R2SM: a package for the analytic computation of the R_2 Rational terms in the Standard Model of the Electroweak interactions

M.V. Garzelli[1] and I. Malamos[2]

[1] *INFN, Italia & Departamento de Física Teórica y del Cosmos y CAFPE, Universidad de Granada, E–18071 Granada, Spain*

[2] *Department of Theoretical High Energy Physics, Institute for Mathematics, Astrophysics and Particle Physics, Radboud Universiteit Nijmegen, 6525 AJ Nijmegen, the Netherlands.*

e-mail: `garzelli@to.infn.it`, `J.Malamos@science.ru.nl`

Abstract

The analytical package written in FORM presented in this paper allows the computation of the complete set of Feynman Rules producing the Rational terms of kind R_2 contributing to the virtual part of NLO amplitudes in the Standard Model of the Electroweak interactions. Building block topologies filled by means of generic scalars, vectors and fermions, allowing to build these Feynman Rules in terms of specific elementary particles, are explicitly given in the R_ξ gauge class, together with the automatic dressing procedure to obtain the Feynman Rules from them. The results in more specific gauges, like the 't Hooft Feynman one, follow as particular cases, in both the HV and the FDH dimensional regularization schemes. As a check on our formulas, the gauge independence of the total Rational contribution (R_1 + R_2) to renormalized S-matrix elements is verified by considering the specific example of the H $\rightarrow \gamma\gamma$ decay process at 1-loop. This package can be of interest for people aiming at a better understanding of the nature of the Rational terms. It is organized in a modular way, allowing a further use of some its files even in different contexts. Furthermore, it can be considered as a first seed in the effort towards a complete automation of the process of the analytical calculation of the R_2 effective vertices, given the Lagrangian of a generic gauge theory of particle interactions.

Keywords: Electroweak interactions, NLO, Virtual Radiative Corrections, Unitarity, Rational terms

1 Introduction

The calculation of NLO radiative corrections to multiparticle production processes has achieved significant progresses in the last few years, thanks to refinements to the traditional techniques [1, 2, 3] on the one hand, and to the introduction of new methods, mainly based on Unitarity [4, 5] and Generalized Unitarity [6] principles, on the other. This has already allowed the computation of several (differential) cross-sections at NLO, especially for key signal and background particle scatterings and decays of interest at colliders, whose signatures can potentially through light on the mechanism underlying the Electroweak (EW) Symmetry Breaking process [7, 8].

A generic 1-loop amplitude can be decomposed as a linear combination of known scalar integrals, with up to 4 external legs, plus a residual Rational part R. In the framework of the approaches based on the Unitarity of the S-matrix, the first piece is Cut-Constructible (CC), i.e. it can be reconstructed by properly cutting the loop amplitude in tree-level like sub-amplitudes, thus conceptually reducing the complexity of the calculation of 1 - loop integrals to a simpler computation of tree-level diagrams. On the other hand, when working in approaches based on 4 integer dimensions, the R terms, or at least some of them as clarified in the following, cannot indeed be calculated just in terms of tree-level diagrams, but a full 1-loop computation is needed. This can be performed according to the traditional Feynman diagram approach [9, 10]. An alternative strategy consists in making use of Unitarity-based on-shell recursion relations [11], leading to a recursive evaluation of R by means of bootstrapping techniques [12]. On the other hand, it is worth observing that the R terms can be put on the same footing as the CC part at the price of introducing a larger number of integer dimensions, as worked out in the d-dimensional extensions of the Unitarity approach [13]. Despite the attractive elegance of the d-dimensional Unitarity formulations, in this paper we choose to follow a variation of the first approach, thus avoiding to introduce extra integer dimensions (and, as a consequence, to extend to these higher dimensions the external particle wave functions), and just allowing $d = 4 + \epsilon$ dimensions in the dimensional regularization scheme we use to regularize the divergencies appearing during the computation of 1-loop integrals (even in case of finite tensor integrals, singularities may arise in the tensor reduction process and need to be regularized). We worked in the framework of the OPP method [14], one of the Unitarity inspired algebraic and universal (i.e. independent from the model of particle interactions at hand) procedures first introduced to automatically calculate the CC part of any 1-loop amplitude. In the OPP approach, the R terms can be organized in two classes, according to their origin: the R_1 terms, arising from the mismatch between the 4-dimensional part of the numerator and the d-dimensional denominator including the poles of the propagators in the regularized integrand of 1-loop integrals, and the R_2 terms, arising instead from the ϵ-dimensional part of the numerator. Both these classes are thus a residual effect of the dimensional regularization scheme one introduces on the integrands to calculate 1-loop integrals. In the framework of OPP, it has been shown that the R_1 terms are closely related to the CC part, and can indeed be obtained numerically *at the same time* of the last one [15]. Unfortunately, however, this property does not apply to the R_2 terms, that, instead, need a dedicated computation. Anyway, the problem of the calculation of the R_2 part of a generic 1-loop amplitude can be solved by observing that only 1-particle irreducible diagrams with up to 4 external legs can contribute to R_2, due to the exclusively ultraviolet nature of these terms, as proven in [10]. In other words, R_2 terms will never contribute to the $1/\epsilon$ and $1/\epsilon^2$ pole parts of 1-loop amplitudes, whose nature is instead completely infrared. Thus, taking into account that the number of the contributing diagrams is limited, it is possible to calculate, once and for all for the theory of interaction at hand, all possible R_2 effective vertices, up to 4-external legs, and then use these effective vertices as building blocks for computing the R_2 contribution to each specific 1-loop amplitude [15]. Given the set of all external particles (with their momenta and their quantum numbers) identifying a 1-loop helicity amplitude, it will be enough to consider all tree-level diagrams joining them, which include one (and only one) R_2 effective vertex. This recipe has been adopted in the HELAC-1-loop code [16], where the R_2 effective vertices entering QCD 1-loop corrections [17] have been implemented, and recursion relations have been adopted to evaluate the corresponding contributions to the amplitudes. Furthermore, our R_2 Feynman Rules, together with (off-shell) recursive relations for tensor integrals, have been applied to the numerical computation of the R_2 contribution to 1-loop multigluon amplitudes in a tensor reduction framework [18].

Throughought this paper we denote by *topology* a set of lines (propagators) connecting a set of points (vertices), by *generic diagram* a topology filled by means of generic Standard Model (SM) scalar s, vector v and/or fermion f fields, and by *particle diagram* a generic diagram whose components have been further specified in terms of selected SM particles (like H, Z, e^-, etc.). After a formal definition of the R terms in Section 2 and our comments on the gauge choice in Section 3, we present our analytical results for the R_2 contributions to all leading generic 1-loop 1-particle irreducible diagrams with up to 4 external legs arising in the SM of EW interactions, diagram by diagram, and the FORM code we have written to obtain from them the R_2 effective vertices involving real specific particles, to be used in actual calculation of 1-loop amplitudes, in Section 4. These analytical results are of interest for people aiming at better understanding the nature and the properties of the R_2 terms. We include a test of the reliability of our code together with considerations concerning the gauge invariance of R in Section 5, and we draw our conclusions in Section 6. Additional information about the notation used in our code is available in the Appendix and in the README file within the package.

2 The R contribution to 1-loop amplitudes

In this Section we formally define the R terms, and in particular the R_2 class, briefly reviewing how they appear in the computation of 1-loop amplitudes. Our starting point is the *integrand* of a generic m-point amplitude, written as

$$A(q) = \frac{N(q)}{D_0 D_1 .... D_{m-1}}\,, \quad \text{with} \;\; D_i = (q+p_i)^2 - m_i^2\,, \tag{1}$$

where q is the loop momentum, $q + p_i$ and m_i are the momentum and the mass of the i-th loop particle ($i = 0,\ 1, ... m-1$).

