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Role of the axial vector $a_1$-meson exchange in hypernuclear nonmesonic weak decays

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In the meson-theoretical potential model for the study of the nonmesonic decay rates and asymmetries of hypernuclei, for the first time, the axial-vector $a_1$ meson ($J^{PC} = 1^{++}, m_{a_1} = 1230$ MeV) is introduced. The $a_1$ meson is the chiral partner of the $\rho$ meson and has been treated in the meson-pair exchange framework as $\rho \pi/a_1$ and $\sigma \pi/a_1$. This is analogous to the treatment of $\rho$ and $\sigma$ exchange in our model. The $a_1$-meson exchange is found to give remarkable modifications of the parity-conserving decay potentials ($^{1}S_0 \rightarrow ^{1}S_0$, $^{3}S_1 \rightarrow ^{3}D_1$) at short range $r \leq 1$ fm. As a result, the calculated intrinsic asymmetry parameter $\alpha_{xx}$ for $^3$He becomes very small and positive in good agreement with the recent high-quality experimental data. The calculated small values of $\alpha_{xx}$ are well compared with the data for $^{11}$B and $^{12}$C within error bars. The inclusion of the $a_1$ meson also improves the $\Gamma_n/\Gamma_p$ ratios and leads to a consistent explanation for the existing nonmesonic weak decay data of the light $A$ hypernuclei ($A \leq 12$). The results calculated in the $\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K + \rho \pi/a_1 + \sigma \pi/a_1$ exchange interaction model are presented together with the estimates without $a_1$. Also, the derivation of the expression for the proton asymmetry is described in some detail to elucidate the calculation procedures and phase conventions.

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I. INTRODUCTION

The hypernuclear nonmesonic weak decay provides a unique opportunity for understanding the hyperon-nucleon weak interaction $\Lambda N \rightarrow NN$ with the strangeness change $\Delta S = 1$. High-quality data obtained with advanced techniques have been reported in recent years from KEK and they are challenges for the theoretical understanding of the weak interaction mechanisms. The recent improved data of the ratio of the neutron-stimulated decay rate $\Gamma_n(\Lambda n \rightarrow nn)$ to the proton-stimulated one $\Gamma_p(\Lambda p \rightarrow np)$ ($n/p$ ratio) tend to converge to $\approx 0.5$ for $^3$He and $^{12}$C [1–3]. The data of the asymmetry parameter of the proton emitted in the nonmesonic decay from the polarized hypernuclear states are found to be very small for $^3$He and for the mixture of $^{12}$C and $^{13}$B [1,4–6]. In light of such high-quality and high-statistics data on a variety of observables, it is to be expected that theoretical models need to cover new types of exchanges in terms of the mesonic quantum numbers $J^{PC}$.

In the last decade many theoretical studies on the weak $\Lambda N \rightarrow NN$ interaction have been devoted to understand the decay observables such as the total decay rate $\Gamma_{nn} = \Gamma_p + \Gamma_n$, the $n/p$ ratio, and the decay proton asymmetry on the basis of the meson-exchange models. Among others, the correlated-2$\pi$ exchange mechanism was first proposed in Ref. [7], and various refinements have been carried out [8,9]. The meson-exchange models have been further extended by several authors [10–13], while new attempts have been made in the direct-quark model [14,15] and the effective-field theory [16,17].

Among the nonmesonic decay observables, for a long time it was puzzling to explain the large experimental $n/p$ ratio and the very small proton asymmetry parameter. However, at present there seems to be a consensus that the $\pi$- and $K$-meson exchange interactions play an important role in explaining the large $n/p$ ratio.

On the other hand, nowadays the unraveling of the problem of the decay asymmetry is controversial. It has been difficult to understand both the sign and magnitude of the proton asymmetry parameter in spite of many theoretical attempts such as meson-exchange models and direct-quark model [9,10,12,15,18–20]. Parreño et al. [16] applied the effective field theory in which they introduced the contact interactions with phenomenological parameters together with the $\pi$- and $K$-exchange interactions and attempted to determine the coupling parameters by fitting the experimental data. They pointed out [16] that the scalar-isoscalar interaction of the contact term is important to fit the asymmetry parameters well. Then Sasaki et al. [21] include the $\sigma$-meson exchange in their model of meson exchange plus quark-quark interaction and treat the $\Lambda N\sigma$ weak and $NN\sigma$ strong vertices phenomenologically with adjustable coupling constants so as to understand the asymmetry parameter of $^3$He. Recently Chumillas et al. [22] show that the uncorrelated and correlated $2\pi$ exchange interactions may play a role in understanding the asymmetry parameter. All these articles stress that the scalar-isoscalar...
type of interactions, such as $\sigma$ exchange, the exchange of the correlated $2\pi$ coupled to $\sigma$, and uncorrelated $2\pi$ exchanges, are especially important in explaining the asymmetry.

Barbero et al. [23] adopted as a model pseudoscalar and vector-meson exchanges plus the $\sigma$-meson exchange, and treated the weak and strong coupling constants of the $\sigma$ meson as adjustable parameters in the same manner as in Ref. [21]. However, they could not explain the data of asymmetry and other observables of $^3_\Lambda$He simultaneously.

In Refs. [7–9], we have shown that $2\pi/\sigma$ and $2\pi/\rho$ exchanges are important in understanding the decay rates and the $n/p$ ratios of light $\Lambda$ hypernuclei, although the $2\pi/\sigma$ exchange interaction does not play a favorable role in explaining the asymmetry parameter of $^3_\Lambda$He [9]. Our $\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K$ exchange model gives large negative values in contrast to the very small asymmetry parameters observed for $^3_\Lambda$He and $^{12}_\Lambda$C. In addition to this discrepancy, this model gives large $n/p$ ratios that overshoot the experimental data for those light $\Lambda$ hypernuclei. These results clearly indicate that new ingredients are needed to improve our model.

In the above model, the central forces in the $^1S_0 \rightarrow ^1S_0$ and $^3S_1 \rightarrow ^3S_1$ channels are too strong with a positive sign, whereas the tensor force in the $^3S_1 \rightarrow ^3D_1$ channel is rather strong with a negative sign. The strong central force nature originates dominantly from the $2\pi/\sigma$ exchange interaction, whereas the strong tensor force is due to the total effect of the $\pi$, $K$, and $2\pi/\rho$ exchange interactions.

To try to solve the discrepancy between theory and experiments for the asymmetry, we extend the meson theoretical approach of Refs. [7–9] by taking new types of the exchanged mesons, which we denote as

\[ \pi/\sigma, \pi/\rho, \pi/\omega, \text{and } K. \]


However, they could not explain the data of asymmetry and other observables of $^3_\Lambda$He simultaneously.

The article is organized as follows. In Sec. II, we present the angular distribution of a proton emitted in the nonmesonic decay from the polarized hypernuclei and define the asymmetry parameter of a proton in the framework of shell model, where the transition amplitudes starting from the $\Lambda N$-relative $s$ state are given explicitly on the basis of the adopted phase conventions. In Sec. III, the extended meson-exchange model is presented in detail. The axial-vector $a_1$-meson exchanges are introduced in terms of $\rho \pi/a_1$ and $\sigma \pi/a_1$ exchanges. The coupling Hamiltonians are given and the adopted coupling constants and parameters are explained and shown. In Sec. IV, first the features of the $a_1$-meson exchange interaction are discussed to elucidate its effects on the decay observables. Then, the numerical evaluations of the decay rates, the $n/p$ ratios, and the decay asymmetry parameters are presented and compared with the $^3_\Lambda$He, $^{11}_\Lambda$B, and $^{12}_\Lambda$C hypernuclear decay data. Finally, a summary and outlook are given in Sec. V.

