The effect of stellar metallicity on the sizes of star clusters

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ABSTRACT

Observations indicate that blue globular clusters have half-light radii systematically larger than those of red globular clusters. In this paper, we test whether the different metallicity-dependent stellar evolution time-scales and mass-loss rates within the clusters can impact their early dynamical evolution. By means of N-body simulations including stellar evolution recipes we simulate the early evolution of small centrally concentrated clusters with and without primordial mass segregation. Our simulations include accurate metallicity-dependent mass loss from massive stars. We find blue clusters to be larger than red clusters regardless of whether the clusters have been primordially mass segregated. In addition, the size difference is found to be larger and consistent with observations for initial models with a low central concentration. These results indicate that the systematic size difference found between red and blue clusters can, at least in part, be attributed to the dynamical effects of differing stellar evolution histories, driven by metallicity.

Key words: stars: evolution – globular clusters: general.

1 INTRODUCTION

Globular clusters are often used as probes of formation and evolution of their host galaxies. We use cluster properties such as mass, size, metallicity and age to infer when and where the clusters are formed, and therefore learn about the star formation history and merger history of the current hosts. However, these fundamental cluster properties are not completely understood in their own right. We know little about the initial conditions of cluster formation, and our understanding of the early evolution of star clusters is currently incomplete. In particular, the early evolution of star clusters is complicated as it involves both stellar dynamics and stellar evolution. Understanding the relevant processes driving the evolution is key to explaining many of the current observations regarding the properties and distribution of globular clusters, such as the specific frequency problem (see Harris 2010, for a review). However, the theoretical advancements in this field of study lag behind the observational progress being made. In particular, making the leap from introducing theoretical models which are merely special cases to ones which are realistic and have a general applicability is an ongoing challenge. Thankfully, with the continuous improvement of high-performance computation, the theoretical models are finally beginning to offer invaluable insight into the nature of globular clusters.

The half-light radius, or effective radius (r_{hl}), of a globular cluster is a measurable and theoretically useful quantity. There is increasing evidence that some size and structural properties of globular clusters in cluster systems are intimately related to the properties of the galaxy in which they are located (e.g. Harris 1986, 1991). Among these properties, the half-light radius is particularly interesting because several models have shown that it remains nearly constant throughout the lifetime of a globular cluster (Spitzer & Thuan 1972; Aarseth & Heggie 1998). Therefore, this quantity allows various globular clusters to be compared with little consideration of their exact age. In this way, understanding the factors that determine the half-light radii of clusters could provide helpful insight when comparing the properties of different clusters. The half-light radius can provide important constraints on the formation and subsequent evolution of globular clusters. Another important observable property of clusters is their heavy-element abundance, or metallicity (Z). By studying the metallicities of globular clusters in a system, it is possible to determine a metallicity distribution function for the system. In doing so, a clear bimodal form emerges (see Harris 2010 for a review). In this bimodal distribution, there is a metal-poor mode centred at $Z \simeq 0.00063$ and a metal-rich mode centred at $Z \simeq 0.0080$. Metal-poor and metal-rich globular clusters are typically referred to as blue and red clusters, respectively. Several studies (e.g. Puzia et al. 1999; Barmby, Holland & Huchra 2002; Harris 2009) have studied the half-light radii of red and blue clusters in elliptical galaxies, and report a systematic size difference between the two. Specifically, blue clusters are found to have a half-light radius ~20 per cent (or ~0.5 pc) larger than red clusters

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in many galaxies such as M31 and NGC 4472. However, there is no consensus of the underlying reason behind this discrepancy. Determining the explanation for this observation could provide important insights into the formation and/or evolution of globular clusters.

Larsen & Brodie (2003) have proposed that this result can be attributed purely to a projection effect. They argue that the observed size difference can result from the differing spatial distributions of the two cluster types in combination with a correlation between galactocentric distance and cluster size. The argument is supported by the observation that the size difference becomes insignificant at large galactocentric distances (Larsen & Brodie 2003). However, Harris (2009), in a more detailed survey, reports the metal-poor clusters to be systematically smaller by roughly the same factor at large galactocentric distances as well, favouring the notion of a more intrinsic difference rather than merely a projection effect. He goes on to offer another possibility for the size discrepancy, which concentrates on differences in formation conditions of the two globular cluster subpopulations. Specifically, he suggests that a higher metallicity protocluster may undergo more rapid cooling and cloud contraction before stars form, hence allowing a metal-rich cluster to have a smaller scale size from the beginning. Yet another explanation has been put forth by Jordán (2004), who claims the differences are the result of the combined effects of mass segregation and the dependence of stellar evolution time-scales on metallicity. The models produced by Jordán (2004) yield results consistent with observations. However, in preparing the models, the important assumption is made that the half-mass radii (r_{hm}) of different metallicity clusters are equal.