A dimensional regularization procedure is then introduced, at the aim of regularizing the singularities in the integrand amplitude, in order to be allowed to evaluate its integral. In renormalizable gauge theories, the number of ultraviolet (UV) divergent integrals is finite, and the UV divergencies are re-absorbed, after integral calculation, in renormalized quantities. The infrared (IR) singularities, instead, cancel when combining together the real and the virtual contribution to the amplitude at each fixed order in the perturbative expansion in terms of the coupling constant, according to the KLN theorem [19]. Thus the results for 1-loop amplitudes will, in general, include a finite part plus a divergent part, of IR origin only.

We choose to work in $d = 4 + \epsilon$ dimensions (dim). The extension to d-dim of the integrand of the amplitude, $N(q) \to \bar{N}(\bar{q})$, is achieved through the transformations:

$$\begin{aligned} q_\mu \to \bar{q}_\mu &= q_\mu + \tilde{q}_{\tilde{\mu}} \,, && (2) \\ \gamma_\mu \to \bar{\gamma}_{\bar{\mu}} &= \gamma_\mu + \tilde{\gamma}_{\tilde{\mu}} \,, && (3) \\ g_{\mu\nu} \to \bar{g}_{\bar{\mu}\bar{\nu}} &= g_{\mu\nu} + \tilde{g}_{\tilde{\mu}\tilde{\nu}} \,, && (4) \end{aligned}$$

where we have denoted quantities in d-dim with a bar, and we have explicitly shown the ϵ-dim part of each quantity, including tildes. The quantities in ϵ-dim are orthogonal with respect to the quantities in 4-dim, thus, in developing the expression of $\bar{N}(\bar{q})$, one has to take into account relations like

$$\begin{aligned} \bar{q}_{\bar{\mu}} v^\mu &= q_\mu v^\mu \,, && (5) \\ \tilde{g}_{\tilde{\mu}\tilde{\nu}} g^{\mu\nu} &= 0 \,, && (6) \end{aligned}$$

and similar ones.

One can thus rewrite the numerator in d-dim as a sum of a 4-dim part plus a residual part. The R_2 terms correspond to the integral of the ϵ-dim part of $\bar{N}(\bar{q})$, i.e.

$$\mathrm{R}_2 = lim_{\epsilon \to 0} \frac{i}{16\pi^2} \int d^d\bar{q} \frac{\tilde{N}(q, \tilde{q}^2, \epsilon)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 .... \bar{D}_{m-1}} \,. \quad (7)$$

To understand the origin of the CC and the R_1 terms, one can instead expand the 4-dim part of the numerator, $N(q)$, in terms of 4-dim denominators D_i, according to the universal OPP decomposition [14], and then observe that d-dim denominators instead of 4-dim denominators appear in the dimensionally regularized integrand of the amplitude [20]. By rewriting these d-dim denominators in terms of the 4-dim denominators of the decomposition according to

$$\frac{1}{\bar{D}_i} = \frac{D_i}{\bar{D}_i D_i} = \left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right) \frac{1}{D_i} \,, \quad (8)$$

one can see that the first part of the expression in the parentheses lead to the CC part of the amplitude (all terms in 4-dim, as tree-level diagrams), whereas the second part of the expression in the parantheses causes the appearence of the R_1 contribution to the amplitude:

$$R_1 = lim_{\epsilon \to 0} \frac{i}{16\pi^2} \int d^d q \frac{f(q, \tilde{q}^2)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 .... \bar{D}_{m-1}} \,. \quad (9)$$

3 Choice of a gauge

One of the crucial points to actually perform the computation of the R_2 terms is the choice of a convenient gauge. We work in the R_ξ class of gauges. More precisely, we consider the generalized R_ξ gauges, characterized by a three-parameter ξ_A, ξ_Z, ξ gauge fixing term. The standard R_ξ gauges can be obtained as the particular case corresponding to $\xi_A = \xi_Z = \xi$.

The main difference between the gauges in this class and the unitary one, is the appearence in the first ones of unphysical scalars and Fadeev-Popov-DeWitt ghosts, as loop particles. The unitary (even called physical) gauge instead, comes out as the particular limit ξ, $\xi_Z \to \infty$ and $\xi_A \to$ finite number (these two independent limits are possible because the first one does not fix the electromagnetic gauge invariance), corresponding to unphysical particles becoming extremely heavy and decoupling from the theory.[1] In fact, the unitary gauge condition eliminates 3 of the 4 degrees of freedom in the scalar Higgs doublet, re-emerging as longitudinal spin states of the Z and $W^\pm$ gauge bosons acquiring a mass. Thus, one has to deal with a reduced number of particles, i.e. with the physical ones only. This reduced number of particles is the reason why, when working at tree-level, unitary gauge is often preferred. However, although loop calculations can indeed be performed even in this gauge, this is not convenient, due to the fact that the unitary gauge is not manifestly renormalizable. In particular, the UV behavior of the theory is not properly manifest and appears worse than it really is. The reason is that the expression of the propagators of the massive gauge bosons in the unitary gauge

$$\frac{-i}{q^2 - m_i^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_i^2}\right) \text{ with } i = W,\, Z \tag{10}$$

does not fall to zero for large momenta $q \to \infty$.

On the other hand, R_ξ gauges are covariant gauges, where the unphysical degrees of freedom in the Higgs doublet do not disappear. However, the gauge fixing part of the Lagrangian eliminates the possibility of their mixing with gauge bosons, thus allowing the development of a perturbation theory, and assigns them a mass which is related to the one of the gauge bosons by the conditions $m_\chi = \sqrt{\xi_Z} m_Z$, $m_\phi = \sqrt{\xi} m_W$. The propagators of the vector bosons in the generalized R_ξ gauges have the expression

$$\frac{-i}{q^2 - m_i^2}\left(g_{\mu\nu} + (\xi_i - 1)\frac{q_\mu q_\nu}{q^2 - \xi_i m_i^2}\right) \text{ with } \xi_i = \xi,\, \xi_Z,\, \xi_A \quad (i = W, Z, A) \tag{11}$$

that, in the limit ξ, $\xi_Z \to \infty$, tends to eq. (10) valid in the unitary gauge. Eq. (11) however, when compared to eq. (10), has the advantage of being applicable both to massive and to massless vector bosons. Furthermore, in the limit $q \to \infty$, it goes to zero, thus the UV behaviour of the theory is not spoiled by the choice of this gauge. Actually, we worked both in the generalized R_ξ gauges and in the unitary gauge, and we explicitly verified that the R_2 effective vertices obtained in the latter have expressions far more complicated [22], as expected on the basis of the previous considerations.[2]

[1] Actually, the Fadeev-Popov ghosts do not completely decouple from the theory. As ξ, $\xi_Z \to \infty$, there are some surviving pieces (Lee-Yang terms) coming from the H-ghost-ghost vertices proportional to ξ, ξ_Z [21].

[2] When working in the Unitary gauge, at the purpose of calculating the R_2 (and the R_1) effective

4 Procedure followed in the calculation of the R_2 effective vertices and structure of the code

We worked in the SM of the EW interactions, and we calculated in an analytical way, by using FORM [23], the general R_2 contributions corresponding to all possible 1-particle irreducible graphs, with up to 4 external legs, written in terms of *generic* scalars s, vectors v and fermions f (as explained in the Introduction, we denote the graphs where generic fields, like s, v and f, appear instead of specific particles, like H, Z, e^-, etc., as *generic diagrams* throughout this paper).

