Precise measurement of the top quark mass from lepton+jets events

We measure the mass of the top quark using top quark pair candidate events in the lepton+jets channel from data corresponding to 1 $fb^{-1}$ of integrated luminosity collected by the D0 experiment at the Fermilab Tevatron collider. We use a likelihood technique that reduces the jet energy scale uncertainty by combining an in-situ jet energy calibration with the independent constraint on the jet energy scale (JES) from the calibration derived using photon+jets and dijet samples. We find the mass of the top quark to be $171.5_{-1.8}^{+1.1}$ GeV.

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Since the discovery of the top quark in 1995 \cite{1}, a substantial effort has gone into measuring and understanding its properties. Its large mass suggests a unique role in the mechanism of electroweak symmetry breaking. Through radiative corrections, a precise measurement of the top quark mass, together with that of the W boson, allows indirect constraints to be placed on the mass of the standard model Higgs boson \cite{2}. A precise knowledge of the top quark mass could also provide a useful constraint to possible extensions of the standard model. It is therefore of great importance to continue improving measurements of the top quark mass \cite{3, 4}.

In this Letter, we present the most precise single measurement of the top quark mass from Run II of the Fermilab Tevatron collider. It uses a matrix element (ME) method with an in-situ jet energy calibration based on a global factor used to scale all jet energies and thereby the invariant mass of the hadronic W boson \cite{5, 6}. This mass is constrained to the well known value of 80.4 GeV through the Breit-Wigner function for the hadronic W boson in the ME for $t \bar{t}$ production. The jet energy scale is further constrained to the standard scale derived from photon+jets and dijet samples within its uncertainties through the use of a prior probability distribution. This analysis is based on data collected by the D0 detector \cite{7} from April 2002 to February 2006 comprising 1 fb$^{-1}$ of integrated luminosity from $\sqrt{s} = 1.96$ TeV.

The top quark is assumed to always decay into a W boson and a b quark producing a $W^+W^−b\bar{b}$ final state from $t\bar{t}$ production. This analysis is based on the lepton+jets channel with one W boson decaying via $W \rightarrow ℓν$ and the other via $W \rightarrow q\bar{q}$. This channel is characterized by a lepton with large transverse momentum ($p_T$), large momentum imbalance due to the undetected neutrino ($p_\nu$), and four high-$p_T$ jets. Events are selected for this analysis by requiring exactly one isolated electron (muon) with $p_T > 20$ GeV and $|η| < 1.1$ ($|η| < 2$), $p_T > 20$ GeV, and exactly four jets with $p_T > 20$ GeV and $|η| < 2.5$, where the pseudorapidity $η = −\ln [\tan (θ/2)]$, and θ is the polar angle with respect to the proton beam direction. Multijet background, typically originating from lepton or jet energy mismeasurements, is further suppressed by requiring the lepton direction and $p_\nu$ vector to be separated in azimuth. Jets are reconstructed using a cone algorithm \cite{8} with radius $R = \sqrt{(Δy)^2 + (Δφ)^2} = 0.5$ where the $y$ is the rapidity. Jet energies are corrected to the particle level using corrections derived from photon+jet and dijet samples. Jets containing a muon are assumed to originate from semileptonic b quark decays and corrected by the muon momentum and average neutrino energy. At least one jet is required to be tagged by a neural-network based algorithm \cite{9} as a b-jet candidate. The tagging efficiency for b jets is $\sim 50\%$ with a misidentification rate of $\sim 1\%$ from light jets. A total of 220 events, split equally between the electron and muon channels, satisfying these criteria is selected.

