The Dutch mathematical landscape

The 16th and 17th centuries

The Dutch mathematical landscape took shape in the 16th century. The first university in the Netherlands was Leiden (1575), followed by Franeker (1585), Groningen (1614), Amsterdam (1632), and Utrecht (1636). All were places of mathematics teaching and research, as was the University of Leuven (founded in 1425 as the oldest university in the Low Countries, with a mathematical Chair established in 1563). Teaching was still based on the *quadrivium*: arithmetic and geometry were seen as *mathematica pura*, whilst astronomy, and music belonged to *mathematica mixta*, i.e. applied mathematics. The latter also included fields like mechanics, (mathematical) optics, cartography, etc. Outside the universities (where the written language was Latin), everyday use of mathematics - like calculating with fractions - by practical people like merchants was promoted through books written in Dutch, like *Die maniere om te leeren cyffren na die rechte consten Algorismi. Int gheheele ende int ghebroken* (1508) by the well-known publisher Thomas van der Noot (1475-1525) in Brussels. The following brief history will concentrate on the Netherlands, but in any case, with a few exceptions like Newton's distinguished predecessor René-François de Sluse (1622-1685), Belgian mathematics went into decline already at the end of the16th century, about a hundred years before a similar fate befell Dutch mathematics.

The well-known marxist historian of mathematics Dirk Jan Struik (1894-2000) characterized the Dutch mathematical landscape of the 16th century as follows: "In this period mathematics appeared mainly as an applied science useful for the many and growing needs of the new economic and political system. Pure science of mathematics, as the scholastic age to a certain degree had known and as we know again since the seventeenth century, played only an accidental role in this early period of modern exact science. (...) The main impulse for the study of mathematics came more and more from the direct and indirect requirements of commerce and the related needs of the growing cities. It came from seafaring, bookkeeping, cartography, and astronomy. Cartography was stimulated not only by the sailor, but also by the centralizing state and the city and by the needs of warfare and of administration."

The first significant mathematician and astronomer of the Low Countries, Gemma Frisius (1508-1555), was a case in point. His book *Cosmographicus liber Petri Apiani* from 1529 laid the basis of early modern cartography through trigonometrical surveying; the famous cartographer Gerardus Mercator (1512-1594) was a pupil of Frisius. Subsequently, his *De principiis astronomiae et cosmographiae* (1530) related the problem of finding the longitude at sea to the problem of measuring time differences while on board. Impractical as it was at the time, this idea nonetheless had a lasting impact on all later work on the subject. A third influential work by Frisius was *Arithmeticae Practicae methodus facilis* (1540). Reprinted 73 times, this was the most popular textbook on arithmetic of the 16th century, combining theory and applications to commerce.

Barring a few works on practical calculations of the type written by van der Noot, *Vanden cirkel* by Ludolph van Ceulen (1540-1610), published in 1596, was the first notable mathematical book written in the vernacular, i.e., the Dutch language. This book breaks with the tradition described by Struik in describing a totally useless but amazing (and correct) computation of π to 20 decimal places, done in four different ways. Van Ceulen later even went up to 35 decimal places, still without any mistake: the result is carved in his tombstone in the Pieterskerk at Leiden. A better-known contemporary of his, publishing in Dutch as well, was Simon Stevin (1548-1620), undoubtedly one of the most impressive scientists in the history of the Low Countries. Stevin was active in every conceivable field of pure and applied mathematics, and also in engineering, physics, astronomy, geography, linguistics, and logic. Apart from the decimal notation for fractions, Stevin also invented the typical Dutch word 'wiskunde' for mathematics. His crowning achievement was *Wisconstige gedachtenissen (*published in 1608), a veritable bible for mathematicians, physicists, engineers, astronomers, and surveyors, which covered and made available most knowledge about the exact sciences at the time.