The starting point for the evaluation of the R_2 contribution to each generic diagram is the corresponding R_2 integrand expression (under the integral sign in eq. (7)), built as suggested in Section 2, once fixed a gauge, on the basis of the considerations in Section 3. Feynman rules written in terms of generic s, v and f according to Denner conventions [24], enter the numerator of this integrand. The integration can then be performed analitically by using conventional techniques, involving Feynman parametrization followed by Wick rotation. The basic formulas for the angular and radial integrations are given by [4]

$$\int d\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}, \tag{12}$$

$$\int_0^\infty dq \frac{q^\beta}{(q^2+X)^\alpha} = \frac{1}{2}\frac{\Gamma\left(\frac{\beta+1}{2}\right)\Gamma\left(\alpha-\frac{\beta+1}{2}\right)}{\Gamma(\alpha)X^{\alpha-\frac{\beta+1}{2}}}. \tag{13}$$

The integration always leads to the disappearence of all divergencies involving negative powers of ϵ. One can thus safely take the limit $\epsilon \to 0$ just after it.

By following this procedure, we obtain the R_2 analytical expressions corresponding to the generic diagrams listed in the `xxxxgentop.h` files present in the package, with `xxxx` = `ss`, `vs`, `vv`, `ff`, `sff`, `vff`, `sss`, `vss`, `svv`, `vvv`, `ssss`, `ssvv` and `vvvv`. The name of each of these files refers to the nature of the external particles and the results are presented generic diagram by generic diagram (e.g. the file `vssgentop.h` includes all R_2 contributions to the 3-point functions including one vector and two scalars, generic diagram by generic diagram, where each generic diagram differs from the others according to the topology and/or to the nature of the fields internal to the loop).

Only the generic diagrams giving rise to an R_2 contribution different from zero are listed. The main criterion for establishing if a diagram can contribute or not to R_2 is power counting in the loop momentum. To every integral of the type

$$\int d^d\bar{q}\,\tilde{q}^{2l}\,\frac{q_{\mu_1}\cdots q_{\mu_{2s}}}{\bar{D}_0..\bar{D}_m} \tag{14}$$

we associated an index $d' = l + s + 1 - m$. Integrals with $d' \geq 0$ can give a contribution, while integrals with $d' < 0$ vanish [25]. By checking in each diagram the number of available loop momenta in the numerator and by counting the number of denominators, we could say beforehand if they may contribute or not. All gauges in the R_ξ class, included the 't Hooft Feynman one, have the same power counting properties. On the other hand,

vertices, we took the limit ξ, $\xi_Z \to \infty$ and $\xi_A \to 1$ before the integration, according to the observations in [22].

in the unitary gauge the situation is different and more diagrams can contribute than in the previous case, at least as for power counting, since the piece of the propagator (10) proportional to $q^\mu q^\nu$ just includes two more powers of momentum in the numerator and thus passes more easily the power counting test. This is counterbalanced by the fact that, due to the reduced number of degrees of freedom, many diagrams are absent (e.g. all those including a `ssv` vertex, since two physical s don't couple with a v and diagrams including unphysical Goldstone s are absent). Another criterion is the existence or not of terms proportional to ϵ or $\tilde{q}^2$ in $\bar{N}(\bar{q})$. In the case of ghosts for example, the loop momenta in the ghost-ghost-vector Feynman rules have an index that comes from an external vector and therefore is 4 dimensional. When such an object is contracted with a loop momentum, it cannot produce $\tilde{q}^2$ terms and thus can not contribute to R_2 terms.[3]

The generic diagrams giving a non-null R_2 contribution in the R_ξ gauges are shown in Fig. 1, 2 and 3, where each of them is presented in a single topological configuration, just for compactness. In fact, only selected topologies have been explicitly dressed to generate generic diagrams, i.e. we explicitly considered only one topology associated to each fixed number of internal and external legs, and then different topologies (and the corresponding generic diagrams) have been obtained from the first ones by proper non-cyclic permutations of the momenta of the external particles, at a fixed configuration of the internal particles. In case of external bosons, besides the momenta, even the Lorentz indices have to be properly permuted, to obtain one topology/diagram from another. The schemes showing our convention corresponding to the topologies explicitly considered for 2-leg (bubbles and tadpoles), 3-leg (triangles and bubbles) and 4-leg (boxes, triangles and bubbles) diagrams, are shown in Fig. 4.

Each R_2 contribution included in a `xxxxgentop.h` file is labelled by `EFFVERyyyyLzzzz`, where `yyyy` correspond to the *ordered* list of the generic external particles (i.e. the first particle corresponds to the p_1 momentum, the second one to p_2, etc.....), `L` stays for "loop", and `zzzz` is the *ordered* list of the generic particles running inside the loop, where the first internal particle is, by convention, the one joining external particle 1 with external particle 2, and so on, up to the last internal particle joining the last external particle to the first external one.

Furthermore, the R_2 contribution corresponding to each generic diagram is presented as an explicit FORM function of the external and internal particles. As shown in Fig. 4, we associated two labels to every internal particle, each of them in correspondence with the different vertex to which it is joined. The reason of this choice is better accomodating charged particles and fermions (an internal fermionic line corresponds to both an antifermion emerging from a vertex and to a fermion entering in another vertex), by taking into account that in each vertex all momenta are supposed to be incoming. This notation seems a little bit redundant, however we found it very useful in making clearer the code.

Furthermore, in case of fermionic loops, just for practical purposes, we explicitly distinguish two generic diagrams corresponding to the same topology, obtained one from the other at a fixed configuration of external legs, by simply changing the direction of the fermion flow (clockwise in the `ff`, `fff`, `ffff` diagrams, and anticlockwise in the `ffac`, `fffac`, `ffffac` diagrams). In case of the `ssvv` diagrams, an additional couple of

[3] Ghost loops do not contribute to R_2 terms. These loops however may contribute to the R_1 part of the amplitude, where have to be taken properly into account.

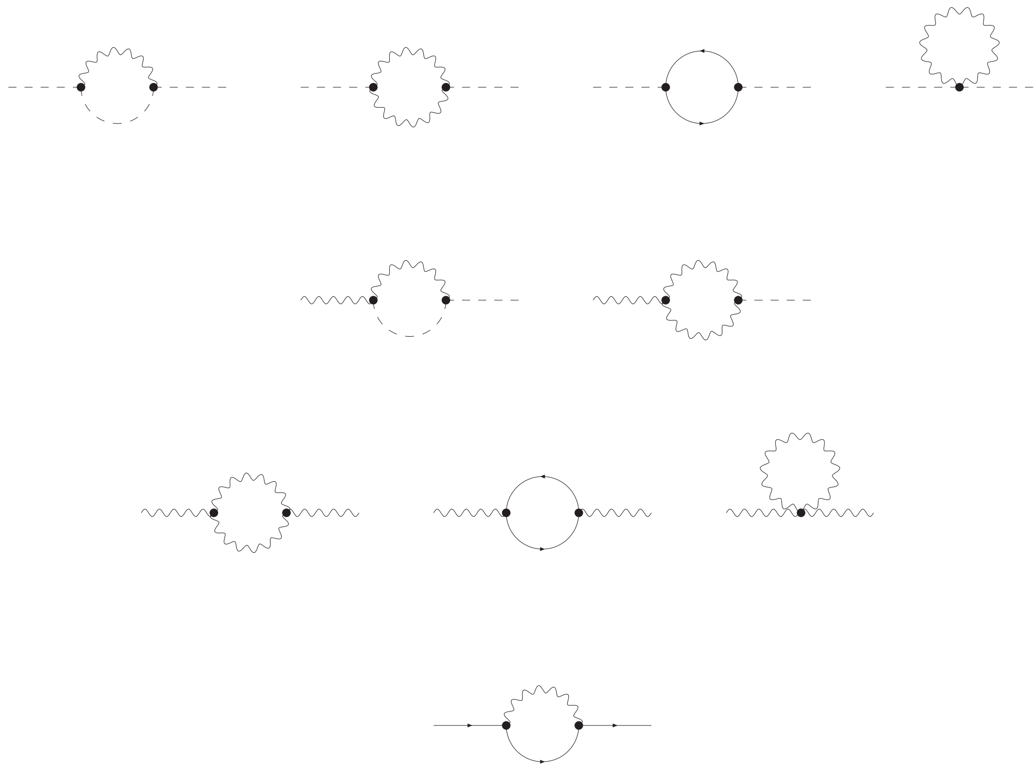

Figure 1: Non null contributions to the `ss`, `vs`, `vv` and `ff` R_2 effective vertices in the generalized R_ξ gauges, with generic finite ξ, ξ_Z, ξ_A. The corresponding analytical formulas associated to selected topologies of each generic diagram are included in the files `xxxxgentop.h`, with `xxxx` = `ss`, `vs`, `vv` and `ff`, respectively.

topologies (clockwise and anticlockwise) appears, corresponding to the exchange of the second scalar with the first vector, that has to be considered separately (see Fig. 3).

