KNOWLEDGE SPILLOVERS FROM FDI: TOWARDS A GENERAL FRAMEWORK*

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May 17, 2005

Abstract

This paper develops a conceptual framework that allows for a consistent and coherent treatment of knowledge spillovers from FDI. In this framework, three aspects play a crucial role. First, FDI is a general notion that actually comprises a host of different investment types. Second, the number and nature of the knowledge spillover transfer channels differs per type of FDI. Third, the fact that knowledge spillovers only accrue when there is a sufficient absorptive capacity in the host country. Confronting empirical research on knowledge spillovers with these observations reveals that they have largely been ignored. We argue that this is the reason for the extremely mixed evidence on knowledge spillovers, and that the empirical research strategy should be altered in order to take proper account of theoretical notions.

Keywords: knowledge spillovers, FDI, multinationals, ownership

JEL - code(s): F 23, O33, R30

* This paper has benefited from useful comments from and discussions with Sjoerd Beugelsdijk, George Gelauff, Arjen Lejour and Paul Tang.
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1. Introduction

Many national governments believe that luring Multinational Enterprises (MNEs) into their economies will have substantial positive economic effects. Aside from direct effects (e.g. a rise in employment), increased attention has been directed towards the existence of knowledge spillovers. A large literature on knowledge spillovers from Foreign Direct Investment (FDI) has subsequently emerged, both theoretically (e.g. Romer, 1990; Grossman and Helpman, 1991) as well as empirically (see e.g. Cincera and van Pottelsberghe de la Potterie, 2001; Saggi, 2002; Keller, 2004). Conceptual grounds on which these knowledge spillovers are believed to occur appear intuitively plausible, but empirical research has presented ambiguous evidence. This also applies to evidence concerning the alleged spatial nature of knowledge spillovers. The efforts of many countries to attract FDIs, validated by the existence of knowledge spillovers, are thus questionable (see e.g. Blomström and Kokko, 2003).

In this paper we develop a conceptual framework that allows for a consistent and coherent treatment of knowledge spillovers from FDI. In our framework three aspects play a crucial role. First, FDI is a general notion that actually comprises a host of different investment types, where investment types are distinguished mainly by degree of ownership. Second, the number and nature of the knowledge spillover transfer channels differs per type of FDI. In fact, as we will show a hump-back shaped relation between ownership and spillover potential will generally appear. This implies that an interesting paradox arises. Whereas the prime reason for firms to undertake FDI is to keep crucial, firm-specific knowledge within the boundaries of the firm, at the same time the inclination to increase the level of ownership goes up when more firm-specific knowledge is at hand. The paradox is then that an increase in (firm-specific) knowledge increases the incentive to organize at a higher level of integration (in order to prevent knowledge from spilling over), which in turn increases the potential for knowledge spillovers given the large degree of (firm-specific) knowledge that has crossed national borders. The third aspect to consider is that spillover potential of FDI on the one hand and actual spillovers on the other are two very distinct concepts. Whereas the former relates mainly to the two aspects mentioned above, the latter has to do with the absorptive capacity of the recipient country. We will refer to this as the technological distance between the investor and the recipient.

To yield further analytical insight on these matters, we also formalize our conceptual framework into a model of knowledge spillovers and FDI. When we confront our theoretical framework with the empirical literature on FDI-related knowledge spillovers, we find that the
fundamental problem with this literature is that it has by and large ignored theoretical concepts.\footnote{Grossman and Helpman (1990) and Hoppe (2005) make a similar point, be it in a different context.} Specifically, most previous empirical modeling efforts have disregarded the fact that FDI covers a wide variety of different investment types. We therefore also discuss the consequences of our conceptual framework for the empirical modeling of knowledge spillovers by FDI. As these are many, we conclude by presenting a strategy how to sensibly validate the empirical relation between FDI and knowledge spillovers.

The remainder of this paper is structured as follows. Section 2 discusses taxonomies and conditions for knowledge spillovers. Section 3 specifically deals with FDI-related knowledge spillovers and develops our conceptual framework. In section 4 we present a formalization of this framework and derive analytical insights as to the relation between (types of) FDI and knowledge spillovers. Section 5 confronts the empirical research on knowledge spillovers with our framework. Section 6 concludes and presents our agenda for further research.

2. Taxonomies and conditions for knowledge spillovers

The fact that knowledge can spill over from one country, industry or firm to another is due to the public good nature of knowledge, as first recognized by Arrow (1962) and later extensively discussed by e.g. Keller (2001) and Blomström and Kokko (2003). The non-rivalry and non-exclusivity of knowledge invoke free-riding behavior, i.e. parties other than the innovator may be able to costlessly benefit from the innovation, so that the actual innovator may not be able to fully appropriate all the rents originating from his innovation. Yet it is also exactly this characteristic of knowledge that is often used to validate government intervention, as it implies that the social returns of innovation may be higher than the private returns.