In this paper, we propose that the observed difference is indeed linked to the dependence of stellar lifetimes and mass loss on metallicity, and the impact this effect has on the dynamical evolution of the cluster (similar to the explanation proposed by Jordán 2004). We expand upon this idea by performing simple dynamical simulations, in which the metallicity-dependent stellar mass-loss rates and stellar lifetimes are also included. In the early evolution of the clusters, there are chiefly two mechanisms that drive the expansion: stellar mass loss and two-body relaxation. In the first of these, stellar mass that is shed as the stars evolve leaves the cluster and effectively reduces the total cluster mass. The decreased mass results in outer stars feeling less attraction to the core of the cluster, which in turn results in a bulk expansion (Baumgardt 2009). In the second mechanism, when massive stars undergo close encounters with low-mass stars, the low-mass stars will tend to be ejected from the encounter at high velocities, often driving them to the outskirts of the cluster. This effectively causes the cluster to expand (Baumgardt 2009). Although stellar mass loss boosts expansion by the first mechanism, it also opposes expansion by greatly reducing the strength of two-body relaxation as the massive stars are lost. In this sense, these two mechanisms are opposing processes. The process that is dominant will drive the expansion. The two processes can also be thought of as causing energy flow and production, respectively (Gieles et al. 2010). A central energy source is present, as a consequence of active stellar mass loss, and is in balance with the energy flow outwards caused by two-body relaxation. Gieles et al. (2010) suggests that this interplay results in no sharp transition between a stellar evolution dominated phase and a dynamics dominated phase. It is important to note that high-mass metal-rich stars lose more mass in stellar winds and also have reduced main-sequence lifetimes (Hurley, Pols & Tout 2000). Thus, at any instant in time during the cluster evolution, a high-metallicity cluster will have shed more of its mass through stellar evolution processes and contain fewer high-mass stars than its low-metallicity counterpart.

This leads us to expect that if stellar mass loss is the dominant expansion mechanism, a metal-rich cluster will be more prone to expanding, whereas if two-body relaxation is dominant, the opposite will be true. Thus, for metal-poor clusters to become larger, we expect expansion due to two-body relaxation to be dominant for the majority of the evolution. We explore this line of reasoning with an *N*-body code and stellar evolution recipes through MUSE (Portegies Zwart 2009). MUSE is a multiphysics, multiscale software environment for modelling astrophysical systems. It consists of a series of modules for stellar dynamics, stellar evolution and stellar hydrodynamics, written in different languages and often based on commonly available codes which were specifically written for one of those tasks. The stellar evolution fits used in this research are functions of metallicity and yield a unique mass-loss history for each metallicity. We employ a modest number of stars to test whether this effect can be important for the evolution of clusters. The impact of differential stellar evolution and mass loss caused by variations in metallicity witnessed in these small clusters should, in general, be applicable to clusters with any number of stars.

2 METHOD

Simulating the early evolution of star clusters is a complex task, requiring detailed treatment of both stellar evolution and gravitational interactions. MUSE is well suited for this problem as it combines stellar evolution and dynamics modules into one package and allows calculations to be carried over between modules with ease. We use NBODY-1H (Makino & Aarseth 1992), a direct *N*-body integrator, to follow the stellar dynamics. This early version of the NBODY series of dynamics codes provides the most consistent treatment of collisional stellar dynamics of the modules available in MUSE. NBODY-1H employs a softening parameter, ϵ, in the gravitational force so that very close encounters between point masses do not dominate the computation. We choose the softening parameter to be small, such that $\epsilon^2 = 10^{-11}$ (in dynamical units), which corresponds to $\epsilon = 0.65$ au in our simulation. We use single-star evolution (SSE) fitting formulae (Hurley et al. 2000) to perform stellar evolution calculations. SSE is continuously updated, and the version implemented into MUSE was available in early 2009 (Hurley, private communication). SSE is composed of a series of analytical formulae that fit observational stellar evolution tracks and accurately model the effect of metallicity on the mass loss and stellar lifetimes of stars. In particular, high-metallicity massive stars have shorter main-sequence lifetimes and a higher mass-loss rate from stellar winds. We assume that any stellar mass that is shed leaves the cluster instantaneously. Additionally, we neglect hydrodynamics, radiative transfer and collisions between stars. The initial stellar population does not contain any primordial binaries. Recent simulations of young star clusters are starting to include many of these effects (e.g. Goodwin & Bastian 2006; Portegies Zwart, McMillan & Makino 2007). However, to test the effect of metallicity, we have chosen to be true to the history of stellar dynamics, and are looking at a simplified system first.