II. ANGULAR DISTRIBUTION OF A PROTON AND ASYMMETRY PARAMETER

When the hypernucleus is produced in the $(\pi^+, K^+), (K^-, \pi^-)$, and $(\gamma, K^+)$ reactions, the produced excited states and/or the ground state are generally polarized. Excited states below the particle emission threshold de-excite through the $\gamma$ transitions and cascade down to the ground state. Thus the hypernuclear ground state is populated and acquires the certain amount of polarization $P_H$ [26,27]. Then the hypernucleus undergoes weak decay such as either mesonic or the nonmesonic decay. In this article we confine ourselves to the nonmesonic weak decays from the polarized hypernuclear ground state.

The vector polarized hypernuclear state is expressed in the density matrix as

\[ \rho = \frac{1}{2J_H + 1}
\begin{bmatrix}
1 + \frac{3}{J_H + 1} (P_H \cdot J_H)
\end{bmatrix}, \]

where $J_H$ is a hypernuclear spin and $P_H$ is a hypernuclear polarization that is directed to the unit vector $\hat{n}$ which is normal to the reaction plane, that is, $P_H = P_H \hat{n}$.

The angular distribution of the proton emitted in the nonmesonic decay of the polarized hypernucleus is evaluated by taking the trace of the density matrix of Eq. (1), with fixing the angles of the outgoing proton momentum $\hat{k}$, as

\[ \frac{d\Gamma(J_H, T_H, P_H \rightarrow \hat{k}_p, v_p)}{d\Omega_{k_p}} \equiv \frac{2\pi}{\hbar} \text{tr}(M\rho M^\dagger). \]

Here $M$ is the nonmesonic weak decay transition matrix connecting the initial hypernuclear state and the final residual nucleus plus outgoing neutron and proton state. Inserting
Eq. (1) into Eq. (2), we obtain
\[
dfrac{d\Gamma(J_H,T_H,P_H \to \hat{k}_p,v_p)}{d\Omega_{k_p}} = \Gamma_{0} + P_H\Gamma_{1} \cos \theta_{kp} \]
\[
= \Gamma_{0}(1 + P_H\alpha_{1} \cos \theta_{kp}) \quad (3)
\]
and
\[
\alpha_{1} = \dfrac{\Gamma_{1}}{\Gamma_{0}} \quad (4)
\]
is the asymmetry parameter of a proton in the nonmesonic decay \cite{18,20}. \(\theta_{kp}\) is the angle between the direction of emitted proton momentum \(k_p\) and the polarization direction \(\hat{n}\). \(\Gamma_{0}\) is equivalent to \(\Gamma_{p}/4\pi\) and \(\Gamma_{p}\) is the proton-stimulated nonmesonic decay rate from the unpolarized hypernuclear state. \(\Gamma_{1}\) is given below. Hereafter we adopt the notation \(k_p = k_2\) and use convention that the outgoing neutron and proton have the momentum \(k_1\) and \(k_2\), respectively, corresponding to the elementary process of \(\Lambda + p \to n + p\).

\[
\Gamma_{1} = \dfrac{2\pi}{2J_{H} + 1} J_{H} + 1 \sum_{j',m',j,M'} \sum_{j,M} \int \frac{dk_{1}}{(2\pi)^{5}} \int \frac{dk_{2}}{(2\pi)^{3}} \delta(E_{f} - E_{i}) \bigg| \sum_{i,M}\begin{array}{c}
\sum_{j,m} \sum_{j',m'} V_{nm}(i,k) \left| A^{\Lambda}_{\Lambda}; J_{H}M_{H}, T_{H}M_{T_H} \right|^{2} \times M_{H}.
\end{array}
\]

The energy conservation \(\delta E\) function is expressed as
\[
\delta(E_{f} - E_{i}) = \delta \left( \dfrac{k_{1}^{2}}{2M_{N}} + \dfrac{k_{2}^{2}}{2M_{N}} + E_{s}(A - 2, J'_{T}T'_{s}\alpha'_{1}) \right.
\]
\[
+ \left( \dfrac{(k_{1} + k_{2})^{2}}{2M_{A-2}} - P_{s} - M_{N} - M_{\Lambda} - \varepsilon_{N} - \varepsilon_{\Lambda} \right) \quad (6)
\]
in which the recoil energy of the residual nucleus is taken into account and the residual nuclear mass \(M_{A-2} = (A - 2)M_{N}\) is used for simplicity. The \(E_{s}(A - 2, J'_{T}T'_{s}\alpha'_{1})\) means the excitation energy of the residual nucleus and \(\varepsilon_{N}\) and \(\varepsilon_{\Lambda}\) are minus separation energy of a nucleon and a \(\Lambda\) hyperon in the hypernucleus, respectively. It is assumed that the hypernuclear state \(|A^{\Lambda}_{\Lambda}; J_{H}M_{H}, T_{H}M_{T_H}\rangle\) is written in the weak coupling model as
\[
|A^{\Lambda}_{\Lambda}; J_{H}M_{H}, T_{H}M_{T_H}\rangle = \left| \begin{array}{c}
\sum_{j,m} \sum_{j',m'} V_{nm}(i,k) \left| A^{\Lambda}_{\Lambda}; J_{H}M_{H}, T_{H}M_{T_H}\right|^{2} \times M_{H}.
\end{array}\right.
\]

By taking into account the energy conservation [see Eq. (6)] and by fixing the direction of the outgoing proton momentum \(k_2\), we adopt three independent integral variables \cite{20} as
\[
k_{1} = k_{1} - k_{2}, \quad \theta_{k_{1}}, \quad \phi_{k_{1}}.
\]

The \(\Gamma_{1}\) of Eq. (5) is rewritten as
\[
\Gamma_{1} = \dfrac{2\pi}{2J_{H} + 1} J_{H} + 1 \sum_{j',m',j,M'} \sum_{j,M} \int \frac{dk_{1}}{(2\pi)^{5}} \int \frac{dk_{2}}{(2\pi)^{3}} \sqrt{\delta(E_{f} - E_{i})} \bigg| \sum_{i,M}\begin{array}{c}
\sum_{j,m} \sum_{j',m'} V_{nm}(i,k) \left| A^{\Lambda}_{\Lambda}; J_{H}M_{H}, T_{H}M_{T_H}\right|^{2} \times M_{H}.
\end{array}
\]

Here, the following relations are used.
\[
k_{Q} = \sqrt{2M_{N}[M_{\Lambda} - M_{N} + \varepsilon_{N} + \varepsilon_{\Lambda} - E_{s}(A - 2, J'_{T}T'_{s}\alpha'_{1})]},
\]
\[
k_{1} = \sqrt{ \dfrac{A - 2}{A - 1} k_{Q}},
\]
\[
k_{2}^{\max} = \sqrt{ \dfrac{A - 2}{A - 1} k_{Q}},
\]
\[
4k_{2}^{2} = k_{1}^{2} + k_{2}^{2} - 2k_{1}k_{2}\cos \theta_{k_{1}}.
\]
The $\Gamma_1$ is expressed in terms of the two-body transition amplitudes and in the case of $\Lambda N$ relative $s$ wave in the shell-model framework, which is given in the Appendix.