Previous literature provides a host of possible taxonomies according to which knowledge spillovers can be classified. Grilliches (1979) talks about innovation externalities and distinguishes between knowledge spillovers on the one hand and rent spillovers on the other, the latter resulting from imperfect price-adjustments following quality improvements of goods and services (also see Cincera and van Pottelsberghe de la Potterie (2001)). Grünfeld (2002a) differentiates between embodied versus disembodied knowledge spillovers, where the former are mainly related to tangibles (e.g. goods) and the latter to intangibles (e.g. services).
Keller (2001) adheres to a somewhat similar taxonomy of active versus passive knowledge spillovers, although he bases this classification mainly on spillover transfer mechanisms: (international) trade, FDI, personal communication patterns, articles in (scientific) journals and patents (Keller, 2001; 2004). Accordingly, spillovers are termed active when they provide the receiver with a kind of “blueprint” of the knowledge (e.g. patents), whereas they are termed passive when it allows the receiver only to apply certain pre-designed elements of the knowledge or technology (e.g. trade in intermediate goods). Note that these spillover mechanisms point out that knowledge spillovers can be both national as well as international in scope.

Although these transfer mechanisms thus provide very general vehicles for knowledge, it is possible to identify more specific transfer channels that can in turn be related to the transfer mechanisms. With respect to FDI for instance, Saggi (2002) presents three transfer channels that are specifically related to FDI: (1) Demonstration effects, which may lead to reverse engineering and imitation, (2) Labor turnover, which means that workers employed by the Multinational Enterprise (MNE) undertaking the FDI may switch jobs and hence take with them firm-specific knowledge, and (3) Vertical linkages, which means that the MNE may transfer knowledge or technology to its suppliers of intermediate goods (upstream linkages) or its customers (downstream linkages). Other authors have introduced some other possible spillover channels (e.g. Blomström and Kokko, 2003; Grünfeld, 2002a), but the three identified by Saggi are generally considered to be the most important. Moreover, demonstration effects are not solely related to FDI, but can be bound up with other mechanisms as well, for instance with trade. Furthermore, Grossman and Helpman (1991) argue that communication is an important transfer channel for trade as well.

Although knowledge may spill over from one end, we still have to consider the other end of the channel: the knowledge-receiving party. For it is not immediately obvious that knowledge will also be received automatically once it spills over. Cohen and Levinthal (1989) were one of the first to address this issue. Consider the following function governing a firm’s (i) stock of technological and scientific knowledge ($z_i$):

$$z_i = M_i + \gamma_i (\Theta \sum_{i \neq j} M_j + T)$$  \[1\]

2 Usually, downstream linkages imply that the MNE is itself in fact a producer of intermediate goods and its customers are hence other firms in stead of (private) consumers.

3 For more variations on vertical (backward) linkages as well as specific examples, see e.g. United Nations (2001), Chapter 4.
Here, $M$ denotes investment in Research and Development (R&D), $\theta$ represents the degree of intra-industry spillovers and $T$ is the level of extra-industry knowledge. The variable of interest is $\gamma$ which denotes the fraction of knowledge in the public domain that the firm is able to assimilate and exploit. The authors define $\gamma$ to be a firm's absorptive capacity, which is in turn governed by:

$$\gamma_i \equiv \gamma(M_i, \beta) \quad \text{s.t.} \quad \gamma_M > 0 \quad \gamma_{MM} < 0$$ \hspace{1cm} [2]

In [2], the variable $\beta$ reflects certain characteristics of outside knowledge. The point to note here is that absorptive capacity is also positively dependent on the firm's own investments in R&D (although decreasingly so). Hence we observe a direct positive influence of investments in R&D from [1], as well as an indirect positive influence from [2] since more investment in R&D also allow the firm to better exploit external knowledge.\footnote{The authors work out this model more extensively and show that the common proposition that increasing spillovers reduce the incentive to invest in R&D is not straightforwardly applicable in their model due to the offsetting direct and indirect effects of R&D.} Authors such as Lankhuizen (1998), Keller (2001) and Grünfeld (2002b;2003) have subsequently substantiated this claim empirically. They have also noted that human capital may be another determinant of absorptive capacity.\footnote{An interesting paper specifically with regard to the absorptive capacity function of human capital is by Acemoglu and Zilibotti (2001). These authors show that even if two countries have access to exactly the same technologies and are different only in (labor-)skill composition, still large productivity differences may arise because of technology-skill mismatch.}

Although important, absorptive capacity alone may not be enough to absorb knowledge spillovers. A growing strand of literature has been arguing that geography matters for knowledge spillovers as well (e.g. Audretsch and Feldman, 2003; Jaffe et al., 1993; Asheim and Gertler, 2005). According to this view, it is important to be spatially located near to the source of knowledge spillovers.\footnote{We would like to point out the similarity between the absorptive capacity discussion and the issues surrounding geography by noting that the former relates to technological (technical) distance between two firms, whereas the latter relates to spatial distance.} Authors like Girma and Wakelin (2001) have argued that some of the distinguished spillover transfer channels are somewhat geographically bounded.\footnote{For instance, the authors argue that labor is more mobile within than between regions, which implies that labor turnover as a transfer channel for knowledge is spatially bounded.} However, the primary reason for the relevance of location addressed in the
literature relates to the *tacit* nature of knowledge, which requires some kind of *face-to-face* interaction in order to be properly transferred from the sender to the receiver.

