) is output-determined, and
6. if \( I \) is behavior-deterministic then \( I^\delta \) is behavior-deterministic.

Write \( \text{out}_I(q) \), or just \( \text{out}(q) \) if \( I \) is clear from the context, for \( \{ a \in O \mid q a \rightarrow \} \), the set of output actions enabled in state \( q \). For \( S \subseteq Q \) a set of states, write \( \text{out}_I(S) \) for \( \bigcup \{ \text{out}_I(q) \mid q \in S \} \). Write \( I \text{ after } \sigma \) for the set \( \{ q \in Q \mid q^0 \sigma \rightarrow \ast q \} \) of states of \( I \) that can be reached via trace \( \sigma \).

The next technical lemma easily follows by induction on the length of trace \( \sigma \).

Lemma 7.1.2 Suppose \( I \) is a determinate IA. Then, for each \( \sigma \in \text{Traces}(I^\delta) \), all states in \( I^\delta \text{ after } \sigma \) are pairwise bisimilar.

Let \( I_1 = \langle I_1, O_1, Q_1, q_1^0, \rightarrow_1 \rangle \) and \( I_2 = \langle I_2, O_2, Q_2, q_2^0, \rightarrow_2 \rangle \) be IAs with the same sets of input and output actions. Write \( A = I \cup O \) and let \( X, Y \subseteq A \). An \( XY \)-simulation from \( I_1 \) to \( I_2 \) is a binary relation \( R \subseteq Q_1 \times Q_2 \) that satisfies, for all \( (q, r) \in R \) and \( a \in A \),

7.1.3 \( XY \)-simulations

In the technical development of this chapter, a major role is played by the notion of an \( XY \)-simulation. Below we recall the definition of \( XY \)-simulation, as introduced in [3], and establish three (new) technical lemma’s.

Let \( I_1 = \langle I, O, Q_1, q_1^0, \rightarrow_1 \rangle \) and \( I_2 = \langle I, O, Q_2, q_2^0, \rightarrow_2 \rangle \) be IAs with the same sets of input and output actions. Write \( A = I \cup O \) and let \( X, Y \subseteq A \). An \( XY \)-simulation from \( I_1 \) to \( I_2 \) is a binary relation \( R \subseteq Q_1 \times Q_2 \) that satisfies, for all \( (q, r) \in R \) and \( a \in A \),
Suppose Lemma 7.1.3 symmetry properties of $q \in Q \subseteq V$. We write $I \equiv_1 I$ and $O \equiv_1 O$.

Let $\pi$.

Proof Suppose Lemma 7.1.4 $XY$-simulation from $I_1$. Then $XY$-simulation from $I_1 \sim I_2$. Hence $XY$-simulation from $I_1$.

The following lemma is used to link alternating simulations and the $\mathit{ioco}$ relation.

Lemma 7.1.4 Suppose $I_1$ and $I_2$ are IAs with $I_1 \sim I_2$. Let $R$ be the maximal $XY$-simulation from $I_1$ to $I_2$. Let $q_1, q_2 \in Q_1$ and $q_3, q_4 \in Q_2$, where $Q_1$ and $Q_2$ are the state sets of $I_1$ and $I_2$, respectively. Then $q_1 \sim q_2 \land q_2$.

Proof Let $R' = \{ (q_1, q_4) \mid \exists q_2, q_3 : q_1 \sim q_2 \land q_2 R q_3 \land q_3 \sim q_4 \}$. It is routine to prove that $R'$ is the maximal $XY$-simulation from $I_1$ to $I_2$. Since $R$ is the maximal $XY$-simulation from $I_1$ to $I_2$, $R' \subseteq R$. Now suppose $q_1 \sim q_2 \land q_2 R q_3 \land q_3 \sim q_4$. By definition, $(q_1, q_4) \in R'$. Hence $(q_1, q_4) \in R$, as required.

The following lemma is used to link alternating simulations and the $\mathit{ioco}$ relation.

Lemma 7.1.5 Let $I_1$ and $I_2$ be determinate IAs such that $I_1 \sim I_2$. Assume that $X \cup Y = A$, where $A$ is the set of all (input and output) actions. Let $R$ be the maximal $XY$-simulation from $I_1$ to $I_2$. Let $\sigma \in A^*$, $q_1 \in Q_1$ and $q_2 \in Q_2$ such that $q_1 \overset{\sigma}{\rightarrow} q_1$ and $q_2 \overset{\sigma}{\rightarrow} q_2$. Then $(q_1, q_2) \in R$.

Proof By induction on the length of $\sigma$.

If $|\sigma| = 0$ then $\sigma = \epsilon$, $q_1 = q_1^0$ and $q_2 = q_2^0$. Since $I_1 \sim I_2$ and $R$ is the maximal $XY$-simulation, $(q_1^0, q_2^0) \in R$. Hence $(q_1, q_2) \in R$.

Now suppose $|\sigma| > 0$. Then there exist $\rho \in A^*$ and $a \in A$ such that $\sigma = \rho a$. Hence there exists states $q_1' \in Q_1$ and $q_2' \in Q_2$ such that $q_1^0 \overset{\rho}{\rightarrow} q_1'$ and $q_2^0 \overset{\rho}{\rightarrow} q_2'$.

By induction hypothesis, $(q_1', q_2') \in R$. Since $X \cup Y = A$, either $a \in X$ or $a \in Y$. We consider two cases:
7.1. Preliminaries

Let these results occur in [3, 98]. The results below link alternating simulation and the \( ioco \) relation. Similarly, in general, \( \sim \) implies \( \preceq \), and hence \( \preceq \) after \( \sigma \), and hence \( o \in out(\mathcal{I}_2) \) after \( \sigma \), by Lemma 7.1.5. Hence we can use Lemma 7.1.4 to obtain \((q_1, q_2) \in R\). It follows that \( q_2 \sim q_1 \), and hence \( o \in out(\mathcal{I}_2) \) after \( \sigma \), as required.

**Lemma 7.1.6** Let \( \mathcal{I}_1 \) and \( \mathcal{I}_2 \) be determinate IAs. Then \( \mathcal{I}_1 \preceq \mathcal{I}_2 \) implies \( \mathcal{I}_1 \ ioco \ \mathcal{I}_2 \).

**Proof** Suppose that \( \mathcal{I}_1 \preceq \mathcal{I}_2 \). Let \( \sigma \in \text{Traces}(\mathcal{I}_2) \) and \( o \in out(\mathcal{I}_1) \) after \( \sigma \). We must prove \( o \in out(\mathcal{I}_1) \) after \( \sigma \). By the definitions, there exists \( q_1 \in Q_1 \) and \( q_2 \in Q_2 \) such that \( q_1^0 \overset{\sigma}{\rightarrow}_* q_1 \) and \( q_2^0 \overset{\sigma}{\rightarrow}_* q_2 \). Let \( R \) be the maximal alternating simulation from \( \mathcal{I}_1^\delta \) to \( \mathcal{I}_2^\delta \). Since both \( \mathcal{I}_1 \) and \( \mathcal{I}_2 \) are determinate, \( \mathcal{I}_1^\delta \) and \( \mathcal{I}_2^\delta \) are determinate, by Lemma 7.1.4. Hence we can use Lemma 7.1.5 to obtain \((q_1, q_2) \in R\). It follows that \( q_2 \sim q_1 \), and hence \( o \in out(\mathcal{I}_1^\delta) \) after \( \sigma \), as required.

**Lemma 7.1.7** Let \( \mathcal{I}_1 \) be an IOA and let \( \mathcal{I}_2 \) be a determinate IA. Then \( \mathcal{I}_1 \ ioco \ \mathcal{I}_2 \) implies \( \mathcal{I}_1 \preceq \mathcal{I}_2 \).

**Proof** Suppose \( \mathcal{I}_1 \ ioco \ \mathcal{I}_2 \). Let \( \mathcal{I}_1 = \langle I, O, Q_1, q_1^0, \rightarrow_1 \rangle \) and \( \mathcal{I}_2 = \langle I, O, Q_2, q_2^0, \rightarrow_2 \rangle \). Define

\[
R = \{(q_1, q_2) \in Q_1 \times Q_2 \mid \exists \sigma \in (I \cup O^\delta)^* : q_1^0 \overset{\sigma}{\rightarrow}_{1*} q_1 \land q_2^0 \overset{\sigma}{\rightarrow}_{2*} q_2\}.
\]

Figure 7.1: Relations Hierarchy

- \( a \in X \). Since \( q_1^0 \overset{a}{\rightarrow} q_1 \), \((q_1^0, q_2^0) \in R \) and \( R \) is an XY-simulation, there exists a \( q_2^0 \) such that \( q_2^0 \overset{a}{\rightarrow} q_2^0 \) and \((q_1, q_2^0) \in R \). Since \( \mathcal{I}_2 \) is determinate, \( q_2^0 \sim q_2 \) and thus \((q_1, q_2) \in R \), by Lemma 7.1.4.

- \( a \in Y \). Since \( q_2^0 \overset{a}{\rightarrow} q_2 \), \((q_1^0, q_2^0) \in R \) and \( R \) is an XY-simulation, there exists a \( q_1^0 \) such that \( q_1^0 \overset{a}{\rightarrow} q_1^0 \) and \((q_1^0, q_2^0) \in R \). Since \( \mathcal{I}_1 \) is determinate, \( q_1 \sim q_1^0 \) and thus \((q_1, q_2) \in R \), by Lemma 7.1.4.

7.1.4 Relating alternating simulations and \textit{ioco} relation

The results below link alternating simulation and the \textit{ioco} relation. Variations of these results occur in [3, 98].

**Definition** Let \( \mathcal{I}_1 \) and \( \mathcal{I}_2 \) be IAs with inputs \( I \) and outputs \( O \), and let \( A = I \cup O \) and \( A^\delta = A \cup \{\delta\} \). Then \( \mathcal{I}_1 \preceq \mathcal{I}_2 \iff \mathcal{I}_1^\delta \sim_{O^\delta I} \mathcal{I}_2^\delta \) and \( \mathcal{I}_1 \preceq \mathcal{I}_2 \iff \mathcal{I}_1^\delta \sim_{A^\delta I} \mathcal{I}_2^\delta \).

In general, \( \mathcal{I}_1 \preceq \mathcal{I}_2 \) implies \( \mathcal{I}_1 \sim_{OI} \mathcal{I}_2 \), but the converse implication does not hold. Similarly, \( \mathcal{I}_1 \preceq \mathcal{I}_2 \) implies \( \mathcal{I}_1 \sim_{AI} \mathcal{I}_2 \), but not vice versa.

Figure 7.1 depicts the hierarchy of these relations.
We claim that \( R \) is an alternating simulation relation from \( \mathcal{I}_1^\delta \) to \( \mathcal{I}_2^\delta \).

Suppose that \( (q_1, q_2) \in R \) and \( q_1 \xrightarrow{o} q_1' \), for some \( o \in O^\delta \). Then there exists a \( \sigma \in (I \cup O^\delta)^* \) such that \( q_1^0 \xrightarrow{\sigma} q_1 \) and \( q_2^0 \xrightarrow{\sigma} q_2 \). Thus \( \sigma \in \text{Traces}^* (\mathcal{I}_1^\delta) \) and \( o \in \text{out}(\mathcal{I}_1^\delta) \) after \( \sigma \). Using that \( \mathcal{I}_1 \xoco \mathcal{I}_2 \), we obtain \( o \in \text{out}(\mathcal{I}_2^\delta) \) after \( \sigma \). This means that there exists a state \( q_3 \) such that \( q_2^0 \xrightarrow{\sigma} q_3 \) and \( q_3 \xrightarrow{o} q_3 \). Hence there exists a state \( q_2' \) such that \( q_2 \xrightarrow{o} q_2' \). By definition of \( R \), \( (q_1', q_2') \in R \).

Now suppose that \( (q_1, q_2) \in R \) and \( q_2 \xrightarrow{i} q_2' \), for some \( i \in I \). As \( \mathcal{I}_1 \) is input-enabled, there exists a state \( q_1' \) such that \( q_1 \xrightarrow{i} q_1' \). By definition of \( R \), \( (q_1', q_2') \in R \).

By taking \( \sigma = \epsilon \) in the definition of \( R \), we obtain \( (q_1', q_2') \in R \). Hence \( \mathcal{I}_1 \xoco \mathcal{I}_2 \), as required.

### 7.2 Basic Framework for Inference of Automata

This section presents (a slight generalization of) the framework of \cite{3} for learning interface automata. We assume there is a teacher, who knows a determinate IA \( \mathcal{T} = \langle I, O, Q, q_0, \rightarrow \rangle \), called the system under test (SUT). There is also a learner, who has the task to learn about the behavior of \( \mathcal{T} \) through experiments. The type of experiments which the learner may do is restricted by a learning purpose \cite{3, 87, 99, 51}, which is a determinate IA \( \mathcal{P} = \langle I, O^\delta, P, p_0^0, \rightarrow_P \rangle \), satisfying \( \mathcal{T} \xoco \mathcal{P} \).

In practice, there are various ways to ensure that \( \mathcal{T} \xoco \mathcal{P} \). If \( \mathcal{T} \) is an IOA then \( \mathcal{T} \xoco \mathcal{P} \) is equivalent to \( \mathcal{T} \xoco \mathcal{P} \) by Lemmas \ref{lem:7.1.6} and \ref{lem:7.1.7} and so we may use model-based black-box testing to obtain evidence for \( \mathcal{T} \xoco \mathcal{P} \). Alternatively, if \( \mathcal{T} \) is an IOA and \( \mathcal{P} \) is output-enabled then \( \mathcal{T} \xoco \mathcal{P} \) trivially holds.

After doing a number of experiments, the learner may formulate a hypothesis, which is a determinate IA \( \mathcal{H} \) with outputs \( O^\delta \) satisfying \( \mathcal{H} \xoco \mathcal{P} \). Informally, the requirement \( \mathcal{H} \xoco \mathcal{P} \) expresses that \( \mathcal{H} \) only displays behaviors that are allowed by \( \mathcal{P} \), but that any input action that must be explored according to \( \mathcal{P} \) is indeed present in \( \mathcal{H} \). Hypothesis \( \mathcal{H} \) is correct if \( \mathcal{T} \xoco \mathcal{H} \). In practice, we will use black-box testing to obtain evidence for the correctness of the hypothesis. In general, there will be many \( \mathcal{H} \)'s satisfying \( \mathcal{T} \xoco \mathcal{H} \xoco \mathcal{P} \) (for instance, we may take \( \mathcal{H} = \mathcal{P} \), and additional conditions will be imposed on \( \mathcal{H} \), such as behavior-determinacy.

In fact, section \ref{sec:7.2.1} establishes that if \( \mathcal{T} \) is behavior-deterministic there always exists a behavior-deterministic IA \( \mathcal{H} \) such that \( \mathcal{T} \xoco \mathcal{H} \xoco \mathcal{P} \). If, in addition, \( \mathcal{T} \) is an IOA then this \( \mathcal{H} \) is unique up to bisimulation equivalence.

**Learning purpose** A trivial learning purpose \( \mathcal{P}_{\text{triv}} \) is displayed in Figure \ref{fig:7.2} (left). Here notation \( i : I \) means that we have an instance of the transition for each input \( i \in I \). Notation \( o : O \) is defined similarly. Since \( \mathcal{P}_{\text{triv}} \) is output-enabled, \( \mathcal{T} \xoco \mathcal{P}_{\text{triv}} \) holds for each IOA \( \mathcal{T} \). If \( \mathcal{H} \) is a hypothesis, then \( \mathcal{H} \xoco \mathcal{P}_{\text{triv}} \) just means that \( \mathcal{H} \) is input enabled.
The learning purpose $P_{\text{wait}}$ displayed in Figure 7.2 (right) contains a nontrivial $\delta$-transition. It expresses that after each input the learner has to wait until the SUT enters a quiescent state before offering the next input. It is straightforward to check that $T \prec P_{\text{wait}}$ holds if $T$ is an IOA.

We now present the protocol that learner and teacher must follow. At any time, the teacher records the current state of $T$, initially $q^0$, and the learner records the current state of $P$, initially $p^0$. Suppose the teacher is in state $q$ and the learner is in state $p$. In order to learn about the behavior of $T$, the learner may engage in four types of interactions with the teacher:

1. **Input.** If a transition $p \xrightarrow{i} p'$ is enabled in $P$, then the learner may present input $i$ to the teacher. If $i$ is enabled in $q$ then the teacher jumps to a state $q'$ with $q \xrightarrow{i} q'$ and returns reply $\top$ to the learner. Otherwise, the teacher returns reply $\bot$. If the learner receives reply $\top$ it jumps to $p'$, otherwise it stays in $p$.

2. **Output.** The learner may send an output query $\Delta$ to the teacher. Now there are two possibilities. If state $q$ is quiescent, the teacher remains in $q$ and returns answer $\delta$. Otherwise, the teacher selects an output transition $q \xrightarrow{o} q'$, jumps to $q'$, and returns $o$. The learner jumps to a state $p'$ that can be reached by the answer $o$ or $\delta$.

3. **Reset.** The learner may send a reset to the teacher. In this case, both learner and teacher return to their respective initial states.

