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Precise measurement of the top-quark mass from lepton + jets data at D0


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I. INTRODUCTION

The observation of the top quark in 1995 [1,2] confirmed the existence of the six quarks in three generations of fermions expected in the standard model (SM) of particle interactions. Because of its mass, the lifetime of the top quark is much shorter than the time scale of hadronization. The large mass of the top quark (making it the only quark whose characteristics can be studied in isolation), the existence of the six quarks in three generations of fermions expected in the standard model (SM) of particle interactions, and the corresponding to a Yukawa coupling to the Higgs boson, also provides a useful constraint on contributions from physics beyond the standard model [4].

The statistical uncertainty on the world average value of $m_t$ is 0.3%, and the accuracy of the measurement of $m_t$ is now dominated by systematic uncertainties [5]. The main systematic contributions arise from uncertainties on the jet energy calibration and on the Monte Carlo (MC) simulation of $t\bar{t}$ events.

We present a new measurement of the mass of the top quark based on 2.6 fb$^{-1}$ of integrated luminosity from $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV, collected with the D0 detector at the Fermilab Tevatron Collider. The analysis focuses on $t\bar{t}$ events identified in lepton + jets ($\ell +$ jets) final states (with $\ell$ representing either an electron or a muon) [6], in which the top and antitop quark are assumed to decay into...

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a $W$ boson and $b$ quark [7], with one of the $W$ bosons in the $W^+W^-$ $b\bar{b}$ final system decaying via $W \rightarrow \ell\nu$ into a lepton and neutrino and the other via $W \rightarrow q\bar{q'}$ into two quarks, and all four quarks ($q\bar{q}'b\bar{b}$) into jets. Such events are characterized by an isolated electron or muon with large transverse momentum ($p_T$), an undetected neutrino that causes a large imbalance in transverse momentum, and four high-$p_T$ jets. In selecting candidate events, we exploit this distinct signature, which helps distinguish these events from background.

Compared to the previous measurement based on data corresponding to $1$ fb$^{-1}$ of integrated luminosity [8], we use a larger data set and an improved evaluation of systematic uncertainties. The analysis uses the same matrix element (ME) analysis technique, with an in situ jet energy calibration based on constraining the invariant mass of the two jets from the decay of the $W$ boson to the world average value of $M_W = 80.4$ GeV [9]. As in the previous measurement, the standard jet energy scale (JES), derived from $\gamma +$ jet and dijet data samples, is used as an additional constraint and implemented through a Gaussian prior on its absolute value and uncertainty. A major improvement in this new measurement is the significant reduction of the uncertainty associated with the modeling of differences in the calorimeter response to $b$-quark and light-quark jets originating from the introduction of a new flavor-dependent jet energy response correction.

This measurement, like all direct measurements of $m_t$, relies on MC $t\bar{t}$ events for absolute calibration. It is therefore important to understand the precise definition of the input mass $m_{t_{\text{gen}}}$ in MC $t\bar{t}$ event generators, such as ALPGEN [10] and PYTHIA [11], used to calibrate the direct measurements. Although $m_{t_{\text{gen}}}$ is not well defined in leading order (LO) generators that use parton showers to model higher-order effects and hadronization, it has been argued that $m_{t_{\text{gen}}}$ should be viewed as being close to the pole mass [12]. In Ref. [13], the D0 Collaboration has extracted $m_t$ from a comparison of the measured $t\bar{t}$ production cross section with predictions from higher-order quantum chromodynamics (QCD) by equating $m_{t_{\text{gen}}}$ both with the pole mass ($m_{t_{\text{pole}}}$) and with the $\overline{\text{MS}}$ mass ($m_{t_{\overline{\text{MS}}}}$). The extracted $m_t$ under the assumption $m_{t_{\text{gen}}} = m_{t_{\text{pole}}}$, is found to agree with the average value of $m_t$ from the Tevatron, while the $m_t$ extracted assuming $m_{t_{\text{gen}}} = m_{t_{\overline{\text{MS}}}}$ is found to be different from the average value of $m_t$. These results favor the pole mass interpretation of $m_{t_{\text{gen}}}$.

This paper is arranged as follows. A brief description of the D0 detector is given in Sec. II, which is followed by a discussion of the selection and reconstruction of the physical objects in this analysis in Sec. III. Section IV summarizes the MC samples used to simulate the events of interest, and Sec. V discusses the technique used to extract the value of $m_t$. This is followed by a description of the calibration of the response of the analysis method in Sec. VI and a discussion of the flavor-dependent jet-response correction used to bring the simulation of the calorimeter response to jets into agreement with data in Sec. VII. The result of the calibration is applied to the data in Sec. VIII, where the measured value of $m_t$ and its statistical uncertainty are also presented. Section IX describes the evaluation of systematic uncertainties and the final result is given in Sec. X. We combine this new measurement in Sec. XI with an updated version of that from Ref. [8] in which the flavor-dependent jet-response correction mentioned above has been applied and the systematic uncertainties have been updated.

II. THE D0 DETECTOR

The D0 detector consists primarily of a magnetic central tracking system, calorimetry, and a muon system. The central tracking system comprises a silicon microstrip tracker (SMT) and a central fiber tracker (CFT), both located within a 1.9 T superconducting solenoidal magnet [14]. The SMT [15] has $\approx 800,000$ individual strips, with typical pitch of 50–80 $\mu$m, and a design optimized for track and vertex finding at $|\eta| < 2.5$, where the pseudorapidity $\eta = -\ln(\tan(\theta/2))$, and is the polar angle with respect to the proton beam direction relative to the center of the detector. The system has a six-barrel longitudinal structure, each with a set of four layers arranged axially around the beam pipe and interspersed with 16 radial disks. In 2006, a fifth layer, referred to as Layer 0, was installed close to the beam pipe [16,17]. The CFT has eight thin coaxial barrels, each supporting two doublets of overlapping scintillating fibers of 0.835 mm diameter, one doublet being parallel to the collision axis and the other alternating by $\pm 3^\circ$ relative to the axis. Light signals are transferred via clear fibers to solid-state photon counters (VLPCs) that have 80% quantum efficiency.

Central and forward preshower detectors, located just outside of the superconducting coil (in front of the calorimetry), are constructed of several layers of extruded triangular scintillator strips that are read out using wavelength-shifting fibers and the VLPC. These detectors provide initial sampling of electromagnetic showers, and thereby help distinguish incident photons from electrons. The next layer of detection involves three liquidargon/uranium calorimeters: a central section (CC) covering $|\eta|$ up to $\approx 1.1$ and two end calorimeters (EC) that extend coverage to $|\eta| \approx 4.2$, all housed in separate cryostats. The electromagnetic (EM) section of the calorimeter is segmented into four layers, with transverse segmentation of the cells in pseudorapidity and azimuth of $\Delta \eta \times \Delta \phi = 0.1 \times 0.1$, except for the third layer, where the segmentation is $0.05 \times 0.05$. The hadronic portion of the calorimeter is located after the EM sections, and consists of fine hadron-sampling layers, followed by more coarse hadronic layers. In addition, scintillators between the CC and EC cryostats provide sampling of developing showers for $1.1 < |\eta| < 1.4$. A muon system [18] is located beyond
the calorimetry and consists of a layer of tracking detectors and scintillation trigger counters before 1.9 T toroids, followed by two similar layers after the toroids. Tracking for $|\eta| < 1$ relies on 10 cm wide drift tubes, while 1 cm mini-drift tubes are used for $1 < |\eta| < 2$.

Luminosity is measured using plastic scintillator arrays located in front of the EC cryostats, covering $2.7 < |\eta| < 4.4$. The trigger and data acquisition systems are designed to accommodate the high instantaneous luminosities of the Tevatron [14,19]. Based on preliminary information from tracking, calorimetry, and muon systems, the output of the first level of the trigger is used to limit the rate for accepted events to 2 kHz. At the next trigger stage, with more refined information, the rate is reduced further to 1 kHz. These first two levels of triggering rely mainly on hardware and firmware. The third and final level of the trigger, with access to all of the event information, uses software algorithms and a computing farm and reduces the output rate to 100 Hz, which is written to tape.

III. OBJECT RECONSTRUCTION AND EVENT SELECTION

In the following sections, we summarize how the physical objects in data and MC events are reconstructed from information in the detector and the criteria applied to these objects to select the $\ell + jets \bar{t}t$ candidate events.

A. Object reconstruction

This section describes the reconstruction of electrons, muons, missing transverse momentum, and jets, and the identification of $b$ jets.