The symmetry factors have not been included at this level of the calculation. They have instead been restored at the following step, i.e. when replacing generic fields with specific particles, accomplished by means of do-loop procedures as explained below.

All global variables, functions and constants used in the package are declared in the file `variables.h`, including some explanations of their meaning. In particular all SM particles, with their mass, charge and isospin, are listed (see also Section 8 on our notations).

The expressions of all EW vertices, taken from Ref. [24], are encoded in a specific file, named `fillvertices.h`, organized in procedures. Each procedure dresses (i.e. fills) a particular generic vertex (e.g. `svv`) with real specific particles. Special care has been taken in the treatment of the `vss`, `vvv` and `vvvv` vertices. In fact, these vertices are not symmetric under the exchange of two generic particles, so one has to distinguish different possibilities. In case of generic diagrams involving `vvvv` vertices, we explicitly obtain different R_2 formulas according to the nature (charged W or neutral V) of the four vectors entering the vertex, and to their position in the diagram as external or internal particles. In general, one has to distinguish 4 cases corresponding to the same topology, as shown in Fig. 5: the case of an internal (W^+, W^-) and an external (W^+, W^-) couple, the one of an internal (W, V) and an external (W, V) couple, the one of an internal

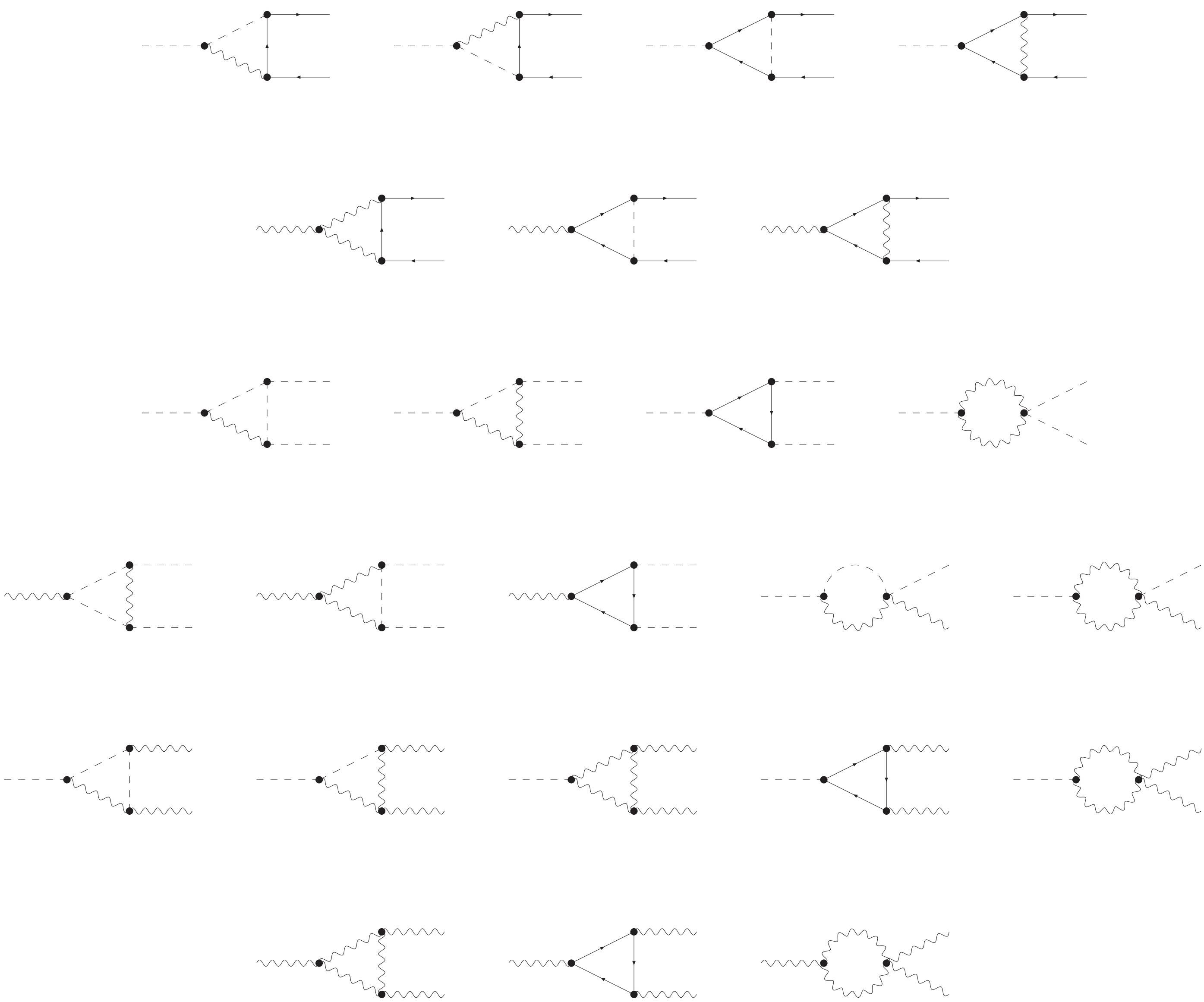

Figure 2: Non null contributions to the `sff`, `vff`, `sss`, `vss`, `svv` and `vvv` R_2 effective vertices in the generalized R_ξ gauges, with generic finite ξ, ξ_Z, ξ_A. The corresponding analytical formulas associated to selected topologies of each generic diagram are included in the files `xxxxgentop.h`, with `xxxx` = `sff`, `vff`, `sss`, `vss`, `svv` and `vvv`, respectively. For all diagrams including a `vvvv` vertex see the comments in Fig. 5.

(V, V) and an external (W^+, W^-) couple, and the one of an internal (W^+, W^-) and an external (V, V) couple. In practice, the first two cases and the second two cases can be grouped together, corresponding to the same formulas. We explicitly distinguish these possibilities in the formulas presented in the files. A further configuration has also to be taken into account in building the $W^+W^-W^+W^-$ R_2 effective vertex, corresponding to an internal (W^+, W^+) and an external (W^-, W^-) couple or viceversa.