The initial distribution of stars within the cluster is given by a King model (King 1966) in which the stars are assumed to be in virial equilibrium. The models are generated using STARLAB (Portegies Zwart et al. 2001). To accentuate dynamical effects, we employ a high initial central potential ($W_0 = 12$) for the majority of our models, but also perform tests at $W_0 = 6$. We use a Kroupa (2001) mass function to generate an appropriate initial mass spectrum for the cluster. All stars begin as zero-age main-sequence stars. The computations are chiefly performed on clusters that

contain 8192 stars initially. Further simulations are performed on 1024-, 4192- and 16 384-star clusters to test for convergence of results. The tidal radius for these clusters is set to 20 pc, which is appropriate for open clusters of a few thousand stars in the Milky Way. Unbound stars outside the tidal radius of the cluster are removed from the simulation. Removing escapers affects the simulation in two ways: once several stars are removed, the computational time decreases; and the half-mass radius is effectively pushed inwards. Simulations are performed at five different metallicities: $Z = 0.02$, 0.007 96, 0.001, 0.000 632 and 0.0001. These values were chosen to span the range from solar metallicity ($Z = 0.02$) to a typical globular cluster value ($Z = 0.001$) to the most metal-poor clusters ($Z = 0.0001$). The most important metallicities to investigate are $Z = 0.007 96$ and 0.000 632, as these correspond to the mean metallicities of red and blue clusters, respectively (Harris 2010).

To test the effects of primordial mass segregation on a given cluster, it is first evolved dynamically for 1.2 half-mass relaxation times (t_{rh}), that is, without allowing the stars to undergo stellar evolution. The primordial dynamical evolution leaves the cluster mass segregated at the time stellar evolution is initiated. We choose this method for simulating primordial mass segregation, as was done by Vesperini, McMillan & Portegies Zwart (2009), since it is a self-consistent and convenient method of generating mass-segregated clusters. However, there are alternate methods for simulating primordial mass segregation (e.g. Baumgardt, De Marchi & Kroupa 2008; Šubr, Kroupa & Baumgardt 2008). Several have suggested that mass segregation occurs on a time-scale of the order of a relaxation time (e.g. Bonnell & Davies 1998). A useful quantity to describe the relaxation time of a cluster is the half-mass relaxation time. We investigated the strength of the mass segregation for clusters primordially mass segregated between $0.2t_{rh}$ and $1.5t_{rh}$. We found the majority of the segregation to occur within the first $1.2t_{rh}$, that is, by this point in time, most massive stars would have migrated to the core of the cluster. Therefore, we chose this length of time as the standard for all subsequent primordial mass segregation. We do not believe that our method of simulating primordial mass segregation will have a significant impact on the results that we are interested in, because the differences between metallicities should only be sensitive to whether or not the cluster has been primordially segregated, and not to the method that has been used to achieve the segregated state.

Table 1 lists all the simulations that have been performed along with their specific information and alias. We use simulation (e) as a standard and will compare other runs to this particular one. It is important to note that in all mass-segregated models, the central potential refers to that of the initial model. That is, the specified W_0 applies to the initial model before mass segregation has occurred. In fact, the central potential at the point of initiation of stellar evolution is considerably lower. Similarly, the values of N listed for all primordially mass-segregated models also apply to the initial model. On the other hand, all half-mass relaxation times shown are valid at the point of initiation of stellar evolution, as well as the initial mass, M_0, and the initial half-mass radius, r_{h0}. Simulation (m) is the result of an average of three separate runs. This averaging is done to reduce the statistical noise present at such a low number of particles. We evolve the simulations for 100 Myr or $5t_{rh}$, whichever is longer. By 100 Myr, stars more massive than $\sim 7\,M_\odot$ have completed their evolution and have become remnants, so the impact of any metallicity dependence will already be present. $5t_{rh}$ is sufficiently long for the cluster to relax dynamically.