For the shell-model configurations of $^5_2$He ($A = 5, J_H = 1/2^+$) and $^{12}_3$C ($A = 12, J_H = 1^-$), further manipulation of $\Gamma_1$ of Eq. (A1) in the Appendix leads to the expression, as follows, in terms of Block-Dalitz notations of the transition energy is used in place of $\gamma_1$ of Eq. (7) in which a $\Lambda$ hyperon in the $j_{\Lambda} = s_{\Lambda}^{1/2}$ state, the following relation applies \[26,27\]. If one assumes the weak-coupling structure for the hypernuclear state of Eq. (7) in which a $\Lambda$ hyperon is in the $j_{\Lambda} = s_{\Lambda}^{1/2}$ state, the following relation applies \[26,29\].

\[
P_{\Lambda} = - \frac{J_H}{J_H + 1} P_H, \quad \text{if } J_H = J_c - 1/2
\]

\[
P_{\Lambda} = P_H, \quad \text{if } J_H = J_c + 1/2.
\]

Then it follows from Eq. (24)

\[
\alpha_{\Lambda} = - \frac{J_H + 1}{J_H} \alpha_1, \quad \text{if } J_H = J_c - 1/2
\]

\[
\alpha_{\Lambda} = \alpha_1, \quad \text{if } J_H = J_c + 1/2.
\]

The intrinsic asymmetry parameter $\alpha_\Lambda$ gives a more intuitive interpretation for the asymmetry, because the asymmetry originates dominantly from the elementary weak interaction between a $\Lambda$ hyperon and a proton inside the hypernucleus.

### III. DESCRIPTION OF $A_1$-MESON EXCHANGE WEAK INTERACTION

The motivation of introducing the axial-vector $a_1$ meson into the $\Lambda N \rightarrow NN$ weak interaction is described in the second half of the Introduction. The $a_1$ meson has $J^{PC} = 1^{++}$ and the mass is 1230.0 MeV. The $a_1$ meson is a chiral partner of the $\rho$ meson just like the $\sigma$ meson is a chiral partner of the $\pi$ meson. Therefore, it is natural to consider the chiral meson pair properly in the meson theoretical weak baryon-baryon interaction. Because for $a_1$ exchange we include only diagrams with the $\rho \pi$ and $\sigma \pi$ decay interactions, the range is comparable with that of, e.g., $\omega$ exchange. The potential behavior at short range is very influential in the nonmesonic weak decays due to the high momentum transfer involved. It is especially noted that the proton asymmetry originates from the interferences between the parity-conserving and the parity-violating amplitudes and, therefore, signs and magnitudes of the decay interaction potentials are crucial for the asymmetry parameter.

We treat the $a_1$-meson exchange through the $\rho \pi/a_1$ exchange and $\sigma \pi/a_1$ exchange processes between $\Lambda$ and the nucleon. Two types of the exchange diagrams are considered for $\rho \pi/a_1$($A$) and $\rho \pi/a_1$($B$) exchanges as shown in Figs. 1(a) and 1(b), respectively. In Fig. 1(a), a nucleon is considered in the intermediate baryon state, while in Fig. 1(b) a $\Sigma$ baryon is considered. The $\sigma \pi/a_1$ exchange diagram is depicted in Fig. 2.
We adopt a minus (−) sign for $g_{\rho\pi a_1}$ which will be explained later.

The $g_{\rho\pi a_1}$ is determined from considering the axial-vector current; namely the hadronic axial-vector current is coupled to the $a_1$ via the Lagrangian density as

$$L^{(A)}_1 = 2g_A \left[ \bar{\psi}_N \gamma_\mu \frac{i}{2} \psi_N + (\partial^\mu \phi_\sigma \psi_\pi - \phi_\sigma \partial^\mu \psi_\pi) \right] \cdot \phi_{a_1,\mu},$$

(33)

So, we take the relation,

$$g_{\rho\pi a_1} = 2g_A = 2g_{NNa_1}.$$

(34)

Now, when one calculates the invariant amplitude $M_{fi}(\Lambda N \rightarrow NN)$ of the $\rho\pi/a_1$ exchange diagrams in Figs. 1(a) and 1(b), one must evaluate the loop integral over the intermediate momentum $k$. After the Feynman integral is introduced, the loop integral has a form symbolically as

$$\int d^4k G(k)$$

(35)

and

$$G(k) = \frac{F_0 + F_1 k^2 + F_2 k^4}{k^2 - c^2}.$$  

(36)

This integral diverges at large $k^2$. To circumvent the divergence we introduce the regularization factor of the form

$$\left[ \frac{\Lambda_n^2}{k^2 + \Lambda_n^2} \right]^n, \quad n = 1, 2.$$  

(37)

The $\Lambda_n$’s are parameters to be determined and $k$ is a three-momentum.

We adopt at the $NNa_1$ vertex the form factor

$$FF = \frac{\Lambda_n^2}{q^2 + \Lambda_n^2},$$

(38)

where $q = p'_i - p_{1\Lambda}$ is the momentum transfer and $\Lambda_n$ is a cutoff mass.

The $\rho\pi/a_1(A)$ exchange potential $V_{\rho\pi a_1(A)}$ is obtained as the Fourier transform of the invariant amplitude $iM_{fi}$ for the diagram of Fig. 1(a). The parameters $\Lambda_1$ and $\Lambda_2$ and the coupling constant $g_{\rho\pi a_1}$ are determined in the following way. We construct the strong $NN$-force version from the similar $\rho\pi/a_1$ exchange diagram of $NN \rightarrow NN$ in which a $\Lambda$ hyperon in Fig. 1(a) is replaced by a nucleon. We call it $V_{\rho\pi a_1}^{\text{strong}}(NN \rightarrow NN)$. Then we equate $V_{\rho\pi a_1}^{\text{strong}}(NN \rightarrow NN)$ approximately to $V_{a_1}^{\text{strong}}(NN \rightarrow NN)$ of the $a_1$-exchange $NN$ force of the ESC04 model [24] as

$$V_{\rho\pi a_1}^{\text{strong}}(NN \rightarrow NN) \approx V_{a_1}^{\text{strong}}(NN \rightarrow NN).$$

(39)

The $\Lambda_1$, $\Lambda_2$, and $g_{\rho\pi a_1}$ are determined so that the approximate relation of Eq. (39) holds as well as possible. The potential $V_{\rho\pi a_1(A)}$ is expressed in the isospin basis as

$$V_{\rho\pi a_1(A)}(r) = \{V_\rho(r)(\sigma_1 \cdot \sigma_2) + V_\pi(r)(L \cdot S) + i V_\Sigma(r) [(\sigma_1 \times \sigma_2) \cdot \hat{r}] + i V_\Lambda(r)(\sigma_2 \cdot \hat{r})](\tau_1 \cdot \tau_2).$$

(40)
The particle 1 refers to a Λ hyperon. The parity-violating vector part of the potential is expressed in the imaginary form.