1. Identification of electrons

Electron candidates are defined by narrow clusters of energy deposited in towers of the electromagnetic calorimeter located within a cone of radius $R = \sqrt{\Delta \eta^2 + (\Delta \phi)^2} = 0.2$, where $\Delta \eta$ is the pseudorapidity and $\Delta \phi$ is the azimuthal angle of each cluster relative to the seed cluster. At least 90% of the total energy measured within this cone is required to be located within the electromagnetic section to be consistent with expectations for electromagnetic showers. Isolation from energy deposited by hadrons is imposed by requiring $(E_{\text{tot}} - E_{\text{EM}})/E_{\text{EM}} < 0.15$, where $E_{\text{tot}}$ ($E_{\text{EM}}$) is the total (electromagnetic) energy in a cone of radius $R = 0.4$ ($R = 0.2$). Candidate electrons are required to have longitudinal and transverse shower profiles compatible with those of electromagnetic showers and to be spatially matched to a track reconstructed in the central tracking system. Electron candidates meeting these criteria are referred to as loose electrons. Finally, (i) the value of a multivariable likelihood discriminant based on tracking system and calorimeter information is required to be consistent with that for an electron, and (ii) a neural network, trained using information from the tracking system, calorimeter, and central preshower detector is used to further reject background from jets misidentified as electrons. Electron candidates meeting these criteria are referred to as tight electrons, and are those used to obtain the final selection.

2. Identification of muons

Muons are identified by requiring a minimum number of wire and scintillator hits on both sides of the toroidal magnets in the muon detector [18]. Cosmic ray background is rejected by requiring scintillator signals consistent in time with muons originating from the $p\bar{p}$ collision. Tracks in the muon system are required to match a reconstructed track in the central tracker having a small impact parameter (par.) with respect to the $p\bar{p}$ interaction vertex (PV) to reject muons from cosmic rays and decays in flight of kaons and pions. Muon candidates must also be isolated from jets with $p_T > 15$ GeV by requiring a separation in $\eta-\phi$ space between the muon and jet of $\Delta R(\mu, \text{jet}) > 0.5$ [20]. Candidates satisfying these requirements are referred to as loosely isolated muons. The following two variables are used to impose additional isolation requirements: $E_{\text{halo}}^{\text{scaled}}$ is defined as the ratio of calorimeter energy within an annulus of $0.1 < R < 0.4$ around the muon direction to the $p_T$ of the muon; $p_{T,\text{cone}}^{\text{scaled}}$ is defined as the ratio of the total $p_T$ of all tracks within a cone of $R = 0.5$, excluding the muon, to the $p_T$ of the muon. Muon candidates meeting all the requirements above that satisfy $E_{\text{halo}}^{\text{scaled}} < 0.12$ and $p_{T,\text{cone}}^{\text{scaled}} < 0.12$ are referred to as veto muons. Further tightening these requirements to $E_{\text{halo}}^{\text{scaled}} < 0.08$ and $p_{T,\text{cone}}^{\text{scaled}} < 0.06$ selects candidates referred to as tightly isolated muons.

3. Measurement of the imbalance in transverse momentum

We use the conservation of momentum to measure the momentum imbalance in the transverse plane ($p_T$). From that, we infer the presence of the neutrino. The $p_T$ is determined from the vector sum of the energies of all cells in the electromagnetic and hadronic calorimeters. Subsequent energy corrections applied to reconstructed objects such as jets and muons are also propagated to the missing transverse momentum.

4. Identification of jets

Jet candidates are reconstructed using the iterative midpoint cone algorithm with a cone radius of $R = 0.5$ [21]. Only calorimeter cells with energies that are 2.5 standard deviations above the mean of the noise distribution are considered in the reconstruction. Isolated cells with energies less than 4 standard deviations above the mean of the noise distribution are also discarded. Among the jet candidates with $p_T > 8$ GeV, the following selection criteria are
imposed. The electromagnetic fraction of the jet energy is required to be below 0.95 to reject electrons and above 0.05 to suppress jets dominated by noise from the hadronic part of the calorimeter. Jets with a large fraction of their energy deposited in the coarse hadronic layers of the calorimeter are rejected to suppress jets dominated by noise typical for those layers. To minimize background from jet candidates arising from noise in the precision readout of the calorimeter, confirmation from the readout system of the first level trigger is required for reconstructed jets. Jets matched to loose electrons with $p_T > 20$ GeV and $\Delta R(e, \text{jet}) < 0.5$ [20] are also rejected. Energies of jets containing muons are corrected with the measured muon momentum after accounting for the typical energy deposited by a minimum ionizing particle.

The energy of a reconstructed jet is corrected, on average, to that of a particle jet [22] containing the final-state particles within a cone of radius $R = 0.5$ corresponding to the reconstructed jet. The first step involves the subtraction of the offset energy due to calorimeter noise and contributions from previous and following beam crossings and multiple interactions within the same beam crossing. This is followed by an absolute response correction determined from $\gamma + \text{jet}$ events and a relative $\eta$-dependent correction based on $\gamma + \text{jet}$ and dijet events. Finally, a showering correction is applied to account for the lateral leakage of energy across the jet cone boundary.

5. Identification of $b$ jets

The lifetime of the $b$ quark, unlike that of the top quark, is far longer than the time-scale for hadronization. This means that, during QCD evolution, the $b$ quark can form short lived $b$ hadrons that travel $\geq 1$ mm before decaying through the weak interaction. We identify the $b$ jets among the candidates satisfying the jet selection criteria described in the previous section by using a neural network (NN) $b$-tagging algorithm that selects jets with displaced vertices and tracks relative to the PV [23]. The NN tagger is based on nine input variables that can be separated into two categories. The first category is related to the reconstructed secondary vertex and includes the vertex quality, the number of associated tracks, the invariant mass of the vertex, the number of secondary vertices reconstructed within the jet, the spatial separation between the jet axis and the position vector of the secondary vertex relative to the PV, and the length of the flight path projected on the transverse plane divided by its uncertainty (which provides a measure of the decay length significance in terms of standard deviations). The second category relies only on the characteristics of the tracks within the jets such as impact parameters, transverse momentum, and track quality. The $b$-jet candidates are also required to have at least two good quality tracks originating from the PV. The tagging efficiency for $b$ jets is $\approx 65\%$ for a misidentification rate of $\approx 3\%$ for $u, d, s$ quark, or gluon jets [24].

IV. MONTE CARLO SAMPLES

The MC events used to model the $t\bar{t}$ signal and the $W + \text{jets}$ background needed for the calibration of the measurement (described in Sec. VI) are generated using ALPGEN [10] to simulate the hard-scattering process and PYTHIA [11] to simulate hadronization and shower evolution. The MLM matching scheme [25] is employed to avoid overlaps between components of the event belonging to the hard process, implemented through a matrix element, and parton evolution (showering) into jets. The $W + \text{jets}$ background samples are divided into two categories: (i) $W + l\bar{p}$ and (ii) $W + (c\bar{c}, l\bar{p})$ and $W + (b\bar{b}, l\bar{p})$, where $l\bar{p}$ (light partons) denotes $u, d, s$ quarks, or gluons. Although the individual processes are produced with ALPGEN which is a LO generator, the relative contributions
between the two categories are determined using next-to-leading order (NLO) calculations, with next-to-leading logarithmic (NLL) corrections based on the MCFM MC generator [26]. The MC samples used to derive jet transfer functions that correlate jet energies with those of partons in the \( t\bar{t} \) events (described in Sec. V B 1) are generated using PYTHIA to simulate both the hard-scattering process and the subsequent hadronization and shower evolution for the events. All MC samples are generated with CTEQ6L1 parton distribution functions (PDFs) [27] and passed through a full GEANT3-based [28] simulation of the DØ detector. To simulate the effects from additional \( p\bar{p} \) interactions, events with no trigger requirements selected from random \( p\bar{p} \) crossings in the collider data having the same instantaneous luminosity profile as the data are overlayed on the fully simulated MC events. This is then followed by the same reconstruction and analysis chain as applied to data.

V. METHOD OF ANALYSIS

The top-quark mass is measured using all kinematic information with a likelihood technique based on probability densities (PDs) constructed, for each individual event, from matrix elements of the processes contributing to the observed final state. This analysis technique, referred to as the matrix-element (ME) method [29], is described below.