3 RESULTS

In Fig. 1 we show the early size evolution of clusters (a) to (e). The half-mass radius as a function of time is plotted versus both a dynamical and physical time-scale. All clusters experience a rapid expansion initially. The early evolution differs for metal-rich and metal-poor clusters. In particular, over the course of the evolution, the metal-poor clusters (c), (d) and (e) become gradually larger over time, in comparison to the metal-rich clusters. In the evolution, the metal-rich clusters are seen to expand more rapidly at first, but are soon overtaken by the metal-poor clusters. Past this crossover point, the metal-poor clusters remain larger. Although Gieles et al. (2010) indicates that there is no sharp transition between a stellar dominated phase and a dynamics dominated phase, in Fig. 1 we notice a gradual transition between the two. From this, we can conclude that in the early evolution (first $\sim$20 Myr), stellar mass loss is the important expansion mechanism, but for the rest of the evolution, two-body relaxation dominates. At the end of the simulation, the physical age of the cluster is 100 Myr. Thus, most of the stellar evolution of the massive stars would have been completed at this point. Due to this, we expect effects of the differing stellar

Table 1. List of simulations performed.

| Alias	N	Z	Primordial mass segregation	W_0	t_{rh} (Myr)	M_0 (M$_\odot$)	r_{h0} (pc)
(a)	8192	0.02	No	12	19.1	14 340.1	0.94
(b)	8192	0.007 96	No	12	19.1	14 340.1	0.94
(c)	8192	0.001	No	12	19.1	14 340.1	0.94
(d)	8192	0.000 632	No	12	19.1	14 340.1	0.94
(e)	8192	0.0001	No	12	19.1	14 340.1	0.94
(f)	8192	0.02	Yes	12	49.5	13 308.4	1.77
(g)	8192	0.00 796	Yes	12	49.5	13 308.4	1.77
(h)	8192	0.001	Yes	12	49.5	13 308.4	1.77
(i)	8192	0.000 632	Yes	12	49.5	13 308.4	1.77
(j)	8192	0.0001	Yes	12	49.5	13 308.4	1.77
(k)	16 384	0.0001	No	12	26.0	26 598.0	0.96
(l)	4096	0.0001	No	12	15.4	6578.3	0.94
(m)	1024	0.0001	No	12	10.5	1669.2	0.96
(n)	8192	0.00 796	No	6	15.5	13 055.0	0.80
(o)	8192	0.000 632	No	6	15.5	13 055.0	0.80
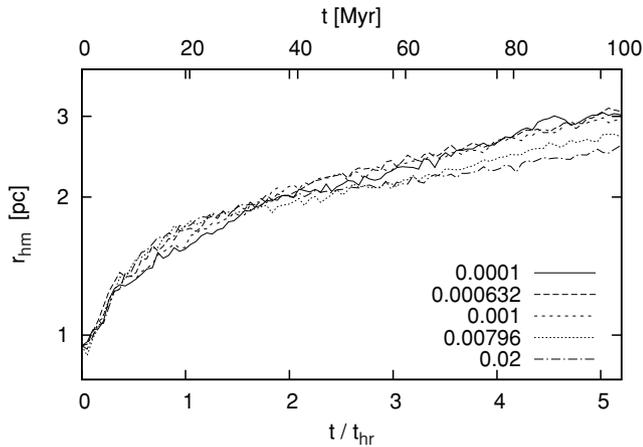
Figure 1. The half-mass radius of an 8192-star cluster evolved at various metallicities without primordial mass segregation. The clusters experience an initial rapid period expansion and then continue to expand until the end of the evolution. The low-metallicity clusters become systematically larger than the high-metallicity clusters over the course of the evolution.

evolutions between metallicities to have impacted the dynamical evolution and be clearly evident. The fact that our results indicate the half-mass radius of the clusters to depend on the metallicity contradicts the fundamental assumption made in Jordán (2004). He assumes that the half-mass radius of clusters is independent of metallicity to determine the dependence of the observed half-light radius on metallicity. We calculated the half-light radii of a number of our clusters at $5t_{\rm rh}$, and compared them to the half-mass radii. In these young stellar systems (unlike Jordan's 13-Gyr-old populations), we do not see a systematic dependence of the half-light radii on metallicity, independent of the half-mass radius of the cluster. The ratio of half-mass to half-light radii was constant for all our clusters, within 10 per cent.

