The Current Status of Asteroseismology

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Abstract Stellar evolution, a fundamental bedrock of modern astrophysics, is driven by the physical processes in stellar interiors. While we understand these processes in general terms, we lack some important ingredients. Seemingly small uncertainties in the input physics of the models, e.g., the opacities or the amount of mixing and of interior rotation, have large consequences for the evolution of stars. The goal of asteroseismology is to improve the description of the interior physics of stars by means of their oscillations, just as global helioseismology led to a huge step forward in our knowledge about the internal structure of the Sun. In this paper we present the current status of asteroseismology by considering case studies of stars with a variety of masses and evolutionary stages. In particular, we outline how the confrontation between the observed oscillation frequencies and those predicted by the models allows us to pinpoint limitations of the input physics of current models and improve them to a level that cannot be reached with any other current method.

Keywords: Oscillations, Stellar; Interior, Convective Zone; Interior, Core; Instrumentation and Data Management

1. Introduction

Despite extensive research in recent decades, we lack detailed knowledge of some important physical processes relevant for the description of stellar interiors. The reason is that, in general, the existing observations do not yet allow a detailed confrontation with the description of the physical properties of either the stellar material in the deepest internal layers, or of the dynamics of the outer stellar envelope. At first sight, seemingly small uncertainties in the input physics of the models have large...
consequences for the whole duration and end of the stellar life cycle. The lack of a good understanding of interior transport processes, caused by different phenomena such as rotation, gravitational settling, radiative levitation, magnetic diffusion, etc., is particularly acute when it comes to precise predictions of stellar evolution, and the galactic chemical enrichment accompanying it.

Given that global helioseismology led to a huge step forward in the accuracy of the internal structure model and of the transport processes inside the Sun, asteroseismology aims to obtain similar improvements for different types of stars by means of their oscillations. Stellar oscillations indeed offer a unique opportunity to probe the internal properties and processes, because these affect the observable frequencies. The confrontation between the frequencies measured with high accuracy and those predicted by models gives insight into the limitations of the input physics of models and improves them. In fact, stellar oscillation frequencies are the best diagnostic known that can reach the required precision in the derivation of interior stellar properties.

At present, the unknown aspects of the physics and dynamics are dealt with by using parameterized descriptions, where the parameters are tuned from observational constraints. These concern, e.g., the treatments of convection, the equation of state, diffusion and settling of elements. When a lack of observational constraints occurs, solar values are often assigned, e.g., to the mixing length parameter in the description of convection, or phenomena are ignored, e.g., convective overshooting and the diffusion of heavy elements. It is hard to imagine, however, that one single set of parameters is appropriate for very different types of stars. Similarly, rotation is either not included or is included only with a simplified treatment of the evolution of the rotation law in stellar models (e.g., Maeder and Meynet, 2000). Fortunately, rotation also modifies the frequencies of the star’s modes of oscillation (e.g., Gough, 1981; Saio, 1981). An adequate seismic modelling of rotation inside stars therefore is within reach with high-precision measurements and mode identifications of stellar oscillations.

The origin and physical nature of stellar oscillations, as well as their mathematical properties, were thoroughly discussed by Cunha et al. (2007), to which we refer the reader for the theoretical considerations of asteroseismology. Extensive recent overviews of the occurrence of stellar oscillations across the entire HR diagram, and asteroseismic applications thereof, are already available in Kurtz (2004, 2006), Cunha et al. (2007), and Aerts, Christensen-Dalsgaard, and Kurtz (in preparation for Springer-Verlag). Rather than repeating such an observational overview here in a far more concise format, we have opted to outline the current status of asteroseismology by focusing on a few carefully chosen examples that show the merits this research field has brought to the improvement of stellar modelling, i.e., we confine ourselves to cases where quantitative measures of internal structure parameters have been achieved. We start by considering examples of stars that oscillate similarly to the Sun, and then move on to pulsators excited by a heat mechanism for a discussion of convective overshooting and rotation inside stars. The rapidly oscillating Ap stars, being pulsators with very strong magnetic fields, are of particular interest to solar astronomers, so are discussed in a separate paper in this volume (Kurtz, 2008).
2. From the Sun to Stars: the Properties of Solar-Like Pulsators

As the oscillations of the Sun are caused by turbulent convective motions near its surface, we expect such oscillations to be excited in all stars with significant outer convection zones. Solar-like oscillations are indeed predicted for the lowest-mass main-sequence stars up to objects near the cool edge of the classical instability strip with masses near 1.6 $M_\odot$ (e.g., Christensen-Dalsgaard, 1982; Christensen-Dalsgaard and Frandsen, 1983; Houdek et al., 1999) as well as in red giants (Dziembowski et al., 2001). Such stochastically excited oscillations have very tiny amplitudes, which makes them hard to detect, particularly for the low-mass stars. Velocity amplitudes were predicted to scale roughly as $L/M$, where $L$ and $M$ are the luminosity and mass of the star, before the first firm discoveries of such oscillations in stars other than the Sun (Kjeldsen and Bedding, 1995). This scaling law was later modified to $(L/M)^{0.8}$ from excitation predictions based on 3D computations of the outer convection zones.
of the stars (Samadi et al., 2005), resulting in lower amplitudes compared with those found for 1D models.