In very general terms, the literature distinguishes between *codified knowledge* and *tacit knowledge* (see e.g. Powell and Grodal, 2005). Codified knowledge is preserved in a more tangible form (such as books, CD-Roms, data-files etc.) whereas tacit knowledge is preserved in intangibles (such as experience, routines, norms etc.). It is possible to relate these different types of knowledge to different transfer mechanisms and channels, and to the relevance of geography. This is done in Table 1 below.

A glance at the table reveals several things: first of all, there seems to be no straightforward relationship between (dis)embodied and active/passive spillovers. This also goes for the relationship between classifications of spillovers and the involved types of knowledge. Furthermore, location indeed matters mainly for spillover mechanisms that incorporate some degree of tacit knowledge. The table also hints to the fact that FDI comprises an interesting mix of spillover transfer channels, types of knowledge and spatiality. This is also the prime reason for the focus on FDI in this paper.

<table>
<thead>
<tr>
<th>Transfer mechanism</th>
<th>Transfer channel(s)</th>
<th>Classification</th>
<th>Involved Knowledge</th>
<th>Geographical relevance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade</td>
<td>a -demonstration effects</td>
<td>a -embodied/passive</td>
<td>A -codified</td>
<td>Relevant/Irrelevant</td>
</tr>
<tr>
<td></td>
<td>b -communication</td>
<td>b -disembodied/active</td>
<td>b -tacit</td>
<td></td>
</tr>
<tr>
<td>Foreign Direct Investment</td>
<td>a -demonstration effects</td>
<td>a -(dis)embodied/passive</td>
<td>a -codified/tacit</td>
<td>Relevant</td>
</tr>
<tr>
<td></td>
<td>b -labor turnover</td>
<td>b -disembodied/active</td>
<td>b -tacit</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c -vertical linkages</td>
<td>c -(dis)embodied/passive</td>
<td>c -codified/tacit</td>
<td></td>
</tr>
<tr>
<td>Communication Patterns</td>
<td>a -communication</td>
<td>a -disembodied/active</td>
<td>a -tacit</td>
<td>Relevant</td>
</tr>
<tr>
<td>Articles in (scientific) journals</td>
<td>a -demonstration effects</td>
<td>a -embodied/active</td>
<td>a -codified</td>
<td>Irrelevant</td>
</tr>
<tr>
<td>Patents</td>
<td>a -demonstration effects</td>
<td>a -embodied/active</td>
<td>a -codified</td>
<td>Irrelevant</td>
</tr>
</tbody>
</table>

*Table 1: Knowledge spillover taxonomies*
3. FDI and knowledge spillovers: a paradox

A well-established theoretical framework of the MNE is Dunning’s (1977) **OLI-paradigm**. The OLI-paradigm argues that there are three main motives to undertake FDI which are related to ownership-, location- and/or internalization advantages.\(^8\) Hence one reason for a firm to engage in FDI in stead of international trade is when it has a critical degree of **firm-specific knowledge** attached to its activities. In this way, the firm is able to better control the application of this knowledge and to prevent it from spilling over.\(^9\) This view thus assumes that knowledge can spill over through international trade, but not through FDI. However, in the previous section we have argued that FDI may very well function as a spillover mechanism for knowledge, albeit relying on different (or even more) **transfer channels** than international trade (Table1). Here a paradox presents itself: An increase in (firm-specific) knowledge increases incentives for FDI in order to prevent knowledge from spilling over, which in turn may actually serve to increase spillover potential given the large degree of (firm-specific) knowledge.\(^10\) We will refer to this paradox as the **OLI-paradox**.

It is evident that the OLI-paradox hinges crucially on the assumption that FDI functions as a knowledge spillover mechanism. Referring back to Table 1, the spillover potential of FDI is largely determined by the specific spillover transfer channels. These transfer channels are therefore of prime importance for substantiating the OLI-paradox, or more generally, for testing the spillover potential of FDI. Specifically, we can distinguish two effects that are helpful in analyzing the OLI-paradox.

First of all, the relevance of distinguishing specific transfer channels becomes (even more) evident when we recognize that FDI is mainly a theoretical notion (see e.g. Lipsey, 2001). It actually comprises various types of foreign investment. Van Marrewijk (2002) shows that classifying and sub-classifying FDI can lead to a host of different types of FDI, many of which are characterized by different **degrees of ownership**.\(^11\) As it turns out, this

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\(^8\) The validity of **OLI-motives** for FDI have also been empirically substantiated (see e.g. Brouthers et al. 1999; Arora and Fosfuri, 2000).

