Direct measurement of the mass difference between top and antitop quarks


(The D0 Collaboration)

1 Universidad de Buenos Aires, Buenos Aires, Argentina
2 LAIFEX, Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, Brazil
3 Universidade do Estado do Rio de Janeiro, Rio de Janeiro, Brazil
4 Universidade Federal do ABC, Santo André, Brazil
5 Instituto de Física Teórica, Universidade Estadual Paulista, São Paulo, Brazil
6 University of Alberta, Edmonton, Alberta, Canada; Simon Fraser University, Burnaby, British Columbia, Canada and McGill University, Montreal, Quebec, Canada
7 University of Science and Technology of China, Hefei, People's Republic of China
8 Universidad de los Andes, Bogotá, Colombia
9 Center for Particle Physics, Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic
10 Czech Technical University in Prague, Prague, Czech Republic
11 Center for Particle Physics, Institute of Physics, Academy of Sciences of the Czech Republic, Prague, Czech Republic
12 Universidad San Francisco de Quito, Quito, Ecuador
13 LPC, Université Blaise Pascal, CNRS/IN2P3, Clermont, France
14 LPSC, Université Joseph Fourier Grenoble 1, CNRS/IN2P3, Institut National Polytechnique de Grenoble, Grenoble, France
15 CPPM, Aix-Marseille Université, CNRS/IN2P3, Marseille, France
16 LAL, Université Paris-Sud, IN2P3/CNRS, Orsay, France
17 LPNHE, IN2P3/CNRS, Universités Paris VI and VII, Paris, France
18 CEA, Saclay, France
19 IPHC, Université de Strasbourg, CNRS/IN2P3, Strasbourg, France
20 IPNL, Université Lyon 1, CNRS/IN2P3, Villeurbanne, France and Université de Lyon, Lyon, France
21 III. Physikalisches Institut A, RWTH Aachen University, Aachen, Germany
22 Physikalisches Institut, Universität Bonn, Bonn, Germany
23 Physikalisches Institut, Universität Freiburg, Freiburg, Germany
24 II. Physikalisches Institut, Georg-August Universität Göttingen, Göttingen, Germany
25 Institut für Physik, Universität Mainz, Mainz, Germany
26 Ludwig-Maximilians-Universität München, München, Germany
27 Fachbereich Physik, Universität Wuppertal, Wuppertal, Germany
28 Panjab University, Chandigarh, India
29 Delhi University, Delhi, India
30 Tata Institute of Fundamental Research, Mumbai, India
31 University College Dublin, Dublin, Ireland
32 Korea Detector Laboratory, Korea University, Seoul, Korea
33 SungKyunKwan University, Suwon, Korea
34 CINVESTAV, Mexico City, Mexico
We present a measurement of the mass difference between $t$ and $\bar{t}$ quarks in lepton+jets final states of $\text{tt}$ events in 1 fb$^{-1}$ of data collected with the D0 detector from Fermilab Tevatron Collider $pp$ collisions at $\sqrt{s} = 1.96$ TeV. The measured mass difference of $3.8 \pm 3.7$ GeV is consistent with the equality of $t$ and $\bar{t}$ masses. This is the first direct measurement of a mass difference between a quark and its antiquark partner.

PACS numbers: 14.65.Ha, 11.30.Er, 12.15.Ff

The CPT theorem [1], which is fundamental to any local Lorentz-invariant quantum field theory, requires that the mass of a particle and that of its antiparticle be identical. Tests of CPT invariance for many of the elementary particles accommodated within the standard model (SM) are available in the literature [2]. Despite the fact that no violations have ever been observed, it is important to search for the possibility of CPT violation in all sectors of the standard model. Because quarks carry color, they cannot be observed directly, but must first
evolve through quantum chromodynamic (QCD) interac-
tions into jets of colorless particles. These jet rem-
nants reflect the characteristics of the initially produced
quarks, such as their charges, spin states, and masses.
If the lifetimes of quarks are much longer than the time
scale for QCD processes, the quarks form hadrons be-
fore they emerge from collisions, and decay from within
bound hadronic states. This makes it difficult to measure
a $q - \bar{q}$ mass difference because of the model dependence
of QCD binding and evolution processes. However, since
the lifetime of the top quark is far shorter than the time
scale for QCD interactions, the top-quark sector provides
a way to measure the mass difference less ambiguously [3].