The procedure to fill generic masses with the ones of real particles and some procedures that relate particles with their charge conjugated, just for internal use, are also included in

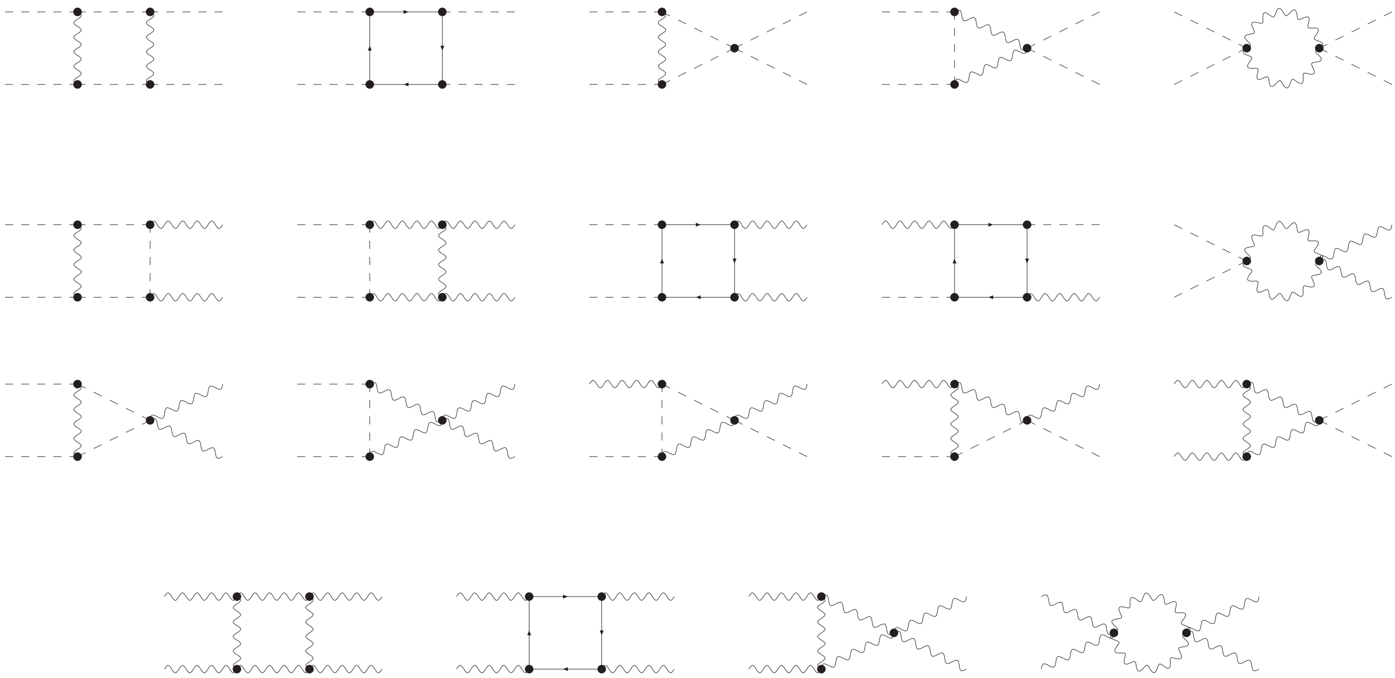

Figure 3: Non null contributions to the `ssss`, `ssvv` and `vvvv` R_2 effective vertices in the generalized R_ξ gauges, with generic finite ξ, ξ_Z, ξ_A. The corresponding analytical formulas associated to selected topologies of each generic diagram are included in the files `xxxxgentop.h`, with `xxxx` = `ssss`, `ssvv` and `vvvv`, respectively. For all diagrams including at least a `vvvv` vertex see the comments in Fig. 5.

the `fillvertices.h` file. All procedures included in the `fillvertices.h` file are valid in the 't Hooft Feynman gauge. The additional multiplicative ξ, ξ_Z factors appearing in some of the expressions of particle masses when considering the generalized R_ξ gauges instead of the 't Hooft Feynman one, are always already included directly into the R_2 formulas associated to the generic diagrams listed in the `xxxxgentop.h` files. In particular, we introduced a `csif` function, which gives the ξ parameter associated to each particle under consideration. To control its values the user is allowed to modify the procedure `fillcsi` in the `fillvertices.h` file. Further warnings about the validity and the applicability of some of these formulas are explicitly mentioned in the files.

The generic diagrams are then dressed by means of specific particles, in such a way to sum together all 1-loop contributions corresponding to the same real external particles, by considering all different non-equivalent topologies. The R_2 effective vertices are built this way, by using the do-loop procedures included in the `xxxxfeynrules.frm` files, with `xxxx`= `ss`, `vs`, `vv`, `ff`, `sff`, `vff`, `sss`, `vss`, `svv`, `vvv`, `ssss`, `ssvv`, `vvvv`.

These do-loop procedures have the advantage of leading to all results for each class of effective vertices joining specific particles in an automatic way, without the need of considering each effective vertex as a particular case, to be treated separately from the others according to the specific external particles attached, and of explicitly drawing all Feynman particle diagrams contributing to it. The most external do-loops run on the external particles, whereas the other ones run on the internal particles. All possible combinations

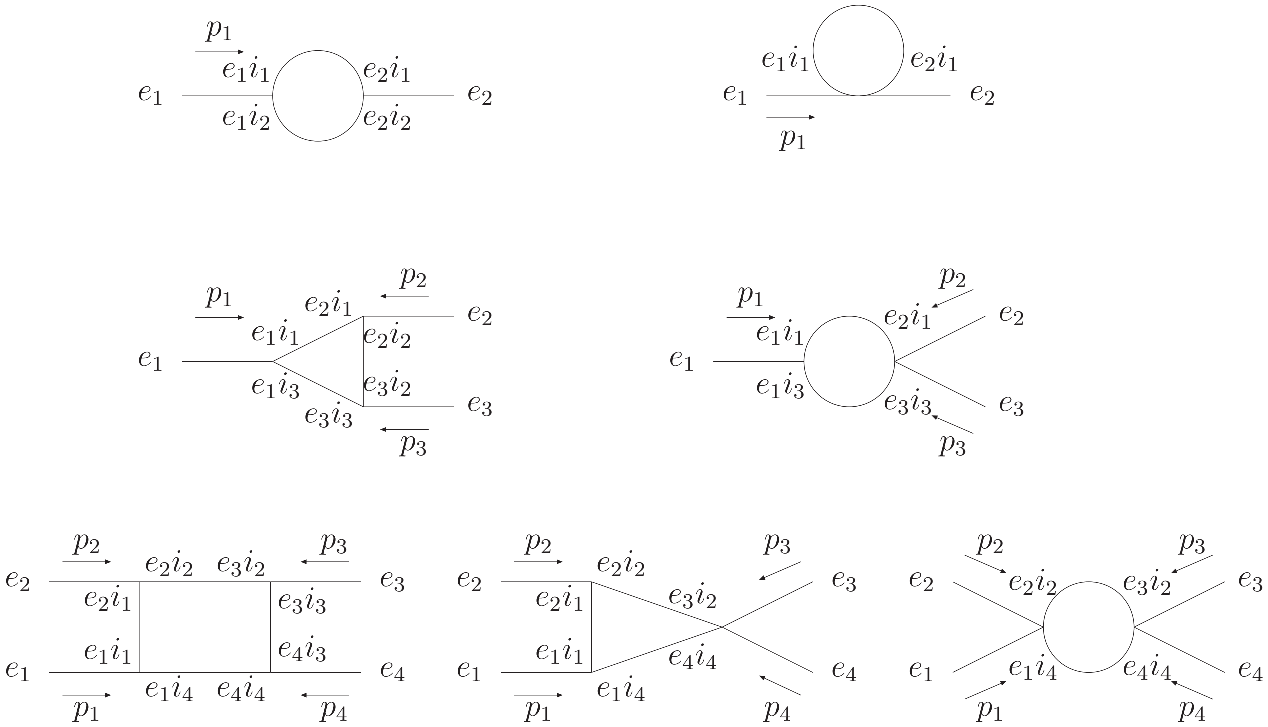

Figure 4: 2-,3- and 4-point 1-particle irreducible topologies *explicitly* considered for the calculation of the R_2 contributions included in the `xxxxgentop.h` files. The symbols e_j (`ext`*j*`fla` in our files), with j = 1,2,3,4, denote external particles. Two indices are associated to each internal particle i_k (k = 1,2,3,4), corresponding to the two vertices to which it is connected. So $e_j i_k$ (`e`*j*`int`*k*`fla` in our files) denotes the k-th internal particle *incoming* in the vertex in which also the j-th external particle is entering. According to the notation used to write down the Feynman rules, all momenta in all vertices are supposed to be incoming. Different topologies have been obtained from these ones by non-cyclic permutations of the external particles, together with their momenta and their Lorentz indices (omitted for simplicity in this figure), if present, for each fixed configuration of the internal ones.

of internal and external particles are considered at the do-loop level, then, the ones that do not exist in nature, are simply discarded due to the lack of the corresponding vertices in the `fillvertices.h` procedures. The do-loop procedures have been tested against a completely independent filling procedure, that works effective vertex by effective vertex, merely consisting in hand writing down, one by one, all diagrams contributing to each effective vertex, on the basis of the output of a FeynArts [26] computation, and summing the corresponding R_2 contributions together.

