Scaling of the Quantum-Hall plateau-plateau transition in graphene

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The temperature dependence of the magneto-conductivity in graphene shows that the widths of the longitudinal conductivity peaks, for the $N = 1$ Landau level of electrons and holes, display a power-law behavior following $\Delta\nu \propto T^\kappa$ with a scaling exponent $\kappa = 0.37 \pm 0.05$. Similarly the maximum derivative of the quantum Hall plateau transitions $(d\sigma_{xy}/d\nu)^{max}$ scales as $T^{-\kappa}$ with a scaling exponent $\kappa = 0.41 \pm 0.04$ for both the first and second electron and hole Landau level. These results confirm the universality of a critical scaling exponent. In the zeroth Landau level, however, the width and derivative are essentially temperature independent, which we explain by a temperature independent intrinsic length that obscures the expected universal scaling behavior of the zeroth Landau level.

The integer quantum Hall effect in two-dimensional systems (2DES) is a direct consequence of the density of states of the Landau levels. Electronic states in the tails of individual Landau levels are localized and give rise to the quantized plateaus in the Hall resistance. The states in the center of the Landau levels are extended, because their wave functions are delocalized. The delocalization in 2DESs is shown to be governed by a localization length, which decays exponentially away from the Landau level centers.[1,2] The critical scaling exponent, is found to be universal for traditional 2DESs.[3]

Recently a new type of 2DES, graphene, joined the playground of the quantum Hall physics. Its unconventional Landau level spectrum of massless chiral Dirac fermions leads to a new half-integer quantum Hall effect,[4,5] which remains visible up to room temperature.[6]

Here we investigate the scaling behavior of the quantum Hall plateau-plateau transitions in graphene. When changing the carrier concentration n at a constant field, the peak width of the longitudinal conductivity, $\Delta\sigma_{xx}$, as well as the inverse slope of the Hall conductivity $d\nu/d\sigma_{xy}$ scale as T^κ. Our experimentally measured scaling exponent $\kappa = 0.40 \pm 0.02$ is consistent with universal scaling theory.[1,2,3,7,8,9] The transition through the zeroth Landau level, however, shows no clear scaling behavior which we explain by a temperature independent intrinsic length scale governing scaling.

The single layer graphene sample was made by micromechanical exfoliation of a crystal of natural graphite and subsequently contacted by gold contacts and patterned into a 1 μm wide Hall-bar by electron-beam lithography and reactive plasma etching.[10] The structure was deposited on a 300 nm Si/SiO$_2$ substrate thereby forming a graphene ambipolar field effect transistor. Prior to the measurements the sample was annealed at 400 K, placing its charge neutrality point (CNP) at zero gate voltage with a mobility of $\mu = 1.0$ m^2(Vs)$^{-1}$.

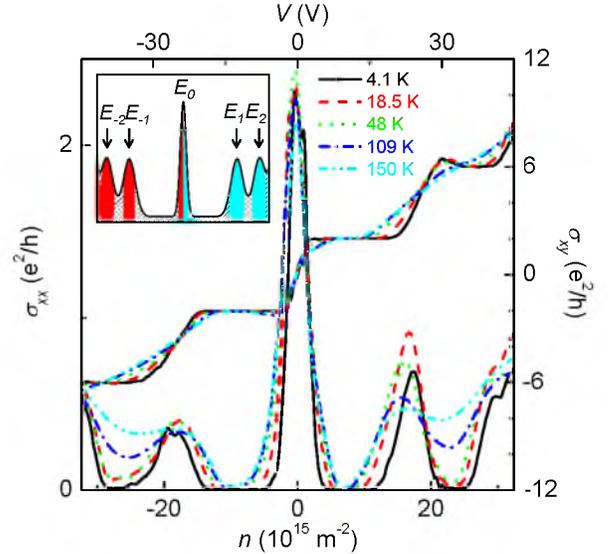In a magnetic field the density of states in graphene splits up into non-equidistant Landau levels[4,5,11,12] ac-

FIG. 1: (Color online) Longitudinal conductivity σ_{xx} and Hall conductivity σ_{xy} as a function of concentration n (bottom-axis) and voltage V (top-axis) at $B = 20$ T for different temperatures. The inset shows the Landau level spectrum in graphene.

cording to $E_n = sgn(N)\sqrt{2e\hbar v_F^2 B|N|}$ (see inset Fig. 1), where N identifies the 4-fold degenerate Landau levels and is given by $N = 0$, ± 1, ± 2, ... This Landau level spectrum leads to the half-integer quantum Hall effect as presented in Fig. 1, where we show the Hall conductivity σ_{xy} and longitudinal conductivity σ_{xx} as calculated from the symmetrized resistivities by a tensor inversion. The electronic states between two Landau levels are localized (dashed regions in the inset of Fig. 1) and lead to Hall plateaus quantized to $\sigma_{xy} = 4ie^2/h$, with $i = N + 1/2$ a half integer. The plateaus are accompanied by zero minima in the longitudinal conductivity σ_{xx}. Around the

centers of the Landau levels the states are extended and the Hall conductivity changes to its next plateau, while the longitudinal conductivity displays a peak.[13]