A. Matrix-element method

If the processes contributing to an observed event do not interfere, the total PD for observing a given event is the sum of all contributing probabilities for that specific final state. Assuming that \( t\bar{t} \) and \( W + \) jets production are the only two contributions, the PD for observing each event is given in terms of the top-quark mass \( m_t \), the jet energy scale factor \( k_{\text{JES}} \) dividing the energies of all jets, and the fractions of \( t\bar{t} \) signal \((f)\) and of \( W + \) jets background \((1 - f)\) in the data by

\[
P_{\text{evi}} = A(x) [f P_{\text{sig}}(x; m_t, k_{\text{JES}}) + (1 - f) P_{\text{bkg}}(x; k_{\text{JES}})],
\]

where \( x \) represents the measured jet and lepton energies and angles; \( A(x) \), which depends only on \( x \), accounts for the geometrical acceptance and efficiencies; and \( P_{\text{sig}} \) and \( P_{\text{bkg}} \) are the PDs for \( t\bar{t} \) and \( W + \) jets production, respectively. For events satisfying \( P_{\text{bkg}} \gg P_{\text{sig}} \), the relative contribution of \( P_{\text{sig}} \) to \( P_{\text{evi}} \) is negligible and has minimal influence on the determination of \( m_t \). Multijet events satisfy this condition and can therefore be represented by \( P_{\text{bkg}} \) as the event kinematics are far closer to \( W + \)jets than to \( t\bar{t} \) production.

Because of the finite detector resolution and the hadronization process, the measured set \( x \) for the observed events will not, in general, be identical to the corresponding set \( y \) of the original final-state partons and the relationship between \( x \) and \( y \) is described by a transfer function. In addition, the initial partons carry momenta \( q_1 \) and \( q_2 \) in the colliding \( p \) and \( \bar{p} \). To account for this complication, \( P_{\text{sig}} \) and \( P_{\text{bkg}} \) must be integrated over all parton states contributing to the observed set \( x \). This involves a convolution of the partonic differential cross section \( d\sigma(y) \) with the PDFs and a transfer function \( W(x; y; k_{\text{JES}}) \) that relates \( x \) and \( y \):

\[
P(x, \alpha) = \frac{1}{\sigma(\alpha)} \int \sum_{\text{flavors}} d\sigma(y, \alpha) dq_1 dq_2 f(q_1) 
\times f(q_2) W(x; y; k_{\text{JES}}),
\]

where \( \sigma(\alpha) \) represents the parameters to be determined in the analysis, the sum runs over all possible initial-state parton flavors, and \( f(q_i) \) are CTEQ6L1 PDFs for finding a parton of a given flavor and longitudinal momentum fraction \( q_i \) in the \( p \) or \( \bar{p} \). Detector resolution is taken into account in \( W(x; y; k_{\text{JES}}) \), representing the probability density for the measured set \( x \) to have arisen from the partonic set \( y \). Dividing by the total observed cross section for the process, \( \sigma(\alpha) \), ensures \( P(x; \alpha) \) is normalized to unity.

The differential cross section term for \( P_{\text{sig}} \) is calculated using the LO ME of the quark-antiquark annihilation process \((\mathcal{M}_q)\). A total of 24 integration variables are associated with the two initial-state partons and the six particles in the final state. Since the angles for the four jets and the charged lepton are sufficiently well measured, the angular resolution terms in \( W(x; y; k_{\text{JES}}) \) can be approximated by Dirac \( \delta \) functions. Integrating over these and four more \( \delta \) functions that impose energy-momentum conservation leaves 10 integrals to evaluate the probability density that represents the \( t\bar{t} \) production probability for a given \( m_t \) and \( k_{\text{JES}} \) [30]:

\[
P_{\text{sig}} = \frac{1}{\sigma_{\text{obs}}} \sum_{i=1}^{24} w_i \int d\rho dm_1^2 dm_3^2 dm_2^2 d\rho_1 dq_1^2 dq_2^2 dq_3^2 dq_4^2 dq_5^2 dq_6^2 \sum_{\text{flavors},r} |\mathcal{M}_q|^2 \frac{f'(q_1)f'(q_2)}{\sqrt{(\eta_{\alpha r} q_1^a q_2^b)^2 - m_{q_1}^2 m_{q_2}^2}} \Phi_6 W(x; y; k_{\text{JES}}).
\]

where, in addition to the CTEQ6L1 PDF given by \( f(q_i) \), the \( f'(q_i) \) also include parameterizations of the probability distributions for the transverse momenta \( q_{iX}^Y \) of the initial-state partons in PYTHIA [11]. The masses of the initial-state partons are given by \( m_{q_i} \), and \( \Phi_6 \) includes the 6-body phase-space factor and other constants. The first sum is over all 24 jet permutations, each carrying a weight \( w_i \), which is the product of four jet weights. The weight for
a $b$-tagged jet with a given $p_T$ and $\eta$ is the average tagging efficiency $\epsilon_a(p_T, \eta)$ for a given parton hypothesis $\alpha(= b, c, lightq, or gluon)$. The weight for a jet that is not $b$-tagged is $1 - \epsilon_a(p_T, \eta)$. The second sum includes up to eight solutions for neutrino kinematics and conservation of transverse momentum used to calculate the transverse momentum of the neutrino. The parameter $\rho$ represents the fraction of the energy carried by one of the quarks from the $W \to q\bar{q}$ decay. The masses of the two $W$ bosons ($M_1, M_2$) and of the pair of top quarks ($m_1, m_2$) are chosen as integration variables because of computational efficiency related to the four Breit-Wigner mass terms that make the ME negligible everywhere except at the mass peaks. The energy ($1/p_T$) of the electron (muon) is defined by $\rho_\mu$. The integration over $q_i$ involves only transverse components. $W(x, y; k_{\text{RES}})$ is the product of five terms for the four jets and one charged lepton, described below. The normalization $\sigma_\beta^2 = \int A(x)P_{\text{sig}} \, dx = \sigma^2(x) \times (A(m_i, k_{\text{RES}}))$ is calculated from the product of the total cross section corresponding to the ME used and the mean acceptance for events whose dependences on $m_t$ and $k_{\text{RES}}$ are determined from MC events. The mean acceptance is shown in Fig. 1 as a function of $m_t$ for different values of $k_{\text{RES}}$.

The differential cross section in $P_{\text{bkg}}$ is calculated using the $W + 4$ jets matrix elements from the VECBOS [31] MC program. The initial-state partons are assumed to have no transverse momenta. The integration is performed over the $W$ boson mass, the energy ($1/p_T$) of the electron (muon), and the energies of the four partons producing the jets, summing over the 24 jet permutations and all neutrino solutions.

The top-quark mass is extracted from $n$ events with a measured set of variables $\tilde{x} = (x_1, x_2, \ldots, x_n)$ through a likelihood function for individual event probabilities $P_{\text{evt}}$ according to

$$L(\tilde{x}; m_t, k_{\text{RES}}, f) = \prod_{i=1}^{n} P_{\text{evt}}(x_i; m_t, k_{\text{RES}}, f).$$

For every assumed pair of $(m_t, k_{\text{RES}})$ values, the value of $f$ that maximizes the likelihood is determined. To obtain the best estimate of $m_t$ and $k_{\text{RES}}$, the two-dimensional likelihood

$$L(\tilde{x}; m_t, k_{\text{RES}}) = L[\tilde{x}; m_t, k_{\text{RES}}, f_{\text{best}}(m_t, k_{\text{RES}})]$$

is projected onto the $m_t$ and $k_{\text{RES}}$ axes according to

$$L(\tilde{x}; m_t) = \int L(\tilde{x}; m_t, k_{\text{RES}})G(k_{\text{RES}})dk_{\text{RES}}$$

and

$$L(\tilde{x}; k_{\text{RES}}) = \int L(\tilde{x}; m_t, k_{\text{RES}})dm_t,$$

using Simpson’s rule [32], where the prior probability distribution $G(k_{\text{RES}})$ is a Gaussian function centered at $k_{\text{RES}} = 1$ with standard deviation (sd) 0.02 determined from the mean of the fractional uncertainty of the standard jet energy scale corrections applied to all jets in the MC samples used in this analysis. The best estimates and the uncertainties on the mass of the top quark and the jet energy scale are then extracted using the mean and the rms of $L(\tilde{x}; m_t)$ and $L(\tilde{x}; k_{\text{RES}})$, respectively. The mean is calculated from $\tilde{m}_t = \int m_t L(\tilde{x}; m_t) \, dm_t$ and the rms from $\tilde{\sigma}^2(m_t) = \int (m_t - \tilde{m}_t)^2 L(\tilde{x}; m_t) \, dm_t$, where $m_t$ corresponds to $m_t$ or $k_{\text{RES}}$, also using Simpson’s rule.

The fit parameter $k_{\text{RES}}$, associated with the in situ jet energy calibration, has the effect of rescaling the energies of all the jets and thereby the 2-jet invariant mass of the hadronically decaying $W$ boson, with the jet energy scale factor $k_{\text{RES}}$. The presence of the Breit-Wigner mass term associated with the hadronically decaying $W$ boson in the ME of Eq. (3) maximizes the likelihood in Eq. (4) when the 2-jet invariant mass coincides with the Breit-Wigner pole fixed at the world average of $M_W = 80.4$ GeV [9]. The additional constraint to the standard scale derived from $\gamma +$ jet and dijet samples is applied through the prior probability distribution $G(k_{\text{RES}})$ in Eq. (6).