Other outstanding personalities of the early modern period include Rudolf Snel van Royen or 'Snellius' (1546-1613), the first professor of mathematics at Leiden (from 1601 onwards), and his famous son Willebrord (1580-1626), who was active in mathematics, surveying, hydrography, astronomy, and physics. Snel's law is named after Willebrord, but his greatest achievement was his book *Eratosthenes Batavus* (1617), in which he computed the distance between Alkmaar and Bergen op Zoom by a triangulation and thereby became the first to determine the length of the arc of the meridian. Rudolf's pupil Adriaan Metius (1571-1635) is remembered for a number of books on applications of geometry and arithmetic to navigation, surveying, and astronomy, and also for teaching at Franeker; his most notable student there was René Descartes (1596-1650).

Descartes lived in the Netherlands from 1628-1648, during which years he profoundly shook up the entire intellectual landscape of the 17th century. His mathematical ideas were most notably spread by Frans van Schooten (1615-1660), who published a renowned Latin edition of Descartes' *Géometrie,* as well as an original two-volume work *Geometria a Renato Des Cartes* (1659-1661) on analytical geometry. These books had a major impact in Europe, deeply influencing, for example, young Isaac Newton (1642-1727) at Cambridge. Furthermore, van Schooten had a number of direct students at Leiden, among whom Christiaan Huygens (1629-1695) was one of the towering figures of 17th century European science. Building his own telescopes, Huygens discovered Saturn's moon Titan as well as its rings in 1655, and in 1656 he invented the pendulum clock (largely motivated by the longitude problem as reformulated by Frisius). Subsequently, his small volume *Van rekeningh in spelen van geluck* (1657) was the first text on probability theory. Huygens's greatest achievements, however, were in mathematical physics, including his masterpieces *Horologium oscillatorium* (1673) and *Traité de la Lumière* (1690). Although the *Horologium* was subsequently eclipsed by Newton's *Principia* (1687), it formed an important predecessor to it, which Newton himself much admired. The *Traité*, on the other hand, arguably surpasses Newton's *Opticks* from 1704, and is still far from obsolete as the source of the wave theory of light.

In addition, van Schooten's students Johannes Hudde (1628-1704), and Johan de Witt (1625-1672) deserve to be mentioned. Until his gruesome assassination by a mob probably stirred up by Prince William III of Orange, de Witt was the most powerful politician of the Dutch Republic in its Golden Age. As a student he had elaborated the geometrical work of Descartes, and he subsequently founded the field of actuarial mathematics and life-table statistics. Similarly, Hudde (who died peacefully) was mayor of Amsterdam from 1672-1703, but during the first half of his life he made important contributions to pre-Newtonian calculus, which for example influenced Leibniz (who once ranked Hudde with Huygens and Newton, an assessment that may have failed to stand the test of time). Finally, Johann Bernoulli (1667-1748) was a professor of mathematics at Groningen from 1695-1705, exerting considerable influence in the country.

The 18th and 19th centuries

Following the end of the Golden Age (i.e., the 17th century), Dutch mathematics went into decline. Whereas other European countries produced mathematicians like Euler, Gauss, Lagrange, Galois, Abel, Cauchy, Riemann, Dirichlet, and a few others, who, in the wake of Newton, initiated much of today's mathematics and often created intellectual traditions in their countries that remain influential to the present time, Holland (like Belgium a century before) fell back to medieval levels as far as mathematics was concerned. With a few exceptions, this meant that a small group of dedicated people kept track of mathematical developments abroad and taught some of these, without making any original contributions.