The ρπ/ω (A) exchange potential \( V_{\rho \pi/\omega}(A) \) for the diagram depicted in Fig. 1(b) is evaluated in the same way as in the case of \( V_{\rho \pi/\omega}(A) \) potential. The potential \( V_{\rho \pi/\omega}(A) \) is expressed in the charge basis as

\[
V_{\rho \pi/\omega}(A)(r) = \{\text{same potential type except } \tau_1, \tau_2 \text{ as in Eq. (40)}\} \mathcal{O}^T_{\rho \pi/\omega}(\Lambda n \to NN).
\]

The operator \( \mathcal{O}^T \) refers to the isospin-dependent factor for the two-nucleon state, which reads

\[
\mathcal{O}^T_{\rho \pi/\omega}(\Lambda n \to nn) = 1 \quad (T = 1),
\]

\[
\mathcal{O}^T_{\rho \pi/\omega}(\Lambda p \to np) = -\frac{3}{\sqrt{2}} \quad (T = 0),
\]

\[
= \frac{1}{\sqrt{2}} \quad (T = 1).
\]

The explicit form of the potential terms in Eqs. (40) and (41) will be given elsewhere [31].

The σπ/ω exchange potential for the diagram of Fig. 2 is similarly evaluated. The loop integral of Fig. 2 has a similar form as Eq. (35) with

\[
G(k) = \frac{F_{0\omega} + F_{1\omega}k^2}{k^2 - c_1^2}.
\]

Because the integrals diverge at large momentum \( k^2 \), we introduce the regularization factor of the form \( \Lambda_1^2 / (k^2 + \Lambda_1^2) \), where \( \Lambda_1 \) is a parameter to be determined. The potential \( V_{\sigma \pi/\omega} \) is expressed as

\[
V_{\sigma \pi/\omega}(r) = \{V_C(r) + V_S(r)(\sigma_1 \cdot \sigma_2) + V_T(r)S_{12} + V_{LS}(r)(\mathbf{L} \cdot \mathbf{S}) + iv\nu_2(r)[(\sigma_1 \times \sigma_2) \cdot \hat{r}] + iv\nu_2(r)(\sigma_1 \cdot \hat{r})(\tau_1 \cdot \tau_2)\}.
\]

Again, the explicit form of the potential terms in Eq. (45) will be given elsewhere [31].

The potentials of \( V_{2\pi/\rho}(A,B) \) and \( V_{2\pi/\rho}(A,B) \) are modified from the previous version [8] in the following points. For the \( V_{2\pi/\rho} \) potential, the \( NN\rho \) vertex form factor of the \( \Lambda_1^2 / (q^2 + \Lambda_1^2) \) type is introduced and the regularization factor of the \( \Lambda_1^2 / (k^2 + \Lambda_1^2) \) type is employed in the loop integral of the intermediate momentum \( k \). The \( \rho \)-meson mass \( m_\rho = 771.1 \text{ MeV} \) is used and the coupling constant is a bit changed.

For the \( V_{2\pi/\rho} \) potential, the \( NN\sigma \) vertex form factor of the \( 1 - q^2 / M_0^2 \) type is introduced [24]. The first factor in the form factor has a zero at \( q^2 = M_0^2 \), which reflects fact that the \( \sigma \) is internally a \( p \)-wave quark-antiquark state [24].

IV. RESULTS AND DISCUSSIONS

A. Features of \( a_1 \)-meson exchange and total nonmesonic decay potential

The strong coupling constants and parameters adopted for the \( \rho \pi/\omega, \sigma \pi/\omega, 2\pi/\rho, \) and \( 2\pi/\sigma \) exchange potentials are listed in Table I, where the coupling constants for the \( a_1 \) meson are taken from the ESC04 model [24,25]. We adopt the sign of \( g_{NN\omega} \) to be the same as that of \( g_{NN\rho} \) and adopt also the sign of \( g_{\rho \pi a_1} \) that of \( g_{\rho \pi \sigma} \), which are suggested from the chiral invariance argument by Schwinger [32]. Weak coupling constants and parameters are the same as those adopted in Ref. [8].

Three types of \( a_1 \)-meson exchange potentials, \( V_{\rho \pi/\omega}(A) \), \( V_{\rho \pi/\omega}(B) \), and \( V_{\sigma \pi/\omega} \), have been calculated. In the following, \( V_{\rho \pi/\omega} \) means the addition of \( V_{\rho \pi/\omega}(A) \) and \( V_{\rho \pi/\omega}(B) \) potentials. The characteristic features of \( V_{\sigma \pi/\omega} \) are as follows: (i) the central and the tensor forces are strong at short range; (ii) the parity-violating force of the \( i[(\sigma_1 \times \sigma_2) \cdot \hat{r}] \) type is strong, whereas that of \( (\sigma_2 \cdot \hat{r}) \) type is weak; and (iii) the potentials \( V_{\rho \pi/\omega}(A) \) and \( V_{\rho \pi/\omega}(B) \) show almost similar \( r \)-dependent behavior though the former potential strength is quite a bit stronger than the latter. The potential features of \( V_{\sigma \pi/\omega} \) are as follows: (i) the potential is dominantly of the central-force type and the potential strength is weak compared with that of the \( \rho \pi/\omega \) exchange and (ii) the parity-violating forces are very weak.

Figure 3 shows the central-type \( \Delta p \to np \) transition potentials in the \( ^1S_0 \to ^1S_0 \) channel. Before introducing the \( a_1 \)-meson exchange [8,9], the strongly positive \( V_{2\pi/\rho} \) determines the total potential behavior to be positive in the most effective region \( r > 0.5 \text{ fm} \). It is remarkable here that the new potential of \( V_{\rho \pi/\omega} \) is very strongly negative at the short range of \( r < 1 \text{ fm} \), which overwhelsms the \( V_{2\pi/\rho} \) potential. The negative \( V_{\rho \pi/\omega} \) behaves additively. As a results, when the \( a_1 \)-meson exchange is incorporated, the summed potential of \( ^1S_0 \to ^1S_0 \) channel is modified drastically to be strongly negative at the

| TABLE I. Strong coupling constants and parameters adopted in the \( \rho \pi/\omega, \sigma \pi/\omega, 2\pi/\rho, \) and \( 2\pi/\sigma \) exchange potentials. |
|---|---|---|
| \( g_{NN\omega} \) | 8.5893 | \( f_{NN\omega} \) | 0.0 |
| \( g_{NN\rho} \) | 0.0 | \( f_{NN\rho} \) | 7.6719 |
| \( g_{\rho \omega} \) | -5020 MeV | \( m_{\omega} \) | 1230.0 MeV |
| \( \Lambda_1(\rho \pi/\omega) \) | 2000 MeV | \( \Lambda_1(\rho \pi/\omega) \) | 1650 MeV |
| \( g_{NN\sigma} \) | 6.30 | \( g_{NN\sigma} \) | 17.1786 |
| \( g_{NN\rho} \) | 2.9870 | \( f_{NN\rho} \) | 14.7557 |
| \( g_{NN\rho} \) | 7.66 | \( g_{NN\rho} \) | -8.0 |
| \( \Lambda_1 \) | 900 MeV | \( \Lambda_1(2\pi/\rho) \) | 920 MeV |
| \( \Lambda_1(2\pi/\sigma) \) | -1850 MeV | \( m_\pi \) | 600.0 MeV |
| \( \Lambda_1 \) | 1130 MeV | \( M_\pi \) | 760 MeV |

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short range region. The individual potentials in the $^3S_1 \rightarrow ^1S_1$ channel (not shown here) behave almost similarly as the corresponding ones in the $^1S_0 \rightarrow ^1S_0$ channel. Therefore, the summed potential for the $^3S_1 \rightarrow ^1S_1$ channel is also modified to be strongly negative at short range.