4. **Hypothesis.** The learner may present a hypothesis to the teacher: a determinate IA $H$ with outputs $O^\delta$ such that $H \preceq P$. If $T \ioco H$ then the teacher returns answer yes. Otherwise, by definition, $H^\delta$ has a trace $\sigma$ such that an output $o$ that is enabled by $T^\delta$ after $\sigma$, is not enabled by $H^\delta$ after $\sigma$. In this case, the teacher returns answer no together with counterexample $\sigma o$, and learner and teacher return to their respective initial states.

The next lemma, which is easy to prove, implies that the teacher never returns $\bot$ to the learner: whenever the learner performs an input transition $p \xrightarrow{i} p'$, the teacher can perform a matching transition $q \xrightarrow{i} q'$. Moreover, whenever the
teacher performs an output transition $q \xrightarrow{o} q'$, the learner can perform a matching transition $p \xrightarrow{o} p'$.

**Lemma 7.2.1** Let $R$ be the maximal alternating simulation from $T^\delta$ to $P^\delta$. Then, for any configuration of states $q$ and $p$ of teacher and learner, respectively, that can be reached after a finite number of steps (1)-(4) of the learning protocol, we have $(q, p) \in R$.

**Proof** Routine, by induction on the number of steps, using Lemma 7.1.4 and the assumption that both $T$ and $P$ are determinate.

We are interested in effective procedures which, for any finite (and some infinite) $T$ and $P$ satisfying the above conditions, allow a learner to come up with a correct, behavior-deterministic hypothesis $H$ after a finite number of interactions with the teacher. In [3], it is shown that any algorithm for learning Mealy machines can be transformed into an algorithm for learning finite, behavior-deterministic IOAs. Efficient algorithms for learning Mealy machines have been implemented in the tool Learnlib [83].

### 7.2.1 Existence and Uniqueness of Correct Hypothesis

In this section, it is established that if $T$ is a behavior deterministic IOA and $P$ is a determinate IA with $T \preceq P$, there exists a unique behavior-deterministic IA $H$ (up to bisimulation) such that $T \ioco H \preceq P$.

**Lemma 7.2.2** Suppose $I_1, I_2, I_3$ and $I_4$ are determinate IAs with the same sets $I$ and $O$ of inputs and outputs, respectively, such that $I_1$ is active, and $I_3$ and $I_4$ are output-determined. Then $I_1 \sim_{OI} I_3 \sim_{AI} I_2$ and $I_1 \sim_{OI} I_4 \sim_{AI} I_2$ implies $I_3 \sim I_4$.

**Proof** Let $R_1$ be the maximal alternating simulation from $I_1$ to $I_3$, $R_2$ be the maximal alternating simulation from $I_1$ to $I_4$, $S_1$ be the maximal $AI$-simulation from $I_3$ to $I_2$, $S_2$ be the maximal $AI$-simulation from $I_4$ to $I_2$, and let $R$ be the relation between states of $I_3$ and $I_4$ given by:

$$(q_3, q_4) \in R \iff \exists q_1, q_2 : \begin{align*}
(q_1, q_3) &\in R_1 \land \\
(q_3, q_2) &\in S_1 \land \\
(q_1, q_4) &\in R_2 \land \\
(q_4, q_2) &\in S_2.
\end{align*}$$

We claim that $R$ is a bisimulation ($AA$-simulation) from $I_3$ to $I_4$.

Suppose $(q_3, q_4) \in R$ and $q_3 \xrightarrow{o} q'_3$. The there exist $q_1$ and $q_2$ such that $(q_1, q_3) \in R_1$, $(q_3, q_2) \in S_1$, $(q_1, q_4) \in R_2$, and $(q_4, q_2) \in S_2$. We consider two cases:
• $a \in I$. Since $S_1$ is an AI-simulation, there exists a state $q'_2$ such that $q_2 \xrightarrow{a} q'_2$ and $(q_3', q'_2) \in S_1$. Since $S_2$ is an AI-simulation, there exists a state $q'_4$ such that $q_4 \xrightarrow{a} q'_4$ and $(q'_1, q'_2) \in S_2$. Since $R_2$ is an OI-simulation, there exists a state $q'_1$ such that $q_1 \xrightarrow{a} q'_1$ and $(q'_1, q'_3) \in S_1$. Since $I_1$ is determinate, $q'_1 \sim q''_1$. Combination of $q'_1 \sim q''_1$ and $(q''_1, q'_3) \in R_1$ gives $(q'_1, q'_3) \in R_1$, using Lemma 7.1.4 and the assumption that $R_1$ is maximal. Hence $(q'_3, q'_4) \in R$, by definition of $R$.

• $a \in O$. Since $I_1$ is active, there exists a transition $q_1 \xrightarrow{a} q'_1$, for some output $o$. Since $R_1$ is an OI-simulation, there exists a state $q''_3$ such that $q_3 \xrightarrow{o} q''_3$ and $(q'_1, q''_3) \in R_1$. Since $I_3$ is behavior-deterministic, $o = a$ and $q''_3 \sim q'_3$. Hence $(q'_1, q'_3) \in R_1$. Since $R_2$ is an OI-simulation, there exists a state $q'_4$ such that $q_4 \xrightarrow{a} q'_4$ and $(q'_1, q'_4) \in R_2$. Since $S_2$ is an AI-simulation, there exists a state $q'_2$ such that $q_2 \xrightarrow{a} q'_2$ and $(q'_4, q'_2) \in S_2$. Since $S_1$ is an AI-simulation, there exists a state $q''_4$ such that $q_2 \xrightarrow{a} q''_4$ and $(q''_4, q'_2) \in S_1$. Since $I_2$ is determinate, $q'_2 \sim q''_2$. Combination of $(q''_4, q''_2) \in S_1$ and $q''_2 \sim q'_2$ gives $(q'_3, q'_2) \in S_1$, using Lemma 7.1.4 and the assumption that $S_1$ is maximal. Hence $(q'_3, q'_4) \in R$, by definition of $R$.

The proof of the case that $(q'_3, q'_4) \in R$ and $q_4 \xrightarrow{o} q'_4$ is fully symmetric.

It is immediate from the definitions that $(q'_3, q'_4) \in R$. Hence $I_3 \sim I_4$, as required.

Suppose that $I_1 \sim_{XY} I_2$. $XY(I_1, I_2)$ is defined to be the product interface automaton induced by $\sim_{XY}$, as the structure $\langle I, O, R, (q'_0, q'_2), \rightarrow \rangle$ where $R$ is the maximal $XY$-simulation relation from $I_1$ to $I_2$ and $(q, r) \xrightarrow{a} (q', r') \iff q \xrightarrow{a} q' \land r \xrightarrow{a} r'$.

**Lemma 7.2.3** Suppose that $I_1 \sim_{XY} I_2$. Then $I_1 \sim_{XA} XY(I_1, I_2) \sim_{AY} I_2$.

**Proof** Let $R$ be the maximal $XY$-simulation from $I_1$ to $I_2$. Let $R_1 = \{(q, (q, r)) \mid (q, r) \in R\}$ and $R_2 = \{(((q, r), r)) \mid (q, r) \in R\}$. It is straightforward to check that $R_1$ is an $XA$-simulation from $I_1$ to $XY(I_1, I_2)$, and $R_2$ is an $AY$-simulation from $XY(I_1, I_2)$ to $I_2$. Since $R$ is an $XY$-simulation from $I_1$ to $I_2$, it contains the pair $(q'_0, q'_2)$. Hence $R_1$ contains $(q'_0, (q'_0, q'_2))$ and $R_2$ contains $((q'_0, q'_2), q'_2)$, so the initial states are related, as required.

**Lemma 7.2.4** Suppose that $I_1 \sim_{XY} I_2$.

1. If $I_1$ and $I_2$ are determinate, then $XY(I_1, I_2)$ is determinate.
2. If $I_1$ or $I_2$ is output-determined, then $XY(I_1, I_2)$ is output-determined.

The following theorem proves that for a given behavior-deterministic SUT and a determinate learning purpose, learner will necessarily find a behavior-deterministic learning hypothesis.
Theorem 7.2.5 Suppose $\mathcal{T}$ is a behavior-deterministic IA and $\mathcal{P}$ is a determinate IA such that $\mathcal{T} \preceq \mathcal{P}$. Then there exists a behavior-deterministic IA $\mathcal{H}$ such that $\mathcal{T} \ioco \mathcal{H} \preceq \mathcal{P}$.

Proof By expanding the definition of $\preceq$, we obtain $\mathcal{T}^\delta \sim_{O^\delta I} \mathcal{P}^\delta$. Thus, by Lemma 7.2.3

$$\mathcal{T}^\delta \sim_{O^\delta I} O^\delta I(\mathcal{T}^\delta, \mathcal{P}^\delta) \sim_{A^\delta I} \mathcal{P}^\delta.$$ 

Let $\mathcal{H} = O^\delta I(\mathcal{T}^\delta, \mathcal{P}^\delta)$. Since $\mathcal{H}$ is active, $\mathcal{H}^\delta = \mathcal{H}$. Hence, by Lemma 7.1.3(2),

$$\mathcal{T}^\delta \sim_{O^\delta I} \mathcal{H}^\delta \sim_{A^\delta I} \mathcal{P}^\delta.$$ 

By the definitions of $\preceq$ and $\ioco$, we obtain $\mathcal{T} \preceq \mathcal{H} \preceq \mathcal{P}$. Lemma 7.1.1 and Lemma 7.2.4 imply that $\mathcal{H}$ is behavior-deterministic, as required. By Lemma 7.1.6, $\mathcal{T} \ioco \mathcal{H}$.

Next theorem, proves that for a given behavior-deterministic SUT and a determinate learning purpose, learning hypothesis is unique.

Theorem 7.2.6 Let $\mathcal{T}$ be a behavior-deterministic IOA over $I$ and $O$, $\mathcal{H}_1$ and $\mathcal{H}_2$ behavior-deterministic IAs over $I$ and $O^\delta$, and $\mathcal{P}$ a determinate IA over $I$ and $O^\delta$ such that $\mathcal{T} \ioco \mathcal{H}_1 \preceq \mathcal{P}$ and $\mathcal{T} \ioco \mathcal{H}_2 \preceq \mathcal{P}$. Then $\mathcal{H}_1^\delta \sim \mathcal{H}_2^\delta$.

Proof Since $\mathcal{T}$ is an IOA, $\mathcal{H}_1$ is determinate and $\mathcal{T} \ioco \mathcal{H}_1$, it follows by Lemma 7.1.7 that $\mathcal{T} \preceq \mathcal{H}_1$. Similarly, we derive $\mathcal{T} \preceq \mathcal{H}_2$. By expanding the definitions of $\preceq$ and $\ioco$, we obtain:

$$\mathcal{T}^\delta \sim_{O^\delta I} \mathcal{H}_1^\delta \sim_{A^\delta I} \mathcal{P}^\delta$$ and $$\mathcal{T}^\delta \sim_{O^\delta I} \mathcal{H}_2^\delta \sim_{A^\delta I} \mathcal{P}^\delta.$$ 

From the assumptions and Lemma 7.1.1 it follows that $\mathcal{T}^\delta$, $\mathcal{H}_1^\delta$, $\mathcal{H}_2^\delta$ and $\mathcal{P}^\delta$ are determinate, $\mathcal{T}^\delta$ is active, and $\mathcal{H}_1^\delta$ and $\mathcal{H}_2^\delta$ are output-determined. Hence we may apply Lemma 7.2.2 to obtain $\mathcal{H}_1^\delta \sim \mathcal{H}_2^\delta$.

7.3 Mappers

In order to learn a “large” IA $\mathcal{T}$, with inputs $I$ and outputs $O$, a mapper is placed between the teacher and the learner, which translates concrete actions in $I$ and $O$ to abstract actions in (typically smaller) sets $X$ and $Y$, and vice versa. The task of the learner is then reduced to inferring a “small” IA with alphabet $X$ and $Y$.

Our notion of mapper is essentially the same as the one of [1].

Mapper A mapper for a set of inputs $I$ and a set of outputs $O$ is a tuple $\mathcal{A} = (\mathcal{I}, X, Y, \Upsilon)$, where

- $\mathcal{I} = (I, O^\delta, R, r^0, \rightarrow)$ is a deterministic IA that is input- and output-enabled and has trivial $\delta$-transitions: $r \xrightarrow{\delta} r' \iff r = r'$.
7.3. Mappers

- $X$ and $Y$ are disjoint sets of abstract input and output actions with $\delta \in Y$.

- $\Upsilon : R \times A^\delta \to Z$, where $A = I \cup O$ and $Z = X \cup Y$, maps concrete actions to abstract ones. We write $\Upsilon_r(a)$ for $\Upsilon(r, a)$ and require that $\Upsilon_r$ respects inputs, outputs and quiescence:

\[(\Upsilon_r(a) \in X \Leftrightarrow a \in I) \land (\Upsilon_r(a) = \delta \Leftrightarrow a = \delta).\]

Mapper $A$ is output-predicting if $\forall o, o' \in O : \Upsilon_r(o) = \Upsilon_r(o') \Rightarrow o = o'$, that is, $\Upsilon_r$ is injective on outputs, for each $r \in R$. Mapper $A$ is surjective if $\forall z \in Z \exists a \in A^\delta : \Upsilon_r(a) = z$, that is, $\Upsilon_r$ is surjective, for each $r \in R$. Mapper $A$ is state-free if $R$ is a singleton set.

**Example** Consider a system with input actions $LOGIN(p_1), SET(p_2)$ and $LOGOUT$. Assume that the system only triggers certain outputs when a user is properly logged in. Then we may not abstract from the password parameters $p_1$ and $p_2$ entirely, since this will lead to nondeterminism. We may preserve behavior-determinism by considering just two abstract values for $p_1$: $ok$ and $nok$. Since passwords can be changed using the input $SET(p_2)$ when a user is logged in, the mapper may not be state-free: it has to record the current password and whether or not the user is logged ($T$ and $F$, respectively). The input transitions are defined by:

\[
\begin{align*}
p \neq p_1 & \Rightarrow (p, b) \xrightarrow{LOGIN(p_1)} (p, T) \\
(p, b) & \xrightarrow{LOGIN(p)} (p, b) \\
(p, T) & \xrightarrow{SET(p_2)} (p_2, T) \\
(p, F) & \xrightarrow{SET(p_2)} (p_2, F) \\
(p, b) & \xrightarrow{LOGOUT} (p, F)
\end{align*}
\]

For input actions, abstraction $\Upsilon$ is defined by

\[
\Upsilon_{(p, b)}(LOGIN(p_1)) = \begin{cases} 
LOGIN(ok) & \text{if } p_1 = p \\
LOGIN(nok) & \text{otherwise}
\end{cases}
\]

\[
\Upsilon_{(p, b)}(SET(p_2)) = SET
\]

For input $LOGOUT$ and for output actions, $\Upsilon_{(p, b)}$ is the identity. This mapper is surjective, since no matter how the password has been set, a user may always choose either a correct or an incorrect login.

**Example** Consider a system with three inputs $IN1(n_1), IN2(n_2)$, and $IN3(n_3)$, in which an $IN3(n_3)$ input triggers an output $OK$ if and only if the value of $n_3$ equals either the latest value of $n_1$ or the latest value of $n_2$. In this case, we may not abstract away entirely from the values of the parameters, since that leads to nondeterminism. We may preserve behavior-determinism by a mapper that records the last values of $n_1$ and $n_2$. Thus, if $D$ is the set of parameter values, the
set of mapper states is defined by $R = (D \cup \{\bot\}) \times (D \cup \{\bot\})$, choose $r^0 = (\bot, \bot)$ as initial state, and define the input transitions by

$$
(v_1, v_2) \xrightarrow{\text{	extsc{IN1}(n_1)}} (n_1, v_2) \\
(v_1, v_2) \xrightarrow{\text{	extsc{IN2}(n_2)}} (v_1, n_2) \\
(v_1, v_2) \xrightarrow{\text{	extsc{IN3}(n_3)}} (v_1, v_2)
$$

Abstraction $\Upsilon$ abstracts from the specific value of a parameter, and only records whether it is fresh, or equals the last value of \textsc{IN1} or \textsc{IN2}. For $i = 1, 2, 3$:

$$
\Upsilon_{(v_1, v_2)}(\text{IN}_i(n_i)) = \begin{cases} \\
\text{IN}_i(\text{old}_1) & \text{if } n_i = v_1 \\
\text{IN}_i(\text{old}_2) & \text{if } n_i = v_2 \land n_i \neq v_1 \\
\text{IN}_i(\text{fresh}) & \text{otherwise}
\end{cases}
$$

This abstraction is not surjective: for instance, in the initial state \text{IN1}(\text{old}_1) is not possible as an abstract value, and in any state of the form $(v, v)$, \text{IN1}(\text{old}_2) is not possible.

Each mapper $\mathcal{A}$ induces an abstraction operator on interface automata, which abstracts an IA with actions in $I$ and $O$ into an IA with actions in $X$ and $Y$. This abstraction operator is essentially just a variation of the state operator well-known from process algebras \cite{9}.