B. Detector resolution

In this section, we describe the parameterizations for the jet and electron energy and muon $p_T$ resolutions used in the transfer function $W(x, y; k_{\text{RES}})$ which is the product of four jet transfer functions for a given jet permutation and an electron or muon transfer function.

1. Parameterization of jet energy resolution

The transfer function for jets, $W_{\text{jj}}(E_x, E_y; k_{\text{RES}})$, represents the probability that a measured jet energy $E_x$ in the detector corresponds to a parent quark of energy $E_y$. It is parameterized in terms of a double Gaussian function whose means and widths are dependent on $E_y$. For the case $k_{\text{RES}} = 1$, it is given by

![FIG. 1 (color online). The mean acceptance as a function of $m_t$ and $k_{\text{RES}}$ for the (a) $e^+ +$ jets and (b) $\mu^+ +$ jets channels.](image-url)
TABLE I. Transfer function parameters for light quarks ($a_i$ in GeV).

| Par. | $|\eta| < 0.5$ | $0.5 < |\eta| < 1$ | $1 < |\eta| < 1.5$ | $1.5 < |\eta| < 2.5$ |
|------|----------------|-----------------|-----------------|-----------------|
| $a_1$ | $-2.74 \times 10^0$ | $-8.02 \times 10^{-1}$ | $1.69 \times 10^{-1}$ | $1.52 \times 10^1$ |
| $b_1$ | $1.67 \times 10^{-2}$ | $-3.59 \times 10^{-3}$ | $1.32 \times 10^1$ | $-2.17 \times 10^{-1}$ |
| $a_2$ | $5.44 \times 10^0$ | $5.40 \times 10^0$ | $-3.26 \times 10^{-1}$ | $3.34 \times 10^0$ |
| $b_2$ | $6.29 \times 10^{-2}$ | $8.46 \times 10^{-2}$ | $6.97 \times 10^0$ | $1.45 \times 10^{-1}$ |
| $a_3$ | $4.30 \times 10^{-4}$ | $4.80 \times 10^{-4}$ | $2.52 \times 10^{-2}$ | $4.06 \times 10^{-3}$ |
| $b_3$ | $1.54 \times 10^1$ | $2.00 \times 10^1$ | $4.71 \times 10^0$ | $1.72 \times 10^1$ |
| $a_4$ | $-2.12 \times 10^{-1}$ | $-2.38 \times 10^{-1}$ | $-8.37 \times 10^{-3}$ | $-3.69 \times 10^{-2}$ |
| $b_4$ | $1.77 \times 10^1$ | $-2.38 \times 10^{-1}$ | $1.03 \times 10^1$ | $1.75 \times 10^1$ |
| $a_5$ | $1.96 \times 10^{-1}$ | $1.89 \times 10^1$ | $6.42 \times 10^{-2}$ | $5.34 \times 10^{-2}$ |

where the $p_i$ are functions of the quark energy for quark $i$ and are parameterized as linear functions of the $E_y$:

$$p_i = a_i + E_y \cdot b_i.$$  

The parameters $a_i$ and $b_i$ are determined from fully simulated $t\bar{t}$ events, following all jet energy corrections and smearing to match resolutions in data. These events are generated with PYTHIA at nine values of the top-quark mass ranging from 155 to 195 GeV in 5 GeV intervals. The parton and jet energies are used in an unbinned likelihood fit that minimizes the product of the $W_{\text{jet}}$ terms for each event with respect to $a_i$ and $b_i$. A different set of parameters is derived (i) for three varieties of quarks: light quarks ($u, d, s, c$), $b$ quarks with a soft muon tag in the jet [33], and all other $b$ quarks, and (ii) for four $\eta$ regions: $|\eta| < 0.5$, $0.5 < |\eta| < 1.0$, $1.0 < |\eta| < 1.5$, and $1.5 < |\eta| < 2.5$, to minimize possible effects due to nonuniform calorimeter response. The values for these parameters are shown in Tables I and II for light-quark and $b$-quark jets, respectively. Figure 2 illustrates the transfer functions for light-quark jets as a function of $E_y$ for different values of $E_y$. In Fig. 3, we compare the 2-jet and 3-jet invariant mass distributions for two types of PYTHIA $t\bar{t}$ + jets events: (i) parton-level events with jet energies smeared using the transfer functions and (ii) fully simulated events where all four reconstructed jets are matched to partons with $\Delta R$ (parton, jet) < 0.5. The 2-jet (3-jet) invariant masses are calculated using the two light-quark jets (all three jets) from the hadronic branch of the $t\bar{t}$ + jets events and correspond to the $W$ boson (top-quark) mass. The overlaid distributions in Fig. 3 indicate that the jet transfer functions describe the jet resolutions well.

For $k_{\text{JES}} \neq 1$, the jet transfer function is changed to

$$W_{\text{jet}}(E_x, E_y; k_{\text{JES}}) = \frac{W_{\text{jet}}(E_x, E_y; 1)}{k_{\text{JES}}},$$

where the $k_{\text{JES}}$ factor in the denominator preserves the normalization $\int W_{\text{jet}}(E_x, E_y; k_{\text{JES}}) dE_x = 1$.

TABLE II. Transfer function parameters for $b$-quark jets without and with a muon within the jet cone ($a_i$ in GeV).

<table>
<thead>
<tr>
<th>$b$-quark jets without a muon within the jet cone</th>
<th>$b$-quark jets with a muon within the jet cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\eta</td>
</tr>
<tr>
<td>$a_1$</td>
<td>$3.30 \times 10^0$</td>
</tr>
<tr>
<td>$b_1$</td>
<td>$-2.13 \times 10^{-1}$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$5.02 \times 10^0$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>$1.73 \times 10^{-1}$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$3.48 \times 10^{-2}$</td>
</tr>
<tr>
<td>$b_3$</td>
<td>$-6.68 \times 10^{-2}$</td>
</tr>
<tr>
<td>$a_4$</td>
<td>$2.38 \times 10^{-2}$</td>
</tr>
<tr>
<td>$b_4$</td>
<td>$5.06 \times 10^0$</td>
</tr>
<tr>
<td>$a_5$</td>
<td>$4.71 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
2. Parameterization of energy resolution for electrons

The electron energy resolution is parameterized by the transfer function

$$W_e(E_x, E_y) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2} \left( \frac{E_x - E_y'}{\sigma} \right)^2 \right].$$ (11)

where $E_x$ is the reconstructed electron energy,

$$E_y' = 1.000 \cdot E_x + 0.324 \text{ GeV},$$ (12)

$$\sigma = \sqrt{(0.028 \cdot E_y')^2 + (S \cdot E_y')^2 + (0.4 \text{ GeV})^2}.$$ (13)

FIG. 2 (color online). Transfer functions for $k_{\text{JES}} = 1$ light-quark jets as a function of measured jet energy for different parton energies in $\eta$ regions: (a) $|\eta| < 0.5$, (b) $0.5 < |\eta| < 1.0$, (c) $1.0 < |\eta| < 1.5$, and (d) $1.5 < |\eta| < 2.5$.}

FIG. 3 (color online). Comparison of (a) 2-jet and (b) 3-jet invariant mass distributions for parton-level $t\bar{t}$ MC events with energies smeared using the transfer functions (open histogram) and fully simulated $t\bar{t}$ MC events with all four jets spatially matched to partons (filled histogram).

3. Parameterization of momentum resolution for muons

We describe the resolution of the central tracker through the uncertainty on the signed curvature of a track, the ratio of the electric charge and of the transverse momentum of a particle, parameterized as a function of pseudorapidity. The muon transfer function is parameterized as

$$W_\mu(k_x, k_y) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2} \left( \frac{k_x - k_y}{\sigma} \right)^2 \right].$$ (16)

where $k_x = (q/p_T)$, and $k_y = (q/p_T)$, with the charge $q$ and transverse momentum $p_T$ of the original muon ($y$) or its reconstructed track ($x$). The resolution

$$\sigma = \begin{cases} \bar{\sigma} & \text{for } |\eta| \leq 1.4 \\ \sqrt{\bar{\sigma}^2 + (c \cdot (|\eta| - 1.4))^2} & \text{for } |\eta| > 1.4 \end{cases}$$ (17)

is obtained from muon tracks in simulated events where the $\bar{\sigma}$ and $c$ parameters are linear functions of $1/p_T$:

$$\bar{\sigma} = \bar{\sigma}_0 + \bar{\sigma}_1 \cdot 1/p_T,$$ (18)

$$c = c_0 + c_1 \cdot 1/p_T.$$ (19)

The values of the coefficients are given in Table III for muon tracks with associated and no associated hits in the silicon tracker. This simplified parameterization of the momentum resolution is valid at high transverse momenta ($p_T > 20$ GeV) where the limitations in coordinate resolution dominate over the effects of multiple scattering.