\(^9\) Markusen and Venables (1998) construct a more formal model of the development of FDI. These authors show that MNEs are more likely to arise when (1) overall markets are large, (2) different national markets are similar in size and factor rewards, (3) firm-level scale economies are large relative to plant-level scale economies and (4) transport costs are high.

\(^10\) Later on in this paper, we will argue that it is the relative knowledge intensity of the FDI that matters for spillover potential. For the moment however, the presented intuition should suffice.

\(^11\) Principally, van Marrewijk makes a distinction between Greenfield FDI, Mergers and Acquisitions. The latter may be subdivided in full, majority and minority acquisitions.
observation is very important for our analysis. Hence we will officially term it the level of control-effect and refer to it as such henceforth.

The second effect is closely related to the first one. Ethier and Markusen (1996) construct a formal two-country, two-period model in order to analyze a firm’s choice of how to service a foreign market. The options range from exporting, to a combination of licensing and exporting to setting up a fully-owned subsidiary. The authors show that a firm’s choice for a specific type of FDI depends on, inter alia, the importance of knowledge capital relative to physical capital. From their discussion it follows that different types of FDI are likely to have different spillover potentials. Let us illustrate this argument by means of an example, presented in Table 2 below. The table combines three different types of FDI (different mainly in the degree of ownership) with the three spillover transfer channels from Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Licensing</th>
<th>Joint Venture</th>
<th>Full Ownership</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstration</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Effects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor</td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Turnover</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Linkages</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Combining different types of FDI with spillover channels

A cross-mark denotes that a transfer channel is relevant for a specific type of FDI. For example, a wholly owned subsidiary may not provide strong spillover effects through demonstration effects as most of the knowledge is kept internal to the firm. However, when the subsidiary hires and trains local personnel, another transfer channel presents itself: labor turnover. In time, some of the personnel may leave the MNE's subsidiary and find work elsewhere, so that knowledge is transferred. A joint venture possibly presents the highest potential for knowledge to spill over: the MNE brings in firm specific knowledge which induces demonstration effects to the partner firm, it may train local personnel which in time can lead to spillovers through labor turnover, and finally it may spill over knowledge to the

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12 It should be noted that other conditions apply to this decision as well, e.g. the applied profit discounting factor and trade (export) costs. An interesting feature of the model is moreover that potential foreign partner firms are able to exploit the newly acquired knowledge in the second period, thus posing a competitive threat to the original inventor.
(already established) supplier- and buyer-network of the partner firm. This establishes the second effect, which we will henceforth refer to as the *channel variety-effect*. It establishes that specific transfer channels have different impacts on different types of FDI, as well as that some types of FDI may have a greater variety of transfer channels ‘available’ than do other types. Both the level of control-effect and the channel variety-effect play a crucial role in explaining the OLI-paradox, to which we will now turn.

The OLI-paradigm holds that, *inter alia*, more firm-specific knowledge leads to a preference of FDI over exports. This paradigm is based on the belief that exports suffer from spillover potential whereas FDIs do not. Based on the discussion in section 2 we already know that this belief is probably false. However, let us reason along the lines of the OLI-paradigm for the moment in order to clarify our argument.

![Figure 1: The OLI-paradigm](image)

Figure 1 denotes the mechanisms underlying the OLI-paradigm. Figure 1a denotes the alleged relationship between spillover potential and degree of ownership (integration)\(^{14}\). Note that exporting implies the lowest possible degree of integration. Accordingly, the OLI-paradigm states that exports incorporate the highest spillover potential. Hence if there is a high degree of firm-specific knowledge embodied in production, exporting is too risky and a firm should

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\(^{13}\) Note that the table implies a rather short time horizon, since even a wholly owned subsidiary will develop long-term supplier- and customer-relationships over time. However, it may take a while before such relationships are well established so that the relationship may be of an arms-length nature at first and vertical linkages are not immediately established.

\(^{14}\) We follow the common practice where a higher degree of integrations denotes more ownership.
in stead choose to produce abroad. The reason for this is that in this case there is no spillover potential. From Table 1 we observe that the main spillover transfer channel for trade is demonstration effects. According to Table 2, the only type of FDI that is not plagued by demonstration effects is fully integrated FDI. We should thus recognize that the OLI-paradigm implicitly refers only to fully owned types of FDI (i.e. degree of integration = 1, which relates to 0 spillover potential in Figure 1a). It thus appears as if the OLI-paradigm only considers the bold endpoints of the Figure 1a, by only considering demonstration effects as a valid transfer channel.