In this Letter, we report a measurement of the dif-
ference between the mass of the top quark ($t$) and that
of its antiparticle ($\bar{t}$) produced in $p\bar{p}$ collisions at $\sqrt{s} =
1.96$ TeV. Our measurement is based on data correspond-
ting to $\sim 1$ fb$^{-1}$ of integrated luminosity collected with
the D0 detector [4] during Run II of the Fermilab Tevatron
Collider. The events used in this analysis, identical
to those in Ref. [5], are top quark pair ($t\bar{t}$) events in
the lepton + jets channel ($\ell$+jets) where each top quark
is assumed to always decay into a $W$ boson and a $b$
quark. One of the $W$ bosons decays via $W \rightarrow \ell\nu$ into
two leptons, and the other one through $W \rightarrow q\bar{q}'$ into
two quarks, and all four quarks ($q\bar{q}'bb$) evolve into jets.

We select events having one isolated electron (muon)
with transverse momentum $p_T > 20$ GeV and $|\eta| < 1.1
(|\eta| < 2)$, missing transverse momentum $p_T > 20$ GeV,
and exactly four jets with $p_T > 20$ GeV and $|\eta| < 2.5,$
where the pseudorapidity $\eta = -\ln \tan(\theta/2)$, and $\theta$
is the polar angle with respect to the proton beam direc-
tion. At least one of the jets is required to be identi-
fied as a $b$-jet candidate. A minimum azimuthal sep-
eparation is required between lepton $p_T$ and $p_T$ vectors to
further reduce multijet background arising from lepton or
jet energy mismeasurements. The positively (negatively)
charged leptons are used to tag the $t$ ($\bar{t}$) in each event. To
reduce instrumental effects that can cause charge depend-
ent asymmetries in lepton energy scale and resolution,
solenoid and toroid magnetic field polarities are routinely
reversed.

The selected data sample consists of 110 $\ell$+jets and
110 $\mu$+jets events. The $W^+$ ($W^-$) boson decays into
hadrons in 105 (115) events and into leptons in 115 (105)
events, consistent with invariance under charge conjuga-
tion. The fraction of $t\bar{t}$ events in this sample is estimated
to be 74%. The background consists of $W$+jets and mul-
tijet events, with the latter comprising 12% of the entire
background.

This analysis uses the matrix element (ME) method
which relies on the extraction of the properties of the
top quark (e.g., the mass) through a likelihood technique
based on probability densities (PD) for each event, cal-
culated from the ME for the two major processes ($t\bar{t}$
and $W$+jets production) that contribute to the selected
$\ell$+jets sample. In calculating the PD for $t\bar{t}$ production,
we include only the leading order (LO) ME from $q\bar{q} \rightarrow t\bar{t}$
production [6]. We assume SM-like $t\bar{t}$ production and
decay, where identical particle and antiparticle masses
are assumed for $b$ quarks and $W$ bosons but not for top
quarks. For $W$+jets production, we use the ME provided
in VECBOS [7]. The PD for each event is given in terms
of the fraction of signal ($f$) and of background ($1 - f$) in
the data and the masses of the $t$ ($m_t$) and the $\bar{t}$ ($m_{\bar{t}}$):

$$P_{\text{evt}} = A(x)[fP_{\text{sig}}(x; m_t, m_{\bar{t}}) + (1 - f)P_{\text{bkg}}(x)],$$

where $x$ denotes the measured jet and lepton energies
and angles, $A(x)$ is a function only of $x$ and accounts
for the geometrical acceptance and efficiencies, and $P_{\text{sig}}$
and $P_{\text{bkg}}$ represent the PD for $t\bar{t}$ and $W$+jets production,
respectively. Multijet events are also represented by $P_{\text{bkg}}$
since $P_{\text{bkg}} \geq P_{\text{sig}}$ for such events [8].

The free parameters in Eq. 1 are determined from a
likelihood $L(x; m_t, m_{\bar{t}}, f)$ constructed from the product
of the $P_{\text{evt}}$ for all events. Jet energies are scaled by an
overall jet energy scale (JES) calibration factor derived
by constraining the reconstructed mass of the two jets
from $W \rightarrow q\bar{q}'$ decays in $t\bar{t}$ events to 80.4 GeV [2, 5].
The likelihood is maximized as a function of $f$ for each
$(m_t, m_{\bar{t}})$ hypothesis to determine $f_{\text{best}}$. An integration
of the likelihood for $f = f_{\text{best}}$ over the sum $m_{\text{sum}} = (m_t +
m_{\bar{t}})/2$ results in a one-dimensional likelihood $L(x; \Delta)$
as a function of mass difference $\Delta = m_t - m_{\bar{t}}$. This is used
to extract the mean value of $\Delta$ and its uncertainty.
A similar procedure involving an integration over $\Delta$ gives
$L(x; m_{\text{sum}})$ which is used to extract the mean value
of $m_{\text{sum}}$ and its uncertainty.