By simply changing the values of some control variables, the user has the possibility to modify the output, e.g. to study the impact of different contributions to each effective vertex joining specific particles, by selecting the box, triangle or bubble diagram contributions only, or the fermionic loop contributions only, or to choose among different gauges in the R_ξ class, in particular the 't Hooft Feynman and the Landau gauge, or among different dimensional regularization schemes, i.e. the Four Dimensional Helicity (FDH) scheme, or the 't Hooft Veltman (HV) one (different schemes differ for the treatment of the polarization vectors of the particles: in the FDH scheme all helicities are treated in

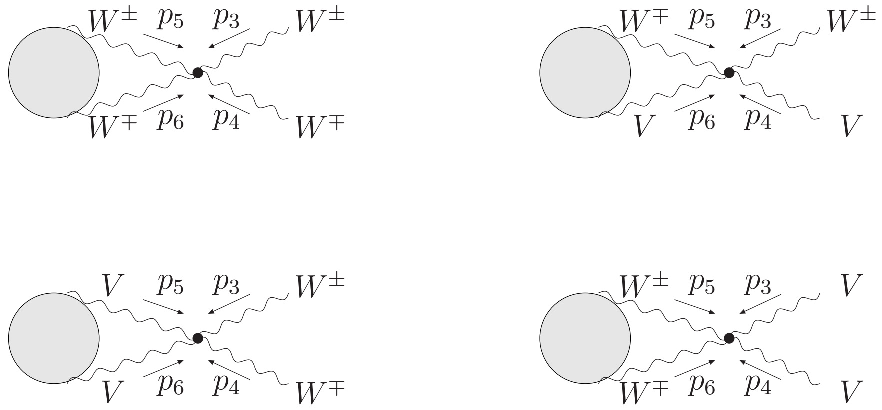

Figure 5: Diagrams in terms of generic fields, including a `vvvv` vertex, considered separately in our work. The charged vector bosons are denoted by $W^{\pm}$ whereas the neutral vector bosons are denoted by V. The first two diagrams are grouped together since they give rise to the same R_2 formulas in the `xxvvgentop.h` files, as well as the second two.

4 dimensions, in the HV scheme unobserved polarization vectors are continued to d dim, whereas the observed ones are kept in 4 dim [27]).

The particular cases here mentioned can be obtained by simply changing the values of specific variables included at the end of each do-loop procedure, as suggested in Table 1, which shows the already implemented options.

The fermionic contribution is gauge independent, i.e. it is the same in all gauges, and this is the reason why we allow its selection separately from the other contributions. The part of the results proportional to λ_{HV} is also gauge independent, i.e. the choice of a regularization scheme can be safely performed in an independent way with respect to the gauge choice.

The package is available as a zipped tar archive at the web address `http://www.ugr.es/~garzelli/R2SM`. After decompressing it, all files with a `.frm` extension can be runned by using a working installation of FORM [23], downloadable from the web at the address `http://www.nikhef.nl/~form`, that needs to be preinstalled by the user. In particular, we extensively used the version 3.2 (May 2008) of the FORM package. We verified that the same output is produced when running the code with FORM version 3.3 (August 2010), now available on the web.

choice of the dimensional regularization scheme	
parameter	*option*
`lambdahv generic`	default
`lambdahv = 0`	FDH scheme
`lambdahv = 1`	HV scheme

choice of the gauge	
parameter	*option*
`csia, csiz, csi generic`	generalized R_ξ gauges, default
`csia = csiz = csi`	1-parameter R_ξ gauges
`csia = csiz = csi = 1`	't Hooft-Feynman gauge

selection of different R_2 contributions				
parameter				*option*
`dummyb`	`dummyc`	`dummyd`	`dummyf`	
1	1	1	1	default, all contributions (boxes, triangles, bubbles) to each effective vertex are kept
1	0	0	0	only bubble contributions are selected (excluding bubble fermionic loops, if any)
0	1	0	0	only triangle contributions are selected (excluding fermionic triangles, if any)
0	0	1	0	only boxes contributions are selected (excluding fermionic boxes, if any)
0	0	0	1	only fermionic-loop contributions are selected
1	1	1	0	only non-fermionic-loop contributions are selected

Table 1: Options already implemented for the control variables the user is allowed to tune at the end of the do-loop procedures in the `xxxxfeynrul.frm` files, to select a particular gauge and regularization scheme, or to enlighten particular contributions to the R_2 effective vertices coming from specific set of diagrams.

5 A check of the gauge invariance of the R contribution to (renormalized) S-matrix elements: the H $\rightarrow \gamma\gamma$ decay

The Green functions in general depend on the gauge choice, but renormalized S-matrix elements must be gauge independent, since they correspond to physical observable quantities.

The contributions to renormalized S-matrix elements of the CC terms and of the R terms are separately gauge invariant. As a particular check of this point and of the correctness of our calculations, we have explicitly proven that the total R contribution, $\mathrm{R}_1 + \mathrm{R}_2$, to the S-matrix element corresponding to the $H \rightarrow \gamma\gamma$ physical decay process at 1-loop is gauge independent. The code we have written to produce this result is included in the subdirectory `RtotHAA` of our package. All new variables specific to R_1 calculations, are included in the `additionalvariables.h` file. The non trivial R_1 contributions to 1-loop generic diagrams including as external particles a generic scalar and two generic vectors, are presented in the `svvR1gentop.h` file. The calculation of the R_1 terms has been performed in a similar way to the one of the R_2 terms, the main difference being the fact that only the 4-dim part of the numerator $\bar{N}(\bar{q})$ can contribute to R_1. The R_1 part of the tensor integrals involved has been extracted by applying the Passarino-Veltman reduction technique [1] in a straightforward way, by disregarding the contribution of the scalar functions A, B, C, D which, by definition, already contribute to the CC part of the amplitude. R_1 effective vertices corresponding to specific `svv` configurations involving physical external particles, can be obtained by dressing the generic diagrams mentioned above with specific particles, thanks to a do-loop procedure, and by conveniently summing the corresponding contributions together, in a way analogous to the one followed to build the R_2 effective vertices. The do-loop procedure we have used to calculate the $\mathrm{R} = \mathrm{R}_1 + \mathrm{R}_2$ effective vertices for all specific `svv` combinations is presented in the file `svvRfeynrules.frm`. Finally, in the file `haa.frm`, we show that the R contribution to the S-matrix element of the $H\gamma\gamma$ decay process is gauge invariant, i.e. the result is independent of any gauge parameter (and it remains the same even in the limit ξ, $\xi_Z \to \infty$ and $\xi_A \to 1$: we have repeated this calculation in the unitary gauge, making these limits before the integration, and obtained the same final result for the R part of the S-matrix element, also in agreement with Ref. [28]).