The modes observed in the Sun at low spherical harmonic degree \( l \), and hence solar-like oscillations observable in distant stars, are high-order acoustic modes. They satisfy an approximate asymptotic relation (e.g., Vandakurov, 1967; Tassoul, 1980; Gough, 1993), according to which, to leading order,

\[
\nu_{n\ell} \sim \Delta \nu \left( n + \frac{\ell}{2} + \epsilon \right),
\]

where \( \nu_{n\ell} \) is the cyclic frequency of a mode of radial order \( n \) and degree \( \ell \) and \( \epsilon \) is a function of frequency determined mainly by the conditions near the stellar surface. Also,

\[
\Delta \nu = \left( 2 \int_0^R \frac{\mathrm{d}r}{c} \right)^{-1}
\]

is a measure of the inverse sound travel time over a stellar diameter, \( r \) being the distance to the stellar centre, \( R \) the surface radius of the star and \( c \) is the adiabatic sound speed. From simple considerations it follows that the large frequency separation satisfies \( \Delta \nu \propto (M/R^3)^{1/2} \) and hence is a measure of the mean density of the star.

Departures from Equation (1) can be characterized by the small frequency separation:

\[
\delta \nu_{n\ell} = \nu_{n\ell} - \nu_{n-1\ell+2}.
\]

This quantity is mainly sensitive to the sound speed in the core of the star and hence provides a measure of the evolutionary state. The sensitivity of \( \Delta \nu \) and \( \delta \nu \) on stellar properties allows a calibration of stellar models in terms of \( (\Delta \nu, \delta \nu) \) to estimate the mass and evolutionary stage of the star (e.g., Christensen-Dalsgaard, 1984, 1988; Ulrich, 1986).

The search for solar-like oscillations in stars in the solar neighbourhood has been ongoing since the early 1980s. The first indication of stellar power with a frequency dependence similar to that of the Sun was obtained by Brown et al. (1991) in a CM1 (Procyon, F5IV). The first detection of individual frequencies of solar-like oscillations was achieved from high-precision time-resolved spectroscopic measurements only in 1995 for the G5IV star \( \eta \) Boo (Kjeldsen et al., 1995); Brown et al. (1997), however, could not establish a confirmation of this detection from independent measurements, but it was subsequently confirmed by Carrier, Bouchy, and Eggenberger (2003) and Kjeldsen et al. (2003). It took another four years before solar-like oscillations were definitely established in Procyon (Martic et al., 1999). While there was a recent controversy about this detection (Matthews et al., 2004; Bedding et al., 2005) which we do not discuss in detail here, the results of Martic et al. (1999) have been confirmed (Mosser et al., 2008) and an in-depth asteroseismic investigation based on a large multisite campaign is presently being conducted. Subsequent to Martic et al.’s results, solar-like oscillations were found in two more stars: the G2IV star \( \beta \) Hyi (Bedding et al., 2001) and the solar twin \( \alpha \) Cen A (Bouchy and Carrier, 2001). These important discoveries led to several more subsequent detections, a summary of which was provided by Bedding and Kjeldsen (2007). The positions of confirmed solar-like pulsators in the HR diagram are displayed in Figure 1. The detected frequencies and frequency separations for all stars behave as expected from theoretical predictions and from scaling relations based on extrapolations from helioseismology.

The oscillation frequencies and frequency separations detected in solar-like pulsators provide additional constraints with which to test models of stellar structure.
and evolution in conditions slightly different from those provided by the Sun. Such studies generally involve a fit of theoretical models, characterized by a number of model parameters, to the set of seismic and non-seismic data available for a given pulsator. Theoretical modelling of solar-like pulsators using this direct fitting approach has been carried out for several stars, including η Boo (Carrier, Eggenberger, and Bouchy, 2005; Guenther et al., 2005), Procyon (Eggenberger et al., 2004a; Eggenberger, Carrier, and Bouchy, 2005; Provost et al., 2006), and α Cen A and B (e.g., Eggenberger et al., 2004b; Miglio and Montalban, 2005; Yildiz, 2007). So far, the main results of these fits are estimates of the stellar masses, ages, and initial metallicities, even though in some cases the results are still controversial, and call for better sets of data.

Among the solar-like pulsators, the binary star α Cen A and B provides a particularly interesting test-bed for studies of stellar structure and evolution, due to the numerous and precise seismic and non-seismic data that are available for both components of the binary. Studies of α Cen A and B, including seismic and non-seismic data for both components, indicate that the age of the system is likely to be between 5.6 and 7.0 Gyr, the value derived being dependent, in particular, on the seismic observables that are included in the fits. Moreover, the same studies point to a significant difference in the values of the mixing-length parameter (αMLT, see Section 3 for a definition), for the two stars, although the sign is uncertain and possibly dependent on the detailed treatment of the effects of the near-surface layers in the analysis of the observed frequencies. Eggenberger et al. (2004b) and Miglio and Montalban (2005) found that the value for α Cen B is larger than that for α Cen A, whereas Teixeira et al. (in preparation) found that αMLT was slightly smaller for α Cen B than for α Cen A. The latter study also found that the best-fitting model for α Cen A was on the border of having a convective core (see Christensen-Dalsgaard, 2005): even a slight increase in the mass of the model led to a significant convective core and hence a model that was quite far from matching the observed properties.