One of these exceptions was Nicolaas Struyck (1687-1769), who made notable contributions to statistics and may even be seen as one of the pioniers of this subject. For example, his books *Inleiding to the algemeene geografie* (1739) and its successor *Vervolg van de beschrijving der staartsterren en nadere ontdekkingen omtrent den staat van 't menselyk geslagt* (1753) ranged from the statistics of comets to populations statistics. In the latter context, he also worked in actuarial mathematics. Struyck published in Dutch, and hence his influence abroad was limited, though in 1912 his work was finally translated into French. Cornelis Douwes (1712-1773) acquired an international reputation through his solution, published in an article called *Verhandeling om buiten den Middag op Zee de waare Middags-Breedte te vinden* (1754), of the problem of finding latitude at sea in situations where solar observations at mid-day were impossible. Rehuel Lobatto (1797-1866) further improved Douwes's method in 1827, but he spent most of his subsequent career in statistics, a field he had mastered under the tutelage of the famous Belgian statistician Lambert Adolphe Quetelet (1796-1874). His name also survives through the Gauss-Lobatto method of numerical integration. The last representative of the era of decline worth mentioning is Christophorus Buys Ballot (1817-1890), who was a professor of mathematics at Utrecht from 1857 onwards. Typically, however, his international reputation derives from the well-known meteorological law named after him - whose mathematical content is somewhat limited.

A typical example of those who at least kept a small mathematical flame burning was Jacob de Gelder (1765-1848). De Gelder had a major impact on national math education, partly through his numerous textbooks (in Dutch) and partly through political manoeuvring, by means of which he managed to make mathematics compulsory at various school types. Higher education in mathematics took place at Universities and, as far as practical applications were concerned, also at nautical schools and military academies. From 1842 onwards, the *Royal Academy for the education of civilian engineers, for serving both nation and industry, and of apprentices for trade* at Delft (i.e., the precursor of Delft University of Technology) also became a major place for applied mathematics. (The Technical Universities of Eindhoven and Twente date from a much later period, being founded in 1956 and 1964, respectively). Living half a century after de Gelder, the name of David Bierens de Haan (1822-1895) has survived: his achievements include the publication of tables of all definite integrals known at the time (1858-1867) and the leading editorial role in the monumental *Oeuvres complètes* of Christiaan Huygens (1880-1895).

This period of stagnation lasted for almost two centuries. As in the other sciences, it came to an end near the close of the 19th century, for reasons still debated among historians. In the natural sciences, the recovery in question led to five Nobel Prizes: Jacobus van 't Hoff (Chemistry, 1901), Hendrik Antoon Lorentz and Pieter Zeeman (Physics, 1902), Johannes Diederik van der Waals (Physics, 1910), and Heike Kamerlingh Onnes (Physics, 1913).

Furthermore, Paul Ehrenfest turned Leiden into a leading center for theoretical physics, counting his close friend Albert Einstein (among others) as a regular visitor.

Thus the way upward for mathematics was led by physics. Indeed, the first notable mathematician of this so-called ‘Second Golden Age’ in the Netherlands, Diederik Johannes Korteweg (1848-1941), was a PhD student of van der Waals, with a thesis on the propagation of waves in elastic tubes. Most of his later mathematical work was in partial differential equations as well, famously including the discovery of the nonlinear PDE named after him and his student Gustav de Vries (1866-1934) - not to be confused with Hendrik de Vries (1867-1954), another PhD student of Korteweg’s.

The 20th century

Korteweg’s most famous student, of course, was Luitzen Egbertus Jan Brouwer (1881-1966), the first Dutch mathematician of truly international stature to arise after Huygens. Repeating the like manoeuvre of Isaac Barrow with respect to Isaac Newton at Cambridge in 1673, Korteweg even vacated his Chair at the University of Amsterdam to Brouwer in 1913. One of the founders of (algebraic) topology, Brouwer also singlehandedly created intuitionism, a constructive approach to mathematics that remains influential, especially in logic and computer science. His most important PhDstudent was Arend Heyting (1898-1980), who formalized intuitionistic logic and has remained a household name in the subject. From Brouwer’s wider circle, Hans Freudenthal (1905-1990) - who had been a PhD student of Heinz Hopf (1894-1971) at Berlin - was the main personality who continued Brouwer’s work in topology and neighbouring areas (like Lie groups). In his later years Freudenthal also published extensively on mathematics education, in which field he has had a lasting influence, especially in the Netherlands.