**Abstraction** Let $\mathcal{T} = \langle I, O, Q, q^0, \rightarrow \rangle$ be an IA and let $\mathcal{A} = \langle \mathcal{I}, X, Y, \Upsilon \rangle$ be a mapper with $\mathcal{I} = \langle I, O^\delta, R, r^0, \rightarrow \rangle$. Then $\alpha_\mathcal{A}(\mathcal{T})$, the abstraction of $\mathcal{T}$, is the IA $\langle X, Y, Q \times R, (q^0, r^0), \rightarrow_{\text{abst}} \rangle$, where transition relation $\rightarrow_{\text{abst}}$ is given by the rule:

$$
q \xrightarrow{a} q' \quad r \xrightarrow{a_i} r' \quad \Upsilon_{r}(a) = z \quad \Rightarrow \quad (q, r) \xrightarrow{z}_{\text{abst}} (q', r')
$$

Observe that if $\mathcal{T}$ is determinate then $\alpha_\mathcal{A}(\mathcal{T})$ does not have to be determinate. Also, if $\mathcal{T}$ is an IOA then $\alpha_\mathcal{A}(\mathcal{T})$ does not have to be an IOA (if $\mathcal{A}$ is not surjective, as in Example \ref{example:non_surjective_mapper}, then an abstract input will not be enabled if there is no corresponding concrete input). If $\mathcal{T}$ is output-determined then $\alpha_\mathcal{A}(\mathcal{T})$ is output-determined, but the converse implication does not hold. The following lemma gives a positive result: abstraction is monotone with respect to the alternating simulation preorder.

**Lemma 7.3.1** If $\mathcal{T}_1 \preceq \mathcal{T}_2$ then $\alpha_\mathcal{A}(\mathcal{T}_1) \preceq \alpha_\mathcal{A}(\mathcal{T}_2)$.

**Proof** Suppose $\mathcal{T}_1 \preceq \mathcal{T}_2$. Let $R$ be the maximal alternating simulation from $\mathcal{T}_1^\delta$ to $\mathcal{T}_2^\delta$. Define the relation $R'$ between states of $\alpha_\mathcal{A}(\mathcal{T}_1)$ and $\alpha_\mathcal{A}(\mathcal{T}_2)$ as follows:

$$(q_1, r_1) R' (q_2, r_2) \iff q_1 R q_2 \land r_1 = r_2.$$

It is routine to prove that $R'$ is an alternating simulation from $(\alpha_\mathcal{A}(\mathcal{T}_1))^\delta$ to $(\alpha_\mathcal{A}(\mathcal{T}_2))^\delta$. Hence $\alpha_\mathcal{A}(\mathcal{T}_1) \preceq \alpha_\mathcal{A}(\mathcal{T}_2)$, as required.
The concretization operator is the dual of the abstraction operator. It transforms each IA with abstract actions in $X$ and $Y$ into an IA with concrete actions in $I$ and $O$.

Concretization  Let $\mathcal{H} = \langle X, Y, S, s^0, \rightarrow \rangle$ be an IA and let $\mathcal{A} = \langle I, X, Y, Y \rangle$ be a mapper for $I$ and $O$. Then $\gamma_{\mathcal{A}}(\mathcal{H})$, the concretization of $\mathcal{H}$, is the IA $\langle I, O^\delta, R \times S, (r^0, s^0), \rightarrow^{\text{conc}} \rangle$, where transition relation $\rightarrow^{\text{conc}}$ is given by the rule:

\[
\frac{r \xrightarrow{a} r' \quad s \xrightarrow{y} s' \quad \Upsilon_r(a) = z}{(r, s) \xrightarrow{a^{\text{conc}}} (r', s')}
\]

Whereas the abstraction operator does not preserve determinacy in general, the concretization of a determinate IA is always determinate. Also, the concretization of an output-determined IA is output-determined, provided the mapper is output-predicting.

**Lemma 7.3.2** If $\mathcal{H}$ is determinate then $\gamma_{\mathcal{A}}(\mathcal{H})$ is determinate.

**Proof** Routine. It is easy to show that the relation $R = \{(r, s), (r, s') \mid s \sim s'\}$ is a bisimulation on $\gamma_{\mathcal{A}}(\mathcal{H})$. Now suppose that $(r, s)$ is a reachable state of $\gamma_{\mathcal{A}}(\mathcal{H})$ with outgoing transitions $(r, s) \xrightarrow{a^{\text{conc}}} (r_1, s_1)$ and $(r, s) \xrightarrow{a^{\text{conc}}} (r_2, s_2)$. Then, by definition of $\gamma_{\mathcal{A}}(\mathcal{H})$, $r \xrightarrow{a} r_1$, $s \xrightarrow{z} s_1$, where $z = \Upsilon_r(a)$, $r \xrightarrow{a} r_2$ and $s \xrightarrow{z} s_2$. Since the IA of $\mathcal{A}$ is deterministic, $r_1 = r_2$. Since $(r, s)$ is reachable in $\gamma_{\mathcal{A}}(\mathcal{H})$, $s$ is reachable in $\mathcal{H}$. Hence, because $\mathcal{H}$ is determinate, $s_1 \sim s_2$. It follows that $((r_1, s_1), (r_2, s_2)) \in R$. Since $R$ is a bisimulation, we conclude $(r_1, s_1) \sim (r_2, s_2)$, as required.

**Lemma 7.3.3** If $\mathcal{A}$ is output-predicting and $\mathcal{H}$ is output-determined then $\gamma_{\mathcal{A}}(\mathcal{H})$ is output-determined.

**Proof** Suppose that $\mathcal{A}$ is output-predicting and $\mathcal{H}$ is output-determined. Let $(r, s)$ be a reachable state of $\gamma_{\mathcal{A}}(\mathcal{H})$ such that, for concrete outputs $o$ and $o'$, $(r, s) \xrightarrow{o^{\text{conc}}} \text{ and } (r, s) \xrightarrow{o'^{\text{conc}}}$. Then it follows from the definition of $\gamma_{\mathcal{A}}(\mathcal{H})$ that there exists abstract outputs $y$ and $y'$ such that $s \xrightarrow{y}$, $s \xrightarrow{y'}$, $\Upsilon_r(o) = y$ and $\Upsilon_r(o') = y'$. Since $(r, s)$ is reachable in $\gamma_{\mathcal{A}}(\mathcal{H})$, it follows that $s$ is reachable in $\mathcal{H}$. Hence, by the assumption that $\mathcal{H}$ is output-determined, $y = y'$. Next, using that $\mathcal{A}$ is output-predicting, we conclude $o = o'$.

In an abstraction of the form $\gamma_{\mathcal{A}}(\mathcal{H})$ it may occur that a reachable state $(r, s)$ is quiescent, even though the contained state $s$ of $\mathcal{H}$ enables some abstract output $y$: this happens if there exists no concrete concrete output $o$ such that $\Upsilon_r(o) = y$. This situation is ruled out by following definition.

**Definition** $\gamma_{\mathcal{A}}(\mathcal{H})$ is quiescence preserving if, for each reachable state $(r, s)$, $(r, s)$ quiescent implies $s$ quiescent.
Concretization is monotone with respect to the \( \preceq \) preorder, provided the concretization of the first argument is quiescence preserving.

**Lemma 7.3.4** Suppose \( \gamma_A(H_1) \) is quiescence preserving. Then \( H_1 \preceq H_2 \) implies \( \gamma_A(H_1) \preceq \gamma_A(H_2) \).

**Proof** Suppose \( H_1 \preceq H_2 \). Let \( R \) be the maximal \( A^\delta I \)-simulation from \( H_1^\delta \) to \( H_2^\delta \). Define relation \( R' \) between states of \( \gamma_A(H_1) \) and \( \gamma_A(H_2) \) as follows:

\[
(r_1, s_1) R' (r_2, s_2) \iff r_1 = r_2 \land s_1 R s_2.
\]

We check that \( R' \) is an \( A^\delta I \)-simulation from \( (\gamma_A(H_1))^\delta \) to \( (\gamma_A(H_2))^\delta \). Suppose \((r, s_1) R' (r, s_2)\).

- Suppose \((r, s_2) \xrightarrow{i} (r', s_2')\) for some \( i \in I \). Let \( \Upsilon_i(i) = x \). Then, by definition of concretization, \( r \xrightarrow{i} r' \) and \( s_2 \xrightarrow{\gamma_i} s_2' \). Using that \( s_1 R s_2 \), we infer that there exists a state \( s_1' \) such that \( s_1 \xrightarrow{x} s_1' \) and \( s_1' R s_2' \). Hence \((r, s_1) \xrightarrow{i} (r', s_1') \) and \((r', s_1') R' (r', s_2')\), as required.

- Suppose \((r, s_1) \overset{a}{\rightarrow} (r', s_1')\) for some \( a \in A^\delta \), \( \Upsilon_r(a) = z \), \( r \xrightarrow{a} r' \) and \( s_1 \xrightarrow{z} s_1' \). Using that \( s_1 R s_2 \), we infer that there exists a state \( s_2' \) such that \( s_2 \xrightarrow{\delta} s_2' \) and \( s_1' R s_2' \). Hence \((r, s_2) \overset{a}{\rightarrow} (r', s_2') \) and \((r', s_1') R' (r', s_2')\), as required.

- Suppose \((r, s_1)\) is quiescent. Then, since \( \gamma_A(H_1) \) is quiescence preserving, \( s_1 \) is quiescent. Since \( s_1 R s_2 \) and \( R \) is an \( A^\delta I \)-simulation from \( H_1^\delta \) to \( H_2^\delta \), it follows that \( s_2 \) is quiescent. Hence, by definition of concretization, \((r, s_2)\) is quiescent.

Since \( R \) is an \( A^\delta I \)-simulation from \( H_1^\delta \) to \( H_2^\delta \), \( s_1 R s_2 \). Hence we have \((r_0, s_0) R' (r_0, s_2)\) and so \( R' \) relates the initial states of \( \gamma_A(H_1) \) and \( \gamma_A(H_2) \). Thus \( \gamma_A(H_1) \preceq \gamma_A(H_2) \), as required.

The lemma below is a key result of this chapter. It says that if \( T \) is \( ioco \)-conforming to the concretization of an hypothesis \( H \), and this concretization is quiescence preserving, then the abstraction of \( T \) is \( ioco \)-conforming to \( H \) itself.

**Lemma 7.3.5** Suppose \( \gamma_A(H) \) is quiescence preserving. Then \( T \ ioco \ \gamma_A(H) \) implies \( \alpha_A(T) \ ioco \ H \).

**Proof** Suppose \( T \ ioco \ \gamma_A(H) \). Let \( \sigma \in Traces(H^\delta) \) and let \( y \in out((\alpha_A(T))^\delta \ after \ \sigma) \). We must show that \( y \in out(H^\delta \ after \ \sigma) \). Let \( \sigma = z_1 \cdots z_n \). Then \( H^\delta \) has a run

\[
s_0 \xrightarrow{z_1} s_1 \xrightarrow{z_2} \cdots \xrightarrow{z_n} s_n
\]

with \( s_0 = s^0 \), and \( (\alpha_A(T))^\delta \) has a run

\[
(q_0, r_0) \xrightarrow{z_1} (q_1, r_1) \xrightarrow{z_2} \cdots \xrightarrow{z_n} (q_n, r_n) \overset{y}{\rightarrow}
\]
with \((q_0, r_0) = (q^0, r^0)\). Then, by definition of \((\alpha_A(T))^{\delta}\), there exists runs

\[
\begin{align*}
q_0 & \xrightarrow{a_1} q_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} q_n \\
r_0 & \xrightarrow{a_1} r_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} r_n
\end{align*}
\]

of \(T^{\delta}\) and \(A\), respectively, such that, for all \(1 \leq i \leq n\), \(\Upsilon_{r_{i-1}}(a_i) = z_i\) and \(\Upsilon_{r_n}(o) = y\). By definition of \((\gamma_A(H))^{\delta}\), this IA has a run

\[
(r_0, s_0) \xrightarrow{a_1} (r_1, s_1) \xrightarrow{a_2} \cdots \xrightarrow{a_n} (r_n, s_n)
\]

Let \(\rho = a_1 \cdots a_n\). Then \(\rho \in Traces((\gamma_A(H))^{\delta})\). Moreover, \(o \in out(T^{\delta} \text{ after } \rho)\).

Using \(\ioco\) \(\gamma_A(H)\), we obtain \(o \in out((\gamma_A(H))^{\delta} \text{ after } \rho)\). Hence \((\gamma_A(H))^{\delta}\) has a run

\[
(r_0, s'_0) \xrightarrow{a_1} (r_1, s'_1) \xrightarrow{a_2} \cdots \xrightarrow{a_n} (r_n, s'_n)
\]

By definition of \((\gamma_A(H))^{\delta}\) and using that \(\gamma_A(H)\) is quiescent preserving, we may infer that \(H^{\delta}\) has a run

\[
s'_0 \xrightarrow{z_1} s'_1 \xrightarrow{z_2} \cdots \xrightarrow{z_n} s'_n \xrightarrow{y}
\]

Hence, \(y \in out(H^{\delta} \text{ after } \sigma)\), as required.

By using a mapper \(A\), we may reduce the task of learning an IA \(H\) such that \(\ioco H \preceq P\) to the simpler task of learning an IA \(H'\) such that \(\alpha_A(T) \ioco H' \preceq \alpha_A(P)\). However, in order to establish the correctness of this reduction, we need two technical lemmas that require some additional assumptions on \(P\) and \(A\). It is straightforward to check that these assumptions are met by the mappers of Examples 7.3 and 7.3, and the learning purposes of Example 7.2.

**Definition** Let \(A = (I, X, Y, \Upsilon)\) be a mapper for \(I\) and \(O\). \(\equiv_A\) is defined to be the equivalence relation on \(I \cup O^{\delta}\) which declares two concrete actions equivalent if, for some states of the mapper, they are mapped to the same abstract action:

\[a \equiv_A b \iff \exists r, r' : \Upsilon_r(a) = \Upsilon_{r'}(b)\]

Let \(T = (I, O, Q, q^0, \rightarrow)\) be an IA. \(P\) and \(A\) are called compatible if, for all concrete actions \(a, b\) with \(a \equiv_A b\) and for all \(p, p_1, p_2 \in P\), \((p \xrightarrow{a} p_1 \land p \xrightarrow{b} p_2) \Rightarrow p_1 \sim p_2\).

**Lemma 7.3.6** Suppose \(\alpha_A(P)\) is determinate and \(P\) and \(A\) are compatible. Then \(\gamma_A(\alpha_A(P)) \preceq P\).

**Proof** We claim \(\gamma_A(\alpha_A(P)) \sim P\). In order to prove this, consider the relation

\[S = \{((r_2, (p_1, r_1)), p_2) \mid p_1 \sim p_2 \land (p_1, r_1) \sim (p_2, r_2)\}\]

It is easy to check that \(S\) relates the initial states of \(\gamma_A(\alpha_A(P))\) and \(P\). We show that \(S\) is a bisimulation.

Suppose \(((r_2, (p_1, r_1)), p_2) \in S\) and \(p_2 \xrightarrow{a} p_2'\). Since the IA for \(A\) is input- and output-enabled, there exist a state \(r_2'\) such that \(r_2 \xrightarrow{a} r_2'\). Let \(\Upsilon_{r_2}(a) = z\). Then, by
definition of the abstraction operator, \((p_2, r_2) \xrightarrow{z} (p'_2, r'_2)\). Since \((p_1, r_1) \sim (p_2, r_2)\), there exists a pair \((p'_1, r'_1)\) such that \((p_1, r_1) \xrightarrow{z} (p'_1, r'_1)\) and \((p'_1, r'_1) \sim (p'_2, r'_2)\). By definition of the abstraction operator, there exists a concrete action \(b\) such that \(\Upsilon_{r_1}(b) = z, p_1 \xrightarrow{b} p'_1\) and \(r_1 \xrightarrow{b} r'_1\). Since \(p_1 \sim p_2\), there exists a \(p''_2\) such that \(p_2 \xrightarrow{b} p''_2\) and \(p'_1 \sim p''_2\). Since \(\mathcal{P}\) and \(\mathcal{A}\) are compatible and \(a \equiv \mathcal{A} b, p'_2 \sim p''_2\). By Lemma 7.1.4, \(p'_1 \sim p'_2\). By definition of the concretization operator, \((r_2, (p_1, r_1)) \xrightarrow{a} (r'_2, (p'_1, r'_1))\). Moreover, \(((r'_2, (p'_1, r'_1)), p'_2) \in S\), as required.

Suppose \(((r_2, (p_1, r_1)), p_2) \in S\) and \((r_2, (p_1, r_1)) \xrightarrow{a} (r'_2, (p'_1, r'_1))\). Let \(\Upsilon_{r_2}(a) = z\). Then, by definition of the concretization operator, \(r_2 \xrightarrow{a} r'_2\) and \((p_1, r_1) \xrightarrow{z} (p'_1, r'_1)\). By definition of the abstraction operator, there exists a concrete action \(b\) such that \(\Upsilon_{r_1}(b) = z, p_1 \xrightarrow{b} p'_1\) and \(r_1 \xrightarrow{b} r'_1\). Since \(\mathcal{P}\) and \(\mathcal{A}\) are compatible and \(a \equiv \mathcal{A} b\), there exists a \(p''_1\) such that \(p_1 \xrightarrow{a} p''_1\) and \(p'_1 \sim p''_1\). Since \(p_1 \sim p_2\), by Lemma 7.1.4 there exists a \(p''_2\) such that \(p_2 \xrightarrow{a} p''_2\) and \(p'_1 \sim p''_2\). By definition of the abstraction operator, \((p_2, r_2) \xrightarrow{z} (p'_2, r'_2)\). Since \(\alpha_{\mathcal{A}}(\mathcal{P})\) is determinate, it follows by Lemma 7.1.4 that \((p'_1, r'_1) \sim (p'_2, r'_2)\). Hence, \(((r'_2, (p'_1, r'_1)), p'_2) \in S\), as required.

