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# Combining Abduction with Conflict-based Diagnosis

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**Abstract.** Conflict-based diagnosis is a recently proposed probabilistic method for model-based diagnosis, inspired by consistency-based diagnosis, that uses a measure of data conflict, called the diagnostic conflict measure, to rank diagnoses. In this paper, this method is refined using an abductive method that reuses part of the computation of the diagnostic conflict measure.

## 1 INTRODUCTION

Conflict-based diagnosis is a recently proposed probabilistic method for model-based diagnosis that is inspired by consistency-based diagnosis, and uses a measure of data conflict, called the diagnostic conflict measure, to rank diagnoses. The probabilistic information that is required to compute the diagnostic conflict measure is represented by means of a Bayesian network. This Bayesian network contains sufficient information to compute abductive diagnoses as well.

In this paper, conflict-based diagnosis is augmented with an abductive method, similar in spirit to the probabilistic method employed by GDE [2]. The method reuses part of the computation of the diagnostic conflict measure. In essence, abductive diagnosis is used to rank conflict-based diagnoses with equal conflict-based rankings.

## 2 PRELIMINARIES

### 2.1 Model-based Diagnosis

In model-based diagnosis, the structure and behaviour of a system is represented by a *logical diagnostic system* $\mathcal{S}_L = (\mathrm{SD}, \mathrm{COMPS})$, where (*i*) SD denotes the *system description*, which is a finite set of logical formulae, specifying structure and behaviour, and (*ii*) COMPS is a finite set of constants, corresponding to the *components* of the system; these components can be faulty. The system description consists of *behaviour descriptions* specifying *normal* and *abnormal* (faulty) functionalities of the components, and of *connections of inputs and outputs of components*.

A *logical diagnostic problem* is defined as a pair $\mathcal{P}_L = (\mathcal{S}_L, \mathrm{OBS})$, where $\mathcal{S}_L$ is a logical diagnostic system and OBS is a finite set of logical formulae, representing *observations*.

Two types of model-based diagnosis are distinguished: (*i*) consistency-based diagnosis [2, 6], and (*ii*) abductive diagnosis [1]. Let $\Delta_C$ consist of the assignment of abnormal behaviour to the set of components $C \subseteq \mathrm{COMPS}$ and normal behaviour to the remaining components $\mathrm{COMPS} - C$, then, adopting the definition from [3], $\Delta_C$ is a *consistency-based diagnosis* of the logical diagnostic problem $\mathcal{P}_L$ iff the observations are consistent with both the system description and the diagnosis; formally: $\mathrm{SD} \cup \Delta_C \cup \mathrm{OBS} \nvDash \bot$.

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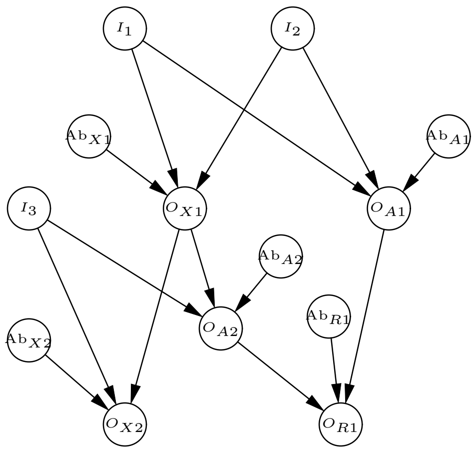


**Figure 1.** The graphical representation of a Bayesian diagnostic system corresponding to the full-adder in [6].

In the abductive approach, the behavioural assumptions $\Delta_C$ are called an *abductive diagnosis* if the system description SD and the behavioural assumptions $\Delta_C$ *imply* the set of observations OBS; formally: $\mathrm{SD} \cup \Delta_C \vDash \mathrm{OBS}$.

### 2.2 Bayesian Diagnostic Problems

Let $P(X)$ be a joint probability distributions of the set of discrete binary random variables $X$, where for a singleton $\mathrm{x}$ and $\bar{\mathrm{x}}$ denote the values 'true' and 'false', respectively. A *Bayesian network* $\mathcal{B}$ is then defined as a pair $\mathcal{B} = (G, P)$, where the acyclic directed graph $G = (V, E)$ represents the relations between the random variables defined in $P(X)$, where each random variable corresponds to a unique vertex.