Further to Brouwer’s influence on 20th century pure mathematics, two Dutchmen were similarly towering figures in applied mathematical fields. First, Jan Tinbergen (1903-1994) founded the field of econometrics and was the first winner of the Nobel Prize for Economics in 1969. Tinbergen studied mathematics and physics, but was an autodidact in economics. His PhD thesis (supervised by Ehrenfest) concerned the mathematical interplay of statistical mechanics and economics. His most famous student - jointly supervised by the well-known quantum physicist Hendrik Anthony Kramers (1894-1952) - was Tjalling Charles Koopmans (1910-1985), who worked in optimization theory and mathematical economics. Koopmans moved to the US in 1940 and received the Nobel Prize for Economics in 1975.

Second, Edsger Wybe Dijkstra (1930-2002), a PhD student of Adriaan van Wijngaarden (1916-1987) in computer science, is credited for having turned this field into an activity actually worthy of the name ‘science’, with contributions ranging from mathematical foundations and algorithms to compilers and operating systems. Before his move to Austin, Texas in 1984, Dijkstra was a professor at Eindhoven from 1962 onwards. His best-known student was Arie Nicolaas Habermann (1932-1993), although the latter’s influence was mainly exerted in the United States during his tenure at Carnegie Mellon.

Many others contributed to the current Dutch mathematical landscape, directly and through their students. We mention the most influential of them (emphatically including school-forming), with thematic grouping subject to some sort of chronological order.

As already mentioned, mathematical research on partial differential equations in the Netherlands started in the 19th century with Korteweg. Subsequently, also the physicists Balthasar van der Pol (1889-1959) and Johannes Martinus Burgers (1895-1981) - who have well-known equations named after them - heavily influenced applied mathematics. The former was at the Philips Nat Lab, and the second at Delft, in which connection also Reinier Timman (1917-1975) should be mentioned in industrial mathematics. The current Dutch mathematical school in partial differential equations (in an applied direction) has largely been the creation of Wiktor Eckhaus (1930-2000) and Lambertus (Bert) Peletier (1937). Hans Lauwerier (1923-1997), a student of Bottema's (see below), founded another Dutch school in applied analysis, directed towards numerical analysis and dynamical systems. The leading Dutch mathematician in the latter area, however, was Floris Takens (1940-2010), a student of Kuiper's. In the direction of functional analysis, Adriaan Cornelis Zaanen (1913-2003), a student of the physicist Johannes Droste (who himself was a pupil of the great Lorentz), had a large number of students, amongst whom Wilhelmus Anthonius Josephus Luxemburg (1929) and Marinus Adriaan Kaashoek (1937) established their own schools. Similarly, Freudenthal's student Arnoud Casper Maria van Rooij (1936) was an influential professor of Analysis at the Catholic University of Nijmegen (which was founded in 1923, but has included a Faculty of Science containing a mathematics department only since 1960).

The pioniers of differential geometry in the Netherlands were Willem van der Woude (1875-1943) and Jan Arnoldus Schouten (1883-1971). Van der Woude's mathematical origins may be traced back to Bierens de Haan, and in the opposite direction of the time-line one may count Nicolaas Kuiper (1920-1994) and Oene Bottema (1901-1992) among his pupils. Schouten was originally an electrical engineer, who (as an autodidact) moved into tensor calculus in order to understand Einstein's general theory of relativity. His best-known student was Dirk Jan Struik, already mentioned above as a historian of mathematics. Schouten was the president of the International Conference of Mathematicians, held in Amsterdam in 1954. Freudenthal's students Willem Titus van Est (1921-2002) and Johannes Jisse (Hans) Duistermaat (1942-2010) further developed differential geometry; both were also active in related areas like algebraic topology, global analysis, and Lie groups. In the latter area a lively scene developed in the Netherlands, involving e.g. Gerrit van Dijk (1939), a student of Springer's (see below) and Lauwerier's student Tom H. Koornwinder (1943). In the strict field of point-set topology, Johannes de Groot (1914-1972) should be mentioned as a follower of Brouwer.