In Figs 2 and 3, we plot the half-mass radius as a function of metallicity at 95 Myr and $5t_{\rm rh}$, respectively, for runs (a) through (i). The value of the half-mass radius plotted in these figures is the time-averaged quantity over 10 Myr centred at 95 Myr and $5t_{\rm rh}$, respectively. For the clusters without primordial mass segregation (a through e), this corresponds to the same instant in time. For the clusters that are primordially mass segregated, $5t_{\rm rh}$ corresponds to a physical age of roughly 250 Myr. The error bars are based on

Figure 3. The half-mass radius of an 8192-star cluster at $5t_{\rm rh}$ at various metallicities with and without primordial mass segregation. Once again, the half-mass radius of the metal-poor clusters is larger, although the difference is more significant at this point than at 95 Myr for the primordially mass-segregated models.

the range of points over the 10-Myr average. At 95 Myr we are comparing clusters of similar stellar evolution stage, while at $5t_{\rm rh}$ we are comparing clusters of similar dynamical age.

Our results show that metal-rich clusters are smaller in size, regardless of whether the cluster is primordially mass segregated. In fact, metal-poor clusters of varying metallicity are very similar in size. The same property is somewhat present in metal-rich clusters of varying metallicity as well. At 95 Myr (or $5t_{\rm rh}$), the difference in size between the red and blue metallicity clusters, without primordial mass segregation, is 0.32 ± 0.05 pc or (12 ± 2) per cent. With primordial mass segregation, the size difference is 0.29 ± 0.05 pc or (8 ± 1) per cent at 95 Myr. At $5t_{\rm rh}$, the size difference between primordially mass-segregated blue and red metallicity clusters is instead 0.58 ± 0.05 pc or (12 ± 1) per cent. In fact, at $5t_{\rm rh}$ (Fig. 4), the trends with and without primordial mass segregation are very similar. Thus, in the primordially mass-segregated clusters as well, our results indicate a significant relationship between half-mass radius and metallicity.

There have been several observational reports of the size discrepancy between red and blue clusters. A size difference ranging
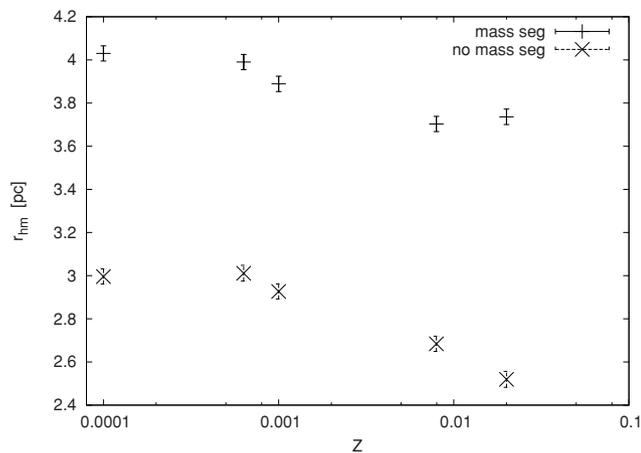

Figure 2. The half-mass radius of an 8192-star cluster at 95 Myr at various metallicites with and without primordial mass segregation. In both cases, the half-mass radius of the metal-poor clusters is larger.
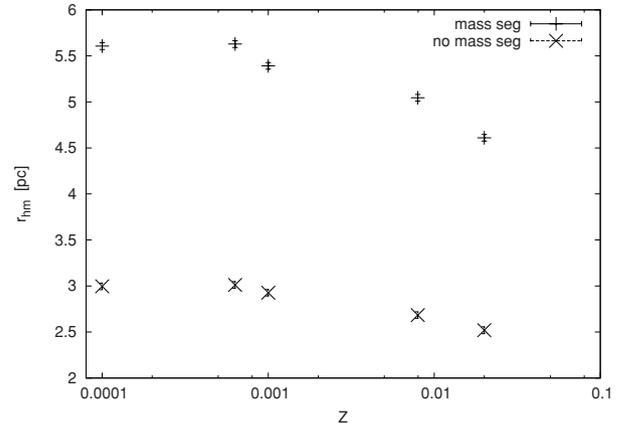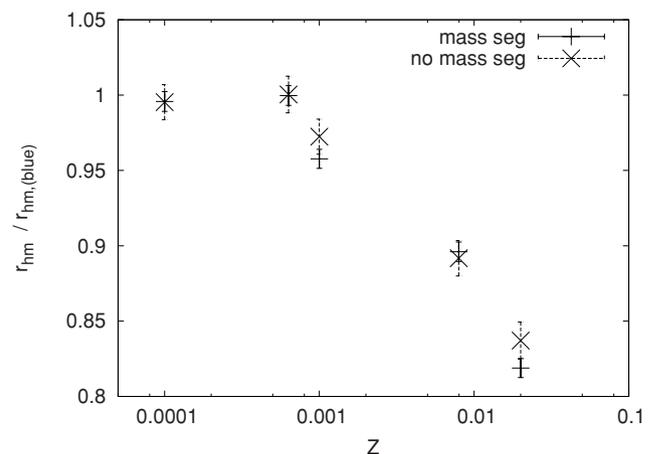