The variables in any ME refer to nascent produced par-
icles (leptons and partons), but the measured quantities
correspond to physical leptons and jets. This difference
is taken into account in the calculation of the event prob-
ability by convoluting over phase space a transfer func-
tion, $W(y, x)$, that provides the resolution for the lepton
in question or a mapping of the observed jet variables in
an event ($x$) to their progenitor parton variables ($y$):

$$P_{\text{sig}} = \frac{1}{\sigma_{\text{ME}}^{\text{norm}}} \times \int \sum d\sigma(y; m_t, m_{\bar{t}}) dq_1 dq_2 F(q_1)F(q_2)W(y, x),$$

(2)

where $d\sigma(y; m_t, m_{\bar{t}})$ is the leading-order partonic differen-
tial cross section, $q_1$ and $q_2$ are the momentum fra-
tions of the colliding partons (assumed to be massless)
within the incident $p$ and $\bar{p}$, and the sum runs over all
possible combinations of initial-state parton flavors, jet-
to-parton assignments, and all $W \rightarrow \ell\nu$ neutrino solu-
tions [9]. In the sum over jet-to-parton assignments in
$P_{\text{sig}}$, each permutation of jets carries a weight $w_1$, which
is the normalized product of probabilities for tagging any
jet under a given parton flavor hypothesis [5]. The $F(q_s)$ include the probability densities for finding a parton of given flavor and longitudinal momentum fraction in the $p$ or $\bar{p}$ assuming the CTEQ6L1 [10] parton distribution functions (PDF), as well as the probability densities for the transverse components of the $q_s$ obtained from the LO event generator PYTHIA [11]. The normalization term $\sigma_{\text{norm}}$ is described below.

The overall detection efficiency for $t\bar{t}$ depends on the values of both $m_t$ and $m_{\tau}$. This is taken into account through the normalization by the observed cross section $\sigma_{\text{norm}} = \int A(x) P_{\text{sig}} dx = \sigma^{tt}(m_t, m_{\tau}) \langle A(m_t, m_{\tau}) \rangle$, where $\sigma^{tt}(m_t, m_{\tau})$ is the total cross section calculated by integrating the partonic cross section $\sigma_{qq}^{tt}$ [12], corresponding to the specific ME used in the analysis, over initial and final parton distributions and summing over initial parton flavors. $\langle A(m_t, m_{\tau}) \rangle$ is the mean acceptance determined from the generated $t\bar{t}$ events. The expressions for $P_{\text{bg}}$ are similar, except that the probability does not depend on $m_t$ or $m_{\tau}$.

Samples of $t\bar{t}$ MC events with different values of $m_t$ and $m_{\tau}$ are required to simulate $t\bar{t}$ production and decay in order to calibrate the results of the analysis. These events are generated with a version of the PYTHIA generator [11] modified to provide independent values of $m_t$ and $m_{\tau}$. The specific values chosen for $(m_t, m_{\tau})$ form a square grid spaced at 5 GeV intervals between $(165,165)$ and $(180,180)$, excluding the two extreme points at $(165,180)$ and $(180,165)$. The MC events for equal values of $m_t$ and $m_{\tau}$ are generated with the default version of PYTHIA.

Approximations made in formulating the likelihood can bias the final result. This issue is examined by comparing the measured and input values of $\Delta$ in pseudo experiments composed of MC $t\bar{t}$ and $W+\text{jets}$ events. The calibration is shown in Fig. 1 in terms of the measured mean $\Delta$ as a function of its input value ($\Delta_{\text{in}}$), separately for the $e+\text{jets}$ and $\mu+\text{jets}$ MC samples, for all MC samples generated at the input reference points on the $(m_t, m_{\tau})$ grid. There are 2, 3, 4, 3, and 2 different $(m_t, m_{\tau})$ points with a common $\Delta_{\text{in}}$ of $-10, -5, 0, +5,$ and $+10$ GeV, respectively. The dispersions in the measured values of mean $\Delta$ for different $(m_t, m_{\tau})$ points, but with same values of $\Delta_{\text{in}}$, are consistent with expected statistical fluctuations, as can be observed in Fig. 1. The fit $\chi^2$/d.o.f. for the points in Figs. 1(a) and 1(b) are 1.8 and 0.84, respectively. The parameterizations shown in Fig. 1 are used to calibrate $L(x; \Delta)$ for the selected data sample.