Despite the successful case studies just outlined, the detailed seismic studies of stars with stochastically-excited modes are currently still in their infancy compared with global helioseismology. However, given the recent detections and the continuing efforts to improve them, we expect very substantial progress in the seismic interpretation of such targets in the coming years. In particular, the CoRoT (e.g., Michel et al., 2006) and Kepler (e.g., Christensen-Dalsgaard et al., 2007) missions will give data of very high quality on solar-like oscillations. As seen in the example of α Cen A above, it is noteworthy that the class of main-sequence solar-like oscillators encompasses transition objects regarding the development of a convective core on the main sequence (1.0 M☉ < M < 1.5 M☉). Asteroseismology will surely refine the details of the yet poorly understood physics that occurs near the core of the objects in this transition region. Also, data of the expected quality will provide information about the depth and helium content of the convective envelope (e.g., Houdek and Gough 2007a), as well as more reliable determinations of stellar ages (Houdek and Gough 2007b).

Additional information from solar-like oscillations is available in the cases of relatively evolved stars, beyond the stage of central hydrogen burning. Here the frequency range of stochastically-excited modes may encompass mixed modes behaving as standing internal gravity waves, or g modes, in the deep chemically inhomogeneous regions, thus providing much higher sensitivity to the properties of this region. In
fact, there is some evidence that such modes have been found in the subgiant $\eta$ Boo (Christensen-Dalsgaard, Bedding, and Kjeldsen, 1995).

Evidence for rotational splitting (see Section 4 for a definition) has been found in $\alpha$ Cen A (Fletcher et al., 2006; Bazot et al., 2007). However, it has not yet been possible to map the interior rotation of a solar-like pulsator, since the present frequency multiplet detections are insufficient. We note that Lochard, Samadi, and Goupil (2004) found, with simulated data, that the presence of mixed modes in a star such as $\eta$ Boo may allow some information to be derived about the variation of the internal rotation with position.

For a few pulsators excited by the heat mechanism, data are already available that provide such information, albeit only very roughly. We discuss this further in Section 4, but first we highlight in the next section some case studies through which the properties of core convection have been tuned by asteroseismology.

3. Seismic Derivation of Convective Overshooting inside Stars

The standard description of convection used in stellar modelling is the Mixing Length Theory (MLT) of Böhm-Vitense (1958). In this theory, the convective motions are treated as being time-independent. In the absence of a rigorous theory of convective motions based on first principles, the convective cells are assumed to have a mean-free-path length of $\alpha_{\text{MLT}} H_p$, where $H_p$ is the local pressure scale height. The mixing-length parameter depends on the physics considered in the model and on the specific formulation of the MLT used. Its value for Model S for the Sun of Christensen-Dalsgaard et al. (1996) is $\alpha_{\text{MLT}} \approx 1.99$, using the Böhm-Vitense (1958) MLT formulation.

In the context of stellar evolution, it is of crucial importance to quantify the amount of matter in the fully mixed central region of the star. This amount is usually derived from the Schwarzschild criterion, which states that convection occurs in regions where the adiabatic temperature gradient is smaller than the radiative gradient. However, from a physical point of view, it is highly unlikely that convective elements stop abruptly at the boundary set by the Schwarzschild criterion. Rather, their inertia causes them to overshoot into the adjacent stable area where radiative energy transport takes place. The amount of such overshooting is, however, largely unknown. For this reason, it is customary to express it as $\alpha_{\text{ov}} H_p$ where $\alpha_{\text{ov}}$ is expected to be a small fraction of $\alpha_{\text{MLT}}$.

The inability to derive a value for $\alpha_{\text{MLT}}$ and $\alpha_{\text{ov}}$ from a rigorous theoretical description is highly unsatisfactory, particularly for stars with a convective core, because the total mass of the well-mixed central region of the star determines its stellar lifetime. This is the reason why great effort has been, and is being, made to quantify $\alpha_{\text{ov}}$, keeping in mind that we have already a fairly good estimate of $\alpha_{\text{MLT}}$ from the Sun. We describe here the power of asteroseismology to determine $\alpha_{\text{ov}}$.

In the solar case helioseismic analyses have provided constraints on the overshoot from the solar convective envelope, assuming that this results in a nearly adiabatic extension of the convection zone followed by an abrupt transition to the radiative temperature gradient (Zahn, 1991). Assuming also, as usual, a spherically symmetric model, such a behaviour introduces a characteristic pattern in the frequencies in the form of an oscillatory variation of the frequencies as functions of the mode order. From the observed amplitude of this signal, an overshoot region
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of the nature considered must have an extent less than around $0.1 H_p$ (Basu, Antia, and Narasimha, 1994; Monteiro, Christensen-Dalsgaard, and Thompson, 1994; Christensen-Dalsgaard, Monteiro, and Thompson, 1995). It was found by Monteiro, Christensen-Dalsgaard, and Thompson (2000) that a similar analysis can be carried out on the basis of just low-degree modes, such as will be observed in distant stars.