Figure 4. The half-mass radius of an 8192-star cluster at $5t_{\rm rh}$ at various metallicities, with and without mass segregation, normalized by the half-mass radius of the $Z = 0.007\,96$ model. The trend of cluster size versus metallicity is independent of the amount of primordial mass segregation.

between 17 and 30 per cent has been found in many galaxies, including NGC 4472 (Puzia et al. 1999) and M31 (Barmby et al. 2002). More recently, Harris (2009) has reported a difference in r_{hl} of (17 ± 2) per cent at all galactocentric distances for the globular cluster populations in six giant elliptical galaxies. In a survey of 43 early-type galaxies, Masters et al. (2010) observe red and blue clusters to have a mean r_{hl} of 2.8 ± 0.3 and 3.4 ± 0.3 pc, respectively. This discrepancy corresponds to a per cent difference within the range listed previously. Harris (2009) points out that the difference in r_{hl} is typically ∼0.5–1 pc. However, since the number of stars in the clusters we have simulated are orders of magnitudes smaller, we believe that a comparison of the per cent difference in size is a more appropriate diagnostic. On the other hand, we see that the observed physical size difference is on par with that indicated by our results. It is possible that this is due to the fact that the results we gather for 8192-star clusters are applicable to much larger clusters as well. That is to say, the results converge for large N. In this way, our clusters do indeed simulate real sized clusters quite accurately. Convergence of results will be discussed later in this section. Although the per cent differences we report fall short of the range that is observed, it is important to note that the clusters we simulate are still expanding at the time that we compare their sizes (see Fig. 1). This implies that the size differences we report at these points may be subject to a small amount of change. As an example, the primordially mass-segregated cluster shows a difference in the size discrepancy when compared at 95 Myr as opposed to $5t_{rh} \simeq 250$ Myr. However, we note that no major differences will develop past the point of ∼100 Myr because all the massive stars have completed their stellar evolution. Thus, the main impact of metallicity differences should already be present in the cluster. Our results clearly indicate the size of clusters to be metallicity dependent. In fact, it is this qualitative agreement between our results and observations that is the most important. The quantitative differences could simply be a consequence of the parameter space we are occupying or the specifics of our simulations and analysis. Although, as previously stated, we should be careful when comparing the physical size differences to observations, it is interesting to note that after $5t_{rh}$, the blue and red primordially mass-segregated clusters differ by roughly 0.6 pc at the half-mass radius. This value is in the range of size differences that Harris (2009) points out as relevant.

Fig. 5 shows the results of runs (e), (k), (l) and (m) on a dynamical time-scale, to illustrate the similarity of the behaviour of the
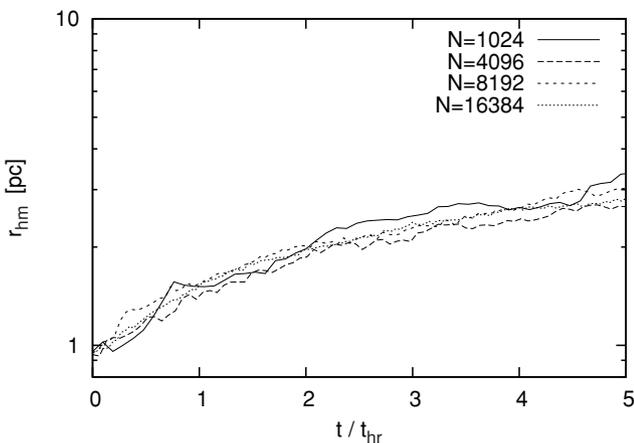
half-mass radius regardless of the number of stars we use in our simulations. Although these are the lowest metallicity clusters in our sample, the behaviour of stellar dynamics is not strongly dependent on metallicity and therefore the convergence tests can be applied to all our metallicities. The convergence of results at higher numbers of stars points to the fact that the results presented in this paper are applicable for a much larger number of stars. Indeed, although we study the metallicity effects in clusters of 8192 stars, the results are applicable to all star clusters, with numbers of stars up to two orders of magnitude larger. This is in agreement with earlier studies, such as those of Baumgardt & Makino (2003), which span a larger range in star number than we used here. The early evolution, and the fractional stellar mass loss from all size of clusters, is largely independent of the number of stars in the cluster.