VI. CALIBRATION OF THE MEASUREMENT

The fully simulated MC samples described in Sec. IV are used in ensemble studies to calibrate the result from the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>With hits in the SMT</th>
<th>No hits in the SMT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$ (GeV$^{-1}$)</td>
<td>$2.082 \times 10^{-3}$</td>
<td>$3.620 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>$1.125 \times 10^{-2}$</td>
<td>$1.388 \times 10^{-2}$</td>
</tr>
<tr>
<td>$c_0$ (GeV$^{-1}$)</td>
<td>$7.668 \times 10^{-3}$</td>
<td>$2.070 \times 10^{-2}$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>$7.851 \times 10^{-2}$</td>
<td>$7.042 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

$$S = \frac{0.164 \text{ GeV}^{12}}{\sqrt{E_y'}} + \frac{0.122 \text{ GeV}^{12}}{E_y'} e^{C/\sin \theta_e} - C,$$ (14)

$$C = 1.3519 - \frac{2.0956 \text{ GeV}}{E_y'} - \frac{6.9858 \text{ GeV}}{E_y'^2}.$$ (15)

$E_y$ is the energy of the original electron, and $\theta_e$ is the polar angle of the electron with respect to the proton beam direction. The parameters above are derived from the detailed modeling of electron energy response and resolution used in Ref. [34].
ME method by determining and correcting for biases in the extracted parameters and their estimated uncertainties. Such biases can be due, for example, to limitations in the LO ME used in Eq. (3) or to the imperfect description of detector resolution using transfer functions with a limited number of parameters. Five $t\bar{t}$ MC samples are generated for $m_t^{\text{gen}} = 165, 170, 172.5, 175,$ and $180$ GeV, with two more produced from the $172.5$ GeV sample by rescaling all jet energies by $\pm 5\%$. $P_{\text{sig}}$ and $P_{\text{bkg}}$ are calculated for these samples and for the $W + \text{jets}$ MC samples. Events are drawn randomly from a $t\bar{t}$ sample with a particular mass and the $W + \text{jets}$ sample to form pseudoexperiments, each with a number of events equal to the one observed in data (before requiring $\geq 1$ $b$-tagged jets), with the signal fraction fluctuated according to a binomial distribution relative to that determined from data. The values of $m_t$ and $k_{\text{JES}}$ are extracted for each pseudoexperiment according to the procedure described in Sec. VA using only events with at least one $b$-tagged jet. A thousand pseudoexperiments are performed for each of the 7 $t\bar{t}$ samples. The means (and their uncertainties) of all 1000 measured values of $m_t$ and $k_{\text{JES}}$ in each sample are determined from Gaussian fits to their distributions and plotted versus the input $m_t^{\text{gen}} - 172.5$ GeV and $k_{\text{JES}} - 1$, respectively. A straight line is fitted to the plotted points, representing the response function used to correct the measurement from data (Fig. 4). For each pseudoexperiment, we also calculate the pulls, defined as $(m_t - \langle m_t \rangle) / \sigma(m_t)$ and $(k_{\text{JES}} - \langle k_{\text{JES}} \rangle) / \sigma(k_{\text{JES}})$, where $\langle m_t \rangle$ and $\langle k_{\text{JES}} \rangle$ are the mean measured $m_t$ and $k_{\text{JES}}$, respectively, for all pseudoexperiments, and $\sigma(m_t)$ and $\sigma(k_{\text{JES}})$ are the rms of $m_t$ and $k_{\text{JES}}$, respectively, for the given pseudoexperiment. The width of the pull distributions for $m_t$ and $k_{\text{JES}}$ are shown as a function of $m_t^{\text{gen}}$ and $k_{\text{JES}}^{\text{gen}}$ in Fig. 5. The average widths of the $m_t$ and $k_{\text{JES}}$ pull distributions are 1.08 and 1.07, respectively.

The signal fraction for the ensemble studies is determined from the selected data sample using the method described in Sec. VA. To correct for biases in the determination of this fraction, a calibration is done using the $W + \text{jets}$ and 172.5 GeV $t\bar{t}$ MC samples, wherein 1000 pseudoexperiments are performed using the same procedure as described in the previous paragraph but with signal fractions set to a different value in each test. The extracted signal fractions as a function of their input values are shown in Figs. 6(a) and 6(b) for the $e + \text{jets}$ and $\mu + \text{jets}$ channels, respectively. Straight lines are fitted to the points in plots representing the response functions used to correct the fractions determined from the selected data sample. The calibration of the signal fraction is performed separately for the $e + \text{jets}$ and $\mu + \text{jets}$ channels. The corrected fractions are $0.35 \pm 0.05$ and $0.41 \pm 0.06$ for the $e + \text{jets}$ and $\mu + \text{jets}$ channels, respectively, prior to requiring at least one $b$-tagged jet. These fractions are $0.71 \pm 0.05$ and $0.75 \pm 0.04$ for the $e + \text{jets}$ and $\mu + \text{jets}$ channels, respectively, after requiring at least one $b$-tagged jet.

**VII. FLAVOR-DEPENDENT JET-RESPONSE CORRECTION FOR MC EVENTS**

The validity of the calibration procedure described in the previous section is based on the assumption of a perfect MC simulation of the events and of the detector. Uncertainties in this assumption are discussed in the section on systematic uncertainties (Sec. IX). The in situ jet energy scale employed in this analysis can account for a global scale discrepancy between data and MC jet energy...
scales (see Sec. IX B 1) by rescaling the energies of the two light jets from the $W \rightarrow q\bar{q}'$ decay of $t\bar{t}$ events to the world average mass of the $W$ boson [9]. This same rescaling is also applied to the two $b$ jets in the event. However, jets originating from different partons have different kinematic characteristics and particle compositions. In particular, $b$ and light jets with different electromagnetic fractions can lead to different responses in a noncompensating calorimeter. Such features, if not properly simulated, can result in a systematic shift in the determination of the top-quark mass. In fact, the largest contribution to the total systematic uncertainty of our previous analysis in Ref. [8] is the $b$/light-quark response ratio which was an estimate of the effect of such a discrepancy.

To bring the simulation of the calorimeter response to jets into agreement with data, and thereby reduce the systematic uncertainty associated with a jet-response difference in data and MC, we determine a flavor-dependent correction factor as follows. We note a discrepancy in the predicted energy deposition in the calorimeter between data and MC when we apply the single-particle responses from data and MC to the individual particles within MC jets that are spatially matched to reconstructed jets [35]:

$$D = \frac{\sum E_i \cdot R^\text{Data}_i}{\sum E_i \cdot R^\text{MC}_i},$$

where the sums run over each particle $i$ in the MC particle jet. $E_i$ is the true energy of particle $i$, and $R^\text{Data}_i$ and $R^\text{MC}_i$ are the single-particle responses in data and MC, respectively. We define a correction factor for a jet of flavor $\beta$ (light quark, gluon or $b$ quark) as the ratio of the discrepancy for jets of flavor $\beta$ to the flavor-averaged discrepancy for jets in $\gamma + \text{jet}$ events, $F^\beta_{\text{cor}} = D^\beta / (D^{\gamma + \text{jet}})$. Defining the correction this way preserves the standard MC jet energy scale that is, strictly speaking, only appropriate for the $\gamma + \text{jet}$ events from which it is derived. At the same time, it brings the relative response difference between jets of flavor $\beta$ and jets in $\gamma + \text{jet}$ events in MC into agreement with that in data. The quantity $F^\beta_{\text{cor}} - 1$ is shown in Fig. 7 as a function of jet $p_T$ and $\eta$ for light-quark, gluon, and $b$ jets. The shaded band at $F^\beta_{\text{cor}} - 1 = 0$ in each plot corresponds to the correction for jets in $\gamma + \text{jet}$ events. We apply these correction factors to the light-quark jets and $b$ jets in a $t\bar{t}$ MC sample generated with $m_{t\bar{t}}^{\text{gen}} = 172.5$ GeV, extract $m_t$ and $k_{JES}$ using our analysis technique, and compare them with the values extracted...
from the same set of events without using this correction. We find shifts of $\Delta m_t = -1.26$ GeV and $\Delta k_{\text{JES}} = 0.005$ relative to the uncorrected sample. Repeating this study on a $t\bar{t}$ MC sample appropriate for the previous analysis [8] yields shifts of $\Delta m_t = -1.28$ GeV and $\Delta k_{\text{JES}} = 0.005$.