Now the lemma follows since, for all IA’s \(I_1\) and \(I_2\), \(I_1 \sim I_2 \Rightarrow I_1^\delta \sim I_2^\delta \Rightarrow I_1 \lesssim I_2\).

**Lemma 7.3.7** Suppose \(\mathcal{A}\) and \(\mathcal{P}\) are compatible, \(\alpha_{\mathcal{A}}(\mathcal{P})\) is determinate and \(\mathcal{H} \lesssim \alpha_{\mathcal{A}}(\mathcal{P})\). Then \(\gamma_{\mathcal{A}}(\mathcal{H})\) is quiescence preserving.

**Proof** By contradiction. Assume that \(\gamma_{\mathcal{A}}(\mathcal{H})\) is not quiescence preserving. Consider a minimal run that shows this, that is, a run

\[(r_0, s_0) \xrightarrow{a_1} (r_1, s_1) \xrightarrow{a_2} \cdots \xrightarrow{a_n} (r_n, s_n) \xrightarrow{a_{n+1}}\]

of \((\gamma_{\mathcal{A}}(\mathcal{H}))^\delta\) with \((r_0, s_0) = (r^0, s^0), (r_n, s_n)\) quiescent in \(\gamma_{\mathcal{A}}(\mathcal{H})\), so \(a_{n+1} = \delta\), and \(s_n\) not quiescent in \(\mathcal{H}\). Let \(z_j = \Upsilon_{r_{j-1}}(a_j)\), for \(1 \leq j \leq n\), and let \(z_{n+1} \neq \delta\) be an output action enabled in state \(s_n\) of \(\mathcal{H}\). Since \((r_n, s_n)\) is quiescent, it follows that there exists no concrete output \(o\) such that \(\Upsilon_{r_n}(o) = z_{n+1}\). From the definition of concretization and the minimality of the run of \((\gamma_{\mathcal{A}}(\mathcal{H}))^\delta\), it follows that

\[r_0 \xrightarrow{a_1} r_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} r_n \xrightarrow{a_{n+1}}\]

is a run of the IA of \(\mathcal{A}\), and

\[s_0 \xrightarrow{z_1} s_1 \xrightarrow{z_2} \cdots \xrightarrow{z_n} s_n \xrightarrow{z_{n+1}}\]

is a run of \(\mathcal{H}\). Let \(S\) be the maximal \(A^\delta I\)-simulation from \(\mathcal{H}^\delta\) to \((\alpha_{\mathcal{A}}(\mathcal{P}))^\delta\). Then, using the assumptions that \(\alpha_{\mathcal{A}}(\mathcal{P})\) is determinate and that \(\mathcal{P}\) and \(\mathcal{A}\) are compatible, we may construct runs

\[p_0 \xrightarrow{a_0} p_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} p_n\]

and

\[(p_0, r_0) \xrightarrow{z_1} (p_1, r_1) \xrightarrow{z_2} \cdots \xrightarrow{z_n} (p_n, r_n) \xrightarrow{z_{n+1}}\]

of \(\mathcal{P}^\delta\) and \((\alpha_{\mathcal{A}}(\mathcal{P}))^\delta\), respectively, such that, for all \(i \leq n, (s_i, (p_i, r_i)) \in S\). But since \((p_n, r_n) \xrightarrow{z_{n+1}}\), it follows that there exists a concrete output \(o\) such that \(\Upsilon_{r_n}(o) = z_{n+1}\). Contradiction.
### 7.4 Inference Using Abstraction

Suppose we have a teacher equipped with a determinate IA $T$, and a learner equipped with a determinate learning purpose $P$ such that $T \preceq P$. The learner has the task to infer some $H$ satisfying $T \ioco H \preceq P$. After the preparations from the previous section, we are now ready to show how, in certain cases, the learner may simplify her task by defining a mapper $A$ such that $\alpha_A(T)$ and $\alpha_A(P)$ are determinate, $P$ and $A$ are compatible, and $T$ respects $A$ in the sense that, for $i, i' \in I$ and $q \in Q$, $i \equiv_A i' \Rightarrow (q \mapsto q)$. Note that if $T$ is an IOA it trivially respects $A$. In these cases, we may reduce the task of the learner to learning an IA $H'$ satisfying $\alpha_A(T) \ioco H' \preceq \alpha_A(P)$. Note that $\alpha_A(P)$ is a proper learning purpose for $\alpha_A(T)$ since it is determinate and, by monotonicity of abstraction (Lemma 7.3.1), $\alpha_A(T) \preceq \alpha_A(P)$.

A teacher for $\alpha_A(T)$ is constructed by placing a mapper component in between the teacher for $T$ and the learner for $P$, which translates concrete and abstract actions to each other in accordance with $A$. Let $T = \langle I, O, Q, q^0, \rightarrow \rangle$, $P = \langle I, O^\delta, P, p^0, \rightarrow_P \rangle$, $A = \langle I, X, Y, Y, \rangle$, and $I = \langle I, O^\delta, R, r^0, \rightarrow \rangle$. The mapper component maintains a state variable of type $R$, which initially is set to $r^0$. The behavior of the mapper component is defined as follows:

1. **Input.** If the mapper is in state $r$ and receives an abstract input $x \in X$ from the learner, it picks a concrete input $i \in I$ such that $Y_r(i) = x$, forwards $i$ to the teacher, and waits for a reply $\top$ or $\bot$ from the teacher. This reply is then forwarded to the learner. In case of a $\top$ reply, the mapper updates its state to the unique $r'$ with $r \xrightarrow{i} r'$. If there is no $i \in I$ such that $Y_r(i) = x$, then the mapper returns a $\bot$ reply to the learner right away.

2. **Output.** If the mapper receives an output query $\Delta$ from the learner, it forwards $\Delta$ to the teacher. It then waits until it receives an output $o \in O^\delta$ from the teacher, and forwards $Y_r(o)$ to the learner.

3. **Reset.** If the mapper receives a **reset** from the learner, it resets its state to $r^0$ and forwards **reset** to the teacher.

4. **Hypothesis.** If the mapper receives a hypothesis $H$ from the learner then, by Lemma 7.3.7, $\gamma_A(H)$ is quiescence preserving. Since $H \preceq \alpha_A(P)$, monotonicity of concretization (Lemma 7.3.4) implies $\gamma_A(H) \preceq \gamma_A(\alpha_A(P))$. Hence, by Lemma 7.3.6 $\gamma_A(H) \preceq P$. This means that the mapper may forward $\gamma_A(H)$ as a hypothesis to the teacher. If the mapper receives response **yes** from the teacher, it forwards **yes** to the learner. If the mapper receives response **no** with counterexample $\sigma o$, where $\sigma = a_1 \cdots a_n$, then it constructs a run

$$(r_0, s_0) \xrightarrow{a_1} (r_1, s_1) \xrightarrow{a_2} \cdots \xrightarrow{a_n} (r_n, s_n)$$

of $(\gamma_A(H))^\delta$ with $(r_0, s_0) = (r^0, s^0)$. It then forwards **no** to the learner, together with counterexample $z_1 \cdots z_n y$, where, for $1 \leq j \leq n$, $z_j = Y_{r_{j-1}}(a_j)$
and \( y = \Upsilon_{r_n}(o) \). Finally, the mapper returns to its initial state.

The next lemma implies that, whenever the learner presents an abstract input \( x \) to the mapper, there exists a concrete input \( i \) such that \( \Upsilon_{r_n}(i) = x \), and the teacher will accept input \( i \) from the mapper. So no \( \perp \) replies will be sent. Moreover, whenever the teacher sends a concrete output \( o \) to the mapper, the learner will accept the corresponding abstract output \( \Upsilon_{r_n}(o) \) generated by the mapper.

**Lemma 7.4.1** Let \( S \) be the maximal alternating simulation from \( T^\delta \) to \( P^\delta \). Then, for any configuration of states \( q, r \) and \( (p, r) \) of teacher, mapper and learner, respectively, that can be reached after a finite number of steps (1)-(5) of the learning protocol, we have \((q, p) \in S \) and \((p, r) \sim (p, r) \) (here \( \sim \) denotes bisimulation equivalence in \( \alpha_A(P) \)).

**Proof** By induction on the number of steps.

Initially, the teacher is in state \( q^0 \), the mapper is in state \( r^0 \), and the learner is in state \( (p^0, r^0) \). Since \( S \) relates the initial states of \( T^\delta \) and \( P^\delta \), \((q^0, p^0) \in S \). Since \( \sim \) is an equivalence relation, \((p^0, r^0) \sim (p^0, r^0) \).

For the induction step, observe that after a reset or hypothesis checking step, teacher, mapper and learner all return to their initial states, which means that we reach a configuration for which, as we observed, the required properties hold. So the interesting cases are the input and output queries.

Suppose that the learner enables an abstract input \( x \in X \), and takes transition \((p, r) \xrightarrow{x} (p', r'_2)\) after presenting \( x \) to the mapper. Since \((p, r) \sim (p, r)\), there exists a transition \((p, r) \xrightarrow{x} (p''', r'_1)\) such that \((p''', r'_1) \sim (p', r'_2)\). Hence, by the definition of the abstraction operator, there exists a concrete input \( i \) such that \( \Upsilon_{r_1}(i) = x \), \( p \xrightarrow{i} p'' \) and \( r_1 \xrightarrow{i} r'_1 \). This means that the mapper accepts the abstract input \( x \), forwards a corresponding concrete input, say \( i \), to the teacher, and jumps to a new state \( r'_1 \). Since \((q, p) \in S \) and \( p \xrightarrow{i} p'' \), there exists a state \( q' \) such that \( q \xrightarrow{i} q' \) and \((q', p'') \in S \). This means that the teacher will accept the input \( i \) from the mapper and jump to a state \( q' \). Since \((p, r) \xrightarrow{x} (p', r'_2)\), there exists an \( i' \) such that \( \Upsilon_{r_2}(i) = x \) and \( p \xrightarrow{i'} p' \). Because \( P \) and \( A \) are compatible and \( i \equiv_A i' \), \( p'' \sim p' \). Hence, by Lemma 6.1.4 \((q', p') \in S \) and so the required properties hold.

Next suppose that the learner sends an output query to the mapper, which is forwarded by the mapper to the teacher. Suppose that the teacher takes transition \( q \xrightarrow{o} q' \) after returning concrete output \( o \in O^\delta \) to the mapper. Then the mapper jumps to the unique state \( r'_1 \) with \( r_1 \xrightarrow{o} r'_1 \) and forwards \( y = \Upsilon_{r_2}(o) \) to the learner. Since \((q, p) \in S \), there exists a state \( p' \) such that \( p \xrightarrow{o} p' \) and \((q', p') \in S \). By definition of the abstraction operator, we have a transition \((p, r_1) \xrightarrow{y} (p', r'_1)\). Since \((p, r) \sim (p, r)\), there exists a transition \((p, r) \xrightarrow{y} (p''', r'_1)\) such that \((p'', r'_1) \sim (p', r'_2)\). This means that the learner will accept the abstract output \( y \) and jump to a state \((p'', r'_2)\). By definition of the abstraction operator, there exists a concrete output \( o' \) such that \( \Upsilon_{r_2}(o') = y \) and \( p \xrightarrow{o'} p'' \). Because \( P \) and \( A \) are compatible
and \( o \equiv_A o', p' \sim p'' \). Hence, by Lemma 7.1.4, \((q', p'') \in S\). It is straightforward to check that bisimulation is a congruence for the abstraction operator (follows also since the defining rules for \( \alpha_A \) are in the De Simone format, see [89, 40]), that is \( p' \sim p'' \) implies \((p', r_1') \sim (p'', r_2')\). Hence, since \( \sim \) is an equivalence, \((p'', r_1') \sim (p'', r_2')\), and so the required properties hold.

We claim that, from the perspective of a learner with learning purpose \( \alpha_A(P) \), a teacher for \( T \) and a mapper for \( A \) together behave exactly like a teacher for \( \alpha_A(T) \). Since the notion of behavior has not been formalized for a teacher and a mapper, the mathematical content of this claim may not be immediately obvious. Clearly, it is routine to describe the behavior of teachers and mappers formally in some concurrency formalism, such as Milner’s CCS [72] or another process algebra [19]. For instance, we may define, for each IA \( T \), a CCS process \( \text{Teacher}(T) \) that describes the behavior of a teacher for \( T \), and for each mapper \( A \) a CCS process \( \text{Mapper}(A) \) that models the behavior of a mapper for \( A \). These two CCS processes may then synchronize via actions taken from \( A^\delta \), actions \( \Delta, \delta, \top, \bot \), and actions \( \text{hypothesis}(H) \), where \( H \) is an interface automaton. If we compose \( \text{Teacher}(T) \) and \( \text{Mapper}(A) \) using the CCS composition operator |, and apply the CCS restriction operator \( \setminus \) to internalize all communications between teacher and mapper, the resulting process is observation equivalent (weakly bisimilar) to the process \( \text{Teacher}(\alpha_A(T)) \):

\[
(\text{Teacher}(T) | \text{Mapper}(A)) \setminus L \approx \text{Teacher}(\alpha_A(T)),
\]

where \( L = A^\delta \cup \{\Delta, \delta, \top, \bot, \text{reset}, \text{hypothesis}\} \). It is in this precise, formal sense that one should read the following theorem. The reason why we do not refer to the CCS formalization in the statement and proof of this theorem is that we feel that the resulting notational overhead would obscure rather than clarify.

**Theorem 7.4.2** Let \( T, A \) and \( P \) be as above. A teacher for \( T \) and a mapper for \( A \) together behave like a teacher for \( \alpha_A(T) \).

**Proof** Initially, the state of the teacher for \( T \) is \( q^0 \) and the state of the mapper for \( A \) is \( r^0 \), which is consistent with the initial state \((q^0, r^0)\) of the teacher for \( \alpha_A(T) \). Suppose the current state of the teacher for \( T \) is \( q \), and the current state of the mapper is \( r \). We consider the possible interactions between the components:

- **Input.** Suppose the learner sends an abstract input \( x \in X \). Using the assumption that \( T \) respects \( A \), it is easy to see that the mapper returns \( \bot \) to the learner exactly if there exists no concrete input \( i \) and state \( q' \) such that \( \Upsilon_r(i) = x \) and \( q \xrightarrow{\bot} q' \). This behavior is consistent with the behavior of a teacher for \( \alpha_A(T) \) from state \((q, r)\),

  Now suppose that \( \Upsilon_r(i) = x \), \( r \xrightarrow{i} r' \), the mapper forwards \( i \) to the teacher, the teacher jumps to a state \( q' \) such that \( q \xrightarrow{i} q' \), sends a reply \( \top \) to the
mapper, who jumps to state $r'$ and forwards $\top$ to the learner. This behavior is consistent with the behavior of a teacher for $\alpha_A(T)$ from state $(q, r)$, which may jump to any state $(q', r')$ such that $(q, r) \xrightarrow{\text{abst}} (q', r')$.

- **Output.** Suppose the learner sends an output query $\Delta$. The mapper will then forwards $\Delta$ to the teacher for $T$. If state $q$ is quiescent then the teacher for $T$ forwards $\delta$ to the mapper, and the mapper forwards $\delta$ to the learner. This behavior is consistent with the behavior of a teacher for $\alpha_A(T)$ from state $(q, r)$. If state $q$ is not quiescent then the teacher for $T$ selects a transition $q \xrightarrow{\alpha} q'$, jumps to $q'$ and returns $o$ to the mapper. The mapper then forwards $\Upsilon_{r}(o)$ to the learner. This behavior is consistent with the behavior of a teacher for $\alpha_A(T)$ from state $(q, r)$, which nondeterministically picks an output $y$ and state $(q', r')$ such that $(q, r) \xrightarrow{y} (q', r')$.

- **Reset.** Suppose the learner sends a reset command. Then the learner returns to its initial state $(p^0, r^0)$. The mapper moves to its initial state $r^0$ and forwards the reset command to the teacher, who also returns to its initial state $q^0$. This behavior is consistent with the behavior of a teacher for $\alpha_A(T)$ which, upon receiving a reset, returns to its initial state $(q^0, r^0)$.

- **Hypothesis.** Suppose that the learner sends a hypothesis $\mathcal{H}$. The mapper will then forward $\gamma_A(\mathcal{H})$ as a hypothesis to the teacher for $T$. If the teacher for $T$ answers yes then the mapper forwards this answer to the learner. In this case $T$ ioco $\gamma_A(\mathcal{H})$ and hence, by Lemma 7.3.5, $\alpha_A(T)$ ioco $\mathcal{H}$. So when the mapper forwards yes to the learner, this is the proper behavior for a teacher for $\alpha_A(T)$.