Owing to their sensitivity to the core structure, the low-degree solar-like oscillations should in principle be sensitive to overshoot from convective cores. Models of $\eta$ Boo without and with overshoot were considered by Di Mauro et al. (2003, 2004). Although the present observed frequencies are not sufficiently accurate to provide direct information about the properties of the core, it was found that for $\alpha_{ov} \geq 0.2$ models could be found in the central hydrogen-burning stage which matched the observed location in the HR diagram. In such models, mixed modes are not expected; thus the definite identification of mixed modes would constrain the extent of overshoot in the star. Straka, Demarque, and Guenther (2005) considered models of Procyon with various types of core overshoot to determine the extent to which overshoot could be asteroseismically constrained.

For the $p$-mode diagnostics considered, little sensitivity to overshoot was found, while the, perhaps unlikely, detection of $g$ modes in Procyon, such as have been claimed in the Sun, would provide much stronger constraints on the overshoot distance. As in the case of $\eta$ Boo, the definite identification of the star as being on the subgiant branch, e.g., from the properties of the oscillation frequencies, would provide strict constraints on the extent of overshoot during the central hydrogen-burning phase. Mazumdar et al. (2006a) made a detailed analysis of the sensitivity of suitable frequency combinations to the properties of stellar cores and found that the mass of the convective core, possibly including overshoot, could be determined with substantial precision, given frequencies with errors that should soon be reached. Cunha and Metcalfe (2007) developed diagnostics of small convective cores that may in principle also provide information about the properties of overshoot; the detailed sensitivity still needs investigation, however.

Quantitative measures of the core convective overshooting parameter have been achieved by fitting the frequencies of some of the $\beta$ Cep stars. This group of young, Population I, near-main-sequence pulsating B stars has been known for more than a century. They have masses in the range $8-18 M_\odot$, and they oscillate in low-order $p$ and $g$ modes with periods in the range 2–8 hours. These oscillations are excited by a heat mechanism acting through opacity features associated with elements of the iron group (e.g., Dziembowski and Pamyatnykh, 1993; Pamyatnykh, 1999; Miglio, Montalban, and Dupret 2007). A recent overview of the observational properties of the class was provided by Stankov and Handler (2005). Most of the $\beta$ Cep stars show multiperiodic light and line-profile variations and most rotate at only a small fraction of the critical velocity.

Significant progress in the detailed seismic modelling of the $\beta$ Cep stars has occurred over the last few years and has led to quantitative estimates of the core overshooting parameter $\alpha_{ov}$ for several class members with slow rotation (see, e.g., Aerts, 2006, for a summary). We illustrate this here for the star $\theta$ Oph, whose frequency spectrum was determined from a multisite photometric campaign and is represented in Figure 2 (Handler, Shobbrook, and Mokgwetsi, 2005). An additional long-term, high-resolution spectroscopic campaign revealed that this star is a member of a spectroscopic binary with an orbital period of 56.71 days and an eccentricity of 0.17 (Briquet et al., 2005), and allowed the identification of the spherical wavenumbers.
Figure 2. The schematic frequency spectrum of the $\beta$ Cep star $\theta$ Oph for the Strömgren $u$ filter as derived from a multisite photometric campaign. The measured photometric amplitude ratios led to an identification of the frequencies $\nu_1, \nu_2, \nu_3,$ and $\nu_4$ as, respectively, $\ell = 2, 2, 0, 1$. (Figure reproduced from Handler, Shobbrook, and Mokgwetsi, 2005).

Figure 3. The rotational kernels defined in Equation (4) as a function of radial distance inside the star ($x = r/R$), for the identified $\ell = 1, p_1$ mode (solid line) and $\ell = 2, g_1$ mode (dashed line) of the $\beta$ Cep star $\theta$ Oph. The vertical dotted line marks the position of the boundary of the convective core, including the overshoot region. Note that the kernels also approximately represent the relative sensitivity of the mode frequencies to other aspects of the stellar interior. (Figure reproduced from Briquet et al., 2007).
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Table 1. The identification of the pulsation modes of the \( \beta \) Cep star \( \theta \) Oph derived from multicolour photometric and high-resolution spectroscopic data. Positive \( m \)-values represent prograde modes. The amplitudes of the modes are given for the Stromgren \( u \) filter and for the radial velocities. Table reproduced from Briquet et al. (2007).

<table>
<thead>
<tr>
<th>ID</th>
<th>Frequency (d(^{-1}))</th>
<th>((\ell, m))</th>
<th>(u) ampl. (mmag)</th>
<th>RV ampl. (km s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\nu_1)</td>
<td>7.1160</td>
<td>(2, -1)</td>
<td>12.7</td>
<td>2.54</td>
</tr>
<tr>
<td>(\nu_5)</td>
<td>7.2881</td>
<td>(2, +1)</td>
<td>2.1</td>
<td>-</td>
</tr>
<tr>
<td>(\nu_2)</td>
<td>7.3697</td>
<td>(2, +2)</td>
<td>3.6</td>
<td>-</td>
</tr>
<tr>
<td>(\nu_3)</td>
<td>7.4677</td>
<td>(0, 0)</td>
<td>4.7</td>
<td>2.08</td>
</tr>
<tr>
<td>(\nu_4)</td>
<td>7.7659</td>
<td>(1, -1)</td>
<td>3.4</td>
<td>-</td>
</tr>
<tr>
<td>(\nu_6)</td>
<td>7.8742</td>
<td>(1, 0)</td>
<td>2.3</td>
<td>-</td>
</tr>
<tr>
<td>(\nu_7)</td>
<td>7.9734</td>
<td>(1, +1)</td>
<td>2.4</td>
<td>-</td>
</tr>
</tbody>
</table>

\((\ell, m)\) of the seven detected frequencies from the line-profile variations induced by the oscillations (see Table 1 reproduced from Briquet et al., 2007).