**VIII. MEASUREMENT OF THE TOP-QUARK MASS**

The likelihoods $L(\tilde{m}; m_t)$ and $L(\tilde{m}; k_{\text{JES}})$ for the selected data, calculated according to Eq. (6) and (7), respectively, are calibrated by replacing $m_t$ and $k_{\text{JES}}$ by parameters fitted to the response plots of Sec. VI:

$$m_t^{\text{calib}} = \frac{(m_t - 172.5 \text{ GeV}) - p_0^m}{p_1^m} + 172.5 \text{ GeV},$$

$$k_{\text{JES}}^{\text{calib}} = \frac{(k_{\text{JES}} - 1) - p_0^{k_{\text{JES}}}}{p_1^{k_{\text{JES}}}} + 1,$$

where $p_i^m$ and $p_i^{k_{\text{JES}}}$ are the parameters of the $m_t$ and $k_{\text{JES}}$ response functions shown in Fig. 4(a) and 4(b), respectively, and $m_t$ and $k_{\text{JES}}$ and their uncertainties are extracted from the mean and rms values of the calibrated likelihoods shown in Figs. 8(a) and 8(b). The extracted uncertainties for $m_t$ and $k_{\text{JES}}$ are multiplied by 1.08 and 1.07, respectively, to correct for deviations of the average pull widths from unity (see Sec. VI). Figure 9 shows the fitted Gaussian contours of equal probability for the two-dimensional likelihood $L(\tilde{m}; m_t, k_{\text{JES}})$.

Distributions in expected uncertainties, determined from 1000 pseudoexperiments performed on the MC $t\bar{t}$ sample for $m_t^{\text{gen}} = 175$ GeV. The measured uncertainties are indicated by the arrows.

**IX. SYSTEMATIC UNCERTAINTIES**

We evaluate systematic uncertainties for three categories. The first category, *modeling of production*, addresses uncertainties in the MC modeling of $t\bar{t}$ and $W +$ jets production. The second category, *modeling of detector*, deals with the uncertainties in jet energy and lepton momentum scales and the simulation of detector response and any associated efficiencies. The third category involves uncertainties in the calibration of both $m_t$ and the signal fraction $f$, and a possible bias from the exclusion of multijet events in MC ensemble studies. The contributions to the systematic uncertainty are summarized in Table IV. In the first three sections below, we describe the evaluation of
TABLE IV. Summary of systematic uncertainties.

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modeling of production:</td>
<td></td>
</tr>
<tr>
<td>Modeling of signal:</td>
<td></td>
</tr>
<tr>
<td>Higher-order effects</td>
<td>±0.25</td>
</tr>
<tr>
<td>ISR/FSR</td>
<td>±0.26</td>
</tr>
<tr>
<td>Hadronization and UE</td>
<td>±0.58</td>
</tr>
<tr>
<td>Color reconnection</td>
<td>±0.28</td>
</tr>
<tr>
<td>Multiple $p\bar{p}$ interactions</td>
<td>±0.07</td>
</tr>
<tr>
<td>Modeling of background</td>
<td>±0.16</td>
</tr>
<tr>
<td>$W +$ jets heavy-flavor scale factor</td>
<td>±0.07</td>
</tr>
<tr>
<td>Modeling of $b$ jets</td>
<td>±0.09</td>
</tr>
<tr>
<td>Choice of PDF</td>
<td>±0.24</td>
</tr>
<tr>
<td>Modeling of detector:</td>
<td></td>
</tr>
<tr>
<td>Residual jet energy scale</td>
<td>±0.21</td>
</tr>
<tr>
<td>Data-MC jet-response difference</td>
<td>±0.28</td>
</tr>
<tr>
<td>$b$-tagging efficiency</td>
<td>±0.08</td>
</tr>
<tr>
<td>Trigger efficiency</td>
<td>±0.01</td>
</tr>
<tr>
<td>Lepton momentum scale</td>
<td>±0.17</td>
</tr>
<tr>
<td>Jet energy resolution</td>
<td>±0.32</td>
</tr>
<tr>
<td>Jet identification efficiency</td>
<td>±0.26</td>
</tr>
<tr>
<td>Method:</td>
<td></td>
</tr>
<tr>
<td>Multijet contamination</td>
<td>±0.14</td>
</tr>
<tr>
<td>Signal fraction</td>
<td>±0.10</td>
</tr>
<tr>
<td>MC calibration</td>
<td>±0.20</td>
</tr>
<tr>
<td>Total</td>
<td>±1.02</td>
</tr>
</tbody>
</table>

each of these contributions in more detail. In the fourth section, we discuss how systematic uncertainties from the previous analysis of 1 fb$^{-1}$ of integrated luminosity [8] are updated to facilitate the combination of the two results presented in Sec. XI. Except for the Data-MC jet-response difference described in Sec. IX B 2, all of the systematic uncertainties described below are calculated prior to the flavor-dependent jet-response corrections of Sec. VII.

We adopt the following convention for systematic uncertainties $\delta m_t$ in $m_t$ and classify them into two types. The first type, referred to as the Type I uncertainty, is the effect of the ±1 sd variation of a relevant quantity. The second type, referred to as the Type II uncertainty, is due to the difference between models. For Type I uncertainties, we refer to the central or default value of the measurement as $m_t^0$ and to the measurement corresponding to the 1 sd ($\pm 1$ sd) variation as $m_t^{+1}$ ($m_t^{-1}$). We compute Type I uncertainties according to $\delta m_t = |m_t^+ - m_t^-|/2$ if $m_t^+ < m_t^0 < m_t^-$, or $m_t^+ < m_t^0$ or $m_t^- < m_t^0$ and according to $\delta m_t = \max(|m_t^+ - m_t^0|/2,|m_t^- - m_t^0|/2)$ if $m_t^+ - m_t^0$ and $m_t^- - m_t^0$ have the same sign. We compute Type II uncertainties by taking the maximal difference between the models as the + and − systematic variations.

Many of our systematic uncertainties are evaluated by comparing two MC $t\bar{t}$ samples generated with the same input mass $m_t^{\text{gen}}$. For these studies, we use samples with a value of $m_t^{\text{gen}}$ close to the world average of $m_t$ such as 172.5 GeV or 170 GeV.

A. Modeling of production

1. Higher-order effects

The MC $t\bar{t}$ samples used to calibrate our measurement are generated using ALPGEN for the hard-scattering process and PYTHIA for shower evolution and hadronization (Sec. IV). We compare the LO generator ALPGEN with the next-to-leading order MC generator MC@NLO [36], in order to evaluate possible contributions from higher-order effects such as additional radiation of hard jets or gg contributions. We compare ALPGEN and MC@NLO MC $t\bar{t}$ samples with identical values of $m_t^{\text{gen}}$ that both use HERWIG [37] for shower evolution and hadronization. HERWIG is used in both cases for consistency because MC@NLO can only be used with HERWIG (ALPGEN can be used with PYTHIA or HERWIG) and we are not interested in comparing different models for shower evolution and hadronization in this study. Ensemble studies are performed on both samples and the difference in the mean extracted $m_t$ from ensembles for the two samples is found to be $m_t^{\text{MC@NLO}} - m_t^{\text{ALPGEN}} = 0.10 \pm 0.25$ GeV. Here, as in all the other systematic sources described below, when a shift in the value of the estimated parameter is statistically dominated, we replace the shift with its statistical uncertainty for the estimate of uncertainty. We, therefore, assign an uncertainty of ±0.25 GeV as the contribution from this source.

2. Initial state radiation/ final state radiation

The uncertainties from this source are in the modeling of additional jets due to initial and final-state radiation (ISR/FSR). To evaluate this contribution, we compare three PYTHIA samples having identical values of $m_t^{\text{gen}}$, with input parameters taken from a CDF ISR/FSR study based on the Drell-Yan process [38]. The three sets of parameters correspond to a fit to data and ±1 sd excursions. Half of the difference between the two excursions corresponds to a change in $m_t$ of 0.26 ± 0.19 GeV.

3. Hadronization and underlying event

In simulating parton evolution and hadronization, PYTHIA and HERWIG model the parton showering, hadronization, and underlying event (UE) differently. To estimate the impact of this difference, we compare two MC $t\bar{t}$ samples with identical values of $m_t^{\text{gen}}$, using ALPGEN for the hard-scattering process, but one sample using PYTHIA and the other using HERWIG for parton showering and hadronization. Ensemble studies indicate a difference in the means of the extracted $m_t$ to be $m_t^{\text{PYTHIA}} - m_t^{\text{HERWIG}} = 0.58 \pm 0.25$ GeV.