Figure 4 shows the tensor potentials in the $^3S_1 \rightarrow ^3D_1$ channel for the various meson exchanges. The $V_{\rho/a_1}$ is strongly positive at short range in contrast to $V_{2\pi/a_1}$. The $V_{\sigma/\pi/a_1}$ has almost no contribution. As a result, when the $a_1$-meson exchange is considered, the summed potential in this tensor channel changes its behavior drastically. The summed potential is almost vanishing at $r \gtrsim 0.9$ fm and changes its sign to be positive at the shorter range. This behavior is quite different from the case of our previous $\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K$ exchange model.

Figure 5 shows the parity-violating vector potentials in the $^3S_1 \rightarrow ^1P_1$ channel. The $V_{\pi/a_1}$ potential of the $i[\sigma_1 \times \sigma_2] \cdot \hat{r}$ type contributes strongly with a positive sign, which behaves additively to $V_{2\pi/\rho}$. The $i[\sigma_2 \cdot \hat{r}]$-type potential does not contribute as much. The summed potential stays strongly positive and is enlarged. In the other parity-violating channels, $^1S_0 \rightarrow ^3P_0$ and $^3S_1 \rightarrow ^3P_1$, the $a_1$-meson exchange does not produce large effects.

It is appropriate to note the typical contributions of the chiral partner mesons, $\pi$ vs. $\sigma$ and $\rho$ vs. $a_1$, to the transition potential. It is well known that $\pi$ exchange produces a strong tensor force, whereas $\sigma$ exchange (2$\pi/\sigma$ exchange in our model) produces a strong central one as seen in Figs. 4 and 3. The central force of $2\pi/\rho$ exchange is weak, whereas that of $\rho/\pi/a_1$ exchange is strong enough at short distances. Another feature is that the tensor forces from $2\pi/\rho$ exchange and $\rho/\pi/a_1$ exchange have similar strength but completely opposite signs at short distances. The parity-violating potentials for the $2\pi/\rho$ and $\rho/\pi/a_1$ exchanges work additively. From these observations, we recognize that it is very important to take the mesonic chiral partners into account simultaneously in evaluating the transition potential. In this respect, an extension of the study of chirality to the strange-meson sector, including $\kappa$, $K$, and $K^*$, might be a forthcoming subject toward a refinement of the present model.

B. Nonmesonic decay rates and asymmetry parameter of $^8_B$He

The nonmesonic weak decay rates and the asymmetry parameters are evaluated for the light $\Lambda$ hypernuclei as $^7_B$He, $^7_L$B, and $^7_C$ in the shell-model framework. It is known from the reaction analyses for ($\pi^+$, $K^+$) and ($K^-$, $\pi^-$) that
these hypernuclei are well described [27] by the product wave functions as shown by Eq. (7).

In evaluating the $\Lambda N \rightarrow NN$ transition matrix element, the initial state correlation for $\Lambda N$ and the final state correlation for the outgoing $NN$ are carefully taken into account. See Ref. [8] for details. Then the $\Lambda N$ wave function $\phi_{\Lambda}(r)$ in Eq. (23) is replaced as

$$
\phi_{\Lambda}(r) \rightarrow f_s(r) \phi_{\Lambda}(r) \chi_{sS,\ell} + \delta_{S1} f_2(r) \chi_{211}
$$

for the relative $s(\ell = 0)$ wave. The $f_s(r)$ is a correlation function for the $s$ wave and $f_2$ is the induced $d$ wave. The relative $p(\ell = 1)$ wave contribution is small and is not considered in the present work. The final state $j_{\ell}(k, r) \chi_{sS,\ell}^\prime$ in Eq. (23) is replaced by the scattering state as

$$
i^0 j_{\ell}(k, r) \chi_{sS,\ell}^\prime \rightarrow i^0 \psi_{sS,\ell}(k, r) \chi_{sS,\ell}^\prime + \delta_{s1} \delta_{\ell1} \left[ \delta_{\ell0} i^2 \chi_2(k, r) \chi_{211} + \delta_{\ell2} \chi_0(k, r) \chi_{011} \right]
$$

The $\psi_{sS,\ell}$ is the scattering wave and the $\chi_2$ and $\chi_0$ are the induced $d$ and $s$ waves, respectively, due to the $NN$ tensor interaction.

Table II lists the calculated results for $^5_\Lambda$He together with the experimental data. In the $\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K$ exchange model the $n/p$ ratio is enhanced strongly mainly due to $\pi$- and $K$-meson exchanges [9,11,12,14], although the theoretical $\Gamma_n/\Gamma_p$ seems too large. Note that the $a_1$-meson exchange gives rise to a much better $n/p$ ratio (0.508) to be relatively comparable with the recent experimental data [4,5].

One of the most important results of the $a_1$-exchange mechanism is that this can naturally give a small asymmetry parameter $\alpha_\Lambda = 0.083$ in good agreement with the recent high-quality data [1,4,5]. This shows the most drastic change, because the previous calculation without $a_1$-meson exchange gives $\alpha_\Lambda = -0.833$ as compared in Table II. To confirm this novel result, we discuss the detailed reason for the change below.

For the present purpose it is illuminating to refer to the expression of the asymmetry parameter in the nonmesonic decay from a polarized $^\Lambda$ hyperon:

$$
\alpha_\Lambda(\text{free}) = \frac{2\sqrt{3} \text{Re}[-ae^* + b(c - \sqrt{2}d)^*/\sqrt{3} + f(\sqrt{2}c + d)^*/\sqrt{3}]}{|a|^2 + |b|^2 + 3|c|^2 + |d|^2 + |e|^2 + |f|^2}.
$$

The numerator $\Gamma_1$ of $\alpha_\Lambda$ for $^5_\Lambda$He contains the similar expression as that of Eq. (48), which is understood in Eq. (20) if one realizes $\cos \theta_k \approx -1$, because the opening angle $\theta_k \approx 180^\circ$ contributes dominantly in the nonmesonic decay. However, one should bear in mind that the amplitudes $a, b, c, d, e$, and $f$ in Eq. (20) are supplemented by the structure factors and are integrated.

In the previous calculation within the $\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K$ exchange model, the large and negative value ($\alpha_\Lambda = -0.833$) comes dominantly from the $-ae^*$ term and next from the $\sqrt{2}fc^*$ term. In this model, both amplitudes $a$ and $e$ are large with a positive sign, which leads $-ae^*$ to be negative and large. The positive and large value of the $a$ amplitude is mainly due to the $2\pi/\sigma$ exchange potential, which is overwhelmingly strong with a positive sign in the $^1S_0 \rightarrow ^1S_0$ channel. The amplitude $e$ is known to be positive and strong because most of the meson exchange potentials contribute additively with a positive sign in the $^3S_1 \rightarrow ^1P_1$ channel. The $\sqrt{2}fc^*$ factor is definitely negative, because $f$ is negative and $c$ is positive. It is noted here that the amplitude $c$ has contributions from $^3S_1 \rightarrow ^1S_1$ (direct amplitude $c_0$) and also from $^3S_1 \rightarrow ^1D_1$ channels (induced amplitude $d$ ) due to the final-state tensor correlation as explained in Eq. (47); that is, $c = c_0 + d$.

Although the summed potential of the direct channel and that of the induced channel have opposite signs, the two channel contributions work additively owing to the phase factor $i^{\ell_0}$ and $\ell_0 = 0$ and 2. Likewise, the $d$ amplitude is composed of the direct $d_0$ and the induced $d'$ amplitudes, i.e., $d = d_0 + d'$. In this way we have a large and negative value ($\alpha_\Lambda = -0.833$) in the $\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K$ exchange model.