Because the frequency spectra of \( \beta \) Cep stars are so sparse for low-order \( p \) and \( g \) modes compared with those of solar-like pulsators (see Figure 2), one does not have many degrees of freedom to fit the securely-identified modes. This led to the identification of the radial order of the modes of \( \theta \) Oph as \( g_1 \) for the frequency quintuplet containing \( \nu_1, \nu_5, \nu_2 \), the radial fundamental for \( \nu_3 \) and \( p_1 \) for the triplet \( \nu_4, \nu_6, \nu_7 \). Fitting the three independent \( m = 0 \) frequencies results in a relation between the metallicity and the core-overshooting parameter, because the stellar models for main-sequence B stars typically depend on the five parameters (\( X, a_{\text{ov}}, Z, M, \) age) if we ignore effects of diffusion. Note that \( a_{\text{MLT}} \) is usually fixed to the solar value; for B stars, with their extremely thin and inefficient outer convection zones, changing \( a_{\text{MLT}} \) within reasonable limits does not change the characteristics of the models. In this way, one finds \( a_{\text{ov}} = 0.44 \pm 0.07 \) from a detailed high-precision abundance determination for \( \theta \) Oph (Briquet et al., 2007).

The reason why we can derive the core overshooting and the rotation (see Section 4), and provide a quantitative measure of these parameters for this star, is the different probing ability of the non-radial modes. This can be illustrated by plotting probing kernels of the modes. Different types of such kernels are used, depending on the kind of behaviour under investigation. This is illustrated in Figure 3, where we show the rotational splitting kernels \( K(x) \) (which will be defined in Equation (4) below) of \( \theta \) Oph for the two non-radial modes. It can be seen that the \( g_1 \) mode’s kernel behaves differently near the boundary of the core region, and thus probes that region in a different way than the \( p_1 \) mode, allowing the derivation of details of the rotational properties as explained below. A similar figure holds for the energy distribution, which allows probing the extent of the core region. A comparable result was obtained for V836 Cen whose frequency spectrum is almost a copy of that of \( \theta \) Oph (see Figure 5) and also for \( \nu \) Eri (Pamyatnykh, Handler, and Dziembowski, 2004).

The combination of low-order \( p \) and \( g \) modes thus turns out to be a very powerful tool to derive the internal structure parameters of massive stars. Additional...
measures of the core overshooting have been obtained for the $\beta$ Cep stars $\beta$ CMa (Mazumdar et al., 2006b) and $\delta$ Ceti (Aerts et al., 2006). For all these $\beta$ Cep stars, $\alpha_{ov}$ ranges from 0.1 to 0.5, although these values depend somewhat on theadopted metal mixture (Thoul et al., 2004). It is remarkable that the frequencies of just two well-identified oscillation modes that have sufficiently different kernels allow one to derive the overshooting parameter with a precision of typically 0.05 expressed in $H_p$. Adding just a few more well-identified modes should drastically reduce this error for specific input physics of the models.

The seismically derived estimates of core overshooting in $\beta$ Cep stars are compatible with the quantitative results for eight detached double-lined eclipsing binaries obtained by Ribas, Jordi, and Giménez (2000), who found $\alpha_{ov}$ to range from 0.1 to 0.6 for primary masses ranging from 1.5 to 9 $M_\odot$. Another way of determining the amount of overshooting from data is by fitting stellar evolutionary tracks to the dereddened colour-magnitude diagrams of clusters, e.g., $\alpha_{ov} = 0.20 \pm 0.05$ for the intermediate age open cluster NGC3680 (Kozhurina-Platais et al., 1997) and $\alpha_{ov} \approx 0.07$ for the old open cluster M67 (VandenBerg and Stetson, 2004). In these two methods, essentially the same five unknown structure parameters occur as for the seismic modelling, since stellar evolution models are used to fit the position of the binary components and of the cluster main-sequence turn-off point in the HR diagram, respectively. The uncertainty on the overshoot distance derived from the light curve analysis of an accurately modelled eclipsing binary or from fitting of a cluster turn-off point is typically between 0.05 and 0.1 $H_p$ provided that the metallicities are known. It is interesting, although perhaps fortuitous, that all these quantitative measures of the amount of overshooting are in agreement with the theoretical predictions by Deupree (2000) from 2D hydrodynamic simulations of zero-age main-sequence stars with a convective core.