If the mapper receives answer no with a counterexample $\sigma o$ then, by definition of a teacher, $\sigma$ is a trace of $(\gamma_A(\mathcal{H}))^\delta$ and $o$ is an output enabled by $T^\delta$ after $\sigma$ but not by $(\gamma_A(\mathcal{H}))^\delta$ after $\sigma$. So if $\sigma = a_1 \cdots a_n$, then the mapper indeed may construct a corresponding run $(r_0, s_0) \xrightarrow{a_1} (r_1, s_1) \xrightarrow{a_2} \cdots \xrightarrow{a_n} (r_n, s_n)$ of $(\gamma_A(\mathcal{H}))^\delta$ with $(r_0, s_0) = (r^0, s^0)$. The mapper then forwards no to the learner, together with counterexample $\rho y$, where $\rho = z_1 \cdots z_n$, $z_j = \Upsilon_{r_ j-1}(a_j)$, for $1 \leq j \leq n$, and $y = \Upsilon_{r_n}(o)$. By construction, $\rho \in \text{Traces}(\mathcal{H}^\delta)$. Since $\sigma o$ is a counterexample, $\sigma o$ is a trace of $T^\delta$. This means that we may construct a run $q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} q_n \xrightarrow{o} q_{n+1}$ of $T^\delta$ with $q_0 = q^0$. Now observe that $(q_0, r_0) \xrightarrow{z_1} (q_1, r_1) \xrightarrow{z_2} \cdots \xrightarrow{z_n} (q_n, r_n) \xrightarrow{y} (q_{n+1}, r_n)$ is a run of $(\alpha_A(T))^\delta$. Hence, $y \in \text{out}((\alpha_A(T))^\delta \text{ after } \rho)$. Since $\sigma o$ is a counterexample generated by the teacher for $T$, $o$ is not enabled by $(\gamma_A(\mathcal{H}))^\delta$ after $\sigma$. In particular, state $(r_n, s_n)$ does not enable $o$. This implies state $s_n$ does not enable $y$. By Lemma 7.1.2 since $\mathcal{H}$ is determinate, no state of $\mathcal{H}^\delta$ reachable via trace $\rho$ enables $y$. We conclude that $y \notin \text{out}(\mathcal{H}^\delta \text{ after } \rho)$, and so $\rho y$ is a counterexample for a teacher for $\alpha_A(T)$. 
Since a teacher for $T$ and a mapper for $A$ together behave like a teacher for $\alpha_A(T)$, it follows that we have reduced the task of learning an $H$ such that $T \text{ioco} H \preceq P$ to the simpler task of learning an $H$ such that $\alpha_A(T) \text{ioco} H \preceq \alpha_A(P)$: whenever the learner receives the answer yes from the mapper, indicating that $\alpha_A(T) \text{ioco} H$ we know, by definition of the behavior of the mapper component, that $\gamma_A(H)$ is quiescent preserving and $T \text{ioco} \gamma_A(H)$. Moreover, by Lemmas 7.3.4 and 7.3.6, $\gamma_A(H) \preceq P$.

Recall that for output-predicting abstractions, if $H$ is behavior-deterministic then $\gamma_A(H)$ is behavior-deterministic. This implies that, for such abstractions, provided $T$ is an IOA, whenever the mapper returns an answer yes to the learner, $\gamma_A(H)$ is in fact the unique interface automaton (up to bisimulation) that satisfies $T \text{ioco} \gamma_A(H) \preceq P$.

### 7.5 Conclusion

This chapter has provided several generalizations of the framework of [1], leading to a general theory of history dependent abstractions for learning interface automata. This work establishes some very interesting links between previous work on concurrency theory, model-based testing, and automata learning.

The theory of abstractions presented in this chapter is not complete yet and deserves further study. The link between this theory and the theory of abstract interpretation [30, 31] needs to be investigated further. Also the concept of $XY$-simulations, which nicely generalizes three fundamental concepts from concurrency theory (bisimulations, simulations and alternating simulations), deserves further study. Finally, an obvious challenge is to generalize the theory of this chapter to SUTs that are not determinate.

Chapter 8 presents the prototype tool Tomte, which is able to automatically construct mappers for a restricted class of scalarset automata, in which one can test for equality of data parameters, but no operations on data are allowed.
Chapter 8

Counterexample-Guided Abstraction Refinement for Learning Scalarset Mealy Machines

This chapter introduces the tool Tomte, which is able to construct mappers similar to those presented in chapter 7, fully automatically for a restricted class of extended finite state machines where one can test for equality of data parameters, but no operations on data are allowed. To fulfill its task, Tomte uses counterexample-guided abstraction refinement: whenever the current abstraction is too coarse and induces nondeterministic behavior, the abstraction is refined automatically. This chapter is organized as follows. In section 8.1 the class of scalarset Mealy Machines is introduced and the algorithm to learn mappers for them is presented. Section 8.3 explains the abstraction learning algorithm in more detail with an example, and reports a summary of the experiments done with Tomte. Finally, the conclusions are presented in section 8.4.

8.1 The World of Tomte

The general approach for using abstraction in automata learning is phrased most naturally at a general, semantic level. And indeed, this is what was done in the previous chapter. However, if we want to devise effective algorithms and implement them, we must restrict attention to a class of automata and mappers that can be finitely represented. This section describes the class of SUTs that the Tomte tool can learn, as well as the classes of mappers and learning purposes that it uses.
8.1.1 Scalarset Mealy Machines

We assume a universe $\mathcal{V}$ of variables. Each variable $v \in \mathcal{V}$ has a type $\text{type}(v) \subseteq \mathbb{N} \cup \{\perp\}$, where $\mathbb{N}$ is the set of natural numbers and $\perp$ denotes the undefined value. If $\mathcal{V}$ is a set of variables, then a valuation for $\mathcal{V}$ is a function $\xi$ that maps each variable in $\mathcal{V}$ to an element of its domain. We write $\text{Val}(\mathcal{V})$ for the set of all valuations for $\mathcal{V}$. We also assume a set $\mathcal{C}$ of constants which contains $\perp$ and a function $\gamma : \mathcal{C} \rightarrow \mathbb{N} \cup \{\perp\}$ that assigns a value to each constant. We define $\gamma(\perp) = \perp$. If $c \in \mathcal{C}$ is a constant then we define $\text{type}(c) = \{\gamma(c)\}$. A term over $\mathcal{V}$ is either a variable or a constant, that is, an element of $\mathcal{C} \cup \mathcal{V}$. We write $\mathcal{T}$ for the set of all terms. If $t$ is a term over $\mathcal{V}$ and $\xi$ is a valuation for $\mathcal{V}$ then we write $\llbracket t \rrbracket_\xi$ for the value to which $t$ evaluates:

$$
\llbracket t \rrbracket_\xi = \begin{cases} 
\xi(t) & \text{if } t \in \mathcal{V} \\
\gamma(t) & \text{if } t \in \mathcal{C}
\end{cases}
$$

A formula or guard $\varphi$ over $\mathcal{V}$ is a Boolean combination of expressions of the form $t = t'$, where $t$ and $t'$ are terms over $\mathcal{V}$. If $\xi$ is a valuation for $\mathcal{V}$ and $\varphi$ is a formula over $\mathcal{V}$, then we write $\xi \models \varphi$ to denote that $\xi$ satisfies $\varphi$. We assume a set $\mathcal{E}$ of event primitives and for each event primitive $\varepsilon$ an arity $\text{arity}(\varepsilon) \in \mathbb{N}$. An event term for $\varepsilon$ is an expression $\varepsilon(t_1, \ldots, t_n)$ where $t_1, \ldots, t_n$ are terms and $n = \text{arity}(\varepsilon)$. We write $\mathcal{ET}$ for the set of event terms.

Event signature An event signature $\Sigma$ is a pair $\langle T_I, T_O \rangle$, where $T_I$ and $T_O$ are finite sets of event terms such that $T_I \cap T_O = \emptyset$ and each term in $T_I \cup T_O$ is of the form $\varepsilon(p_1, \ldots, p_n)$ with $p_1, \ldots, p_n$ pairwise different variables. We require that the event primitives as well as the variables of different event terms in $T_I \cup T_O$ are distinct. We refer to the variables occurring in an event signature as parameters.

Below scalarset Mealy machines are defined. The scalarset datatype was introduced by Ip and Dill [19] as part of their work on symmetry reduction in verification. Operations on scalarsets are restricted so that states are guaranteed to have the same future behaviors, up to permutation of the elements of the scalarsets. Using the symmetries implied by the scalarsets, a verifier can automatically generate a reduced state space. On scalarsets no operations are allowed, we only allow the use of constants in $\mathcal{C}$. The only predicate symbol that may be used is equality.

Definition A scalarset Mealy machine (SMM) $\mathcal{M}$ is a tuple $\langle \Sigma, \mathcal{V}, L, l_0, \Gamma \rangle$, where

- $\Sigma = \langle T_I, T_O \rangle$ is an event signature, with $\perp \not\in \text{type}(p)$, for each parameter $p$ of $\Sigma$,
- $\mathcal{V} \subseteq \mathcal{V}$ is a finite set of state variables, with $\perp \in \text{type}(v)$, for each $v \in \mathcal{V}$,
- $L$ is a finite set of locations,
8.1. The World of Tomte

Figure 8.1: A sample SMM: alternating bit protocol receiver

- \( l_0 \in L \) is the initial location,
- \( \Gamma \subseteq L \times T_I \times G \times (V \rightarrow T) \times E \times T \times L \) is a finite set of transitions. In a transition \( \langle l, e_I(p_1, \ldots, p_k), g, \epsilon_O(u_1, \ldots, u_l), l' \rangle \in \Gamma \), we refer to \( l \) as the source, \( g \) as the guard, \( \epsilon \) as the update and \( l' \) as the target. We require that \( g \) is a formula over \( V \cup \{ p_1, \ldots, p_k \} \), there exists an event term \( \epsilon_O(q_1, \ldots, q_l) \in T_O \) such that, for each \( i \), \( u_i \) is a term over \( V \) with \( \text{type}(u_i) \subseteq \text{type}(q_i) \cup \{ \bot \} \), and, for each \( v \), \( g(v) \in V \cup C \cup \{ p_1, \ldots, p_k \} \) and \( \text{type}(g(v)) \subseteq \text{type}(v) \).

We say that \( \mathcal{M} \) is deterministic if, for all distinct transitions \( \tau_1 = \langle l_1, e_1^I, g_1, e_1^O, l_1' \rangle \) and \( \tau_2 = \langle l_2, e_2^I, g_2, e_2^O, l_2' \rangle \) in \( \Gamma \), \( l_1 = l_2 \) and \( e_1^I = e_2^I \) implies \( g_1 \wedge g_2 \equiv \text{false} \).

Example Figure 8.1 is a model of receiver running alternating bit protocol, which is a SMM.

Semantics of SMM To each SMM \( \mathcal{M} \) we associate an IA \([\mathcal{M}]\) in the obvious way. Transitions of the Mealy machine are turned into pairs of consecutive input and output transitions of the corresponding interface automaton.

The semantics of an event term \( \epsilon(p_1, \ldots, p_k) \) is the set of actions \([\epsilon(p_1, \ldots, p_k)] = \{ \epsilon(d_1, \ldots, d_k) \mid d_i \in \text{type}(p_i), 1 \leq i \leq k \} \). The semantics of a set \( T \) of event terms is defined by pointwise extension: \([T] = \bigcup_{\epsilon \in T} [\epsilon] \).

Let \( \mathcal{M} = \langle \Sigma, V, L, l_0, \Gamma \rangle \) be a SMM with \( \Sigma = \langle T_I, T_O \rangle \). The semantics of \( \mathcal{M} \), denoted \([\mathcal{M}]\), is the interface automaton \( \langle I, O, Q, q_0, \rightarrow \rangle \) where
- \( I = [T_I] \) and \( O = [T_O] \).
Figure 8.2: A sample SMM which is not restricted: a LIFO buffer of size 3

- $Q = L \times \text{Val}(V) \cup L \times \text{Val}(V) \times O$,
- $q^0 = (l_0, \xi_0)$, where $\xi_0(v) = \bot$, for $v \in V$,
- $\rightarrow \subseteq Q \times (I \cup O) \times Q$ is the smallest set that satisfies

\[
\langle l, \varepsilon_I(p_1, \ldots, p_k), g, e, \varepsilon_O(u_1, \ldots, u_\ell), l' \rangle \in \Gamma \\
\forall i \leq k, \iota(p_i) = d_i \\
\xi' = (\xi \cup \gamma \cup \iota) \circ g \\
\forall i \leq \ell, u_i \xi_i = d_i' \neq \bot
\]

\[
(l, \xi) \xrightarrow{\varepsilon_I(d_1, \ldots, d_k)} (l', \xi', \varepsilon_O(d_1', \ldots, d_{\ell}')) \\
(l', \xi', \varepsilon_O(d_1', \ldots, d_{\ell}')) \xrightarrow{\varepsilon_O(d_1', \ldots, d_{\ell}')} (l', \xi')
\]

Observe that if $\mathcal{M}$ is a deterministic SMM then $\llbracket \mathcal{M} \rrbracket$ is a behavior-deterministic IA. Tomte can learn the subclass of deterministic SMMs, which only record the first and the last occurrence of an input parameter:

**Restricted SMMs** Let $\mathcal{M}$ be a SMM. Variable $v$ records the last occurrence of input parameter $p$ of $\mathcal{M}$ if for each transition $\langle l, \varepsilon_I(p_1, \ldots, p_k), g, e, l' \rangle \in \Gamma$, if $p \in \{p_1, \ldots, p_k\}$ then $g(v) = p$ else $g(v) = v$. Moreover, $v$ may not occur in the codomain of $g$. Variable $v$ records the first occurrence of input parameter $p$ if for each transition $\langle l, \varepsilon_I(p_1, \ldots, p_k), g, e, l' \rangle \in \Gamma$, if $p \in \{p_1, \ldots, p_k\}$ and $g \Rightarrow v = \hat{\bot}$ holds then $g(v) = p$ else $g(v) = v$. Moreover, $v$ may not occur in the codomain of $g$.

We say that $\mathcal{M}$ only records the first and the last occurrence of parameters if, whenever $g(v) = p$ in some transition, $v$ either records the first or the last occurrence of $p$.

**Example** Figure 8.1, the alternating bit protocol receiver, shows a restricted SMM, because the state variables do not record any input parameter. But the SMM of Figure 8.2 is not restricted, as the model records three copies of input parameter $d$. A restricted SMM may only record the first and the last occurrences of an input parameter, that are at most two copies.
Symmetry Reduction in Scalarset Mealy Machines

This section provides lemmas and theorems to prove the general bisimilarity of two SMM’s in case of their bisimilarity when their variable domains are substituted with a large enough finite subset of natural numbers. We assume in this section that, all the variables in $V$ and all variables of SMM’s have domain $\mathbb{N} \cup \{\bot\}$.

**Definition** An *automorphism* is a morphism from a mathematical object to itself.

**Definition** We call an automorphism $h : \mathbb{N} \cup \{\bot\} \rightarrow \mathbb{N} \cup \{\bot\}$ *constant respecting* if $h(\bot) = \bot$ and it maps the value of each constant in the language to itself, ie. $\forall c \in C, h(\gamma(c)) = \gamma(c)$.

Lemma 8.1.1 states that applying a constant respecting automorphism on a valuation of a set of variables preserves the logical values of the formulas under that valuation. This lemma is needed to prove that computing the value of a term under a valuation and applying a constant respecting automorphism on that value, is equivalent to computing the value of that term under the composition of the automorphism and the valuation.

**Lemma 8.1.1** If $h$ is a constant respecting automorphism, then

$$\xi \models \varphi \iff h(\xi) \models \varphi$$

where $\varphi$ is a formula over $V$, $\xi$ is a valuation for $V$, and $h(\xi)$ is the valuation for $V$ defined by $h(\xi) = h \circ \xi$.