Figure 1b accordingly shows the relationship between the amount of firm specific knowledge (transferred abroad) and integration. The OLI-paradigm states that when firm specific knowledge surpasses a certain threshold (say $K'$), a firm should decide to produce abroad in stead of export. However, we just stated that the OLI-paradigm appears to only consider endpoints, so that producing abroad means engaging in a fully-owned type of FDI. Accordingly, the relationship between knowledge and ownership cannot be continuous, but instead is dichotomous, as shown in the right graph by the two bold lines, connected by the thin grey line (at which the function is thus not valued).

One obvious objection to our reasoning along the lines of the OLI-paradigm thus far, is that by only considering exports on the one hand and fully owned FDI on the other we are disregarding the level of control-effect. I.e. a firm can also choose to cooperate with a foreign firm when engaging in FDI, choices ranging along a continuum of degrees of integration. Note that allowing for the level of control-effect appears to leave the OLI-paradigm intact at first sight. For the figures displayed above allow for a continuum of different types of FDI since there can exist continuous negative (positive) relationships between ownership and spillover potential (transferred knowledge), as displayed by the dashed lines.15

However, we have shown by means of the simple example in Table 2 that different types of FDI are related to different types and numbers of spillover transfer channels through the channel variety-effect. Hence, if we allow both for the level of control-effect and the channel variety-effect, the relationship between spillover potential and degree of ownership changes dramatically from the one shown in Figure 1a. More specifically, if for the moment we attach equal weights to the transfer channels, we are able to express spillover potential of each type of FDI in terms of number of transfer channels only. As our simple example in

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15 Such relationships are suggested by e.g. Hill et al. (1990) and Dimelis and Louri (2002)
Table 2 already shows, this is relationship is not monotone. In fact, it looks more like an inverted U, such as the SS-curve in Figure 2 below.

![SS-curve](image)

**Figure 2: The OLI-paradox**

Thus we now obtain a hump-back shaped relationship between spillover potential and ownership, which reaches a maximum at say \( I' \). For values of \( I > I' \) the relationship is indeed negative, albeit not linearly so. Still, since this is what the OLI-paradigm implies, we can state that after a certain degree of ownership \( I' \) the OLI-paradigm holds. However, for all degrees of ownership \( I < I' \), we actually observe a positive relationship between ownership and spillover potential, which is exactly the opposite of what the OLI-paradigm conjectures.

The point to note here is that when the MNE adheres to the relationships implied by the OLI-paradigm in Figure 1, it will transfer more knowledge abroad as its degree of FDI-ownership increases (Figure 1b). This is because, while (possibly) recognizing the level of control-effect, it disregards the channel variety-effect. Yet if the *true* relationship is the one we conjecture in Figure 2 (i.e. including the channel variety-effect), this implies that the MNE should actually transfer less knowledge abroad, since up to a certain extent (\( I' \)) spillover potential is increasing. Consequently, the MNE is thus actually increasing spillover potential at first when it moves from exports to FDI. Moreover, as it is simultaneously transferring more knowledge abroad, this in turn stimulates actual spillovers since there is more knowledge to spill over. The OLI-paradox thus unfolds itself: Having more firm specific
knowledge, a firm is more likely to engage in FDI in order to prevent knowledge spillovers, whereas it is thus actually stimulating them (up to a certain extent $I'$).

Obviously, the precise position and curvature of the $SS$-curve depends on the precise number and nature of the underlying spillover transfer channels. If these indeed have different weights attached to them (and we expect that they do), then the curve may become steeper or flatter and $I'$ may shift to the left or right. The way we have drawn it in the figure is just to illustrate that it will typically be of a non-linear, non-monotone nature.

Moreover, we note that a high level of transferred knowledge and high spillover potential (the combination of which reaches a maximum at $I'$) does not automatically imply that the actual knowledge spillovers are high as well. In line with our discussion in the previous section, we argue that this also depends on the absorption capacity of the recipient. The local (partner) firm(s) has to have some stock of knowledge itself to be able to absorb knowledge spillover from the MNE. Hence, next to a high potential for knowledge spillovers, the extent of the actual spillovers also depends on the degree of knowledge embodied in the FDI relative to the degree of knowledge present in the local (partner) firm(s). We will refer to this as the technological distance between the investor and recipient. In the formal model we develop in the next section, it will play a central role.

We conclude our exposition of how ownership relates to knowledge spillovers by discussing the role geography plays. Surely, spatiality has been at the heart of our analysis all over, as we consider FDI. However, so far we have taken the location decision of the MNE as given, since the focus has been on the level of ownership. We note, however, that the OLI-paradigm also stresses location as an important motivation for FDI. We have already noted that previous literature has argued that FDI-related knowledge spillovers have a crucial geographical dimension (see Table 1). Hence, if knowledge spillovers are indeed spatially bounded, the basis for the OLI-paradox is strengthened. Accordingly, any theory or empirical test of the OLI-paradox should take account of geography. Theoretically, we see prospects here for applying insights and modeling techniques of geographical economics. This is especially relevant if one realizes that the spatiality of knowledge spillovers is typically understood to mean that they most likely appear in economic clusters.

\[\text{16 Evidence for the spatiality of knowledge spillovers is rather mixed however (see e.g. Jaffe et al., 1993; Audretsch and Feldman, 1996; Cornish, 1997; Girma and Wakelin, 2002).}\]
4. A formal model of FDI and knowledge spillovers

The previous section provides the basic building blocks for a coherent theory of knowledge spillovers related to FDI. Such a theory should take account of the level of control-effect and at the same time recognize how this induces the channel variety-effect. It also should take account of the fact that turning knowledge spillover potential into an actual knowledge spillover has something to do with the absorptive capacity of the recipient (technological distance). Moreover, if knowledge spillovers are indeed spatially bounded, these processes have to be modeled as well.