4. Seismic Derivation of the Internal Rotation Profile of Stars

The rotation of a star implies a splitting of the oscillation frequencies compared with the case without rotation. Hence, rotation becomes apparent in frequency spectra as multiplets of $2\ell + 1$ components for each mode of degree $\ell$. Ignoring rotational effects higher than order one in the rotational frequency as well as the influence of a magnetic field, the frequency splitting becomes

$$\nu_m = \nu_0 + m \int_0^R K(r) \frac{\Omega(r)}{2\pi} \frac{dr}{R},$$

where $\nu_m$ is the cyclic frequency of a mode of azimuthal-order $m$, and $\Omega$ is the angular velocity which we here assume to depend only on the distance ($r$) to the centre. The rotational kernels are defined as

$$K(r) = \frac{(\xi_r^2 - 2\xi_r \xi_t + (\ell(\ell + 1) - 1)\xi_t^2) r^2 \rho}{\int_0^R \frac{dr}{R} \left[ \xi_r^2 + \ell(\ell + 1)\xi_t^2 \right] r^2 \rho},$$

with $\xi_r$ and $\xi_t$ the radial and tangential components of the displacement vector

$$\xi = (\xi_r e_r + \xi_t \nabla_t) Y_\ell^m.$$
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Figure 4. Frequency splittings $|\Delta \nu| = |\nu_m - \nu_0|$ of the triplets detected for GD 358 as a function of radial overtone $n$. The full line connects the values for the $m = +1$ components corresponding to prograde modes and the dotted line those for $m = -1$ representing retrograde modes. (Figure reproduced from Winget et al., 1994).

If such multiplets are observed, their structures are a great help in mode identification, as non-radial modes with a given value of $\ell$ have $2\ell + 1$ multiplet peaks corresponding to the different values of $m$, although not all peaks may be visible owing to the geometry of the modes or excitation of the components, while radial modes show no multiplet structure. The recognition of the multiplet structure is far easier for very slow rotators, where “slow” here means that the rotational frequency is far lower than the frequency spacing for $m = 0$ components of modes of adjacent radial order $n$.

Below, we describe two types of pulsators for which a quantitative measure of differential interior rotation has been established.

4.1. White Dwarfs

The first detection of differential (i.e., non-rigid) rotation inside a star besides the Sun was achieved for the DBV white dwarf GD 358 from a multisite campaign by the Whole Earth Telescope organisation (Winget et al., 1994). The multiperiodic variations of DBV white dwarfs are due to low-degree, high-order $g$ modes, excited by the heat mechanism active in the second partial ionization zone of helium. Their oscillation periods range from 4 to 12 minutes and their photometric amplitudes are relatively large, from a few mmag to 0.2 mag (e.g., Bradley, 1995). Among the more than 180 significant frequency peaks detected in the white-light photometric lightcurve of GD 358 covering 154 hours of data, 27 are the components of well-identified triplets. These frequency splittings are shown as a function of radial order $n$ in Figure 4. It can be seen that larger splittings occur for higher radial order,
while one would expect these splittings to be constant for rigid rotation inside the white dwarf. Since the modes of higher radial-order probe predominantly the outer layers and those of lower radial-order the inner parts, the rotation of GD 358 must be radially differential. The mean splitting for the modes of \( n = 16 \) and 17 leads to a rotation period of 0.89 days through Equations (3) and (4), while for \( n = 8,9 \) the rotation period is 1.6 days. Winget et al. (1994) therefore concluded that the inner parts of GD 358 rotate 0.6 times more slowly than its outer layers, where “inner” and “outer” refer to those regions probed by the detected triplets. However, Kawaler, Sekii, and Gough (1999) found that the data were not yet of sufficient quality to allow a more detailed inversion for the variation of the internal rotation with depth.

The detailed seismic modelling of GD 358 followed that achieved previously for the prototypical DOV white dwarf PG1159-035 (GW Vir) described in the seminal work by Winget et al. (1991), which was again based on data collected by the Whole Earth Telescope consortium (Nather et al., 1990). This led to 125 significant frequencies for GW Vir, of which 101 were identified as components of rotationally split triplets and quintuplets. Unlike the case for GD 358, all multiplets for a given \( \ell \) showed the same frequency splitting within the measurement errors, allowing Winget et al. (1991) to deduce a constant rotation period of approximately 1.38 ± 0.01 day throughout the white dwarf. Classical spectroscopy can in no way reveal the rotation periods of single compact stellar remnants with such high precision, not even in the case where the inclination angle can be estimated from independent information.

The deviation of GD 358’s splittings for \( m = +1 \) with respect to those for \( m = -1 \) in Figure 4 was interpreted by Winget et al. (1994) in terms of a weak magnetic field of 1300 ± 300 G, which causes splittings \( \sim |m^2| \) in addition to the rotational splitting given in Equation (3) (e.g., Dziembowski and Goode, 1984; Jones et al., 1989). The effect of a magnetic field could not be established for the frequency multiplets of PG 1159-035, which led to an upper limit of 6000 G for that object’s magnetic field (Winget et al., 1991). It is noteworthy that the magnetic-field strength that can be probed by classical spectroscopy of white dwarfs through the Zeeman effect requires fields roughly a factor of 1000 stronger than what can be found from asteroseismology.

The case studies of GD 358 and PG 1159-035 by the Whole Earth Telescope consortium implied not only a first test case for the technique of asteroseismology, but at the same time a real breakthrough in the derivation of white dwarf structure models. It not only led to estimates of internal rotation and magnetic field strength, but also allowed a high-precision mass estimate (0.586 ± 0.003 \( M_\odot \) for PG1159-035 and 0.61 ± 0.03 \( M_\odot \) for GD 358). It also proved that the outer layers of white dwarfs are compositionally stratified. This was derived from deviations of the frequency spacings due to mode trapping compared with spacings for unstratified models. Mass estimates with such high precision cannot be achieved from other means, except for relativistic effects in binary pulsars. These two seismic studies of white dwarfs paved the road for many others of their kind, but none of the more recent ones have led to more accurate internal rotation rates than those for PG1159-035 and GD 358. We refer to Kepler (2007) and Fontaine and Brassard (in preparation) for recent review papers on white-dwarf seismology.