A *Bayesian diagnostic system* is denoted by $\mathcal{S}_B = (G, P)$, where $P$ is a joint probability distribution of the vertices of $G$, interpreted as random variables, and $G$ is obtained by mapping a logical diagnostic system $\mathcal{S}_L$ to a Bayesian diagnostic system as follows: (*i*) component $c$ is represented by its *input* $I_c$ and *output* $O_c$, where each arc points from input to the output, (*ii*) to each component $c$ there belongs an *abnormality vertex* $\mathrm{Ab}_c$. An example is given in Figure 1.

Let the set of values of the abnormality variables $\mathrm{Ab}_c$, with $c \in \mathrm{COMPS}$, be denoted by

$$\delta_C = \{\mathrm{ab}_c \mid c \in C\} \cup \{\overline{\mathrm{ab}}_c \mid c \in \mathrm{COMPS} - C\},$$

which establishes a link to $\Delta_C$ in logical diagnostic systems.

In this paper, the set of observed input and output variables are referred to as $I_\omega$ and $O_\omega$, whereas the unobserved input and output variables will be referred to as $I_u$ and $O_u$ respectively. Let $i_\omega$ denote the values of the observed inputs, and $o_\omega$ the observed output values.

The set of observations is then denoted as $\omega = i_\omega \cup o_\omega$. The following assumptions are used in the remainder of this paper: (*i*) the probabilistic behaviour of a component that is faulty is independent of its inputs, and (*ii*) normal components behave deterministically. These are realistic assumptions, as it is unlikely that detailed functional behaviour is known for a component that is faulty, whereas when the component is not faulty, it is certain it behaves as intended.

A Bayesian diagnostic problem, denoted by $\mathcal{P}_B = (\mathcal{S}_B, \omega)$, consists of (*i*) a Bayesian diagnostic system and (*ii*) a set of observations $\omega$ [5, 4].

### 2.3 Conflict-based Diagnosis

The theory of conflict-based diagnosis uses the diagnostic conflict measure to solve Bayesian diagnostic problems [4], where a numeric value is assigned to each diagnosis to order them. Define $\omega = i_\omega \cup o_\omega$ as the observations, then the *diagnostic conflict measure* (DCM), denoted by $\mathrm{conf}_{\delta_C}(\omega)$, is defined as

$$\mathrm{conf}_{\delta_C}(\omega) = \log \frac{P(i_\omega \mid \delta_C) P(o_\omega \mid \delta_C)}{P(i_\omega, o_\omega \mid \delta_C)}. \tag{1}$$

Using the independence properties from Bayesian diagnostic problems we obtain:

$$\mathrm{conf}_{\delta_C}(\omega) = \log \frac{\sum_i P(i) \sum_{o_u} \prod_c P(O_c \mid \pi(O_c))}{\sum_{i_u} P(i_u) \sum_{o_u} \prod_c P(O_c \mid \pi(O_c))}. \tag{2}$$

Intuitively, if the probability of the individual occurrence of the observations is smaller than that of the joint occurrence (if the numerator is smaller than the denominator), then the observations do 'like' or support each other. Thus, a smaller value of the DCM indicates a better fit between observations and component behaviours. Therefore, the DCM imposes an ordering on diagnoses, where the lower the DCM for a diagnosis is, the better the diagnosis fits the diagnostic problem. A diagnosis is a *conflict-based diagnosis*, if its DCM is non-positive, and it is also called *minimal*, if it has the least DCM value in comparison to the other conflict-based diagnoses.

## 3 ABDUCTIVE CONFLICT-BASED DIAGNOSIS

In the ranking obtained by conflict-based diagnosis there may be cases, where the diagnoses have the same DCM. This has motivated us to develop a method which offers a way to distinguish such diagnoses. This method makes use of abductive computations, for which parts of the computation of the DCM are reused.

### 3.1 The Relation between Abductive and Consistency-based Reasoning

In our probabilistic setting, the consistency condition requires that the probability of the occurrence of the observations given the diagnosis is non-zero. Formally, in consistency-based reasoning, we are searching for diagnoses $\delta_C$ with $P(i_\omega, o_\omega \mid \delta_C) > 0$. Note that the set of abnormality assumptions $\delta_C$ is given knowledge.

In abductive reasoning, on the other hand, the observations have to be implied by the system descriptions and the abnormality assumptions $\delta_C$. This means that we are looking for abnormality assumption $\delta_C$ that can be explained by the observations; formally: $P(\delta_C \mid i_\omega, o_\omega)$. Using Bayes' rule the following relationship between consistency-based and abductive reasoning can be established:

$$P(\delta_C \mid i_\omega, o_\omega) = \frac{P(i_\omega, o_\omega \mid \delta_C) P(\delta_C)}{P(i_\omega, o_\omega)}, \tag{3}$$

where $1/P(i_\omega, o_\omega)$ is a normalisation constant. The maximum a-posteriori assignment (MAP) diagnosis, defined as $\delta_C^\star = \mathrm{argmax}_{\delta_C} P(\delta_C \mid i_\omega, o_\omega)$, is the natural probabilistic analogue to the concept of subset-minimal abductive diagnosis [7].