The region of extended states or delocalized states can be described by the energy interval where the localization length $\xi(E)$, i.e., the spatial extention of the wave function, increases beyond some characteristic length, $\xi(E) > L$. States remain localized if their localization length remains below this characteristic length, $\xi(E) < L$. According to scaling theory the localization length $\xi(E)$ follows a power law behavior[1,2] as a function of energy given by

$$\xi(E) = \xi_0|E - E_c|^{-\gamma}, \qquad (1)$$

with E_c the singular energy at the center of the Landau level and γ the critical exponent. Towards this singular energy the localization length diverges and states become delocalized when $\xi(E) > L$, making the states in the center of the Landau levels extended. To probe this delocalization we have to translate the energy relation to measurable quantities. In a first approximation, we assume a constant density of states (DOS) close to the singular energy.[15] In this case the energy in equation (1) is directly proportional to the measurable filling factor, $\nu = nh/eB$, of the Landau levels, i.e. $E \propto \nu$.

At finite temperatures the characteristic length L is determined by the inelastic scattering rate, $L_{in} \propto T^{-p/2}$, with p the inelastic scattering exponent. We can directly relate this temperature dependent quantity to the scaling of the localization length, and equation (1) becomes a function of temperature, $|\nu - \nu_c| \propto T^{p/2\gamma}$, with ν_c the Landau level center and ν its edge. This scaling behavior has therefore a direct relation to the quantum Hall transitions measured in the magneto-conductivity at finite temperatures through the quantities $(d\sigma_{xy}/d\nu)^{max}$ and $\Delta\nu$.[1,2] The width $\Delta\nu$ is defined as the distance between two extrema in the derivative $d\sigma_{xx}/d\nu$ and $(d\sigma_{xy}/d\nu)^{max}$ is the maximum in the derivative of σ_{xy}. Both quantities show a power law behavior $\Delta\nu \propto T^{\kappa}$ and $(d\sigma_{xy}/d\nu)^{max} \propto T^{-\kappa}$ with the critical exponents κ and γ related according to $\kappa = p/2\gamma$. An exact value for p can not be determined from our data, however $p = 2$ is a commonly used value for two-dimensional systems governed by short range scattering,[16,17,18] which is indeed expected in our high mobility graphene sample.[19,20,21,22]

The higher Landau levels in graphene have been shown to behave similar to the Landau level structure in traditional two-dimensional electron systems (2DEGs),[23] and are therefore a good starting point for scaling measurements. The width $\Delta\nu$ and derivative $(d\sigma_{xy}/d\nu)^{max}$ as a function of temperature for the first and second electron and hole Landau level are shown in Figure 2 at fixed magnetic fields between 5 and 30 T.

The widths $\Delta\nu$ and derivatives $(d\sigma_{xy}/d\nu)^{max}$ of these higher Landau levels show a clear dependence on temperature as can be seen from Fig. 2. The widths $\Delta\nu$ of the first electron and hole level at magnetic fields between 10 and 30 T show a power law behavior follow-
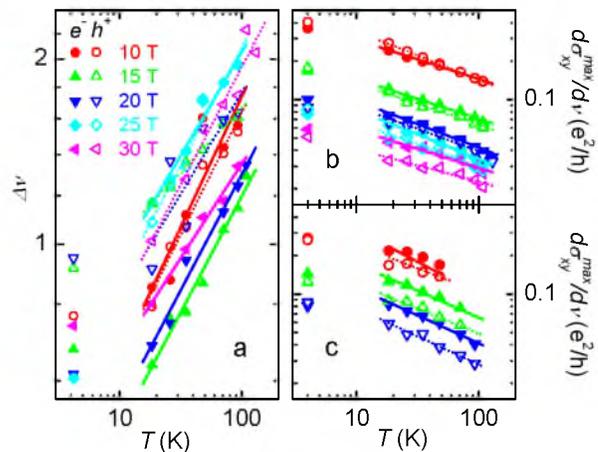

FIG. 2: (Color online) (a) $\Delta\nu$, distance between two extrema in the derivative $d\sigma_{xx}/d\nu$, of the first electron Landau level (solid symbols) and the first hole (open symbols) Landau level for different magnetic fields (b) The derivative $(d\sigma_{xy}/d\nu)^{max}$ as a function of temperature for the same levels. (c) The derivative $(d\sigma_{xy}/d\nu)^{max}$ as a function of temperature for the second electron and hole level.

ing $\Delta\nu \propto T^{\kappa}$. The scaling exponents extracted from these data, for holes $\kappa = 0.37 \pm 0.05$ and for electrons $\kappa = 0.37 \pm 0.06$, are all identical within the error margins and show no evidence of a magnetic field dependence. The error is determined by the scattering of all the individual κ-values, the statistical error for each κ is smaller. At $T < 15$ K the curves in Fig. 2a flatten, which is an indication that the localization length becomes independent on temperature and becomes dominated by an intrinsic length scale,[7] possibly the 1 μm width of the sample.