The top quark mass is determined from the data sample with a likelihood method based on per-event probability densities (p.d.’s) constructed from the MEs of the processes contributing to the observed events. Assuming only two processes, $t\bar{t}$ and $W+ jets$ production, the p.d. to observe an event with measured variables $x$ is

$$P_{\text{evt}} = A(x) \left[ fP_{\text{sig}}(x; m_t, k_{\text{jens}}) + (1 - f)P_{\text{bkg}}(x; k_{\text{jens}}) \right],$$

where the top quark mass $m_t$, jet energy scale factor $k_{\text{jens}}$, dividing the energies of all jets, and observed signal fraction $f$ are the parameters to determine from data. $P_{\text{sig}}$ and $P_{\text{bkg}}$ are, respectively, p.d.’s for $t\bar{t}$ and $W+ jets$ production. Multijet events satisfy $P_{\text{bkg}} \gg P_{\text{sig}}$ and are also represented by $P_{\text{bkg}}$. $A(x)$ is a function only of $x$ and accounts for the geometrical acceptance and efficiencies. $P_{\text{sig}}$ and $P_{\text{bkg}}$ are calculated by integrating over all possible parton states leading to the measured set $x$. In addition to the partonic final state described by the variables $y$, these states include the initial state partons carrying momenta $q_1$ and $q_2$ in the colliding $p$ and $\bar{p}$. The integration involves a convolution of the partonic differential cross section $dσ(y; m_t)$ with the p.d.’s for the initial state partons $f(q_i)$ and the transfer function $W(y, x; k_{\text{jens}})$:

$$P_{\text{sig}} = \frac{1}{N} \int \sum dσ(y; m_t) dq_1 dq_2 f(q_1) f(q_2)W(y, x; k_{\text{jens}}),$$

where the sum runs over all possible initial state parton flavor combinations. $f(q_i)$ includes parton density functions (PDFs) for finding a parton of a given flavor and longitudinal momentum fraction in the $p$ or $\bar{p}$ (CTEQ6L1 \cite{9}) and parameterizations of the p.d.’s for the transverse components of $q_i$ derived from PYTHIA \cite{10}. Jet fragmentation effects and experimental resolution are taken into account by $W(y, x; k_{\text{jens}})$, representing the p.d. for the measured set $x$ to have arisen from the partonic set $y$. The normalization factor $N$, defined as the expected observed cross section for a given $(m_t, k_{\text{jens}})$, ensures $A(x)P_{\text{sig}}$ (and ultimately $P_{\text{evt}}$) is normalized to unity.

The differential cross section term in $P_{\text{sig}}$ is calculated using the leading order ME for $q\bar{q} \rightarrow t\bar{t}$. After all energy and momentum constraints are taken into account, this term is integrated over the energy associated with one of the quarks from the hadronic W boson decay, the masses of the two W bosons and two top quarks, and the energy $(1/p_T)$ of the electron (muon). It is summed over 24 possible jet-parton assignments each carrying a b-jet tagging weight $\frac{1}{p_T}$ and over the neutrino solutions. $W(y, x; k_{\text{jens}})$ is the product of five terms for the four jets and one charged lepton. The jet terms are parameterized in terms of jet energy with a function involving the sum of two Gaussians whose parameters depend linearly on parton energy. The term for the charged lepton is parameterized as a Gaussian distribution in energy $(1/p_T)$ for electrons (muons). All
parameters for $W(y; x; k_{jes})$ are derived using fully simulated Monte Carlo (MC) events. The normalization cross section $\sigma^m_{\text{bs}} = \int A(x) P_{\text{sig}} dx = \sigma^m (m_t) \langle A(m_t, k_{jes}) \rangle$ is calculated using the total cross section corresponding to the ME used and the mean acceptance for events whose dependencies on $m_t$ and $k_{jes}$ are determined from MC events.

The differential cross section term in $P_{\text{bkg}}$ is calculated using the $W+4$ jets MEs provided by VECBOS [12]. The integration is performed over the energies of the four partons producing the jets, the $W$ boson mass, and the energy ($1/p_T$) of the electron (muon) summing over 24 possible jet-parton assignments and two neutrino solutions. The transverse momenta of the initial state partons are assumed to be zero.