4.2. Main-Sequence Stars

There are presently only three main-sequence stars, besides the Sun, for which an observational constraint on the internal-rotation profile has been derived. In all three
cases, it was achieved through asteroseismology of β Cep stars. Several of these are suitable targets to attempt mapping of their interior rotation because their rotational frequencies are well below the frequency spacing between multiplets. These stars are particularly interesting targets for this purpose, because the largest uncertainty in stellar evolution models for massive stars is precisely concerned with rotational mixing effects.

The first seismic proof of differential rotation in a massive star was obtained for the B3V star V836 Cen (HD 129929; Aerts et al., 2003). This result was derived from the well identified (parts) of one rotationally split frequency triplet and one quintuplet, as shown in Figure 5. This star’s detected frequency spectrum is obviously very similar to the one of θ Oph (compare Figures 2 and 5), except that V836 Cen is a slower rotator than θ Oph. Given that only two multiplets were available for V836 Cen, Dupret et al. (2004) assumed a linear rotation law and concluded that the rotational frequency near the stellar core is 3.6 times higher than at the surface. It was possible to derive this because the kernels of the $g_1$ and $p_1$ modes probe differently the rotational behaviour near the stellar core, just as for θ Oph (see Figure 3). A very similar result, the rotation of the deep interior exceeding the surface rotation by a factor between three and five, was obtained by Pamyatnykh, Handler, and Dziembowski (2004) from the $g_1$ and $p_1 \ell = 1$ modes of the B2III β Cep star ν Eri (HD 29248). These results are compatible with the assumption of local angular-momentum conservation. Both V836 Cen and ν Eri are – for upper main-sequence stars – very slow rotators, with surface rotation velocities of $2 \text{ km s}^{-1}$ (V836 Cen) and $6 \text{ km s}^{-1}$ (ν Eri). This made the seismic derivation of the interior rotation possible, because the splitting of the multiplets does not interfere with the

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**Figure 5.** The schematic frequency spectrum of the β Cep star V836 Cen derived from single-site Geneva $U$ data spanning 21 years. The dotted lines are frequencies that are not yet firmly established; these were not used in the seismic modelling. (Figure reproduced from Aerts et al., 2004).
frequency separation between different multiplets. The rotation profile itself could not be tuned further, given that only parts of very few multiplets were available. Classical spectroscopy, even at extremely high resolution, could never have led to the proof of differential interior rotation, as it can only measure the surface rotation. Moreover, the intrinsic line broadening of such stars is typically of order $\approx 10$ km s$^{-1}$, which is larger than the surface rotation velocity of these two stars, preventing a derivation of the projected equatorial rotation velocity to better than 1 km s$^{-1}$.

For the star $\theta$ Oph, which is a twin of V836 Cen as far as the detected frequency spectrum is concerned, rigid interior rotation could not be excluded from comparison of the frequency spacing in its triplet and its quintuplet (Briquet et al., 2007). Its frequency precision is two orders of magnitude lower that for V836 Cen and one order of magnitude lower than for $\nu$ Eri. In any case, strong differential rotation is excluded for that star as well.

5. Expected Future Improvements

5.1. Compact Pulsators and the Tuning of Atomic Diffusion

A field within asteroseismology, which we did not discuss extensively here but which is undergoing rapid growth and may tune our knowledge of microscopic diffusion for stellar structure and of binary star evolution, is the application to pulsating subdwarf B stars (sdBVs). While sdBVs with $p$ modes were discovered a decade ago (Kilkenny et al., 1997), those with $g$ modes were discovered more recently (Green et al., 2003). The existence of sdBVs was predicted independently and simultaneously with their observational discovery (Charpinet et al., 1996). An opacity bump associated primarily with iron-group elements turns out to be efficient driving mechanism. The atomic diffusion processes that are at work in sdB stars – radiative levitation and gravitational settling – cause iron (and also zinc) to become overabundant in the driving zone, thus exciting low-order $p$ and $g$ modes (Charpinet et al., 1997; Jeffery and Saio, 2006). The details of the diffusion processes are, however, still uncertain. These may also be relevant for the SPBs and $\beta$ Cep stars (Bourge et al., 2006), for which diffusion processes have been ignored so far in the seismology. Such processes are dominant in the atmospheres of the roAp stars, as is discussed by Kurtz (2008).

From an evolutionary point of view, the sdB stars are poorly understood. Their effective temperatures are in the range 23,000 – 32,000 K, and their log $g$ in the range 5 – 6. They all have masses below 0.5 $M_\odot$ which implies that they have lost almost their entire hydrogen envelope at the tip of the red-giant branch. Their thin hydrogen layer does not contain enough mass to burn hydrogen, making them evolve immediately from the giant branch towards the extreme horizontal branch. While it is clear they will end their lives as low-mass white dwarfs, it is yet unclear how they expelled their envelopes. All scenarios that have been proposed involve close binary interaction (Han et al., 2003; Hu et al., 2007).