6 Conclusions

We have presented an analytical package, written in FORM and available on the web, to calculate the R_2 effective Feynman rules by means of which one can compute the R_2 contribution to 1-loop amplitudes for whichever process in the SM of the EW interactions. As recommended in the literature, we have chosen to work in the R_ξ class of gauges, identified by finite values of the ξ, ξ_Z, ξ_A parameters, due to the good suitability of this class to loop calculations, since the UV behaviour of the theory is not spoiled by the particular expression of the propagators of the massive gauge bosons in this class. We have explicitly verified that this general recommendation also apply to the calculation of the R_2 contribution alone, to 1-loop amplitudes: even if, from the technical point of view, analytical calculations of the R part of the amplitude can be performed as well in the unitary gauge, the R_ξ gauges have by far better properties in terms of the compactness and the simplicity of the final R_2 analytical formulas with respect to the unitary gauge.

We have also offered a check of the gauge-invariance of the total R contribution, $\mathrm{R}_1 + \mathrm{R}_2$, to the renormalized S-matrix element for the $H\gamma\gamma$ process.

The package is modular, and different pieces can be conveniently reintegrated in other calculations. The user is allowed to play with some parameters, in order to specify a particular gauge in the R_ξ class or a particular dimensional regularization scheme, and to enlighten different partial contribution to the R_2 effective vertices. These options allow everybody to reproduce, as particular cases, some of the results presented in our previous papers [22, 29], in a straightforward way.

Finally, we think that our effort can be considered a first seed towards the more complex task of automatically recovering, given a Lagrangian for whichever model of particle interactions, all R_2 effective Feynman Rules for the theory at hand.

7 Acknowledgments

We are grateful to R. Pittau and R. Kleiss for many useful discussions and suggestions. The work of M. V. G. was supported by the italian INFN, the work of I. M. was supported by the RTN European Programme MRTN-CT-2006-035505 (HEPTOOLS, Tools and Precision Calculations for Physics Discoveries at Colliders). M. V. G. also acknowledges her partecipation in the MEC Project FPA2008-02984, thanks to which a brief stay of Y. M. at the University of Granada, crucial for developing this research together, was possible.

8 Appendix: Notation

We closely follow the conventions in the paper [24]. We adopt the Bjorken-Drell or "mostly minus" metric convention $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$.

We consider 1-loop amplitudes with at most 4 external legs. The momenta of the external particles are denoted by p_1, p_2, p_3, p_4 and are all supposed to be incoming. In the 2 point effective vertices, the Lorentz indices associated to the external bosons whose momenta are p_1 and $p_2(= -p_1)$, are respectively denoted by α and δ. In the 3 point effective vertices, the Lorentz indices associated to the external bosons whose momenta are $p_1(= -p2 - p3)$, p_2 and p_3, are denoted by τ, ω and χ, respectively. Finally, in the 4 point effective vertices, the Lorentz indices associated to the external bosons whose momenta are p_1, p_2, p_3 and $p_4(= -p1 - p2 - p3)$ are denoted by α, β, τ and χ.

We worked in the generalized R_ξ gauge, by allowing 3 different parameters ξ_A, ξ_Z and ξ (`csia`, `csiz` and `csi` in our files) in the gauge fixing term (usually taken to be equal in the standard R_ξ gauge). The λ_{HV} (`lambdahv` in our files) dependence, identifying the dimensional regularization scheme, has been made explicit in all results. We checked that the contribution proportional to λ_{HV} for each effective vertex is always invariant with respect to gauge transformations.

In our FORM files, we labelled the particles of the EW sector of the SM according to the notation of Table 2. Actually, we limited to consider only one weak-isospin family of quarks and one weak-isospin family of leptons. Anyway, the results can be extended to three families in a straightforward way, by just working on the fermionic part of the do-loop procedures in all files with a `.frm` extension. The number of quark colors can be fixed by the user by a proper choice of the value of the variable `ncol` at the end of each do-loop procedure. The contribution of fermionic loops is independent of the gauge choice, since all propagators and coupling constants involving fermions are the same in all gauges. Thus, for the parts of our final results including fermion loops, and for their generalization to three weak-isospin families, the reader can safely refer to the analytical formulas already presented in our previous paper [29].

The mass of each SM particle is denoted by adding an "`m`" before the name of the particle (e.g. `mhsca` is the mass of the Higgs scalar, `mwvec` is the mass of the W gauge boson). The pole masses of the unphysical scalars χ and $\phi^{\pm}$ are reduced to the masses

Particle content of the EW sector of the SM		
generic particle	*specific particles*	*notation in our* FORM *files*
scalars s	H, χ, ϕ^+, ϕ^-	`hsca, chisca, phiplussca, phimensca`
vectors v	A, Z, W^+, W^-	`avec, zvec, wplusvec, wmenvec`
ghosts fp	u_A, u_Z, u_W^+, u_W^- $\bar{u}_A$, $\bar{u}_Z$, $\bar{u}_W^+$, $\bar{u}_W^-$	`gavec, gzvec, gwplusvec, gwmenvec` `gavecbar, gzvecbar, gwplusvecbar, gwmenvecbar`
charged leptons f	e, μ, τ $\bar{e}$, $\bar{\mu}$, $\bar{\tau}$	`ele, muele, tauele` `elebar, muelebar, tauelebar`
neutral leptons f	ν_e, ν_μ, ν_τ $\bar{\nu}_e$, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$	`nuele, numuele, nutauele` `nuelebar, numuelebar, nutauelebar`
up-type quarks f	u, c, t $\bar{u}$ $\bar{c}$, $\bar{t}$	`uquark, cquark, tquark` `ubarquark, cbarquark, tbarquark`
down-type quarks f	d, s, b $\bar{d}$ $\bar{s}$, $\bar{b}$	`dquark, squark, bquark` `dbarquark, sbarquark, bbarquark`

Table 2: Notation used in this work for the particle content of the EW sector of the SM. The Faddeev-Popov-De Witt ghosts are only relevant for the calculation of the R_1 terms, whereas all other particles enter in the calculation of both the R_2 and the R_1 contributions to 1-loop amplitudes.

of the corresponding massive gauge bosons by $m_\chi = \sqrt{\xi_Z} m_Z$ and $m_\phi = \sqrt{\xi} m_W$. We also widely apply the relation $m_Z = m_W / \cos\theta_W$, and thus express our results in terms of m_W. The sine and cosine of the Weinberg angle are denoted by `sinwei` and `coswei`, respectively, and the e coupling constant is denoted by `eem`.

The third component of the weak-isospin of each fermion is denoted by including the prefix "`i3`" before the name of the particle: `i3nuele` (= 1/2) corresponds to the ν_e, `i3ele` (= -1/2) corresponds to the e^-, `i3u` (= 1/2) and `i3d` (= -1/2) correspond to the u and d quark quantum numbers, respectively.

The projector operators $\Omega_+ = \frac{1+\gamma_5}{2}$ and $\Omega_- = \frac{1-\gamma_5}{2}$ are denoted by `omegaplus` and `omegaminus`, respectively. As for the treatment of γ_5, the reader can refer to the comments already presented in Ref. [29].

References

[1] G. Passarino and M. J. G. Veltman, Nucl. Phys. B **160** (1979) 151.

[2] A. Denner, S. Dittmaier, Nucl. Phys. B **734** (2006) 62 - 115 [arXiv:hep-ph/0509141]; A. Denner, S. Dittmaier, [arXiv:1005.2076 [hep-ph]].

[3] T. Binoth, J.P. Guillet, G. Heinrich, E. Pilon and T. Reiter, Comput. Phys. Commun. **180** (2009) 2317 - 2330 [arXiv:0810.0992 [hep-ph]]; G. Heinrich, G. Ossola, T. Reiter and F. Tramontano, [arXiv:1008.2441 [hep-ph]].

[4] G. 't Hooft and M. Veltman, Diagrammar, CERN Report 73-9, Geneva, Switzerland (1973).

[5] Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower, Nucl. Phys. B **425** (1994) 217 [arXiv:hep-ph/9403226] and Nucl. Phys. B **435** (1995) 59 [arXiv:hep-ph/9409265]; Z. Bern, L. J. Dixon and D. A. Kosower, Nucl. Phys. B **513** (1998) 3 [arXiv:hep-ph/9708239].