In this section we take the first steps in constructing such a theory. Our analysis will be far from complete and therefore, at this point, merely serves to illustrate how the insights of the previous section can be formalized. It should be noted that a formal model concerning the extent of knowledge spillovers is not novel (e.g. Grossman and Helpman, 1991; Eaton and Kortum, 1999). However, previous efforts in this area of research have mainly sufficed by introducing some kind of *knowledge diffusion parameter*, say $\phi$. Very generally, knowledge diffusion is then modeled as:

$$K_{i,t+1} = \sum_{j=0}^{\delta} K_{j,t} + \phi K_{j,t+1} \quad \text{s.t. } i \neq j \quad [3]$$

Here, $K$ denotes the stock of knowledge. Hence, a country’s ($i$) stock of knowledge at time $t+1$ depends on its accumulated stock over the past and some amount of knowledge that is spilled over from abroad ($j$) at rate $\phi$. Accordingly, $\phi$ is either assumed to be exogenously determined or to be randomly drawn from some specific probability distribution. Although it is mostly acknowledged that $\phi$ must be determined endogenously, the issue is never explored further. The novel contribution of our model lies in the fact that we aim to do so.\(^{17}\) We seek to find out how $\phi$ is determined by (a) allowing for the level of control-effect, while (b)

\(^{17}\) Although the parallel with the endogenous growth literature is easily made, there is a slight difference. The endogenous growth literature endogenizes technological change by making it the aim of purposeful action by economic agents. In the case of knowledge spillovers, we have to slightly alter this notion in that it is probably not the aim of any economic agent to (purposefully) spill over firm-specific knowledge. Rather, we will argue that knowledge spillovers are a *consequence* (not an aim) of purposeful action, specifically resulting from conscious choices regarding degree of FDI-ownership and transferred knowledge.
recognizing the channel variety-effect and (c) acknowledging that it is actually technological distance that eventually determines actual knowledge spillovers.\(^{18}\)

Consider Figure 3 below which shows a unit circle, (i.e. a circle that covers a distance of 1). The circle represents technological space and the starting point (i.e. 0) is arbitrary. The numbers \(n = 1, \ldots, N\) are potential partner firms in the foreign economy, whereas \(m = 1, \ldots, M\) are the possible locations at which the MNE may enter technology space. Note that the locations of both local firms and the MNE represent technological states. Now, if we denote such a technological state by \(x_i\) - where subscript \(i\) denotes the MNE and subscript \(j\) denotes an arbitrary local firm - \(x = (x_i - x_j)\) thus denotes technological distance. We assume throughout that \(x_j\) is given, so that \(x\) can only be varied by varying \(x_i\).\(^{19}\) This translates into Figure 3 by assuming that the positions of \(n\) are fixed whereas \(m\) is flexible. Accordingly, the distance between two local firms (1/\(n\)) can be thought of as a measure of average absorption capacity. This may be influenced by general knowledge characteristics, market structure and/or government policy. Hence, the situation as shown in Figure 3 is a result of such forces and thus provides a scope for general equilibrium analysis.\(^{20}\)

![Figure 3: Technological distance on the unit circle](image)

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\(^{18}\) Thus note that (a) and (b) together \textit{indirectly} determine actual spillovers through spillover potential, whereas (c) \textit{directly} determines actual spillovers.

\(^{19}\) This may seem to be a rather restrictive assumption. However, we are interested in MNE behavior and FDI spillover potential. Hence, we are modeling from the MNE’s point of view. Since the MNE will be able only to influence \(x_i\), the assumption is not implausible.

\(^{20}\) The construct of the model proposed here is based on Grossman and Helpman (2002a;2002b), although these authors apply it as a model for outsourcing vs. FDI decisions.
Our model implies two other assumptions that we should highlight: first, we assume that the local firms are located at equal distances \((1/n)\) from each other in order to keep the model tractable. Second, note that an MNE, by construction of the model, always locates somewhere in-between two local firms (assuming \(n \geq 2\)). However, we assume that all firms are technologically equal (i.e. different locations on the unit circle are not assigned different cardinal values) and that technological distance \((x)\) is defined with respect to the local firm that is closest to the MNE (so that the remaining distance is \(1/n - x\)).