However, when the $a_1$-meson exchange interaction is introduced, $\rho\pi/a_1$ and $\sigma\pi/a_1$ exchange central potentials work together to modify the inner radial behavior of the summed potentials in the $^1S_0 \rightarrow ^1S_0$ and $^3S_1 \rightarrow ^3S_1$ channels. See Fig. 3 for the $^1S_0 \rightarrow ^1S_0$ channel. The tensor force of the $\rho\pi/a_1$ exchange has a vital role to cancel the tensor force of $2\pi/\rho$ exchange potential as is noted in Sec. IV A and leads to a modification of the summed potential as seen in Fig. 4. In the $^3S_1 \rightarrow ^1P_1$ channel the $\rho\pi/a_1$ exchange works to enlarge the summed potential in Fig. 5. As a result, the $-ae^*$ term turns out to be positive. The combination $f/(\sqrt{2}c + d)^*$ remains, however, negative, because $fd^*$ is negative and dominates over $\sqrt{2}fc^*$. The $b(c - \sqrt{2}d)^*/\sqrt{3}$ term contribution is small and positive. Finally, such cancellation gives rise to a very small $\alpha_\Lambda$.

The calculated decay rate $\Gamma_{nm}$ is 0.358, which is 16% smaller than the experimental data of Ref. [1]. The two-nucleon induced decay process [33] might explain some part of this discrepancy.

### Table II. Calculated decay rates, $\Gamma_n/\Gamma_p$ ratios, and the asymmetry parameters of $^5_\Lambda$He in the three meson-exchange models are compared with the experimental data. The decay rates are shown in units of the free $\Lambda$ decay rate $\Gamma_\Lambda$.

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma_{nm}$</th>
<th>$\Gamma_n/\Gamma_p$</th>
<th>$\alpha_\Lambda = \alpha_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.278</td>
<td>0.109</td>
<td>-0.417</td>
</tr>
<tr>
<td>$\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K$</td>
<td>0.379</td>
<td>0.707</td>
<td>-0.833</td>
</tr>
<tr>
<td>$\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K + \rho\pi/a_1 + \sigma\pi/a_1$</td>
<td>0.358</td>
<td>0.508</td>
<td>0.083</td>
</tr>
<tr>
<td>Exp. [1,2,4]</td>
<td>0.424 ± 0.024</td>
<td>0.45 ± 0.11 ± 0.03</td>
<td>0.11 ± 0.08 ± 0.04</td>
</tr>
<tr>
<td>Exp. [5]</td>
<td>0.07 ± 0.08 ± 0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exp. [6]</td>
<td>0.24 ± 0.22</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE III. Parameter sets, A to Set E, of cutoff masses in the form factors and the coupling constants in evaluating the $\rho\pi/a_1$ and $\sigma\pi/a_1$-exchange transition potentials. Parameters are given in units of MeV except $g_{\rho\pi a_1}$.

<table>
<thead>
<tr>
<th>Set</th>
<th>$\Lambda_{a_1}$</th>
<th>$\Lambda_1(\rho\pi/a_1)$</th>
<th>$\Lambda_2(\rho\pi/a_1)$</th>
<th>$\Lambda_1(\sigma\pi/a_1)$</th>
<th>$g_{\rho\pi a_1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1400</td>
<td>2000</td>
<td>1650</td>
<td>1500</td>
<td>17.18</td>
</tr>
<tr>
<td>B</td>
<td>1400</td>
<td>2000</td>
<td>1650</td>
<td>1500</td>
<td>17.18</td>
</tr>
<tr>
<td>C</td>
<td>1400</td>
<td>2000</td>
<td>1650</td>
<td>1500</td>
<td>17.18</td>
</tr>
<tr>
<td>D</td>
<td>1400</td>
<td>2000</td>
<td>1650</td>
<td>1500</td>
<td>17.18</td>
</tr>
<tr>
<td>E</td>
<td>1400</td>
<td>2000</td>
<td>1650</td>
<td>1500</td>
<td>17.18</td>
</tr>
</tbody>
</table>

Before going to the $^{12}_A$C and $^{11}_A$B cases, it is interesting to examine the sensitivity of the decay observables to the $a_1$-meson exchange potential parameters such as the cutoff masses and the coupling constants. Here we have five options of the parameters as listed in Table III. Then we change three cutoff masses $\Lambda_{a_1}$ in the $NNa_1$ vertex form factor of Eq. (38) within acceptable ranges and select four other combinations (Set B to Set E) as listed in Table III. It is noted that the monopole-type form factor of Eq. (38) with cutoff $\Lambda_{a_1} = 1400$ MeV (1100 MeV, 1600 MeV) simulates well the Gaussian form factor of $\exp[-q^2/\Lambda^2]$ with $\Lambda = 1450$ MeV (1150 MeV, 1650 MeV) at $q \approx 300$–600 MeV/c, which is the most relevant region of momentum transfer in the nonmesonic decay. The following behavior of the potential is noticed. When the smaller cutoff $\Lambda_{a_1}$ is adopted, the potential strength is naturally reduced at the region less than $r \approx h/\Lambda_{a_1}c$, whereas at larger distances up to $r \approx 1$ fm the reduction becomes milder in comparison with the case of adopting the bigger $\Lambda_{a_1}$. Due to such a potential behavior, the coupling constant $g_{\rho\pi a_1} = -4000$ MeV is chosen in Set D to meet the relation of Eq. (39) at $r \geq 0.5$ fm. The calculations of the decay rates and the asymmetry parameter of $^{3}_A$He are performed also with these options of Set B to Set E. In Table IV we compare the calculated results for $^{3}_A$He for the five parameter sets, A–E, of Table III. One sees in Table IV that the calculated asymmetry parameter $\alpha_{a_1}$ is sensitive to the cutoff $\Lambda_{a_1}$ used in the potentials, though the decay rates are not so changed. Because the cutoff masses as $\Lambda_{a_1}$, $\Lambda_1(\rho\pi/a_1)$, $\Lambda_2(\rho\pi/a_1)$, and $\Lambda_1(\sigma\pi/a_1)$ control the short-range behavior of the $a_1$-meson potentials, it is natural that the asymmetry parameter $\alpha_{a_1}$ depends sensitively on the signs and magnitudes of the summed potentials. Thus the $\alpha_{a_1}$ might give us subtle information on the behavior of the weak decay potentials especially at the short distance. Within the present model framework it seems reasonable to adopt the parameter Set A for the $\rho\pi/a_1$ and $\sigma\pi/a_1$ exchange potentials.

C. Nonmesonic decay rates and asymmetry parameters of $^{12}_A$C and $^{11}_A$B

Table V summarizes the results for $^{12}_A$C. In the $\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K$ exchange model, the $\Gamma_n/\Gamma_p$ ratio is large compared with the recent experimental data [1,3]. The calculated proton asymmetry parameter $\alpha_1$ of $^{12}_A$C is 0.377, which is equivalent to the intrinsic asymmetry parameter $\alpha_1 = -0.755$. This value deviates from the experiment [1,4,5] very much. In contrast to these results, when the $a_1$-meson exchange is introduced, the $\Gamma_n/\Gamma_p$ ratio is improved to the value of 0.407 and the intrinsic asymmetry parameter $\alpha_1$ is dramatically changed to be 0.045. These new theoretical values are both well comparable with the recent experimental data [4,5]. The decay rate $\Gamma_{nm}$ is calculated to be 0.754, which is a little smaller than the experiment [1,34] but still within the error bars.