[6] R. Britto, F. Cachazo, B. Feng, Nucl. Phys. B **725** (2005) 275 [arXiv:hep-th/0412103].

[7] J. R. Andersen, J. Archibald, S. Badger *et al.*, Summary Report of the SM and NLO Multileg Working Group, Proc. of the Les Houches Workshop "Physics at TeV Colliders", les Houches, France, 8 - 26 June 2009 [arXiv:1003.1241 [hep-ph]].

[8] C. F. Berger, Z. Bern, L. J. Dixon *et al.* [arXiv:1009.2338 [hep-ph]], Phys. Rev. Lett. **102** (2009) 222001 [arXiv:0902.2760 [hep-ph]], Phys. Rev. D **80** (2009) 074036 [arXiv:0907.1984 [hep-ph]] and [arXiv:1004.1659 [hep-ph]];
R.K. Ellis, K. Melnikov and G. Zanderighi, Phys. Rev. D **80** (2009) 094002 [arXiv:0906.1445 [hep-ph]];
G. Bevilacqua, M. Czakon, C. G. Papadopoulos, R. Pittau and M. Worek, JHEP **0909** (2009) 109 [arXiv:0907.4723 [hep-ph]];
A. Bredenstein, A. Denner, S. Dittmaier, S. Pozzorini, Phys. Rev. Lett. **103** (2009) 012002 [arXiv:0905.0110 [hep-ph] and [arXiv:1001.4006v1 [hep-ph]];
K. Melnikov, G. Zanderighi, Phys. Rev. D **81** (2010), 074025 [arXiv:0910.3671 [hep-ph]];
T. Binoth, N. Greiner, A. Guffanti *et al.*, Phys. Lett. B **685** (2010) 293 - 296 [arXiv:0910.4379 [hep-ph];
G. Bevilacqua, M. Czakon, C.G. Papadopoulos, M. Worek, Phys. Rev. Lett. **104** (2010) 162002 [arXiv:1002.4009 [hep-ph]];
A. Denner, S. Dittmaier, T. Gehrmann, C. Kurz, [arXiv:1003.0986 [hep-ph];
T. Melia, K. Melnikov, R. Rontsch, G. Zanderighi, [arXiv:1007.5313 [hep-ph]];
R. Frederix, S. Frixione, K. Melnikov, G. Zanderighi, [arXiv:1008.5313 [hep-ph]].

[9] Z. G. Xiao, G. Yang and C. J. Zhu, Nucl. Phys. B **758** (2006) 1 - 34 [arXiv:hep-ph/0607015] and Nucl. Phys. B **758** (2006) 53 - 89 [arXiv:hep-ph/0607017].

[10] T. Binoth, J. P. Guillet and G. Heinrich, JHEP **0702** (2007) 013 [arXiv:hep-ph/0609054]. A. Bredenstein, A. Denner, S. Dittmaier and S. Pozzorini, JHEP **0808** (2008) 108 [arXiv:0807.1248 [hep-ph]].

[11] C.F. Berger, D. Forde, Ann. Rev. Nucl. Part. Sci. (2010) [arXiv:0912.3534 [hep-ph]].

[12] Z. Bern, L. J. Dixon and D. A. Kosower, Phys. Rev. D **71**, 105013 (2005) [arXiv:hep-th/0501240], Phys. Rev. D **72**, 125003 (2005) [arXiv:hep-ph/0505055] and Phys. Rev. D **73**, 065013 (2006) [arXiv:hep-ph/0507005];
C. F. Berger, Z. Bern, L. J. Dixon, D. Forde and D. A. Kosower, Phys. Rev. D **74**, 036009 (2006) [arXiv:hep-ph/0604195] and Phys. Rev. D **75**, 016006 (2007) [arXiv:hep-ph/0607014].

[13] C. Anastasiou, R. Britto, B. Feng, Z. Kunszt and P. Mastrolia, Phys. Lett. B **645** (2007) 213 [arXiv:hep-ph/0609191] and JHEP**0703** (2007) 111 [arXiv:hep-ph/0612277];
W. T. Giele, Z. Kunszt and K. Melnikov, JHEP **0804** (2008) 049 [arXiv:0801.2237 [hep-ph]];
R. K. Ellis, W. T. Giele, Z. Kunszt and K. Melnikov, Nucl. Phys. B **822** (2009) 270 [arXiv:0806.3467 [hep-ph]];
P. Mastrolia, G. Ossola, T. Reiter, F. Tramontano, JHEP **1008** (2010) 080 [arXiv:1006.0710 [hep-ph]].

[14] G. Ossola, C. G. Papadopoulos and R. Pittau, Nucl. Phys. B **763** (2007) 147 [arXiv:hep-ph/0609007].

[15] G. Ossola, C. G. Papadopoulos and R. Pittau, JHEP **0805** (2008) 004 [arXiv:0802.1876 [hep-ph]].

[16] A. van Hameren, C. G. Papadopoulos and R. Pittau, JHEP **0909** (2009) 106 [arXiv:0903.4665 [hep-ph]];
G. Bevilacqua, M. Czakon, M.V. Garzelli *et al.*, Proc. of the 10^{th} DESY Workshop on Elementary Particle Theory: Loops and Legs in Quantum Field Theory, Worlitz, Germany, April 25 - 30 2010, [arXiv:1007.4918 [hep-ph]].

[17] P. Draggiotis, M. V. Garzelli, C. G. Papadopoulos and R. Pittau, JHEP **0904** (2009) 072 [arXiv:0903.0356 [hep-ph]].

[18] A. van Hameren, JHEP **0907** (2009) 088 [arXiv:0905.1002 [hep-ph]].

[19] T. Kinoshita, J. Math. Phys. **3** (1962) 650; T. D. Lee and M. Nauenberg, Phys. Rev. **133** (1964) 1549.

[20] G. Ossola, C. G. Papadopoulos and R. Pittau, JHEP **0707** (2007) 085 [arXiv:0704.1271 [hep-ph]].

[21] T. D. Lee and Chen-Ning Yang, Phys. Rev. **128** (1962) 885;
C. Grosse-Knetter and R. Kogerler, Phys. Rev. **D48** (1993) 2865 [arXiv:hep-ph/9212268].

[22] M. V. Garzelli, I. Malamos and R. Pittau, [arXiv:1009.4302 [hep-ph]].

[23] J. A. M. Vermaseren, [arXiv:math-ph/0010025], Nucl. Phys. Proc. Suppl. **183** (2008) 19 [arXiv:0806.4080 [hep-ph]] and Proc. of the 10^{th} DESY Workshop on Elementary Particle Theory: Loops and Legs in Quantum Field Theory, Worlitz, Germany, April 25 - 30 2010, [arXiv:1006.4512 [hep-ph]].

[24] A. Denner, Fortsch. Phys. **41** (1993) 307 [arXiv:0709.1075 [hep-ph]].

[25] F. del Aguila, R. Pittau, JHEP **07** (2004) 017 [arXiv:hep-ph/0404120].

[26] T. Hahn and M. Perez-Victoria, Comput. Phys. Commun. **118** (1999) 153 [arXiv:hep-ph/9807565];
T. Hahn, Comput. Phys. Commun. **140** (2001) 418 [arXiv:hep-ph/0012260], http://www.feynarts.de/

[27] Z. Bern, D. Kosower, Nucl. Phys. B **379** (1992), 451 - 561.

[28] D. Y. Bardin and G. Passarino, Clarendon, Oxford, U.K.(1999), p. 493 - 494.

[29] M. V. Garzelli, I. Malamos and R. Pittau, JHEP **1001** (2010) 040 [arXiv:0910.3130 [hep-ph]].