We seek to model knowledge spillovers \((S)\) as a function of spillover potential (by combining the level of control-effect and channel variety-effect in \(g(I)\)) and of technological distance \(x\):

\[
S = F(g(I), h(x))
\]  

[4]

Let us start by specifying \(h(x)\), which gives the direct relationship between \(S\) and \(x\). The absorptive capacity discussion leads us to believe that there is a hump-back shaped relationship between \(S\) and \(x\): if \(x\) is too small, both firms are technologically very similar and hence spillovers are small as well (since there is not much to learn). However, if \(x\) is too large, the local firm has too few technological means to absorb spillovers so that spillovers are small again. The following formulation respects these conditions:

\[
h(x) = -2x^2 + \frac{x}{n}
\]  

[5]

The first term in [5] simply denotes the double negative squared value of technological distance between the MNE and the local firm, whereas the second term denotes the technological distance between the MNE and a local firm, relative to the total distance between two local firms.

The second relationship we model is that between \(S\) and \(I\) as given by \(g(I)\) in [4]. Let us define \(I\) as follows:

\[
I \equiv \frac{f_i}{f_i + f_j}
\]
Here $f_i$ denotes the amount of investment by firm $i$ in the foreign operation, where we again assume that $i$ represents the MNE and $j$ an arbitrary local firm. Different types of FDI are hence defined in terms of alternative degrees of integration, which takes care of the level of control-effect. As shown in Figure 1b and discussed in the previous section, the view is commonly held (at least by the MNE) that there is a positive relationship between $x_i$ (the amount of knowledge transferred from MNE HQ to its subsidiary) and $I$. But since $x = (x_i - x_j)$ and we have fixed $x_j$, this implies that there is a positive relationship between $x$ and $I$ as well. We assume – like the MNE – that this relationship is linear and denoted by:

$$I = 2xn$$

Note that this formulation respects the fact that the maximum value of $I = 1$. Recall from the previous section that the channel variety-effect states that spillover potential $g(I)$ is non-linearly related to ownership. In our discussion of Figure 2 we attached equal weights to all spillover transfer channels. Here we will make a similar simplification by assuming a symmetric quadratic relationship between spillover potential and the level of integration:

$$g(I) = -I^2 + I \quad \text{(6)}$$

Now, if we recognize that actual spillovers $S$ are positively related to spillover potential $g(I)$, combining [5] and [6], while scaling down the latter by $4n^2$, implies that [4] becomes:

$$S = -3x^2 + \frac{3x}{2n} \quad \text{(7)}$$

From [7] we can calculate the value of $x$ at which spillovers are greatest ($x'$):

$$x' = \frac{1}{4n} \quad \text{(8)}$$

Now, the decision of the MNE concerning what type of FDI is optimal for investing abroad obviously does not depend solely on spillovers: we need a formulation for profits as well,
preferably in terms of $x$. We assume at this point that the relationship between (joint) profits ($\pi$) and technological distance is positive and linear:\(^{21}\)

$$\pi = \frac{x}{n} \quad [9a]$$

Hence joint profits are determined by the technological distance between the MNE and the local firm relative to the total distance between two local firms. MNE profits are then given by:

$$\pi_{MNE} = I \left( \frac{x}{n} \right) - c = 2x^2 - c \quad [9b]$$

Here, $c$ denotes some amount of sunk costs which are always incurred by the MNE when investing abroad.\(^{22}\) Comparing [9b] with [7] allows us to derive the values of $x$ (and $I$) for which spillovers equal MNE profits. Note that this implies that we consider knowledge spillovers as a cost for the MNE, since we directly relate them to profits. The costs of spillovers can be thought of as a loss in ownership advantages, which damages the MNE’s competitiveness.\(^{23}\) Hence we are able to define those values of $x$ – and $I$ – for which MNE profits are larger than spillovers. Proceeding in this fashion (and disregarding the theoretically irrelevant case at which $\hat{x} < 0$) we obtain:

$$\hat{x} = \frac{1}{2} \left[ \frac{3}{10n} + \sqrt{\left( -\frac{3}{10n} \right)^2 + \frac{4}{5} c} \right] \quad [10]$$

Hence, when $x > \hat{x}$ (or equivalently, when $I > \hat{I} = 2\hat{x}n$), MNE profits are greater than spillovers. Accordingly, the MNE will choose to invest abroad only at a level of integration

\(^{21}\) The intuition behind this assumption is that as technological distance between the MNE and the local firm increases, the complementarity of their technologies increases as well, allowing for larger cooperative gains.

\(^{22}\) Here we assume that sunk costs are incurred by the MNE only whereas the remainder of the joint profits is shared proportionally to degree of ownership.

\(^{23}\) Alternatively, one may argue that knowledge spillovers cannot be interpreted as (nominal) costs. Accordingly, it would not be possible to relate them to profits. We may then set marginal spillovers equal to marginal profits in order to derive the optimal technological distance.
of $I > \hat{I}$. Note that [10] thus gives us an (implicit) function of the degree of integration, where spillover potential is endogenous. Note that the critical value $\hat{I}$ (the degree of integration after which the MNE will choose to invest abroad) is positively dependent on sunk costs ($c$) and the number of local firms ($n$).