$P_{\text{sig}}$ and $P_{\text{bkg}}$ are calculated using MC techniques on a grid in $(m_t, k_{jes})$ having spacings of 1.5 GeV and 0.015, respectively. At each grid point, a likelihood function $L(x; m_t, k_{jes}, f)$ is constructed from the product of the individual event p.d.'s ($F_{\text{evt}}$) and $f$ is determined by minimizing $-\ln L$ at that point. $L(x; m_t, k_{jes})$ is then projected onto the $m_t$ and $k_{jes}$ axes according to $L(x; m_t) = \int L(x; m_t, k_{jes}) G(k_{jes}) dk_{jes}$ and $L(x; k_{jes}) = \int L(x; m_t, k_{jes}) dm_t$. The prior $G(k_{jes})$ is a Gaussian function centered at $k_{jes} = 1$ with width $\sigma = 0.019$ determined from the photon+jet and dijet samples used in the standard jet energy scale calibration. Best estimates of $m_t$ and $k_{jes}$ and their uncertainties are then extracted from the mean and rms of $L(x; m_t)$ and $L(x; k_{jes})$.

The measurement technique described above is calibrated using MC events produced with the ALPGEN event generator [13] employing PYTHIA for parton showering and hadronization and implementing the MLM matching scheme [14]. All generated events are processed by a full GEANT [15] detector simulation followed by the same reconstruction and analysis programs used on data. Fig. 1 shows comparisons of the 2-jet and 3-jet invariant mass distributions between data and MC using $t\bar{t}$ events with a true top quark mass ($m_t^{\text{true}}$) of 170 GeV. These are calculated using jets assigned as the decay products of the top quark and $W$ boson from the hadronic branch in the permutation with the largest weight (defined as the product of $P_{\text{sig}}$ and the $b$-jet tagging weight) around the peak of $L(x; m_t, k_{jes})$. MC distributions are normalized to data distributions with $f = 0.74$ determined from data. The background includes simulated $W+$jets events and data events selected from a multijet enriched sample. The latter comprises 12% of the total background based on estimates from data. The estimated number of $t\bar{t}$ events ($e +$jets: $91 \pm 9$, $\mu +$jets: $71 \pm 8$) agrees with the expectation ($e +$jets: $89 \pm 6$, $\mu +$jets: $73 \pm 5$).

Five $t\bar{t}$ MC samples are generated with $m_t^{\text{true}} = 160$, 165, 170, 175, and 180 GeV, with six more produced from three of these by scaling all jet energies by $\pm 5\%$. $P_{\text{sig}}$ and $P_{\text{bkg}}$ are calculated for these events from which pseudo-experiments fixed to the number of data events are randomly drawn with a signal fraction fluctuated according to a binomial distribution around that determined from data. The mean values of $m_t$ and $k_{jes}$ for 1000 pseudo-experiments are shown in Fig. 2 as functions of the true values parameterized by straight lines. The average widths of the $m_t$ and $k_{jes}$ pull distributions are 0.10 and 1.1, respectively. The pull is defined as the deviation of a measurement from the mean of all measurements divided by the uncertainty of the measurement per pseudo-experiment. The measured uncertainties in data are corrected by the deviation of the average pull width from 1.0.

$L(x; m_t)$ and $L(x; k_{jes})$ for the selected data samples are calibrated according to the parameterizations shown in Fig. 2 $L(x; m_t)$ is shown in Fig. 3a with a measured $m_t = 171.5 \pm 1.8$ (stat. + JES) GeV. The measured $k_{jes} = 1.030 \pm 0.017$ represents a $1.2 \sigma$ shift from $k_{jes} = 1$ where $\sigma$ is the sum in quadrature of the width of $G(k_{jes})$ and the uncertainty of the measured $k_{jes}$. Fig. 3b compares the measured uncertainty for $m_t$ with the expected uncertainty distribution from pseudo-experiments in MC assuming $m_t^{\text{true}} = 170$ GeV.