We define the pull as $(\Delta - \langle \Delta \rangle)/\sigma(\Delta)$ where $\Delta$ is the measured mass difference for a given pseudo experiment, $\langle \Delta \rangle$ is the mean measured mass difference for all pseudo experiments, and $\sigma(\Delta)$ is the uncertainty of the measured mass difference for the given pseudo experiment. The mean widths of the pull distributions for all samples used in Fig. 1 are 1.2 and 1.1 for $e+\text{jets}$ and $\mu+\text{jets}$, respectively. The deviations of these widths from 1 are used to correct the measured uncertainties in data.

Fitted two-dimensional Gaussian contours of equal probability (in terms of the standard deviation $\sigma$) for $L(x; m_t, m_{\tau})$ are shown for the electron and muon data samples in Figs. 2(a) and 2(b), respectively. The corresponding $L(x; \Delta)$ for both channels are given in Figs. 3(a) and 3(b). The two sets of data are consistent within their respective uncertainties, and the small correlations $(\rho_{e+\text{jets}} = -0.05, \rho_{\mu+\text{jets}} = -0.01)$ extracted from the fits in Fig. 2 between $m_t$ and $m_{\tau}$ are not statistically significant, nor are the shifts in the projections shown in Fig. 3.

Results from the two channels are combined through a weighted average of the separate electron and muon values. This has the advantage of using their respective pulls to adjust the uncertainties of each measurement before combining the two results. Using this averaging process, we quote the final combined means and their statistical uncertainties as $\Delta = 3.8 \pm 3.4 \text{(stat.) GeV}$ and $m_{\text{sum}} = 170.9 \pm 1.5 \text{(stat.) GeV}$. The latter is consistent

![Figure 1: Values of the measured mean $\Delta$ from MC pseudo experiments as a function of $\Delta_{\text{in}}$, parameterized by straight lines for (a) $e+\text{jets}$ and (b) $\mu+\text{jets}$ MC events. Dotted lines represent complete equality between measured and input values. Results from pseudo experiments with same $\Delta_{\text{in}}$ but different $m_{\text{sum}}$ correspond to the extra points for fixed $\Delta_{\text{in}}$ (see text).](image1)

![Figure 2: Fitted contours of equal probability for the two-dimensional likelihoods as a function of $m_t$ and $m_{\tau}$ for (a) $e+\text{jets}$ and (b) $\mu+\text{jets}$ data. The boxes, representing the bins in the two-dimensional histograms of the likelihoods, have areas proportional to the bin contents, set equal to the value of the likelihood evaluated at the bin center.](image2)
with the previous measurement of Ref. [5] (see also Ref. [13]).

The systematic uncertainties are summarized in Table I. The first category, *Physics modeling*, comprises the uncertainties in MC modeling of $t\bar{t}$ and $W^+\text{jets}$ events. The second category, *Detector modeling*, addresses uncertainties in the calibration of jet energy and simulation of detector response. The last category, *Method*, addresses uncertainties in the calibration and possible systematic effects due to assumptions made in the analysis. Except for two, all systematic uncertainties are identical to those described previously [5]. Many of these uncertainties (e.g., uncertainties in JES, PDF, jet resolution, multijet contamination) are expected to partially cancel in the measurement of the mass difference, but are often dominated by the statistics of the samples used to evaluate them. The two new contributions address the possibilities of (i) reconstructing leptons with the wrong charge, and (ii) uncertainties from modeling differences in the response of the calorimeter to $b$ and $\bar{b}$ jets [14], which can affect the measurement of the mass difference. These were evaluated for (i) by estimating the effect of an increase in charge misidentification in MC simulations that would match that found in data (~1% for both $e$ and $\mu$). For (ii), studies were performed on MC samples and on data seeking any difference in detector response to $b$ and $\bar{b}$ quarks beyond expectations from interactions of their decay products, which are accommodated in the MC simulations. The observed differences were limited by the statistics of both samples. The total systematic uncertainty is 1.2 GeV. Combining the systematic and statistical uncertainties of the measurement in quadrature yields $\Delta = 3.8 \pm 3.7$ GeV, a value consistent with CPT invariance.