Johannes Gualtherus van der Corput (1890-1975) and Hendrik Douwe Kloosterman (1900-1968) pioneered the study of number theory in the Netherlands. Both were students of Jan Cornelis Kluyver (1860-1932), who worked in analysis, differential geometry, and - indeed - number theory. Kloosterman also spent some time abroad with famous mathematicians, e.g., G.H. Hardy at Oxford, E. Landau at Göttingen, and E. Hecke at Hamburg. He had a number of students who later became professors of mathematics in Holland and had numerous descendants themselves. These include Nicolaas Govert de Bruijn (1918) - influential in analysis, combinatorics, automated theorem proving, and other areas of discrete mathematics - and Tonny Albert Springer (1926), active e.g. in algebraic groups. Frans Oort (1935), officially a PhD student of van Est's, turned to algebraic geometry under the influence of Kloosterman, and was instrumental in shaping the current landscape in algebraic number theory and algebraic geometry. One of van der Corput's student was Jurjen Ferdinand Koksma (1904-1964), who in 1930 became the first professor of mathematics at the protestant Free University in Amsterdam (founded in 1880). Another was Jan Popken (1905-1970), who had two particularly influential students. Abraham van der Sluis (1928-2004) became the doyen of numerical mathematics in the

Netherlands, counting e.g. Henk van der Vorst (1944) among his students. Rob Tijdeman (1943), on the other hand, continued the tradition in analytic number theory and Diophantine approximations started by van der Corput.

Evert Beth (1908-1964) made important contributions to (philosophical) logic around the mid-century. Heyting's students Dirk van Dalen (1932) and Anne Sjerp Troelstra (1939) further continued the process of turning the Netherlands into a leading place for logic and constructive mathematics, affecting also theoretical computer science (e.g., though lambda-calculus and functional programming, in which Holland traditionally occupies a strong position). Johan Jacob Seidel (1919-2001) and Jacobus (Jack) van Lint (1932-2004) introduced and developed areas like graph theory and coding theory in the Netherlands, with lasting influence.

David van Dantzig (1900-1959) was originally influenced by Brouwer as well as by Bartel Leendert van der Waerden (1903-1996), one of the pioneers of modern abstract algebra (who left Holland in 1926). Thus van Dantzig's PhD thesis (supervised by the latter) was squarely in pure mathematics, concerning topology and algebra, but he is mainly credited for two quite different contributions. First, jointly with van der Corput, Koksma, and Schouten he founded the *Mathematisch Centrum* (*MC*) in 1946, a research institute in Amsterdam that is still thriving - its current name is *Centrum voor Wiskunde en Informatica,* or *CWI* - at a later stage, de Groot's student Pieter Cornelis Baayen (1934) was an influential director from 1980-1994. Second, van Dantzig introduced (or, in the light of Struyck, perhaps one should say *re*introduced) mathematical statistics in the Netherlands. This field was further developed by his pupil Jan Hemelrijk (1918-2005), and subsequently most notably by the latter's student Willem van Zwet (1934). Another of van Dantzig's students, Johannes Theodorus Runnenburg (1932-2008), was one of the first mathematical probability theorists in Holland. This field was also heavily influenced by theoretical and mathematical physics, particularly through Pieter Willem Kastelyn (1924-1996), Michael Sylvester Keane (1940), and Wim Vervaat (1942-1994). Along with Jacob Willem Cohen (1923-2000), Runnenburg's student Gijsbert de Leve (1926-2009) established the subject of stochastic operations research in the Netherlands. Finally, the foundation of the research institute EURANDOM at Eindhoven in 1997 by Keane, van Zwet, and Jaap Wessels (1939-2009) considerably strengthened the position of stochastics in the Netherlands.

Epilogue

As detailed in an earlier chapter, the mathematics clusters emerged from the landscape just described. It is hard to find specific Dutch specialties like windmills or wooden shoes in this landscape; in other words, like its geographical counterpart, the current Dutch mathematical landscape looks essentially flat as far as the coverage of mathematics is concerned. In particular, there seems to be a healthy balance and cross-fertilization between pure and applied mathematics, so that Struik's comments on 16th century Dutch mathematics quoted above do not seem to apply anymore.