The linear behavior of the maximum derivatives $(d\sigma_{xy}/d\nu)^{max}$ in the Hall transition regions between the $\nu = \pm 2$ and $\nu = \pm 6$ plateaus are consistent with these observations, as shown in Fig. 2b. For both the first hole and the first electron level the curves show a power law behavior with scaling exponents $\kappa = 0.40 \pm 0.03$ and $\kappa = 0.40 \pm 0.04$ respectively for all fields up to $B = 25$ T.

In the high field limit ($B = 30$ T) the data shows a small reduction of the scaling exponent compared to the other fields. At these high fields the degeneracy of the Landau levels is partly lifted,[24] which leads to an additional minimum in σ_{xx} and a developing plateau in σ_{xy} in the center of the Landau levels, here at filling factor $\nu = \pm 4$. Due to the broad Landau levels this splitting remains obscured in σ_{xx} and σ_{xy}. However, their derivatives at the center Landau level positions are much more sensitive to the onset of this splitting, which makes it difficult to extract reliable scaling data at the highest magnetic field.

At low temperatures, the $(d\sigma_{xy}/d\nu)^{max}$-curves measured between 5 and 25 Tesla (see Fig. 2b) also flatten of, similar to what is observed in the width. This confirms
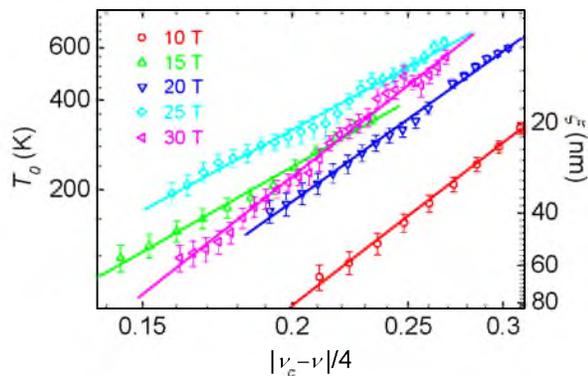
FIG. 3: (Color online) Scaling behavior in the tails of the first electron Landau level for various magnetic fields as a function of relative filling factor $|\nu_c - \nu|/4$ with $\nu = nh/eB$. The factor $1/4$ in the x-axis accounts for the four-fold Landau level degeneracy. The localization length on the right axis was calculated by eq. (3) with $C = 1$.
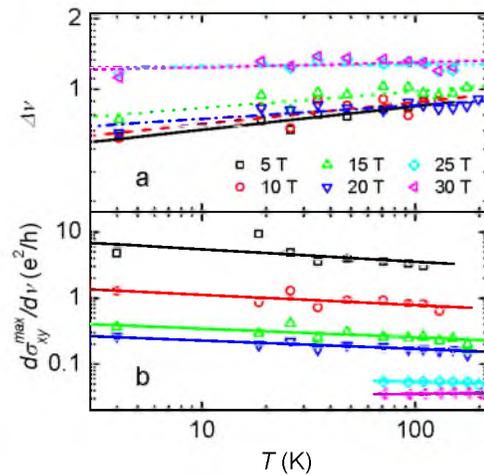

FIG. 4: (Color online) (a) $\Delta\nu$, distance between two extrema in the derivative $d\sigma_{xx}/d\nu$, as a function of temperature for the zeroth Landau level. (b) The derivative $(d\sigma_{xy}/d\nu)^{max}$ as a function of temperature for the zeroth Landau level.

that an intrinsic length apparently starts to dominate the localization length below $T = 15$ K.

Figure 2c shows $(d\sigma_{xy}/d\nu)^{max}$ for the second hole and electron Landau levels. Despite of the limited temperature range it does show a power law behavior with a scaling exponent of $\kappa = 0.40 \pm 0.03$ and $\kappa = 0.41 \pm 0.03$ for holes and electrons respectively. The width at the second Landau level is no longer clearly distinguishable and therefore no data could be obtained on the scaling behavior of these levels from this technique.

When we assume the temperature exponent of the inelastic scattering length to be $p = 2$, we obtain by the relation $\kappa = p/2\gamma$, a critical exponent $\gamma = 2.5 \pm 0.2$ for all the higher Landau levels combined. This value is in good agreement with the universal value $\gamma = 2.39 \pm 0.1$ showing that localization of the higher Landau levels in graphene follows a similar scaling behavior as traditional two-dimensional electron systems.