In summary, we have measured the $t$ and $\bar{t}$ mass difference in $\sim 1 \text{ fb}^{-1}$ of data in $\ell^+\text{jets} \bar{t}t$ events and find the mass difference to be $m_t - m_{\bar{t}} = 3.8 \pm 3.7$ GeV, corresponding to a relative mass difference of $\Delta/m_{\text{sum}} = (2.2 \pm 2.2)\%$. This is the first direct measurement of a mass difference between a quark and its antiquark partner.

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physics modeling</td>
<td></td>
</tr>
<tr>
<td>Signal</td>
<td>±0.85</td>
</tr>
<tr>
<td>PDF uncertainty</td>
<td>±0.26</td>
</tr>
<tr>
<td>Background modeling</td>
<td>±0.03</td>
</tr>
<tr>
<td>Heavy flavor scale factor</td>
<td>±0.07</td>
</tr>
<tr>
<td>$b$ fragmentation</td>
<td>±0.12</td>
</tr>
<tr>
<td>Detector modeling:</td>
<td></td>
</tr>
<tr>
<td>$b$/light response ratio</td>
<td>±0.04</td>
</tr>
<tr>
<td>Jet identification</td>
<td>±0.16</td>
</tr>
<tr>
<td>Jet resolution</td>
<td>±0.39</td>
</tr>
<tr>
<td>Trigger</td>
<td>±0.09</td>
</tr>
<tr>
<td>Overall jet energy scale</td>
<td>±0.08</td>
</tr>
<tr>
<td>Residual jet energy scale</td>
<td>±0.07</td>
</tr>
<tr>
<td>Muon resolution</td>
<td>±0.09</td>
</tr>
<tr>
<td>Wrong charge leptons</td>
<td>±0.07</td>
</tr>
<tr>
<td>Asymmetry in $bb$ response</td>
<td>±0.42</td>
</tr>
<tr>
<td>Method:</td>
<td></td>
</tr>
<tr>
<td>MC calibration</td>
<td>±0.25</td>
</tr>
<tr>
<td>$b$-tagging efficiency</td>
<td>±0.25</td>
</tr>
<tr>
<td>Multijet contamination</td>
<td>±0.40</td>
</tr>
<tr>
<td>Signal fraction</td>
<td>±0.10</td>
</tr>
<tr>
<td>Total (in quadrature)</td>
<td>±1.22</td>
</tr>
</tbody>
</table>

We thank the staffs at Fermilab and collaborating institutions, and acknowledge support from the DOE and NSF (USA); CEA and CNRS/IN2P3 (France); FASI, Rosatom and RFBR (Russia); CNPq, FAPERJ, FAPESP and FUNDUNESP (Brazil); DAE and DST (India); Colciencias (Colombia); CONACyT (Mexico); KRF and KOSEF (Korea); CONICET and UBAcyT (Argentina); FOM (The Netherlands); STFC and the Royal Society (United Kingdom); MSMT and GACR (Czech Republic); CRDF Program, CFI, NSERC and WestGrid Project (Canada); BMBF and DFG (Germany); SFI (Ireland); The Swedish Research Council (Sweden); CAS and CNSF (China); and the Alexander von Humboldt Foundation (Germany).


[8] When this is satisfied, the relative contribution of $P_{\text{sig}}$ to $P_{\text{ext}}$ will be negligible for such events, minimizing their influence in the determination of $m_t$ and $m_{\tilde{t}}$.

[9] The transverse components of the unmeasured $\nu$ momentum $p_\nu$ are determined from the $p_T$ balance of the $t\bar{t}$ event, but the remaining ambiguity in the longitudinal component of $p_\nu$ leaves more than one possibility or “solution” for $p_\nu$.


[12] $\sigma_{\bar{q}q} = \frac{s\alpha_s^2}{2\pi}\left(3E_t^2 + 3E_\perp^2 + 2p^2 + 6m_t\right)$ where $\alpha_s$ is the strong coupling constant, $s$ is the square of the center-of-mass energy in the incoming $q\bar{q}$ rest frame, and $E_t$ and $p$ are the energies of the $t$ or $\bar{t}$ and their common momentum, respectively, in the $q\bar{q}$ frame.

[13] The result in Ref. [5] was obtained by further constraining the jet energy scale to the one derived from photon+jets and dijet samples. This constraint is not used in the present analysis since it shifts $m_t$ and $m_{\tilde{t}}$ in the same way and has no effect on $\Delta$.

[14] Differences in the composition of $b$ and $\bar{b}$ jets, such as different $K^+/K^-$ fractions, can cause differences in calorimeter response because of differences in $K^+/K^-$ interaction cross sections.