To verify the in-situ jet energy calibration, we repeat the analysis on data by fixing $k_{jes}$ to the measured value and removing the $W$ boson mass constraint, replacing $L(x; m_t, k_{jes}, f)$ with $L(x; m_t, m_W, f)$. $P_{\text{sig}}$ and $P_{\text{bkg}}$ are
are calculated in the same way as for the grid in $(m_t, m_W)$ having spacings of 1.5 GeV and 1 GeV, respectively. $L(x; m_t)$ and $L(x; m_W)$ are calculated in the same way as for the grid in $(m_t, k_{jes})$ except that no prior probability distribution is used for $L(x; m_t)$. We find $m_W = 80.3 \pm 1.0 \text{ GeV}$ which is consistent with the constraint of 80.4 GeV.

Systematic uncertainties are evaluated for three categories. The first category involves the modeling of MC $t\bar{t}$ and $W+$jets events and includes uncertainties in the modeling of extra jets due to radiation in $t\bar{t}$ events, the distribution shapes and the heavy flavor fraction in $W+$jets events, $b$ fragmentation, and the PDFs used in generating events. The second category is associated with the simulation of detector response and includes possible effects due to the energy and $|\eta|$ dependence of the jet energy scale unaccounted for by the in-situ calibration, uncertainties in the modeling of the relative calorimeter response to $b$ and light quark jets, and uncertainties associated with the simulation of jet energy resolution and reconstruction efficiency, muon $p_T$ resolution, and trigger efficiency. The third category is related to assumptions made in the method and uncertainties in the calibration and includes possible effects due to the exclusion of multijet events and non-lepton+$jets$ $t\bar{t}$ events from the calibration procedure, uncertainties in the signal fraction used in ensemble tests, and uncertainties associated with the parameters defining the calibration curve. Contributions from all these sources are summarized in Table I and sum in quadrature to $\pm 1.1 \text{ GeV}$.

The leading sources of uncertainty in Table I are those associated with the $b$/light response ratio and signal modeling. The first of these is evaluated by estimating the possible difference in this ratio between data and MC and scaling the energies of all jets matched to $b$ quarks in a MC $t\bar{t}$ sample by this amount. The analysis is repeated for this sample and the difference in $m_t$ from that of the unscaled sample taken as the uncertainty. The uncertainty associated with the modeling of additional jets in $t\bar{t}$ events is evaluated using both data and MC samples. Using MC $t\bar{t}$ events, the fraction of $t\bar{t}$ signal events with $\geq 5$ jets is varied such that the ratio of 4-jet to $\geq 5$-jet events in MC matches that in data including its uncertainties. The difference in the resulting $m_t$ from that of the default sample is taken as the uncertainty. Using data, this is done through ensemble tests in which a fixed number of $\geq 5$-jet events not used in the measurement are randomly drawn for each experiment and combined with the default sample of 4-jet events. The ensemble tests are repeated for different fractions of $\geq 5$-jet events constituting up to 30% of each experiment. $m_t$ for the default sample is compared with the mean from each ensemble test and the largest difference taken as the systematic uncertainty. Both procedures yield consistent estimates for this systematic uncertainty.

In summary, we present a measurement of the top quark mass using $t\bar{t}$ lepton+jets events from 1 fb$^{-1}$ of data collected by the D0 experiment. Using a ME technique combining an in-situ calibration of the jet energy scale with the calibration based on the photon+$jets$ and dijet samples gives us

$$m_t = 171.5 \pm 1.8(\text{stat.}+\text{JES}) \pm 1.1(\text{syst.}) \text{ GeV},$$

representing the most precise single measurement to date.

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The weight for a $b$-tagged jet with a given $p_T$ and $\eta$ under a parton flavor hypothesis $\alpha(=b, c, \text{light } q \text{ or gluon})$ is given by the parameterization of the average tagging efficiency $\epsilon_\alpha(p_T, \eta)$. Consequently, the weight for a jet not $b$-tagged is $1-\epsilon_\alpha(p_T, \eta)$. The event weight is defined as the product of jet weights.