The currently known sdB pulsators have multiple periods in the range 80 – 600 seconds and amplitudes up to 0.3 mag. Their amplitude variability and faintness have prevented unambiguous mode identifications so far, limiting the power of seismic inference to tune the diffusive and rotational processes. Rapidly rotating cores have been claimed for some of the sdBVs in order to explain their dense frequency spectra.
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in terms of low-degree modes (Kawaler and Hostler, 2005). Firm observational proof of that is not yet available, but, given the impressive efforts undertaken to understand the internal and atmospheric structure of these stars as well as their evolutionary status, we expect rapid progress in the near future. For a recent overview of the status of sdB seismology, we refer to Charpinet et al. (2007).

Recently, a seven-year study of the sdBV star V391 Peg (Silvotti et al., 2007) used the extreme frequency stability of two independent pulsation frequencies to show the presence of a \( \approx 3.2 \, \text{M}_{\text{Jupiter}} \) planet that had moved from about 1 AU out to 1.7 AU during the red giant phase of the sdB star precursor, allowing the planet to survive, much as the Earth may survive the Sun’s red giant phase in about 7 Gyr. This novel application of asteroseismology highlights the close relation and mutual interests of helioseismology, asteroseismology, planet-finding and solar system studies.

5.2. Heat-Driven Pulsators along the Main Sequence

Overshooting parameters and internal rotation profiles have not yet been determined for the other heat-driven pulsators known along the main sequence (besides the \( \beta \) Cep stars), such as the slowly pulsating B stars (SPBs) and the A- and F-type \( \delta \) Sct and \( \gamma \) Dor stars. The main obstacles to overcome are the limited number of detected oscillation frequencies of the \( g \) modes for the SPB stars and \( \gamma \) Dor stars, and reliable mode identification for those as well as for the \( \delta \) Sct stars. While the pioneering space missions WIRE (Buzasi, 2002; Bruntt and Southworth, 2008) and MOST (Matthews et al., 2004; Walker, 2008) led to an impressive and unprecedented number of oscillation modes for several such stars, the time base of the data was limited to a few weeks and unique mode identifications are not available for these mission’s target stars. It is to be expected that the uninterrupted photometry obtained by the CoRoT (five months time base, launched 26 December 2006) and Kepler (3.5 years time base, to be launched in 2009) space missions, along with their ground-based spectroscopy programmes, will result in the necessary frequency precision and mode identification. This should imply big steps forward for the seismic modelling of these type of stars.

Thus, even with several space missions in operation, ground-based efforts to increase the number of heat-driven pulsators with (preferably simultaneous) long-term multicolour photometric and high-resolution spectroscopic data for mode identification should definitely be intensified. It was this type of extensive data that yielded sudden and immense progress in the \( \beta \) Cep star seismology discussed in this paper and that also advanced significantly the interpretation of the oscillation spectrum of the prototypical \( \delta \) Sct star FG Vir (Zima et al., 2006). Only systematic and dedicated observing programmes can bring us to the stage of mapping and calibrating the internal-mixing processes inside stars across the HR diagram.

5.3. Solar-Like Pulsators

A new dimension in the progress for stochastically-excited pulsators is expected from the combination of asteroseismic and interferometric data, as explained by Cunha et al. (2007). The CoRoT and Kepler missions will also provide a large improvement in the data for solar-like oscillators. In particular, Kepler will yield data over several years for more than a hundred stars, together with three-month surveys of many more stars. The very extended observations may reveal possible frequency variations...
associated with stellar magnetic cycles, as has been observed in the Sun, and hence improve our understanding of such cycles. Also, the identification and interpretation of mixed modes, which have both a \( p \)-mode and a \( g \)-mode character due to a highly condensed stellar core, would be a great help to tune stellar evolution models towards the end of, and after, the central hydrogen-burning phase.

Data of even higher quality on solar-like oscillators can be obtained with dedicated \( \text{m} \text{s}^{-1} \)-precision radial-velocity campaigns, since the intrinsic stellar noise background is much lower, relative to the oscillations, in velocity than in photometry (e.g. Harvey 1988). This is the goal of the SIAMOIS (Mosser \textit{et al.}, 2007) and SONG (Grundahl \textit{et al.}, 2007) projects. SIAMOIS will operate from the South Pole, while SONG aims at establishing a global network of moderate-sized telescopes. Both are dedicated to obtain high-precision radial-velocity observations. This will increase substantially the number of (rotationally split) detected modes, particularly at relatively low frequency where the mode lifetime is longer and the potential frequency accuracy is higher. With the high-quality data expected from CoRoT, \textit{Kepler}, SIAMOIS and SONG, we may hope to carry out inverse analyses (e.g., Basu \textit{et al.}, 2002, Roxburgh and Vorontsov 2002) to infer the detailed properties of stellar cores.

These projects, and others further into the future, covering stars across the HR diagram will provide an extensive observational basis for investigating stellar interiors. Together with the parallel development of stellar modelling techniques we may finally approach the point, in the words of Eddington (1926), of being 'competent to understand so simple a thing as a star'.

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\textbf{References} 


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Kepler, S.O.: 2007, Comm. in Asteroseismology 150, 221.


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