**Proof** We prove this lemma by induction on the number of operators in $\varphi$.

**Basis** $\varphi$ has no operators, that is $\varphi$ is an atomic formula. An atomic formula has four possible forms

1. $\varphi \equiv c = c'$ where $c, c' \in C$:

   $$h(\xi) \models \phi \iff$$
   $$h(\xi) \models c = c' \iff$$
   $$\gamma(c) = \gamma(c') \iff$$
   $$\xi \models c = c' \iff$$
   $$\xi \models \phi$$

---

1. A morphism is an abstraction derived from structure-preserving mappings between two mathematical structures. The study of morphisms and of the structures (called objects) over which they are defined, is central to category theory.
2. $\varphi \equiv v = c$ where $v \in V$ and $c \in C$:

$$h(\xi) \models \phi$$
$$h(\xi) \models v = c$$
$$h(\xi)(v) = \gamma(c) \iff (h \text{ is constant respecting.})$$
$$h(\xi(v)) = h(\gamma(c)) \iff (h \text{ is a bijection.})$$

$$\xi(v) = \gamma(c)$$
$$\xi \models v = c$$
$$\xi \models \phi$$

3. $\varphi \equiv c = v$ where $v \in V$ and $c \in C$:

$$h(\xi) \models \phi$$
$$h(\xi) \models c = v$$
$$\gamma(c) = h(\xi)(v) \iff (h \text{ is constant respecting.})$$
$$h(\gamma(c)) = h(\xi(v)) \iff (h \text{ is a bijection.})$$

$$\gamma(c) = \xi(v)$$
$$\xi \models c = v$$
$$\xi \models \phi$$

4. $\varphi \equiv v = v'$ where $v, v' \in V$:

$$h(\xi) \models \phi$$
$$h(\xi) \models v = v'$$
$$h(\xi)(v) = h(\xi)(v') \iff (h \text{ is a bijection.})$$

$$\xi(v) = \xi(v')$$
$$\xi \models v = v'$$
$$\xi \models \phi$$

**Induction**  Assume that for all formulas $\varphi$ with at most $k$ operators, we have

$$\xi \models \varphi \iff h(\xi) \models \varphi.$$  

We prove for any formula $\varphi'$ with $k + 1$ operators, we have

$$\xi \models \varphi' \iff h(\xi) \models \varphi'.$$

$\varphi'$ has two possible forms:

1. $\varphi' \equiv \neg(\varphi_0)$
2. $\varphi' \equiv \varphi_1 \land \varphi_2$
For each possible form, we perform the induction assumption on the subformulas of $\varphi'$:

1. $\varphi' \equiv \neg(\varphi_0)$

   $h(\xi) \models \varphi' \iff$
   $h(\xi) \models \neg(\varphi_0) \iff$
   $h(\xi) \nmid \varphi_0 \iff$ (induction assumption)
   $\xi \nmid \varphi_0 \iff$
   $\xi \models \neg(\varphi_0) \iff$
   $\xi \models \varphi'$

2. $\varphi' \equiv \varphi_1 \land \varphi_2$

   $h(\xi) \models \varphi' \iff$
   $h(\xi) \models \varphi_1 \land \varphi_2 \iff$
   $h(\xi) \models \varphi_1 \land h(\xi) \models \varphi_2 \iff$ (induction assumption)
   $\xi \models \varphi_1 \land \xi \models \varphi_2 \iff$
   $\xi \models \varphi'$

Lemma 8.1.2 asserts that applying a constant respecting automorphism on the value of term under a valuation, is the same as computing the value of the term under the composition of the automorphism and the valuation. This assertion is required for proving that a constant respecting automorphism preserves the structure of a SMM which is the semantics of a SMM.

**Lemma 8.1.2** If $h$ is a constant respecting automorphism, then

$$h([t]_{\xi}) = [t]_{h(\xi)}$$

where $t$ is a term such that $t \in C \cup V$, and $\xi$ is a valuation of $V$.

**Proof** There are two possible cases:

1. $t \triangleq c \in C$:

   $$h([t]_{\xi}) = h([c]_{\xi})$$
   $$= h(\gamma(c)) \ (h \text{ is constant respecting.})$$
   $$= \gamma(c)$$
   $$= [c]_{h(\xi)}$$
   $$= [t]_{h(\xi)}$$
2. \( t \triangleq v \in V \)

\[
\begin{align*}
    h([t]_{\xi}) &= h([v]_{\xi}) \\
    &= h(\xi(v)) \\
    &= h(\xi)(v) \\
    &= [v] \cdot h(\xi) \\
    &= [t] \cdot h(\xi)
\end{align*}
\]

Lemma 8.1.3 states that applying a constant preserving automorphism maintains the structure of a SMM which represents the semantics of a SMM. Applying a constant preserving automorphism on a SMM means applying the automorphism on the states of the SMM, that is for each state applying the automorphism on the valuation represents that state, as well as applying the automorphism on the value of the constants and terms appearing in the transitions.

**Lemma 8.1.3** Let \( \mathcal{M} \) be a SMM. If \( h \) is a constant respecting automorphism, then for interface automaton \( \mathcal{I} = \llbracket \mathcal{M} \rrbracket \), we have

\[
q \xrightarrow{\varepsilon_I(d_1,\ldots,d_k)} q' \Rightarrow h(q) \xrightarrow{\varepsilon_I(h(d_1),\ldots,h(d_k))} h(q'),
\]

and

\[
q' \xrightarrow{\varepsilon_O(d'_1,\ldots,d'_{\ell})} q'' \Rightarrow h(q') \xrightarrow{\varepsilon_O(h(d'_1),\ldots,h(d'_{\ell}))} h(q''),
\]

where \( q, q' \) and \( q'' \) are states of \( \mathcal{I} \) and have the form of \((l, \xi), (l', \xi', \varepsilon_O(d'_1,\ldots,d'_{\ell}))\) and \((l', \xi')\), respectively, and \( h(q) = (l, h(\xi)), h(q') = (l', h(\xi'), \varepsilon_O(h(d'_1),\ldots,h(d'_{\ell})))\) and \( h(q'') = (l', h(\xi'))\).

**Proof** Suppose \( q \xrightarrow{\varepsilon_I(d_1,\ldots,d_k)} q' \), then there exists a transition

\[
\langle l, \varepsilon_I(p_1,\ldots,p_k), g, \rho, \varepsilon_O(u_1,\ldots,u_\ell), l' \rangle
\]

of \( \mathcal{M} \) and a valuation \( \iota \) of parameters of \( \{p_1,\ldots,p_k\} \) such that if \( \iota(p_i) = d_i \) for \( 1 \leq i \leq k \),

\[
\begin{align*}
    \xi \cup \iota &\models g \\
    \xi' &\equiv (\xi \cup \gamma \cup \iota) \circ \rho
\end{align*}
\]

If we apply a constant respecting automorphism \( h \), we have:

\[
\iota(p_i) = d_i \Rightarrow h \circ \iota(p_i) = h(\iota)(p_i) = h(d_i) \text{ for } 1 \leq i \leq k \quad (8.1)
\]

\[
\begin{align*}
    \xi \cup \iota &\models g \Rightarrow (\text{lemma 8.1.1}) \\
    h \circ (\xi \cup \iota) &\models g \Rightarrow h(\xi) \cup h(\iota) \models g \\
    h(\xi) \cup h(\iota) &\models g \text{ (8.2)}
\end{align*}
\]
and

\[ \xi' \equiv (\xi \cup \gamma \cup \iota) \circ \rho \Rightarrow \\
\mathcal{h}(\xi') \equiv \mathcal{h} \circ (\xi \cup \gamma \cup \iota) \circ \rho \Rightarrow \\
\mathcal{h}(\xi') \equiv \mathcal{h}(\xi) \cup \gamma \cup \mathcal{h}(\iota) \circ \rho \Rightarrow \\
\mathcal{h}(\xi') \equiv (\mathcal{h}(\xi) \cup \gamma \cup \mathcal{h}(\iota)) \circ \rho \tag{8.3} \]

For \(1 \leq i \leq \ell\),

\[ [u_i]_{\xi'} = d_i' \Rightarrow \\
\mathcal{h}([u_i]_{\xi'}) = \mathcal{h}(d_i') \Rightarrow \text{(lemma 8.1.2)} \tag{8.4} \]

From (8.1), (8.2), (8.3) and (8.4), we conclude

\[ \forall i \leq k, \mathcal{h}(\iota)(p_i) = \mathcal{h}(d_i) \quad \mathcal{h}(\xi) \cup \mathcal{h}(\iota) \models g \\
\mathcal{h}(\xi') = (\mathcal{h}(\xi) \cup \gamma \cup \mathcal{h}(\iota)) \circ \rho \\
\forall i \leq \ell, [u_i]_{\mathcal{h}(\xi')} = \mathcal{h}(d_i') \\
(l, \mathcal{h}(\xi)) \xrightarrow{\varepsilon_1(h(d_1), \ldots, h(d_k))} (l', \mathcal{h}(\xi'), \varepsilon_\mathcal{O}(h(d'_1), \ldots, h(d'_\ell))) \\
\mathcal{h}(q) \xrightarrow{\varepsilon_1(h(d_1), \ldots, h(d_k))} \mathcal{h}(q') \]

Furthermore, there exists state \(q'' = (l', \xi')\) of \(\mathcal{M}\) such that

\[ q' \xrightarrow{\varepsilon_\mathcal{O}(d'_1, \ldots, d'_\ell)} q'' \]

and \(\mathcal{h}(q'') = (l', \mathcal{h}(\xi'))\). Hence,

\[ \mathcal{h}(q') \xrightarrow{\varepsilon_\mathcal{O}(h(d_1), \ldots, h(d_k))} \mathcal{h}(q''). \]

Theorem 8.1.1 is the heart of this section which proves that for bisimilarity checking of two SMM’s with infinite domains, it is enough to substitute the domains by finite subsets of the domains and do the checking.

**Definition** For each SMM \(S = \langle \Sigma, V, L, l_0, \Gamma \rangle\), we define \(S^n\) to be the SMM obtained from \(S\) by replacing the types of the variables with \(\{0, \ldots, n - 1\} \cup \{\perp\}\). Furthermore, we define \(\text{Val}^n(V)\) to be the set of all valuations of \(V\) over \(\{0, \ldots, n - 1\} \cup \{\perp\}\).

**Theorem 8.1.4** Let \(S_1 = \langle \Sigma_1, V_1, L_1, l_0^1, \Gamma_1 \rangle\) and \(S_2 = \langle \Sigma_2, V_2, L_2, l_0^2, \Gamma_2 \rangle\) be SMM’s with \(\Sigma_1 = \Sigma_2\). Let \(n_0\) be large enough (larger than the number of variables of \(S_1\) and \(S_2 + \text{sum of the number of parameters + number of constants}\) \((n_0\) must be larger than the value of the largest constant.) and assume \(S_1^{n_0} \equiv S_2^{n_0}\), that is \(S_1^{n_0}\) and \(S_2^{n_0}\) are bisimilar.

Then, \(S_1 \equiv S_2\).
Proof Let $R \subseteq (L_1 \times \text{Val}^{n_0}(V_1) \cup L_1 \times \text{Val}^{n_0}(V_1) \times O) \times (L_2 \times \text{Val}^{n_0}(V_2) \cup L_2 \times \text{Val}^{n_0}(V_2) \times O)$ be a bisimulation from $[S_1^{n_0}]$ to $[S_2^{n_0}]$.

Define

$$R' = \{(s_1, s_2) \in (L_1 \times \text{Val}(V_1) \cup L_1 \times \text{Val}(V_1) \times O) \times (L_2 \times \text{Val}(V_2) \cup L_2 \times \text{Val}(V_2) \times O) \mid \exists \text{ a constant respecting automorphism } h : (h(s_1), h(s_2)) \in R\}.$$  

We claim that $R'$ is a bisimulation from $[S_1]$ to $[S_2]$.

Since the initial states of $S_1$ and $S_2$ are also the initial states of $S_1^{n_0}$ and $S_2^{n_0}$, and they are related by $R$, they are also related by $R'$ (take the identity function as automorphism).

![Figure 8.3: Bisimulation between finite automata versus infinite automata](image)

Suppose $q_1$ and $q'_1$ be states of $[S_1]$ such that $q_1 \xrightarrow{\varepsilon_I(d_1, \ldots, d_k)} q'_1$, and $q_2$ be a state of $[S_2]$ such that $q_1 R' q_2$, Then

- There exists a constant respecting automorphism $h$ such that
  $$(h(q_1), h(q_2)) \in R.$$  

- There is a transition
  $$\langle l, \varepsilon_I(p_1, \ldots, p_k), g, g, \varepsilon_O(u_1, \ldots, u_l), l' \rangle$$
such that \( q = (l, \xi) \) and \( q' = (l', \xi', \varepsilon_O(d'_1, \ldots, d'_\ell)) \), where \( d'_i = [u_i]_{\xi'} \) for \( 1 \leq i \leq \ell \), and if \( \iota \) is a valuation with \( \iota(p_i) = d_i \) (for \( 1 \leq i \leq k \)),

\[
\begin{align*}
\xi \cup \iota & \models g \\
\xi' & \equiv (\xi \cup \gamma \cup \iota) \circ g
\end{align*}
\]

- There exists a constant respecting automorphism \( h' \) such that
  \[
  \begin{align*}
  h'(q_1) &= h(q_1) \\
  h'(q_2) &= h(q_2) \\
  h'(d_i) &\leq n_0 \text{ for } 1 \leq i \leq k
  \end{align*}
  \]
  (Follows since \( n_0 \) is greater than \( |V_1| + |V_2| + |C| + |P| \))

- By lemma 8.1.3
  \[
  h'(q_1) \xrightarrow{\varepsilon_{I(h'(d_1), \ldots, h'(d_k))}} h'(q'_1).
  \]
  Since \((h'(q_1), h'(q_2)) \in R\), and \( R \) is a bismulation, there exists a state \( r \) such that
  \[
  h'(q_2) \xrightarrow{\varepsilon_{I(h'(d_1), \ldots, h'(d_k))}} r
  \]
  and
  \[
  (h(q'_1), r) \in R.
  \]
  By the lemma 8.1.3 since \( h'^{-1} \) is a constant respecting automorphism such that,
  \[
  q_2 \xrightarrow{\varepsilon_{I(d_1, \ldots, d_k)}} h'^{-1}(r).
  \]
  Observe
  \[
  (q'_1, h'^{-1}(r)) \in R'.
  \]

This construction is depicted in figure 8.3.

Using lemma 8.1.3 case of \( \varepsilon_O \) is similar.

### 8.1.2 Abstraction Table

For each event signature, we define a family of symbolic abstractions, parametrized by what is called an abstraction table. For each parameter \( p \), an abstraction table contains a list of variables and constants. If \( v \) occurs in the list for \( p \) then, intuitively, this means that for the future behavior of the SUT it may be relevant whether \( p \) equals \( v \) or not.

**Abstraction table** Let \( \Sigma = \langle T_I, T_O \rangle \) be an event signature, let \( P \) be the set of parameters that occur in \( T_I \), and let \( U \) be the set of parameters that occur in \( T_O \).

Let \( v_p^f \) and \( v_p^l \) be fresh variables with \( \text{type}(v_p^f) = \text{type}(v_p^l) = \text{type}(p) \cup \{ \bot \} \), and let \( V^f = \{ v_p^f \mid p \in P \} \) and \( V^l = \{ v_p^l \mid p \in P \} \). An abstraction table for \( \Sigma \) is a
function $F : P \cup U \rightarrow (V^f \cup V^l \cup C)^*$, such that, for each $p \in P \cup U$, all elements of sequence $F(p)$ are distinct, and, for each $p \in U$, $F(p)$ lists all the elements of $V^f \cup V^l \cup C$.

**Mapper induced by abstraction table** Let $\Sigma = \langle T_I, T_O \rangle$ be a signature and let $F$ be an abstraction table for $\Sigma$. Let $P$ be the set of parameters in $T_I$ and let $U$ be the set of parameters in $T_O$. Then the mapper $A^F_\Sigma = \langle \mathcal{I}, X, Y, \Upsilon \rangle$ with $\mathcal{I} = \langle I, O^\delta, R, r^0, \rightarrow \rangle$ is defined as follows:

- Let, for $p \in P \cup U$, $p'$ be a fresh variable with $\text{type}(p') = \{0, \ldots, |F(p)| - 1\} \cup \{\bot\}$. Let $T_X = \{\varepsilon(p_1', \ldots, p_k') \mid \varepsilon(p_1, \ldots, p_k) \in T_I \}$ and $T_Y = \{\varepsilon(p_1', \ldots, p_l') \mid \varepsilon(p_1, \ldots, p_l) \in T_O \}$. Then $I = [T_I], O = [T_O], X = [T_X], \text{and } Y = [T_Y]$.

- $R = \text{Val}(V^f \cup V^l)$ and $r^0(v^f_p) = r^0(v^l_p) = \bot$, for all $p \in P$.

- $\rightarrow$ and $\Upsilon$ are defined as follows, for all $r \in R$,
  1. $r \overset{\delta}{\rightarrow} r$ and $\Upsilon_r(\delta) = \delta$.
  2. Let $o = \varepsilon_O(d_1, \ldots, d_k)$ and let $\varepsilon_O(q_1, \ldots, q_k) \in T_O$. Then $r \overset{o}{\rightarrow} r$ and $\Upsilon_r(o) = \varepsilon_O(d_1', \ldots, d_k')$, where, for $1 \leq j \leq k$, $d_j'$ is the smallest index $m$ such that $[F(q_j)_m]_r = d_j$, or $d_j' = \bot$ if there is no such index.
  3. Let $i = \varepsilon_I(d_1, \ldots, d_k)$ and let $\varepsilon_I(p_1, \ldots, p_k) \in T_I$. Let $r_0 = r$ and, for $1 \leq j \leq k$,

$$r_j = \begin{cases} r_{j-1}[d_j/v^f_p][d_j/v^l_p] & \text{if } r_{j-1}(v^f_p) = \bot \\ r_{j-1}[d_j/v^l_p] & \text{otherwise} \end{cases} \quad (8.5)$$

Then $r \overset{i}{\rightarrow} r_k$ and $\Upsilon_r(i) = \varepsilon_I(d_1', \ldots, d_k')$, where, for $1 \leq j \leq k$, $d_j'$ is the smallest index $m$ such that $[F(p_j)_m]_{r_{j-1}} = d_j$, or $d_j' = \bot$ if there is no such index.

Strictly speaking, the mappers $A^F_\Sigma$ introduced above are not output-predicting. In fact, in each state $r$ of the mapper there are infinitely many concrete outputs that are mapped to the abstract output $\bot$. However, SUTs whose behavior can be described by scalarset Mealy machines have a remarkable property: the only possible values for output parameters are constants and values of previously received inputs. As a result, the mapper will never send an abstract output with a parameter $\bot$ to the learner. This in turn implies that in the behavior-deterministic hypothesis $\mathcal{H}$ generated by the learner $\bot$ will not occur as an output parameter. Since $A^F_\Sigma$ is output-predicting for all the other outputs, it follows that the concretization $\gamma_{A^F_\Sigma}(\mathcal{H})$ is behavior-deterministic.