These results are also consistent with the low temperature behavior of the conduction in the Landau level tails.[15] In these tails the conductivity decreases with decreasing temperature and disappears when the temperature is lowered to $T = 0$ (see Fig. 1). When kT is small enough to make the activation to the mobility edge and excitation across potential barriers to neighboring states improbable, conduction is governed by a variable range hopping type of conductivity. In this regime electrons or holes are able to tunnel between states within an energy range kT leading to a slightly increased conductivity. The temperature dependence of the conductivity in this regime is given by[25,26,27,28,29]

$$\sigma_{xx} = \sigma_0 e^{-\sqrt{T_0/T}}, \qquad (2)$$

with a temperature dependent prefactor $\sigma_0 \propto 1/T$. The characteristic temperature T_0 is determined by the Coulomb energy and is inversely proportional to the lo-

calization length $\xi(\nu)$ at a particular filling factor ν

$$T_0(\nu) = C\frac{e^2}{4\pi\epsilon\epsilon_0 k_B \xi(\nu)}, \qquad (3)$$

with C a dimensionless constant in the order of unity and $\epsilon \approx 2.5$ is the effective dielectric constant for graphene on silicon dioxide.

Using equation (2) we can extract a value for the characteristic temperature T_0 from the temperature dependent conductivity measurements, which is directly related to the localization length via equation (3). Figure 3 shows the results for the first electron Landau level as a function of the relative filling factor $|\nu_c - \nu|$. A similar graph is found for the first hole level. Combining these results with equation (1) gives us a direct value for the critical exponent $\gamma = 2.6 \pm 0.5$ for the first electron and hole Landau level. This is indeed in good agreement with the values obtained from the width $\Delta\nu$ and derivative $(d\sigma_{xy}/d\nu)^{max}$.

It is interesting to note that the universality of the critical exponent is preserved despite the four-fold degeneracy of the Landau levels in graphene compared to lower degeneracies measured in other 2DESs. It was argued that degenerate levels would show a change in the critical exponent.[30] The results presented here however show that the Landau level degeneracy does not appear to be relevant.[31]

Let us now focus our attention to the zeroth Landau level. From a similar analysis as described before we obtain $\Delta\nu$ and $(d\sigma_{xy}/d\nu)^{max}$ as a function of temperature for magnetic fields between 5 and 30 T (see Fig. 4). Contrary to the higher Landau levels, at high fields neither the width (Fig. 4a) nor the derivative (Fig. 4b) of this zeroth level shows a clear dependence on temperature.

Note that in the low temperature limit at high magnetic fields scaling measurements of the zeroth Landau level, especially in $(d\sigma_{xy}/d\nu)^{max}$, are obscured, similar to what we observed for the $\nu = \pm 4$ state in the first Landau level. In this regime the Landau level degeneracy is partly lifted and a $\nu = 0$ state appears[14,24,32] and develops into a quantum Hall plateau at the CNP. Consequently $(d\sigma_{xy}/d\nu)^{max}$ will go to zero and is unrelated to any scaling behavior.

The temperature independence of $\Delta\nu$ and $(d\sigma_{xy}/d\nu)^{max}$ in the zeroth Landau level suggest that the localization length is determined by an intrinsic length scale rather than a thermal length scale. The intrinsic length is in this case directly related to the localization length by $L_{int} = \xi = \xi_0|\Delta\nu/2|^{-\gamma}$ where the prefactor ξ_0 is proportional to the magnetic length $l_B = \sqrt{\hbar/eB}$ with a proportionality factor in the order of one.[3] This provides an estimate for the intrinsic length of the order of 10 nm. This size is in reasonable agreement with the size of the intrinsic ripples in graphene, $L_{rip} \approx 10$ nm,[33,34] suggesting that the origin of the temperature independent intrinsic length lies in ripple-induced localization.[35,36] Admittedly, numerous other scenarios, such as the existence of electron-hole puddles,[37] may also be considered as the cause of a different localization mechanism in the zeroth Landau level compared to the higher levels. (see e.g. also Ref. 38 and references therein).

To conclude we have shown that the width $\Delta\nu$ and the derivative $(d\sigma_{xy}/d\nu)^{max}$ in the high Landau levels of graphene show a typical scaling behavior with an average $\kappa = 0.40\pm0.02$ leading to a critical exponent $\gamma = 2.5\pm0.2$, where the temperature exponent was assumed to be $p = 2$. These results are consistent with scaling measurements in the VRH-regime yielding $\gamma = 2.6 \pm 0.5$. In the zeroth Landau level no temperature dependence of the width nor of the derivative was observed in the measured range. We explain this result by a temperature independent intrinsic length leading to the absence of a universal scaling behavior of the zeroth Landau level.

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