Simulations (n) and (o) represent 8192-star clusters with a low initial central potential ($W_0 = 6$) at red and blue metallicity, respectively. The tests were performed to investigate whether the size discrepancy remains if the initial central potential is lower. Indeed, at $5t_{rh} = 77.5$ Myr, the half-mass radius of the blue cluster is 0.39 ± 0.05 pc, or (21 ± 3) per cent larger than that of the red cluster. At 95 Myr, the difference is instead 0.41 ± 0.05 pc, or (20 ± 3) per cent. These results are in the range of the observed size differences. Therefore, we find a better agreement between our results and observations when W_0 is low.

4 SUMMARY AND DISCUSSION

In this paper, we have used an N-body code and stellar evolution recipes including stellar mass loss to simulate the early evolution of star clusters of different metallicities. In our analysis, we include clusters with and without primordial mass segregation. We focus our attention on the half-mass radius of the clusters, as we find it to convey the same results as the half-light radius. Clusters without primordial mass segregation demonstrate roughly a (12 ± 2) per cent difference in half-mass radius between red and blue clusters at $5t_{rh} = 95$ Myr. Clusters with primordial mass segregation exhibit an (8 ± 1) per cent difference in half-mass radius between red and blue metallicities at 95 Myr and a (12 ± 1) per cent difference at $5t_{rh} \simeq 250$ Myr. The simulation was repeated at constant metallicity for $N = 1024$, 4096, 8192 and 16 384. On a physical time-scale, the evolution converges at larger N, indicating that our results are indeed applicable to much larger clusters. On a dynamical time-scale, the evolution is very similar regardless of N, which suggests that the dynamical evolution is typically similar regardless of the number of stars in the cluster. The size difference between blue and red clusters was found not only to persist, but to be even more significant, at a low value of the initial central potential. Therefore, we conclude that the dependence of stellar evolution and mass-loss histories on metallicity impacts the dynamics, and at least in part, is responsible for the size differences observed between red and blue globular clusters.

There is increasing evidence that globular clusters begin their evolution primordially mass segregated. Although mass segregation occurs naturally over the course of the evolution of a globular cluster, the process is disturbed by the stellar evolution that is occurring simultaneously. Thus, a primordially mass-segregated cluster experiences a different dynamical evolution. Therefore, for the sake of generality, we run simulations of both primordially mass-segregated and unsegregated models. Our results demonstrate that the size difference persists in both cases. In a primordially mass-segregated cluster, the initial configuration consists of the majority of massive stars clustered in the core amongst a great number of low-mass

Figure 5. The half-mass radius of clusters with varying star numbers, N, plotted on a dynamical time-scale. The evolution is dynamically similar at all N.

stars, and additional low-mass stars occupying the outer regions of the cluster. It is expected that in this configuration, there will be an increase in the number of close encounters between low- and high-mass stars. Due to this, and since we observe that two-body relaxation is the dominant expansion mechanism for the majority of the evolution in the unsegregated case, that should also be the case for the primordially mass-segregated case. In addition, due to the increase in the number of close encounters between low- and high-mass stars, we anticipate a primordially mass-segregated cluster to produce even greater size discrepancies between metallicities. However, we observe the size difference to be equal at $5t_{rh}$ and even larger for the unsegregated cluster at 95 Myr.

The so-called specific frequency problem in globular cluster systems refers to the inconsistency between the number of metal-poor halo stars per unit globular cluster at the same metallicity and the equivalent ratio for metal-rich halo stars. In fact, there are roughly five times fewer metal-poor halo stars per unit globular cluster at the same metallicity (see Harris 2010, for a review). This discrepancy is inconsistent with the notion that halo stars come from tidally stripped or disrupted globular clusters (if this were the case, one would expect the two ratios to be equal). One solution to this problem is that metal-rich clusters are more easily disrupted, thus accounting for the inflated number of high-metallicity halo stars. If a cluster indeed is to be more easily disrupted, one would expect it to expand more over the course of its evolution, and in doing so, is more susceptible to tidal stripping and evaporation of stars. However, our results indicate that it is the metal-poor clusters that experience a larger expansion over their evolution. Thus, our results are incompatible with this solution to the specific frequency problem.