4. Color reconnection

The MC samples used in this analysis do not simulate color reconnection for the final-state particles [39]. To evaluate the possible effect of color reconnection on the determination of $m_t$, we compare two MC $t\bar{t}$ samples with
identical values of $m^2_{t\bar{t}}$, using PYTHIA 6.4 tunes APRO and ACRPRO, which are identical except for the inclusion of color reconnection in ACRPRO. Ensemble studies of $t\bar{t}$ events performed on both samples yield a difference in the means of the extracted $m_t$ of $m_t^{\text{APRO}} - m_t^{\text{ACRPRO}} = 0.26 \pm 0.28$ GeV. We take the uncertainty on this difference and assign $\pm 0.28$ GeV as the contribution from this source.

5. Modeling of jet mass

Unlike the jet algorithm used in Run I of Tevatron, the iterative midpoint cone algorithm used for Run II defines jets of intrinsic mass [21]. The effect of inaccuracies in the simulation of jet masses on the top-quark mass measurement is found to be negligible and is presently ignored.

6. Multiple $p\bar{p}$ interactions

Effects from additional $p\bar{p}$ interactions are simulated by overlaying on MC events unbiased triggers from random $p\bar{p}$ crossings. These overlaid events are then reweighted according to the number of interaction vertices to assure that the simulation reflects the instantaneous luminosity profile of the data. To evaluate the contribution from the uncertainty associated with the reweighting procedure, we repeat the ensemble studies used to derive the $m_t$ calibration but without the reweighting. The rederived calibration is applied to $L(\tilde{x}; m_t)$ for the selected data sample, $m_t$ is extracted and compared with the value from the default calibration, and found to shift by $-0.07$ GeV. This extreme check of the size of this contribution to the uncertainty shows that our result is not affected significantly by variations in luminosity.

7. Modeling of background

This systematic uncertainty receives contributions from two sources, one based on the data-MC discrepancy in background-dominated distributions and a second from uncertainty in the renormalization scale used to generate the $W + \text{jets}$ samples. For the first source, we identify distributions in which there is poor agreement between data and MC in the modeling of background. Specifically, in both channels, we examine lepton $p_T$ and the $\eta$ of the jet of lowest $p_T$ in the 3-jet multiplicity bin. Ensemble studies are performed on a sample of MC $t\bar{t}$ events using background events reweighted to match the distributions in data. The mean of the extracted $m_t$ for this sample is found to shift by $-0.03$ GeV relative to that of the same MC $t\bar{t}$ events using the default background events.

The $W + \text{jets}$ MC samples used in this analysis (Sec. IV) are generated using identical renormalization and factorization scales of $\mu = M^2_W + \Sigma p_T^2$ where the sum is over the jets in an event. To evaluate the effect of the uncertainty in this scale, we generate two more $W + \text{jets}$ MC samples with modified renormalization and factorization scales of $\mu/2$ and $2\mu$. We perform ensemble studies on a $t\bar{t}$ MC sample using these modified $W + \text{jets}$ samples and find that the means of the extracted $m_t$ shift by 0.13 GeV ($\mu/2$) and 0.32 GeV ($2\mu$) relative to the studies using the default $W + \text{jets}$ sample. We take half of the larger excursion and assign $\pm 0.16$ GeV as the contribution from this source.

The contributions from the above data-MC discrepancy for the background and from the uncertainty on the scales are combined in quadrature for a total of systematic uncertainty of $\pm 0.16$ GeV.

8. $W + \text{jets}$ Heavy-flavor scale factor

The default heavy-flavor content in LO ALPGEN MC $W + \text{jets}$ (Sec. IV) is increased by a factor of 1.47 for the $Wc\bar{c} + \text{jets}$ and $Wb + \text{jets}$ contributions to achieve agreement with NLO calculations of cross sections that include NLL corrections based on the MCFM MC generator [26]. To evaluate the uncertainty from this source, we shift this factor up to 1.97 and down to 0.97 and, for each variation, repeat the ensemble studies described in Sec. VI for the calibration of $m_t$, apply this to $L(\tilde{x}; m_t)$ in data, and reextract $m_t$. The shifts in $m_t$ relative to the default value are found to be $-0.07$ GeV and 0.02 GeV when the scale factors are shifted up and down, respectively. We assign $\pm 0.07$ GeV as the contribution from this source to the uncertainty of $m_t$.

9. Modeling of $b$ jets

Possible effects in modeling $b$-quark fragmentation are studied by reweighting the simulated $t\bar{t}$ events used in the calibration of the measurement to simulate other choices of $b$-quark fragmentation models for the $b$ jets. All the default MC samples used in this analysis consist of events that are reweighted from the default PYTHIA $b$-quark fragmentation function (based on the Bowler model [40]) to a Bowler scheme with parameters tuned to data collected at the LEP $e^+e^-$ collider [41]. To evaluate the systematic uncertainty, these events are reweighted again to account for differences between LEP and SLAC $e^+e^-$ data [41]. The ensemble studies of $m_t$ are repeated using these reweighted events, the new calibration applied to $L(\tilde{x}; m_t)$ for data, and $m_t$ extracted. $m_t$ is found to shift by 0.08 GeV relative to the default value.

Additional differences in the response of $b$ jets can be expected in the presence of semileptonic decays of $b$ or $c$-quarks. The incorrect simulation of semileptonic $b$ and $c$-quark decay branching fractions can therefore lead to a systematic shift in the extracted value of $m_t$. We take an uncertainty of $\pm 0.05$ GeV determined in Ref. [42] as the contribution from this source.

Combining the two above uncertainties in quadrature gives $\pm 0.09$ GeV, which we assign as the systematic uncertainty for the modeling of $b$ jets.
10. Choice of PDF

We evaluate this systematic uncertainty using a PYTHIA MC \( t \bar{t} \) sample that is reweighted to match possible excursions in the PDF parameters represented by the 20 CTEQ6M uncertainty PDFs [27]. Ensemble studies are repeated for each of these variants for only \( t \bar{t} \) events and the uncertainty evaluated using the following formula [27]:

\[
\delta m_{t}^{\text{PDF}} = \frac{1}{2} \left( \sum_{i=1}^{20} [\Delta M(S_{i}^{+}) - \Delta M(S_{i}^{-})]^{2} \right)^{1/2},
\]

where the sum runs over PDF excursions in the positive \( (S_{i}^{+}) \) and negative \( (S_{i}^{-}) \) directions. \( \delta m_{t}^{\text{PDF}} \) is found to be 0.24 GeV.

B. Modeling of detector

1. Residual JES uncertainty

The in situ jet energy calibration employed in this analysis addresses a possible global scale difference in JES between data and MC. Any other discrepancy, such as a dependence on \( p_{T} \) and \( \eta \), can have a systematic effect on the determination of \( m_{t} \). To estimate this, the fractional uncertainty associated with the standard jet energy correction, derived using the \( \gamma + \text{jet} \) and dijet samples, is parameterized as a function of \( p_{T} \) and \( \eta \). This uncertainty includes statistical and systematic contributions from both data and MC added in quadrature. All jet energies in a \( t \bar{t} \) MC sample are then scaled up by the parameterized uncertainty as a function of \( p_{T} \) and \( \eta \). The parameters are then shifted in such a way that the average scale shift applied to all jets vanishes. Ensemble studies are performed on the default and scaled samples, and the extracted \( m_{t} \) found to shift by 0.21 GeV relative to the default sample.

2. Data-MC jet-response difference

The uncertainties in the flavor-dependent jet-response correction for MC events (described in Sec. VII), used to bring the simulation of calorimeter response into agreement with that observed in the data, are associated with uncertainties in single-particle responses in data and MC. To evaluate the effect of these uncertainties on the value of \( m_{t} \), we change the correction factors by \( \pm 1 \) sd and apply them to the light jets and \( b \) jets in a \( t \bar{t} \) MC sample. The value of \( m_{t} \) is extracted and the mean is found to shift by \( \pm 0.28 \) GeV relative to the sample corrected using the central values.

3. \( b \)-Tagging efficiency

Discrepancies in the \( b \)-tagging efficiency between data and MC can lead to a systematic shift in the extracted \( m_{t} \). To evaluate the effect of possible discrepancies, the tag rate functions for \( b \) and \( c \) quarks and the mistag rate function for light quarks are changed by 5% [23] and 20%, respectively, corresponding to the uncertainties on these functions. Ensemble studies for all \( t \bar{t} \) MC samples are then repeated and the \( m_{t} \) calibration rederived and applied to data to extract \( m_{t} \). The result is compared with that from the default calibration and found to shift by \( -0.08 \) GeV.