Table VI shows the calculated results for $^{11}_A$B with $J_H = 5/2^+$ together with the available experimental data. The general trends of the calculations for $^{11}_A$B are very similar to those for $^{12}_A$C especially for the $\Gamma_n/\Gamma_p$ ratios and the asymmetry parameters $\alpha_1$. It is noted particularly for $^{11}_A$B that in the evaluation of $\Gamma_1$ of Eq. (A1) in the Appendix, the sum for the angular momentum $L_0$ extends $L_0 = 0$ and 2, when the nucleon in the orbit ($n_a\ell_a j_a = 0p_3/2$ contributes to the nonmesonic decay rate. The contribution of the $L_0 = 2$ part to $\Gamma_1$ is, however, as small as 2% of the contribution of $L_0 = 0$ part to $\Gamma_1$. The calculated $\alpha_1$ is 0.078, which is in agreement within error bars with the averaged decay data of $^{12}_A$C and $^{11}_A$B hypernuclei [4,5].

It is worth to mention that the calculated intrinsic asymmetry parameters for light $^{3}_A$He, $^{11}_A$B and $^{12}_A$C hypernuclei have very similar numbers, though the numbers should have some uncertainties because of the approximations adopted in the derivations of the decay potentials and also of the simple wave functions used. It can also be noted that the recent experimental data of $\alpha_1$ [4–6] for those light $\Lambda$ hypernuclei have small values with common overlaps within error bars.

In view of these observations, we note that the intrinsic asymmetry parameter $\alpha_1$ is a “good” observable that gives us direct and subtle information of the $\Lambda + p \rightarrow n + p$ transition interaction at short range. Naturally it does not depend much on the detailed hypernuclear structures, spins, and hypernuclear species [6].

The present results have demonstrated an essential role, in the context of our model, of the $a_1$ exchanges in explaining the observed small $\alpha_1$. As mentioned, it was difficult to get small $\alpha_1$ in our previous calculations [9] and also in the present work in which the $\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K$ exchange model.
TABLE V. Calculated decay rates, $\Gamma_\nu/\Gamma_p$ ratios and the asymmetry parameters of $^{12}\Lambda C$ in the three meson-exchange models are compared with the experimental data. The decay rates are shown in units of the free $\Lambda$ decay rate $\Gamma_{\Lambda}$.

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma_{\nu}/\Gamma_p$</th>
<th>$\alpha_{\Lambda} = -\frac{J_{\Lambda}+1}{J_{\Lambda}}\alpha_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.620</td>
<td>0.101</td>
</tr>
<tr>
<td>$\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K$</td>
<td>0.843</td>
<td>0.618</td>
</tr>
<tr>
<td>$\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K + \rho\pi/\alpha_1 + \sigma\pi/\alpha_1$</td>
<td>0.754</td>
<td>0.407</td>
</tr>
<tr>
<td>Exp. [1,4]</td>
<td>0.940 ± 0.035</td>
<td>0.56 ± 0.12 ± 0.04</td>
</tr>
<tr>
<td>Exp. [5]</td>
<td>0.51 ± 0.13 ± 0.05</td>
<td>−0.20 ± 0.26 ± 0.04</td>
</tr>
<tr>
<td>Exp. [34]</td>
<td>0.824 ± 0.056 ± 0.066</td>
<td>−0.16 ± 0.28 ± 0.00</td>
</tr>
</tbody>
</table>

TABLE VI. Calculated decay rates, $\Gamma_\nu/\Gamma_p$ ratios and the asymmetry parameters of $^{11}\Lambda B$ in the three meson-exchange models are compared with the experimental data. The decay rates are shown in units of the free $\Lambda$ decay rate $\Gamma_{\Lambda}$.

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma_{\nu}/\Gamma_p$</th>
<th>$\alpha_{\Lambda} = -\frac{J_{\Lambda}+1}{J_{\Lambda}}\alpha_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.519</td>
<td>0.123</td>
</tr>
<tr>
<td>$\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K$</td>
<td>0.760</td>
<td>0.732</td>
</tr>
<tr>
<td>$\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K + \rho\pi/\alpha_1 + \sigma\pi/\alpha_1$</td>
<td>0.660</td>
<td>0.503</td>
</tr>
<tr>
<td>Exp. [34]</td>
<td>0.861 ± 0.063 ± 0.073</td>
<td>−0.20 ± 0.26 ± 0.04</td>
</tr>
<tr>
<td>Exp. [4]</td>
<td>0.519 ± 0.13 ± 0.05</td>
<td>−0.16 ± 0.28 ± 0.00</td>
</tr>
</tbody>
</table>

has been applied. In this connection, we recall the article by Chumillas et al. [22], who claimed that the $\alpha_{\Lambda}$ of $^{3}\Lambda He$, $^{12}\Lambda C$, and $^{11}\Lambda B$ can be explained well within the octet-meson exchange potentials and the correlated $2\pi(2\pi/\sigma)$ + uncorrelated $2\pi$ exchange potentials calculated by Jido et al. [11]. Their conclusion seems different from that of our calculation. Note that our $2\pi/\sigma$ exchange interaction is regarded to be an effective potential that contains also uncorrelated $2\pi$ exchange contributions to some extent in addition to the correlated ones. In fact the $2\pi/\sigma$ exchange $NN$ force of our model is quite similar to the $NN$ force in Fig. 14 of Ref. [11]. However, it is not always easy to make a direct comparison between these two approaches. Moreover, the intermediate baryons adopted in the calculation of the weak $\Delta N \rightarrow NN$ force are also different: $NN$ and $N\Delta$ channels are considered but $\Sigma N$ channel is not in Refs. [11,22], whereas in our model $[7–9]$ the intermediate baryons $N$ and $\Sigma$ are taken into account but $\Delta$, at least explicitly, is not.

V. SUMMARY AND OUTLOOK

The recent high-quality experiments for hypernuclear nonmesonic weak decays provided us with a converging value of $\Gamma_\nu/\Gamma_p$ ratio $\approx 0.5$. This value deviates much from the basic one-pion-exchange mechanism, and therefore many theoretical efforts have been made for the explanation. Another important observable is the asymmetry parameter $\alpha_{\Lambda}$ of protons emitted in the decay, and the observed values now tend to be very small, differing from almost all the theoretical estimates. Thus how to understand these experimental data consistently from the viewpoint of meson-exchange models for the $\Delta N \rightarrow NN$ weak transition interaction has been a long-standing dispute.

To give an answer to the challenging questions mentioned above, we have introduced the axial-vector $a_1$-meson exchange, for the first time, into the estimate of the $\Delta N$ weak interaction. The chief argument is to investigate the short-range properties of the weak interaction and, for that purpose, to make the set of exchanged mesonic quantum numbers more complete. It is noted, moreover, that the $a_1$ meson is the chiral partner of the $\rho$ meson. Furthermore, the introduction of this new mechanism was stimulated by the success of the ESC04 model for the strong baryon-baryon interactions. The $a_1$-meson exchange has been treated in the form of $\rho\pi/\alpha_1$ and $\sigma\pi/\alpha_1$ combinations within the meson-pair exchange framework.