Numerous assumptions and simplifications have been made to make the problem more tractable. Most importantly, we perform our tests on clusters that contain roughly 8000 stars, whereas globular clusters typically have 10^5–10^6 stars. The main concern here is that the effects witnessed in our smaller clusters are no longer present or relevant in larger clusters. Although our results indicate the cluster evolution to converge at larger values of N, this does not guarantee that the convergence remains at $N \sim 10^5$. On the other hand, we believe that our analysis of the processes causing the size discrepancy to be valid regardless of the size of the cluster since metallicity differences will always alter the main-sequence lifetimes of massive stars. Because of this, large clusters of different metallicity will have different number of massive stars at any point in the early evolution. Metallicity will affect the impact of two-body relaxation, and thus significantly impact the subsequent evolution. Therefore, we are confident that the qualitative results of our simulations will persist at larger N, although the exact value of the size difference may not be consistent.

We perform the bulk of our simulations on clusters with an extremely high initial central potential. This was initially thought to accentuate dynamical effects, and in doing so, would emphasize differences in the evolution of the different metallicity clusters. As was earlier described, the simulations were repeated once with $W_0 = 6$ at red and blue metallicities. In this case, we observe the size discrepancy not only to persist but also be enhanced. This result demonstrates that a high initial central potential does not necessarily accentuate differences in the interplay between stellar and dynamical evolution. Although a W_0 of 6 is considered a fairly low value of the parameter, this result demonstrates that our results are applicable at both low and high values of W_0. Therefore, we anticipate our results to be relevant to clusters within this range and typical values of globular clusters. A possible explanation for seeing an enhanced effect at small W_0 could be attributed to the relative time-scales of the dynamics and the stellar evolution. For the mass-loss differences between metallicities to play the greatest role in altering the dynamical evolution, the typical two-body relaxation time-scale should be of the order of the stellar evolution time-scale. In this regime, one would expect to see the greatest interplay between the effects. Using the central relaxation time, t_c, as a measure of the relevant dynamical time-scale, we postulate that the closer this quantity is to the typical time-scale of the stellar evolution, the greater the size discrepancies between different metallicity clusters will be. In our clusters, the central relaxation times are shorter than the stellar evolution time-scale of even the most massive stars. However, the $W_0 = 6$ cluster we use has a t_c nearly two times larger than the $W_0 = 12$ cluster. This implies that the relevant dynamical time-scale for the $W_0 = 6$ cluster is more comparable to the stellar evolution time-scale. Consequently, this cluster experiences the greater difference between varying metallicities. According to the 2010 update of Harris (1996), the majority of Milky Way globular clusters that have not experienced core collapse have $t_c \sim$ 30–1000 Myr. Stars of mass roughly between 2 and 10 $M_\odot$ have lifetimes in this range. Thus, the dynamical time-scale is comparable to the stellar evolution time-scale for stars in this range. Stars with even higher masses will have stellar lifetimes shorter than 30 Myr, but differences in their mass-loss histories will still have an impact on the dynamics of the cluster. By this, metallicity differences are expected to alter the dynamical evolution of globular clusters, and it is thus consistent with our analysis.

In all our simulations, we assume a constant tidal radius of 20 pc. Although changing this parameter would alter the Lagrangian radii (distances from the cluster centre at which various percentages of the total mass are enclosed) over the course of the evolution, it would affect clusters of different metallicities in the same way, and thus would not change our results. Many additional simplifications have been made to the system including instantaneous ejection of gas, no primordial binaries and no binary stellar evolution. In general, ignoring these processes does impact the evolution significantly. However, most of these processes have been thoroughly studied in the past, and their effects are thought to be well understood. We believe that we are justified in neglecting these because they will not produce effects that are significantly metallicity dependent. Thus, removing these simplifications may alter the evolution of the cluster, but not in ways that will change the size discrepancy between clusters of different metallicities.

Our simulations have not included the effects of a population of primordial binaries in the cluster. Binaries are known to affect the dynamical evolution of clusters, mainly through the slowing or halting of core collapse. In the early dynamical stages of cluster evolution, as presented here, however, the effect of binaries is expected to be minimal. What has yet to be investigated in detail, however, is the effect of binarity on stellar mass loss and the subsequent implications for the dynamical evolution of clusters. Stellar mass loss in binary systems could well be enhanced, due to the presence of the companion; it could also be suppressed in some cases because the presence of a close companion means that the star does not reach a giant stage with enhanced mass loss but instead interacts and possibly merges with its companion. A more careful treatment of binary populations and binary evolution, especially for binaries of low metallicity, is required before we can fully understand the effects of binary populations on the sizes of red and blue globular clusters.

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