**Theorem 8.1.5** Let $\mathcal{M} = \langle \Sigma, V, L, l_0, \Gamma \rangle$ be a SMM that only records the first and last occurrence of parameters. Let $F$ be an abstraction table for $\Sigma$. Then $\alpha_{A^F_\Sigma([\mathcal{M}])}$ is finitary.
Proof (sketch) Let $S$ be the relation that deems two states $s_1, s_2$ of $\alpha_{A_{\Sigma}}([M])$ equivalent iff there exists a constant respecting automorphism $h$ with $h(s_1) = s_2$. Then, clearly, $S$ is an equivalence relation. We claim that $S$ is a bisimulation on $\alpha_{A_{\Sigma}}([M])$. Suppose that $(s_1, s_2) \in S$ with $s_1 = (q_1, r_1)$, $s_2 = (q_2, r_2)$, and $h$ a constant respecting automorphism that maps $s_1$ to $s_2$. Suppose further that $(q_1, r_1) \overset{a}{\rightarrow} (q_1', r_1')$. Then, by definition of the abstraction operator, there exists an $a$ such that $q_1 \overset{a}{\rightarrow} q_1'$, $r_1 \overset{a}{\rightarrow} r_1'$, and $\Upsilon_r(a) = z$. By Lemma 8.1.3 $h(q_1) \overset{h(a)}{\rightarrow} h(q_1')$. Moreover, by the definition of mapper $A_{\Sigma}$, $h(r_1) \overset{h(a)}{\rightarrow} h(r_1')$. Also by definition of mapper $A_{\Sigma}$, $\Upsilon_r(a) = \Upsilon_h(h(a))$. We conclude $(h(q_1), h(r_1)) \overset{h(a)}{\rightarrow} (h(q_1'), h(r_1'))$, that is, $h(s_1) \overset{h(a)}{\rightarrow} h(s_2)$. Hence, $S$ is a bisimulation, as claimed.

To each state $s = ((l, \xi), r)$ or $s = ((l, \xi, o), r)$ of $\alpha_{A_{\Sigma}}([M])$ we associate a partial equivalence relation $\text{PER}(s)$ on $V \cup V^f \cup V^l \cup C$ which puts variables or constants in the same equivalence class whenever they evaluate to the same non-$\bot$ value:

$$\text{PER}(s) = \{t' \in V \cup V^f \cup V^l \cup C \mid [t']_{\xi \cup r} = [t]_{\xi \cup r} \neq \bot \} \mid t \in V \cup V^f \cup V^l \cup C\}.$$

The reader may check that $s' = h(s)$ implies that $\text{PER}(s) = \text{PER}(s')$. Since in all (reachable) states of the form $(l, \xi, \epsilon_O(d'_1, \ldots, d'_l))$ the values of the output parameters are determined by the event term $\epsilon_O(u_1, \ldots, u_l)$ and valuation $\xi$, it follows that bisimulation $S$ has finitely many equivalence classes. Hence the induced quotient structure, which is behaviorally equivalent to $\alpha_{A_{\Sigma}}([M])$, is a finite interface automaton and $\alpha_{A_{\Sigma}}([M])$ is finitary.

Theorem 8.1.6 Let $M = \langle \Sigma, V, L, I_0, \Gamma \rangle$ be a deterministic SMM that only records the first and last occurrence of parameters. Let $\text{Full}(\Sigma)$ be the abstraction table $F$ for $\Sigma$ in which, for each $p$, $F(p)$ has maximal length. Then $\alpha_{A_{\Sigma}}([\text{Full}(\Sigma)])([M])$ is behavior deterministic.

Proof (sketch) Suppose a state $s$ of $\alpha_{A_{\Sigma}}([\text{Full}(\Sigma)])([M])$ has two distinct outgoing transitions. Then $s$ must be of the form $s = ((l, \xi), r)$. Suppose $s$ has outgoing transitions $s \overset{x}{\rightarrow} s'$ and $s \overset{x}{\rightarrow} s''$. Then $s'$ and $s''$ can only be different because in $x$ some abstract parameter has value $\bot$, leading to different concrete values in $s'$ and $s''$. But since these concrete values are fresh, there exists a constant preserving automorphism $h$ such that $h(s') = s''$. Hence, by the proof of the previous theorem, $s'$ and $s''$ are bisimilar, as required.

Definition Let $I$ and $O$ be disjoint sets of input and output actions. Then $\text{Mealy}(I, O)$ is the IA $\langle I, O, \{m_0, m_1\}, m_0, \{(m_0, i, m_1) \mid i \in I\} \cup \{(m_1, o, m_0) \mid o \in O\}\}.$

Lemma 8.1.7 Let $\Sigma = \langle T_I, T_O \rangle$ be an event signature, let $I = [T_I]$ and $O = [T_O]$, let $M$ be a SMM over $\Sigma$, let $F$ be an abstraction table for $\Sigma$, and let $\mathcal{P} = \text{Mealy}(I, O)$. Then $A_{\Sigma}$ and $\mathcal{P}$ are compatible, $\text{ioco } \mathcal{P}$ and $\gamma_{A_{\Sigma}}(\alpha_{A_{\Sigma}}(\mathcal{P})) \preceq P$.
We have now solved the problem of learning deterministic symbolic Mealy machines that only record the first and last occurrence of parameters, at least in theory! By Theorem 8.1.6 and Lemma 8.1.7, we can apply the approach described in Section 7.3 if we use mapper $A_{\Sigma}^{\text{Full}}$ and learning purpose $\text{Mealy}(I,O)$. By Theorem 8.1.5, we know that $I = \alpha_{A_{\Sigma}^{\text{Full}}}(\|M\|)$ is finite state and so if we use our mapper component in combination with any tool that is able to learn finite interface automata, the learning procedure will always terminate. The only problem is that in practice $I$ will be much too large. For instance, if we have an event signature with just 10 parameters including an event type with 4 parameters, then the number of actions of $I$ will be at least $21^4 \approx 2.10^5$, which is far beyond what state-of-the-art learning tools can handle.

### 8.2 Counterexample-Guided Abstraction Refinement

In order to avoid the practical problems that arise with the abstraction table $\text{Full}(\Sigma)$, we take an approach based on counterexample-guided abstraction. We start with the simplest mapper, which is induced by the abstraction table $F$ with $F(p) = \epsilon$, for all $p \in P$, and only refine the abstraction (i.e., add an element to the table) when we have to. Our CEGAR procedure starts with the simplest of these abstractions (essentially the empty table). If using this abstraction we find a correct hypothesis we are done. Otherwise, we refine the abstraction by adding an entry to our table. Since there are only finitely many possible abstractions and we know that the abstraction that corresponds to the full table is sound, our CEGAR approach will always terminate (at least in theory).

The reason why refinement steps may be necessary is that $\alpha_{A_{\Sigma}^F}(\|M\|)$ may exhibit nondeterministic behavior. During the construction of a hypothesis we will not observe nondeterministic behavior, even when table $F$ is not full: due to our choice of the concretization function $\upsilon$, which always chooses fresh values, the mapper induced by $F$ will behave exactly as the mapper induced by $\text{Full}(\Sigma)$, except that the set of abstract actions is smaller. Only if the learner has formulated a hypothesis $H$, the mapper has forwarded this hypothesis to the teacher, and the teacher responds with no with a counterexample $\sigma_0$ we may face a problem: the counterexample may be due to the fact that $H$ is incorrect, but it may also be due to the fact that $\alpha_{A_{\Sigma}^F}(\|M\|)$ is not behavior-deterministic. In order to figure out the nature of the counterexample, we first construct the unique execution of $A_{\Sigma}^F$ with trace $\sigma_0$. Then we assign a color to each occurrence of a parameter value in this execution:

**Definition** Let $r \xrightarrow{i} r'$ be a transition of $A_{\Sigma}^F$ with $i = \epsilon_I(d_1, \ldots, d_k)$ and let $\epsilon_I(p_1, \ldots, p_k) \in T_I$. Let $\Upsilon,(i) = \epsilon_I(d'_1, \ldots, d'_k)$. Then we say that a value $d_j$ is **green** if $d'_j \neq \perp$. Value $d_j$ is **black** if $d'_j = \perp$ and $d_j$ equals the value of some constant
or occurs in the codomain of state $r_{j-1}$ (where $r_{j-1}$ is defined as in equation (8.5) above). Value $d_j$ is red if it is neither green nor black.

Intuitively, a value of an input parameter $p$ is green if it equals a value of a previous parameter or constant that is listed in the abstraction table, a value is black if it equals a previous value that is not listed in the abstraction table, and a value is red if it is fresh. The mapper now does a new experiment on the SUT in which all the black values of input parameters in the trace are converted into fresh “red” values. If, after abstraction, the trace of the original counterexample and the outcome of the new experiment are the same, then hypothesis $H$ is incorrect and we forward the abstract counterexample to the learner. But if they are different then we may conclude that $\alpha_{\mathcal{A}_E}(\mathcal{T})$ is not behavior-deterministic. In this case, the run for the original counterexample contains at least one black value, which determines a new entry that we can add to the abstraction table.

### 8.2.1 Implementation details

In Tomte, LearnLib is employed to do the automata inference. The Tomte tool, which implements the mapper component, sits in between LearnLib and the SUT.

Rather than using a separate model-based testing tool to test the correctness of hypotheses, we used the ability of LearnLib to generate test sequences. Once a hypothesis has been constructed, LearnLib can generate long test sequences to check if the hypothesis is correct, using a library of well-known test generation algorithms. Depending on whether LearnLib is constructing a hypothesis or is testing one, Tomte adjusts its behavior. During the learning phase, Tomte selects fresh concrete values whenever it receives an abstract action with parameter value $\perp$. During the testing phase, instead of selecting fresh concrete values for an abstract parameter value $\perp$, random values are selected. In this way, we ensure that the full concretization $\gamma_{\mathcal{A}}(\mathcal{H})$ is explored. By tuning the probability distribution used by Tomte for selecting random values, we obtained an efficient and reliable way to test the correctness of $\gamma_{\mathcal{A}}(\mathcal{H})$: in none of our experiments we suffered from false positives.

Once Tomte has discovered that the current abstraction is too coarse, it must select a black valued parameter and “make it green” by adding it as a new entry to the abstraction table. This is done via a series of experiments in which black values are converted one by one into fresh values, until a change in observable output is detected.

The algorithm for finding this new abstraction is outlined in Algorithm 1. Here, for an occurrence $b$, param$(b)$ gives the corresponding formal parameter, source$(b)$ gives the previous occurrence $b'$ which, according to the execution of $\mathcal{A}_{E_0}$, is the source of the value of $b$, and variable$(b)$ gives the variable in which the value of $b$ is stored in the execution of $\mathcal{A}_{E_0}$. To keep the presentation simple, the set of constants here is assumed to be empty. A series of experiments in which black occurrences and their sources are converted one by one into fresh values (lines 4
and 5) is run on the SUT (lines 6 and 7), until a change in observable output is detected (lines 8 and 9). When the new abstraction entry has been added to the abstraction table, the learner is restarted with the new abstract alphabet.

**Algorithm 1** Abstraction refinement

**Input:** Counterexample \( c = i_1 \cdots i_n \)

**Output:** Pair \((p, v)\) with \(v\) new entry for \(F(p)\) in abstraction table

1: Add black occurrences of values in \(c\) to queue \(Q\)
2: while abstraction not found do
3: \( b := \text{dequeued black value occurrence from } Q \)
4: \( c' := c, \) where \(b\) is set to a fresh value
5: \( c'' := c, \) where \(\text{source}(b)\) is set to a fresh value
6: \( o' := \text{output from running } c' \text{ on SUT} \)
7: \( o'' := \text{output from running } c'' \text{ on SUT} \)
8: if \(o'\) and \(o''\) are different from output of \(c\) then
9: \( \text{return } (\text{param}(b), \text{variable}(\text{source}(b))) \)
10: end if
11: end while

### 8.3 Experiments

In this section the operation of Tomte is illustrated by means of the Session Initiation Protocol (SIP) \[85\]. SIP is an application layer protocol for controlling multimedia communication sessions, such as voice and video calls over Internet Protocol (IP). The protocol can be used for creating, modifying and terminating two-party (unicast) or multiparty (multicast) sessions. Sessions may consist of one or several media streams.

In this section, SIP protocol is referred as presented in \[1\], and Tomte is used to construct the abstraction for inferring the behavior of the SIP Server entity when setting up connections with a SIP Client. The input messages from the SIP Client to the SIP Server are represented as \(\text{Method(From, To, Contact, CallId, CSeq, Via)}\), where

- **Method** defines the type of request, either INVITE, PRACK, or ACK,
- **From** and **To** are addresses of the originator and receiver of the request,
- **CallId** is a unique session identifier,
- **CSeq** is a sequence number that orders transactions in a session,
- **Contact** is the address where the Client wants to receive input messages, and
8.3. Experiments

- Via indicates the transport path that is used for the transaction.

The output messages from the SIP Server to the SIP Client are represented as $\text{StatusCode}(\text{From}, \text{To}, \text{CallId}, \text{CSeq}, \text{Contact}, \text{Via})$, where $\text{StatusCode}$ is a three digit status code that indicates the outcome of a previous request from the Client, and the other parameters are as for a input message.

Tomte expects the input messages to begin with an “I”, and the output messages to begin with an “O”. Initially, no abstraction for the input is defined in the learner, which means all parameter values are $\bot$. As a result every parameter in every input action is treated in the same way and the mapper selects a fresh concrete value, e.g. the abstract input trace $\text{IINVITE}(\bot, \bot, \bot), \text{IACK}(\bot, \bot, \bot), \text{IPRACK}(\bot, \bot, \bot)$ is translated to the concrete trace $\text{IINVITE}(1, 2, 3), \text{IACK}(4, 5, 6), \text{IPRACK}(7, 8, 9), \text{IPRACK}(10, 11, 12)$. In the learning phase queries with distinct parameter values are sent to the SUT, so that the learner constructs the abstract Mealy machine shown in Figure 8.4. In the testing phase parameter values may be duplicated, which may lead to non-deterministic behavior. The test trace $\text{IINVITE}, \text{IACK}, \text{IPRACK}, \text{IPRACK}$ in Figure 8.5 leads to an 0200 output that is not foreseen by the hypothesis, which produces an 0481.

Rerunning the trace with distinct values as before leads to an 0481 output. Thus, to resolve this problem, the input abstraction must be refined. Therefore, we identify the green and black values in the trace and try to remove black values. The algorithm first successfully removes black value No. 1 by replacing the nine in the IPRACK input with a fresh value and observing the same output as before. However, removing black edge No. 2 changes the final outcome of the trace to an 0481 output. As a result, we need to refine the input abstraction by adding an equality check between the first parameter of the last IINVITE message and the first parameter of an IPRACK message to every IPRACK input. Apart from refining the input alphabet, every concrete output parameter value is abstracted to either a constant or a previous occurrence of a parameter. The abstract value is the index of the corresponding entry in the abstraction table. After every input abstraction refinement, the learning process needs to be restarted. We proceed until the learner finishes the inference process without getting interrupted by a non-deterministic output.

Besides SIP protocol, Tomte was successfully used to learn the following models:

- Alternating bit protocol (ABP) [13], see figure 8.1.
- Biometric Passport [2]
- A simple Login System
- Farmer-Wolf-Goat-Cabbage Puzzle
- Palindrome/Repeated Digit Checker

Table 8.1 gives an overview of the systems learned, with the number of input refinement steps, total learning and testing queries, number of states of the learned abstract model, and time needed for learning and testing (in seconds). All models inferred have been checked to be bisimilar to their SUT. For this purpose the learned model is combined with the abstraction and the CADP tool set, [http://www.inrialpes.fr/vasy/cadp/](http://www.inrialpes.fr/vasy/cadp/) is used for equivalence checking.
Figure 8.4: Hypothesis of SIP protocol

Figure 8.5: Non-determinism in SIP protocol
### Table 8.1: Learning statistics

<table>
<thead>
<tr>
<th>System under test</th>
<th>Input refinements</th>
<th>Learning/Testing queries</th>
<th>States</th>
<th>Learning/Testing time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternating Bit Protocol - Sender</td>
<td>1</td>
<td>193/3001</td>
<td>7</td>
<td>1.3s/104.9s</td>
</tr>
<tr>
<td>Alternating Bit Protocol - Receiver</td>
<td>2</td>
<td>145/3002</td>
<td>4</td>
<td>0.9s/134.5s</td>
</tr>
<tr>
<td>Alternating Bit Protocol - Channel</td>
<td>0</td>
<td>31/3000</td>
<td>2</td>
<td>0.3s/107.5s</td>
</tr>
<tr>
<td>Biometric Passport</td>
<td>3</td>
<td>2199/3582</td>
<td>5</td>
<td>7.7s/94.5s</td>
</tr>
<tr>
<td>Session Initiation Protocol</td>
<td>3</td>
<td>1755/3402</td>
<td>13</td>
<td>8.3s/35.9s</td>
</tr>
<tr>
<td>Login</td>
<td>3</td>
<td>639/3063</td>
<td>5</td>
<td>2.0s/56.8s</td>
</tr>
<tr>
<td>Farmer-Wolf-Goat-Cabbage Puzzle</td>
<td>4</td>
<td>699/3467</td>
<td>10</td>
<td>4.4s/121.8s</td>
</tr>
<tr>
<td>Palindrome/Repdigit Checker</td>
<td>11</td>
<td>3461/3293</td>
<td>1</td>
<td>10.3s/256.4s</td>
</tr>
</tbody>
</table>
8.4 Conclusion

Tomte implements an algorithm which uses the technique of counterexample-guided abstraction refinement: initially, the algorithm starts with a very coarse abstraction $\mathcal{A}$, which is subsequently refined if it turns out that $\alpha_{\mathcal{A}}(T)$ is not behavior-deterministic. The idea to use CEGAR for learning state machines has been explored recently by Howar at al. [47] who developed and implemented a CEGAR procedure for the special case in which the abstraction is static and does not depend on the history.