According to Equation (3), computation of abductive diagnoses requires the computation of consistency-based diagnoses.

### 3.2 Abductive Probabilistic Computations

Next, a formula to compute abductive diagnoses of Bayesian diagnostic problems is derived, which is used to distinguish between equally ranked conflict-based diagnoses.

Note that the numerator $P(i_\omega, o_\omega \mid \delta_C)$ in Equation (3) is also the denominator of the DCM in equations (1) and (2); according to [4]:

$$P(i_\omega, o_\omega \mid \delta_C) = P(i_\omega) \sum_{i_u} P(i_u) \sum_{o_u} \prod_c P(O_c \mid \pi(O_c)).$$

In contrast to Equation (2), the factor $P(i_\omega)$ is not divided out.

The denominator of the abductive formulas is computed as:

$$P(i_\omega, o_\omega) = P(i_\omega) \sum_{\delta_c} P(\delta_c) \sum_{i_u} P(i_u) \sum_{o_u} \prod_c P(\mathrm{o}_c \mid \pi(O_c)).$$

It is now possible to derive the abductive computational form:

$$\begin{aligned} P(\delta_c \mid i_\omega, o_\omega) &= \frac{P(i_\omega, o_\omega \mid \delta_c) P(\delta_c)}{P(i_\omega, o_\omega)} = \\ &= \frac{P(i_\omega) \sum_{i_u} P(i_u) \sum_{o_u} \prod_c P(\mathrm{o}_c \mid \pi(O_c)) P(\delta_c)}{P(i_\omega) \sum_{\delta_c} P(\delta_c) \sum_{i_u} P(i_u) \sum_{o_u} \prod_c P(\mathrm{o}_c \mid \pi(O_c))} \\ &= \frac{P(\delta_c) \sum_{i_u} P(i_u) \sum_{o_u} \prod_c P(\mathrm{o}_c \mid \pi(O_c))}{\sum_{\delta_c} P(\delta_c) \sum_{i_u} P(i_u) \sum_{o_u} \prod_c P(\mathrm{o}_c \mid \pi(O_c))}. \end{aligned} \tag{4}$$

At first sight, it seems computationally infeasible to compute $P(\delta_c \mid i_\omega, o_\omega)$ in this manner. However, the computation can be simplified as $P(\delta_C \mid \omega)$ is only used to rank diagnoses and thus the denominator need not be used as it is the same for all diagnoses; *only* the numerator has to be computed. The computation of the numerator is easy, since the second term $\sum_{i_u} P(i_u) \cdots$ is already computed as part of the denominator of the DCM (see Equation (2)). Only the probability $P(\delta_c)$ needs to be computed, which is a product of the individual probabilities for (ab)normal behaviours of the components.

## 4 CONCLUSIONS

In this paper a method was described to augment conflict-based diagnosis with probabilistic abductive diagnosis. The refinement of conflict-based diagnosis by abduction has the virtue that it reuses part of the computation required for finding conflict-based diagnoses.

## REFERENCES

[1] L. Console and P. Torasso. A Spectrum of Logical Definitions of Model-based Diagnosis, *Computational Intelligence*, 7:133–141, 1991.

[2] J. de Kleer and B. C. Williams. Diagnosing multiple faults, *AIJ*, 32:97–130, 1987.

[3] J. de Kleer, A. K. Mackworth, and R. Reiter. Characterizing diagnoses and systems. *AIJ*, 56:197–222, 1992.

[4] I. Flesch, P.J.F. Lucas and Th. van der Weide. Conflict-based diagnosis: Adding uncertainty to model-based diagnosis. IJCAI, 380–388, 2007.

[5] J. Pearl. *Probabilistic Reasoning in Intelligent Systems:Networks of Plausible Inference*. Morgan Kauffman, San Francisco, CA, 1988.

[6] R. Reiter. A theory of diagnosis from first principles. *AIJ*, 32:57–95, 1987.

[7] S.E. Shimony and E. Charniak. A new algorithm for finding MAP assignments to belief networks. AIJ, Volume 6, pp. 185–193, 1991.