We have presented the calculated results for $^{3}\Lambda He$, $^{12}\Lambda C$, and $^{11}\Lambda B$ in the $\pi + 2\pi/\rho + 2\pi/\sigma + \omega + K + \rho\pi/\alpha_1 + \sigma\pi/\alpha_1$ exchange interaction model. The results are also compared with the estimates without the $a_1$-meson exchange. First, we have found the striking role of $a_1$ exchange in giving rise to remarkable modifications of the parity-conserving decay potentials ($^{1.3}S \rightarrow ^{1.3}S$ and $^{3}S_1 \rightarrow ^{3}D_1$) at short range $r \lesssim 1$ fm.

Second, as a direct result of the short-range behavior mentioned above, we obtained a drastic change in the estimate of the intrinsic asymmetry parameter $\alpha_{\Lambda}$, and the value for $^{3}\Lambda He$ becomes very small and positive (0.08) in good agreement with the recent high-quality experimental data. One notices that the intrinsic asymmetry parameters do not change much for other species: $\alpha_{\Lambda} = 0.08$ and 0.04 for $^{11}\Lambda B$ and $^{12}\Lambda C$, respectively. These theoretical values of $\alpha_{\Lambda}$ compare well with the observed value, being essentially zero within the experimental error bars.
Additionally we have tried to vary the cutoff-mass parameters within acceptable ranges and found that the theoretical $\alpha_\Lambda$ is rather sensitive to their choice, although we think the essential role of the $\alpha_1$-meson exchange is persistent. For the theoretical calculation of $\alpha_1 = \Gamma_\Lambda / \Gamma_0$ (or $\alpha_\Lambda$), which comes from the interference between parity-conserving and parity-violating terms, one should be careful in treating the phases concerned. For this purpose we present the general expression of the proton asymmetry in the outgoing proton helicity frame.

As the third point, we emphasize that the inclusion of the $\alpha_1$ meson improves the $\Gamma_\Lambda / \Gamma_\rho$ ratios such as to become well comparable to the experimental data of $^{3}\text{He}$ and $^{12}\Lambda\text{C}$. Thus, in this article, we have shown the unique and important role that $\alpha_1$ exchange can play when added to the nonmesonic weak decay interaction, noting that it leads to a consistent explanation for the existing experimental data of light $\Lambda$-hypernuclear decays ($A < 12$).

We make several remarks on the underlying behaviors of the $\Lambda N \rightarrow NN$ weak potentials obtained here from various meson- and meson-pair-exchange mechanisms. The $\rho \pi/\alpha_1$ exchange potential is short range in nature and has notable features in the central and tensor forces and the parity-violating vector force of the $l[(\sigma_1 \times \sigma_2) \cdot \vec{p}]$ type. The $\alpha_1$-exchange central force is strong enough (negative sign) to cancel out the $2\pi/\rho$ exchange central force. The tensor force is strong (positive sign) and behaves in opposition to that of the $2\pi/\rho$ exchange one. The parity-violating force in the $^1S_1 \rightarrow ^1P_1$ channel is strong and works additively to other meson-exchange interactions. The $\sigma \pi/\alpha_1$-exchange potential is dominantly of the central type but the effect is relatively small. Such potential behaviors due to the $\alpha_1$-meson exchange are the reason for the remarkable changes in $\alpha_\Lambda$ and $\Gamma_\Lambda / \Gamma_\rho$.

The present study is the first extension of the previous work in Ref. [8] through the proper consideration of the chiral partners in meson exchanges in the non-strange-meson sector and the introduction of the $K$-meson exchange. Although we are successful at present in explaining the weak decay observables of light $\Lambda$ hypernuclei, further elaboration of the theoretical basis is interesting. Because the nonmesonic weak decay is a high-momentum-transfer process, short-range properties of the weak transition potentials should be further explored through the exchanges of chiral partners mesons extending to the strange-meson sector. As another direction of sophistication, the short-range behaviors of the initial-$\Lambda N$ and final-$NN$ states should be solved with the aid of the modern soft-core baryon-baryon interactions in place of the hard-core strong interaction [35] employed in the present treatment. Naturally such improvement along these directions are planned in the next stage.

**ACKNOWLEDGMENTS**

The authors thank Y. Yamamoto for offering us the initial $\Lambda N$ state correlation relations together with useful comments. They are grateful to T. Nagae, T. Maruta, T. Kishimoto, S. Ajimura, and E. Hiyama for their interests and discussions. This work has been done under the support of Grant-in-Aid for Scientific Research (Grant no. 13640294) and Grant-in-Aid for Scientific Research in Priority Area (Multi-Quark System Probed by Strangeness, Grant no. 18042005) from Ministry of Education, Culture, Sports, Science and Technology of Japan.

**APPENDIX: EXPRESSION OF $\Gamma_1$ DEFINED BY EQ. (3)**

The $\Gamma_1$ shown generally by Eq. (11) is expressed in terms of the two-body transition amplitudes and in the case of $\Lambda N$ relative $s$ wave in the shell-model framework as

$$
\Gamma_1 = \frac{2\pi}{2J_H + 1} \times 2M_N \times \frac{3}{J_H + 1} \sum_{J, L, \Lambda, \lambda} \sum_{S, S', S_1, S_2} \sum_{\ell_1, \ell_2} \sum_{k, k'} \int \frac{d(k^{\text{max}}_2) (k_2 \cos(\theta_2)) d\theta_2}{2\pi} \int_0^{k_2^{\text{max}}} dk_2
$$

$$
\times \sum_{M_{\theta_1} = 0} \sum_{\ell_1 \ell_2} \frac{1}{2\pi} \int \frac{d(cos(\theta_2)) d\theta_2}{2\pi} \sum_{M_{\theta_1} = 0} \frac{(A - 2k_2^2)^2}{(A - 2k_2^2)^2 - k_2^4 [(A - 2)^2 - \cos^2(\theta_2)]}
$$

$$
\times \left(4\pi^2 \right) \sum_{M_{\theta_1} = 0} \sum_{\ell_1 \ell_2} \frac{1}{2\pi} \int \frac{d(cos(\theta_2)) d\theta_2}{2\pi} \sum_{M_{\theta_1} = 0} \frac{(A - 2k_2^2)^2}{(A - 2k_2^2)^2 - k_2^4 [(A - 2)^2 - \cos^2(\theta_2)]}
$$

$$
\times \left(4\pi^2 \right) \sum_{M_{\theta_1} = 0} \sum_{\ell_1 \ell_2} \frac{1}{2\pi} \int \frac{d(cos(\theta_2)) d\theta_2}{2\pi} \sum_{M_{\theta_1} = 0}
$$

Conventions and discussions are planned in the next stage.
Here $M_{\ell_0}(n_\lambda\,n_\Lambda; n\ell NL; M_N, M_\Lambda)$ is a generalized Talmi-Moshinsky transformation coefficient for different masses of a nucleon and a $\Lambda$ hyperon. Other notations used are the same as those in Ref. [8].

When the nonmesonic decay takes place from the $\Delta N$ relative $s$ state, the angular momentum $k$ takes 1 only and $L_0$ must have even number, 0, 2, ... It is evident that $\ell_0$ and $\ell_0'$ must have opposite parities with each other because $\ell_0 + \ell_0' + k = \text{even}$. One can put $\phi_k = \phi_K = 0$ in the arguments of the spherical harmonics $Y_{\ell_0, M - M'}$ and $Y_{\ell_0', M - M'}$, because there is no $\phi_k = \phi_K$ dependence because of the relation of $M - M' = -(M - M')$.

[31] K. Itonaga et al. (in preparation).