Tomte is able to construct mappers for a restricted class of extended transition systems, called scalarset automata. In scalarset automata, one can test for equality of data parameters, but no operations on data are allowed. Scalarsets also motivated the recent work of [23], which establishes a canonical form for a variation of scalarset automata.

Even though the class of systems to which our approach currently applies is limited, the fact that we are able to learn models of systems with data fully automatically is a major step towards a practically useful technology for automatic learning of models of software components.

Currently, Tomte can learn SUTs that may only remember the last and first occurrence of a parameter. Apparently, it is easy to dispose this restriction. Furthermore, the CEGAR based approach of this chapter can be further extended to systems that may apply simple or known operations on data, using technology for automatic detection of likely invariants, such as Daikon [34].
Chapter 9

Conclusion of Part Two

During the last two decades, important developments have taken place in the area of automata learning, see e.g. [6, 84, 83, 15, 18, 64, 46, 71]. History dependent abstraction operators are the key for scaling existing methods for active learning of automata to realistic applications. A major challenge is the development of algorithms for the automatic construction of abstraction mappers: the availability of such algorithms will boost the applicability of automata learning technology. In paper [47], a method is presented that is able to automatically construct certain state-free mappers. Aarts, Jonsson & Uijen [1] have proposed a framework for history dependent abstraction operators. Using this framework they succeeded to automatically infer models of several realistic software components with large state spaces, including fragments of the TCP and SIP protocols. Despite this success, the approach of Aarts et al suffers from limitations that hinder its applicability in practice.

Chapter 7 of this dissertation provided several generalizations of the framework of [1], leading to a general theory of history dependent abstractions for learning interface automata. This theory offers four important improvement to the theory of history dependent abstraction operators:

**From Mealy machines to interface automata** The approach of [1] is based on Mealy machines, in which each input induces exactly one output. In practice, however, inputs and outputs often do not alternate: a single input may sometimes be followed by a series of outputs, sometimes by no output at all, etc. For this reason, the approach of chapter 7 is based on interface automata [32], which have separate input and output transitions, rather than the more restricted Mealy machines.

**Learning purposes** In practice, it is often neither feasible nor necessary to learn a model for the complete behavior of the SUT. Typically, it is better to concentrate the learning efforts on certain parts of the state space. This is achieved using the
concept of a learning purpose \cite{3} (known as test purpose within model-based testing theory \cite{57, 51, 99}), which allows one to restrict the learning process to relevant interaction patterns only. In the theory of chapter \cite{7} the concept of a mapper component of \cite{1} is integrated with the concept of a learning purpose of \cite{3}. This integration is nontrivial and constitutes one of the main technical contributions of this thesis.

**Forgetful abstractions** The main result of \cite{1} only applies to abstractions that are output predicting. This means that no information gets lost and we infer a model that is behaviorally equivalent to the model of the teacher: $\mathcal{M} \approx \gamma_A(\mathcal{H})$. In order to deal with the complexity of real systems, we need to support also forgetful abstractions that over-approximate the behavior of the teacher. For this reason, in this thesis, the notion of equivalence $\approx$ is replaced by the $\text{ioco}$ relation, which is one of the main notions of conformance in model-based black-box testing \cite{92, 93} and closely related to the alternating simulations of \cite{5}.

**Handling equivalence queries** Active learning algorithms in the style of Angluin \cite{6} alternate two phases. In the first phase a hypothesis is constructed and in the second phase (called an equivalence query by Angluin \cite{6}) the correctness of this hypothesis is checked. In general, no guarantees can be given that the answer to an equivalence query is correct. Tools such as LearnLib, “approximate” equivalence queries via long test sequences, which are computed using some established algorithms for model-based testing of Mealy machines. In the approach of \cite{1}, one needs to answer equivalence queries of the form $\alpha_A(\mathcal{M}) \approx \mathcal{H}$. In order to do this, a long test sequence for $\mathcal{H}$ that is computed by the learner is concretized by the mapper. The resulting output of the SUT is abstracted again by the mapper and sent back to the learner. Only if the resulting output agrees with the output of $\mathcal{H}$ the hypothesis is accepted. This means that the outcome of an equivalence query depends on the choices of the mapper. If, for instance, the mapper always picks the same concrete action for a given abstract action and a given history, then it may occur that the test sequence does not reveal any problem, even though $\alpha_A(\mathcal{M}) \not\approx \mathcal{H}$. Hence the task of generating a good test sequence is divided between the learner and the mapper, with an unclear division of responsibilities. This makes it extremely difficult to establish good coverage measures for equivalence queries. A more sensible approach, which is elaborated in this thesis, is to test whether the concretization $\gamma_A(\mathcal{H})$ is equivalent to $\mathcal{M}$, using state-of-the-art model based testing algorithms for systems with data, and to translate the outcomes of that experiment back to the abstract setting.

The theoretical advances that are described in chapter \cite{7} of this thesis are important to bring automata learning tools and techniques to a level where they can be used routinely in industrial practice.

Chapter \cite{8} of this thesis presented a prototype tool Tomte, which automatically constructs mappers for a restricted class of extended transition systems, using a
counterexample-guided abstraction refinement approach.

The CEGAR technique is used for learning state machines in a recent research of Howar at al [47], who developed and implemented a CEGAR procedure for the special case in which the abstraction is static and does not depend on the execution history. The approach of this thesis, however, is applicable to a much richer class of systems, which for instance includes the SIP protocol and the various components of the Alternating Bit Protocol.
Chapter 10

Epilogue

This thesis was organized in two parts following two different research areas, specifically, verification of wireless sensor networks and automata learning.

Part one covered my research in 2008 and 2009, when I was working in collaboration with the European project Quasimodo to devise a CEGAR-based method for verification of an arbitrary size wireless sensor network provided by Chess. This research led to considerable results, namely establishing the necessary and sufficient conditions, in the form of constraints on the parameters of a fully-connected WSN in order to guarantee its being synchronized, and discovering a flaw in Chess implementation. Despite the original plan, we did not succeed to use CEGAR approach for conquering the state space explosion problem when verifying the Chess synchronization protocol for a general network of arbitrary size.

Part two covered my research in 2010 and 2011 when I collaborated in design and implementation of a CEGAR-based algorithm for automatically learning a limited class of parametric systems, called scalarset symbolic Mealy machines. Furthermore, we provided a solid theoretical foundation for learning interface automata using a large class of abstractions.

In sum, CEGAR-based parametric model checking of WSN synchronization protocols is difficult, if possible at all. Nevertheless, this challenge is worth more research. CEGAR seems to be a powerful method in extending the available approaches to automata learning.
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Samenvatting

Studies op het Gebied van Verificatie van Draadloze Sensornetwerken en het Construeren van Abstracties voor het Leren van Automaten

Ingebouwde systemen zijn computersystemen die zijn ingebouwd in apparaten en de functionaliteit van deze apparaten voor een belangrijk deel bepalen. Voorbeelden zijn te vinden in smartphones, moderne autos, MRI-scanners en in de zelfscanners in supermarkten. Ingebouwde systemen zijn veelal onzichtbaar, maar hebben desondanks een enorme invloed op de manier waarop de moderne mens de fysieke wereld om zich heen waarnemt en er mee interacteert. Voor ingebouwde systemen waarbij de veiligheid van mensen in het geding is, zoals autos en medische apparatuur, is de betrouwbaarheid van evident belang. Maar ook in niet-kritieke toepassingen, zoals spelcomputers en dvd-spelers, kunnen onverwachte fouten desastreus zijn voor de reputatie van een fabrikant en de verkoop van nieuwe apparaten drastisch verminderen.

Bij het ontwerpen van een betrouwbaar ingebouwde systeem moeten we typisch aantonen dat dit systeem nooit in bepaalde gevaarlijke toestanden kan komen. Verificatie en validatie (V&V) zijn hierbij cruciaal. Deze dissertatie benadert V&V vanuit twee gezichtspunten. In het eerste deel beschrijven we de modellering en verificatie van een realistische casus op het gebied van draadloze sensornetwerken. In het tweede deel onderzoeken we een geheel nieuwe techniek waarbij abstractieverfijning gebruikt wordt voor het automatisch leren van modellen van ingebouwde systemen. Deze modellen kunnen dan vervolgens ingezet worden voor V&V.

In het eerste deel van dit proefschrift onderzoeken we de toepasbaarheid van bestaande verificatiemethoden aan de hand van een industriële casus op het gebied van draadloze sensornetwerken die is aangedragen door het Nederlandse bedrijf Chess eT International BV. Een draadloos sensornetwerk (DSN) bestaat uit een verzameling kleine apparaatjes, knopen genaamd, die verbonden zijn via een netwerk en gezamenlijk een taak uitvoeren. Iedere knoop is in staat berekeningen uit te voeren (gebruikmakend van een of meer micro-controllers, processoren of DSP
chips), beschikt over wat geheugen (programma-, data- en/of flashgeheugen), over een RF transceiver (normaliter met een bi-directionele antenne), een stroombron (bijvoorbeeld batterijen of zonnecellen), en verscheidene sensoren en actuatoren. De Chess casus heeft betrekking op de regels (het protocol) dat knopen gebruiken om met elkaar te communiceren. Een effectief protocol voor draadloze sensor-netwerken moet weinig stroom verbruiken, moet voorkomen dat knopen tegelijk berichten versturen (door elkaar heen praten), moet gememoreerd kunnen worden met zo min mogelijk code en geheugen, moet voldoende bandbreedte leveren voor een gegeven applicatie, en moet om kunnen gaan met wisselende radiofrequenties en netwerkomstandigheden. Chess heeft een DSN platform ontwikkeld op basis van een epidemiisch communicatiemodel. Om aan de strikte vereisten ten aanzien van energieverbruik te kunnen voldoen maakt Chess gebruik van een zogenaamd Time Division Multiple Access (TDMA) protocol, waarbij de knopen slechts een fractie van de tijd actief zijn en zich gedurende de rest van de tijd in een slaaptoestand bevinden waarin ze vrijwel geen energie gebruiken. Voor de goede werking van dit protocol is het cruciaal dat alle knopen gelijk actief zijn: het heeft geen zin wanneer een knoop berichten verstuurt wanneer al zijn buren slapen. Dit vereist dat de klokken van aangrenzende knopen (vrijwel) gelijk lopen. In dit proefschrift wordt de model checker Uppaal gebruikt om twee kloksynchronisatiealgoritmen voor het draadloze sensornetwerk van Chess te modelleren en te verifiëren. Met behulp van Uppaal is het ons gelukt om foutscenarios te vinden die tot toestanden leiden waarin het netwerk niet meer gesynchroniseerd is. Middels experimenten met een draadloos sensornetwerk hebben medewerkers van Chess aangetoond dat deze scenarios zich ook daadwerkelijk kunnen voordoen. Op basis van de door ons gevonden foutscenarios introduceren we drie condities waaraan de protocolparameters moeten voldoen om fouten uit te sluiten. Met behulp van bewijstechnieken die gebruik maken van invarianten bewijzen we dat deze condities noodzakelijk en afdoende zijn voor correctheid in het speciale geval van netwerken waarin alle knopen direct met elkaar verbonden zijn. Deze bewijzen zijn gecheckt met behulp van de bewijsassistent Isabelle/HOL.

Het tweede deel van dit proefschrift beschrijft hoe tegenvoorbeeld-gedreven abstractieverfijning gebruikt kan worden om geheel automatisch modellen (toestandsdiagrammen) te leren van computersystemen puur op basis van testen en observaties van het extern waarneembare gedrag. Bestaande programma's voor het actief leren van toestandsautomaten zijn in staat om modellen te leren met maximaal circa 10.000 toestanden. Dit is onvoldoende voor het leren van modellen van realistische softwarecomponenten die, door het gebruik van programmavariabelen en dataparameters in berichten, dikwijls een veel groter aantal toestanden hebben. Abstractie blijkt cruciaal voor het leren van modellen van dergelijke systemen. In praktische toepassingen waarbij leertecnologie gebruikt wordt om modellen te construeren van softwarecomponenten, definieren gebruikers dikwijls handmatig abstracties waarbij een groot aantal concrete berichten worden afgebeeld op een beperkt aantal abstracte berichten. In deze dissertatie wordt een complete theorie
van abstracties voor het leren van toestanddiagrammen gepresenteerd. Er wordt aangetoond dat zulke abstracties volledig automatisch geconstrueerd kunnen worden voor een bepaalde klasse van toestanddiagrammen waarin getest kan worden op gelijkheid van data parameters, maar geen bewerkingen op data toegestaan zijn. Bij de constructie wordt gebruik gemaakt van tegenvoorbeeld-gedreven abstractieverfijning: indien een abstractie te grof is en non-deterministisch gedrag veroorzaakt in het geleerde model, dan wordt deze abstractie automatisch verfijnd. Met behulp van een prototype implementatie van ons algoritme zijn wij er in geslaagd modellen van verschillende realistische software componenten, zoals het biometrische paspoort en het SIP protocol, volledig automatisch te leren.
Summary

Studies on Verification of Wireless Sensor Networks and Abstraction Learning for System Inference

Embedded systems are redefining how we perceive and interact with the physical world. While mission-critical embedded applications raise obvious reliability concerns, unexpected or premature failures in even noncritical applications such as game boxes and portable video players can erode a manufacturer’s reputation and greatly diminish acceptance of new devices. The design of reliable systems requires assuring that the system never moves through a dangerous state, and verification and validation (V & V) is the key. This dissertation approaches V & V of embedded systems from two different perspectives: in the first part, modeling and verification of a real-world case-study provided by the Chess eT International B.V. is described, whereas the second part investigates automata learning (automatic modeling of systems) using abstraction refinement.

In part one, the industrial case-study of Chess on wireless sensor networks is investigated. A wireless sensor network (WSN) is a collection of nodes organized into a cooperative network. Each node has a processing capability (one or more micro-controllers, CPUs or DSP chips), may contain multiple types of memory (program, data and flash memories), has a RF transceiver (usually with a single omnidirectional antenna), has a power source (e.g., batteries and solar cells), and accommodate various sensors and actuators. An effective protocol for wireless sensor networks must consume little power, avoid collisions, be implemented with a small code size and memory requirements, be efficient for a single application, and be tolerant to changing radio frequency and networking conditions. Chess eT International B.V. has developed a WSN platform using a gossip (epidemic) communication model. In order to meet strict energy constraints, Chess used a Time Division Multiple Access (TDMA) protocol in which the nodes are active only in a limited period and for the remainder of the time, nodes switch to an energy saving mode. In the first part of this thesis, the model checker UPPAAL is used for modeling and verification of two synchronization algorithms for wireless sensor networks. Indeed, UPPAAL is used for extracting the error scenarios representing the situations where the network goes out of synch. The error scenarios are repro-
ducible in reality. Based on such error scenarios, three conditions are introduced for a fully connected network to work correctly. The conditions are proved to be necessary and sufficient using invariant proof techniques. Isabelle/HOL supports the proofs.

Part two describes how counterexample-guided abstraction refinement (CEGAR) can be employed to infer models automatically through observations and test, that is, through black-box reverse engineering. State-of-the-art tools for active learning of state machines are able to learn state machines with at most in the order of 10,000 states. This is not enough for learning models of realistic software components which, due to the presence of program variables and data parameters in events, typically have much larger state spaces. Abstraction is the key when learning behavioral models of realistic systems. Hence, in most practical applications where automata learning is used to construct models of software components, researchers manually define abstractions which, depending on the history, map a large set of concrete events to a small set of abstract events that can be handled by automata learning tools. In the second part of this thesis, a full theory of abstraction for learning interface automata is presented. Moreover, it is shown how such abstractions can be constructed fully automatically for a class of extended finite state machines in which one can test for equality of data parameters, but no operations on data are allowed. This aim is reached through counterexample-guided abstraction refinement: whenever the current abstraction is too coarse and induces nondeterministic behavior in the learned model, the abstraction is refined automatically. Using a prototype implementation of the algorithm, models of several realistic software components, including the biometric passport and the SIP protocol were learned fully automatically.
Curriculum Vitae

Faranak Heidarian was born in Shahrekord, Iran, on August 24, 1981. In 1999, she graduated from Farzangan Education Center in Shahrekord, and went to Sharif University of Technology in Tehran, Iran. She received her bachelor degree in Software Engineering in 2004 and her Masters degree in Computer Science in 2007. In January 2008 she went to the Netherlands to continue her education. She became a PhD student in Radboud University Nijmegen, under supervision of Frits Vaandrager.
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