4. Trigger efficiency

The MC events used in this analysis have associated weights to simulate the effect of trigger efficiencies. To evaluate the effect of the uncertainties in these weights on the top-quark mass, we repeat the ensemble studies on all \( t \bar{t} \) MC samples with the weights set to unity, rederive the \( m_{t} \) calibration, and apply it to the data to extract \( m_{t} \). The result is found to shift by \( -0.01 \) GeV.

5. Lepton momentum scale

A relative difference in the lepton momentum scale between data and MC can have a systematic effect on \( m_{t} \). To evaluate this, we first determine the size of the discrepancy and correct the scale of one \( t \bar{t} \) MC sample. Ensemble studies are repeated on the corrected sample and the mean of the extracted \( m_{t} \) is found to shift by \( 0.17 \) GeV relative to the default sample.

6. Jet energy resolution

Since the jet transfer functions used are derived from MC samples, improper simulation of jet energy resolution can result in a bias in the extracted \( m_{t} \). To evaluate a possible bias, ensemble studies are performed using a \( t \bar{t} \) MC sample with jet energy resolutions degraded by 1 sd. The mean of the extracted \( m_{t} \) in this sample is found to shift by \( 0.32 \) GeV [43].

7. Jet identification efficiency

The uncertainties associated with the scale factors used to achieve data-MC agreement in jet identification efficiencies are propagated to the measurement of \( m_{t} \) by decreasing the jet identification efficiencies in a \( t \bar{t} \) MC sample according to these uncertainties. We can only simulate a decrease and not an increase, as reconstructed jets can be dropped but not created. Ensemble studies indicate that the mean of the extracted \( m_{t} \) shifts by \( 0.26 \) GeV relative to that of the default sample [43,44].

C. Method

1. Multijet contamination

The multijet background is not included in the ensemble studies used to derive the calibrations described in Sec. VI as we have assumed that \( P_{\text{bkg}} \gg P_{\text{sig}} \) for such events (see Sec. VA), resulting in a negligible influence on the determination of \( m_{t} \). To evaluate possible systematic effects due to this assumption, we select a multijet-enriched sample of events from data by inverting the lepton isolation criterion...
in the event selections. We repeat the ensemble studies to derive the \( m_t \) calibration using the multijet-enriched sample in the sample composition. The rederived calibration is applied to data, and the extracted \( m_t \) is found to shift by 0.14 GeV relative to the default calibration [44].

2. Signal fraction

The signal fractions determined from data and used in the ensemble studies have associated statistical uncertainties. These signal fractions are varied by their uncertainties, independently for each decay channel, and the ensemble studies repeated for all MC samples to rederive the \( m_t \) calibration shown in Fig. 4(a). The new calibrations are then applied to the data and results compared with those obtained using the default calibration. The resulting uncertainties in \( m_t \) evaluated by changing the signal fractions in each decay channel are then added in quadrature and divided by two to obtain a total of \( \pm 0.10 \) GeV.

3. MC calibration

We estimate the effect of the statistical uncertainties associated with the offset and slope parameters determined from the fit to the response plot shown in Fig. 4(a). To estimate this uncertainty, we change these two parameters, one at a time, by their uncertainties and apply the modified calibration to the data to extract \( m_t \) and calculate the difference relative to the \( m_t \) extracted using the default calibration. We combine, in quadrature, the differences in \( m_t \), resulting from such changes in each parameter and find an uncertainty of \( \pm 0.20 \) GeV.

D. Treatment of systematic uncertainties in previous analysis

To facilitate the combination of the new measurement with the previous one, we have updated the systematic uncertainties presented in Table I of Ref. [8]. All of the uncertainties in this table are unchanged, except for the uncertainties in the modeling of signal and the relative \( b/\)light-quark response ratio. The uncertainty for the modeling of signal in the previous analysis is replaced with one from the current analysis, which includes contributions from uncertainties in the modeling of higher-order effects, ISR/FSR, hadronization and underlying event, color reconnection, and multiple hadron interactions. The uncertainty on \( b/\)light-quark response is replaced with that associated with differences in jet response in data and MC for the current analysis (see also Sec. VII). The uncertainty in the modeling of background in Table I of Ref. [8] is the sum in quadrature of (i) the uncertainty in the heavy-flavor scale factor, and (ii) the uncertainty associated with discrepancies between data and MC background distributions. Since the uncertainty on the renormalization and factorization scale was not evaluated in the previous analysis, we include the additional contribution described in the second part of Sec. IX A 7. We also evaluate the uncertainty associated with the flavor-dependent jet-response correction factors appropriate for the previous analysis, using the procedure described in Sec. IX B 2. We find the mean of the extracted \( m_t \) shifts by 0.13 GeV \((-0.22 \text{ GeV})\) relative to the sample corrected with the central values when we change the correction factors by 1 sd \((-1 \text{ sd})\). We assign \( \pm 0.22 \text{ GeV} \) as the contribution from this source. Adding the contributions from all sources in quadrature gives a total of \( \pm 0.97 \) GeV.

X. RESULT OF THE CURRENT MEASUREMENT

We measure the mass of the top quark in \( t\bar{t} \) lepton + jets events using a matrix-element method that combines an \textit{in situ} jet energy calibration with additional information from the standard jet energy scale derived from \( \gamma + \text{jet} \) and dijets samples. Using data corresponding to 2.6 \( \text{fb}^{-1} \) of integrated luminosity collected by the D0 experiment from Run II of the Tevatron collider, we extract the value

\[
\begin{align*}
\frac{m_t}{\text{GeV}} &= 176.01 \pm 0.10\text{(stat)} \pm 0.79\text{(JES)} \pm 1.02\text{(syst)} \\
\text{or } m_t &= 176.01 \pm 1.64 \text{ GeV}.
\end{align*}
\]

XI. COMBINATION WITH THE PREVIOUS MEASUREMENT

Our result from a previous measurement using the same analysis technique, and based on earlier data corresponding to 1 \( \text{fb}^{-1} \) of integrated luminosity, is

\[
\begin{align*}
\frac{m_t}{\text{GeV}} &= 171.5 \pm 1.76\text{(stat + JES)} \pm 1.1\text{(syst)} \\
\text{or } m_t &= 171.5 \pm 9.4 \text{ GeV}.
\end{align*}
\]

We combine the two measurements using the Best Linear Unbiased Estimator (BLUE) method [45,46] to get a result equivalent to 3.6 \( \text{fb}^{-1} \) of integrated luminosity. The combined value of the mass is

\[
\begin{align*}
\frac{m_t}{\text{GeV}} &= 172.74 \pm 0.83\text{(stat)} \pm 0.78\text{(JES)} \pm 0.96\text{(syst)} \\
\text{or } m_t &= 172.74 \pm 2.03 \text{ GeV}.
\end{align*}
\]

We combine the two measurements using the Best Linear Unbiased Estimator (BLUE) method [45,46] to get a result equivalent to 3.6 \( \text{fb}^{-1} \) of integrated luminosity. The combined value of the mass is

\[
\begin{align*}
\frac{m_t}{\text{GeV}} &= 174.94 \pm 0.83\text{(stat)} \pm 0.78\text{(JES)} \pm 0.96\text{(syst)} \\
\text{or } m_t &= 174.94 \pm 1.49 \text{ GeV}.
\end{align*}
\]

The procedure we follow uses the same method and classes of uncertainty as used by the Tevatron Electroweak Working Group [5] in combining individual measurements for Tevatron averages of the top-quark mass.

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[6] Events with $W \rightarrow \tau \nu$ decays can pass our selection criteria when the $\tau$ lepton decays leptonically. This is accounted for by including the simulation of $W \rightarrow \tau \nu$ decays in the MC samples used to calibrate the measurement.
[7] In the SM, the top-quark decays via $t \rightarrow Wb$ 100% of the time.
[16] The previous measurement in Ref. [8] is based on data corresponding to 1 fb$^{-1}$ of integrated luminosity recorded before the installation of Layer 0 while the new measurement presented here is based on data corresponding to 2.6 fb$^{-1}$ of integrated luminosity recorded after this installation.
[20] The separation in $\eta - \phi$ space between objects $a$ and $b$ is defined as $2S_\Delta R = \sqrt{(\eta_a - \eta_b)^2 + (\phi_a - \phi_b)^2}$.
[38] CDF collaboration (private communication) with; A. Abulencia et al. (CDF Collaboration), Phys. Rev. D 73, 032003 (2006).


[43] Although this is a Type I uncertainty described in the introduction of Sec. IX, we do not take half of this value as the change is only evaluated in one direction.

[44] These systematic uncertainties are taken from Ref. [8].