As our model is still far from complete we will postpone an in-depth analysis of its implications for the moment. In stead we provide a numerical simulation in the Appendix of this paper in order to further clarify matters. As argued, the relationship between spillover potential and integration $g(I)$ is non-linear due to the channel variety-effect. However, we have assumed that it is symmetric around some mean (in this case 0.5), disregarding the potential different weights attached to the different numbers and varieties of spillover channels. Hence on further aim of our analysis is to extend our model along this line.

Another aim is to embed the spillover model in a general equilibrium framework. As we have already noted, the set-up of the model as depicted in Figure 3 is itself a result of GE modeling, examples of which can be found in the geographical economics literature. Moreover, by incorporating the spillover model in more formal theories of the MNE (e.g. Ethier and Markusen, 1996) we will be able to more formally derive the conditions under which the OLI-paradox holds. All these extensions are part of our current research agenda.

5. Empirical research

So far we have discussed mainly conceptual issues on FDI-related knowledge spillovers. However, there is a large and ever growing strand of empirical research on the subject, which may lead the reader to question most of what has been discussed above. If indeed empirical research on this subject is so wide-spread, has the OLI-paradox not been addressed and established already? Is the theoretical framework on which existing empirical literature frames itself not sufficient? Are we really in need of a(ther) theoretical model? We will show in this section that although these questions are valid, the empirical literature thus far has been largely ignoring the issues that we have raised in the previous sections.

The empirical estimation function that is often encountered in the spillover literature has the following generic form (using matrix-vector notation):

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24 By combining [10] and [8] we can derive for what value(s) of $c$ the MNE will choose to invest abroad albeit spillover potential being at its maximum. It turns out that this is the case when $c = -1/16n^2$, i.e. if a local government wants to maximize spillover potential, it should subsidize the MNE by an amount of $c + 1/16n^2$. 

Here $\gamma$ denotes productivity$^{25}$, $c$ is a constant, $X$ is a matrix of standard control variables (e.g. investments in R&D, Human Capital and (more recently) trade), $Z$ is the variable of interest and $D$ are time/cross-section dummies. In the FDI-related knowledge spillover literature, $Z$ is thus the amount of (inward and/or outward) FDI.$^{26}$

Equation [11] shows the usual estimating strategy. This type of empirical model is encountered at all levels of analysis, i.e. national (e.g. Coe and Helpman, 1995; Lichtenberg and Van Pottelsberghe de la Potterie, 1996;1998), sectoral (e.g. Braconier et al. 2001; Sjöholm, 1997; Barrell and Pain, 1999; Verspagen, 1997), firm (e.g. Aitken and Harrison, 1999; Keller and Yeaple, 2003; Branstetter, 2000) and regional (e.g. Audretsch and Feldman, 1996; Cornish, 1997; Haskel et al., 2002).$^{27}$

The first point to note is that in the FDI-related knowledge spillover literature, the variable $Z$ is the total amount of FDI in either stocks or flows, from country $i \in N$ into country $j$ (s.t. $j \neq i$).$^{28}$ Hence, any effect that is left after controlling for the most obvious determinants of productivity is ascribed to FDI (cf. Görg and Greenaway, 2004). Apart from the fact that this approach is rather crude, a more fundamental problem lies in the fact that the coefficient $\beta$ eventually measures some roughly aggregated effect of FDI on productivity. For instance, although direct (composition) effects of FDI are sometimes controlled for, this is certainly not always the case.

If we confront this empirical strategy with our discussion in section 3 (and 4), two observations arise. First of all, whereas we have – based on previous theoretical efforts – deduced and established that the type of FDI is of prime importance for establishing its

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$^{25}$ This is usually taken to be either Total Factor Productivity (TFP) or Labor Productivity (LP).

$^{26}$ Inward FDI is expected to influence TFP because foreign MNEs will want to exploit their technological advantage in the host country ("technology exploiting FDI"), so that spillovers may occur. Outward FDI is expected to influence home country TFP because the MNE may locate abroad to learn from foreign technology ("technology sourcing FDI"). It should be clear immediately that both sourcing and exploiting motives may underlie both types of FDI, so that it is not obvious a priori whether FDI has a positive influence on home or host country TFP (also see e.g. Criscuolo et al. (2002) or Narula and Zanfei (2005) for related discussions).

$^{27}$ For some excellent surveys on the empirical (FDI-related) knowledge spillovers literature, the reader is referred to, inter alia, Cincera and Van Pottelsberghe de la Potterie (2001), Grünfeld (2002a), Keller (2004), Görg and Greenaway (2004) and Hoppe (2005).

$^{28}$ There has been a long-standing debate in the FDI-related knowledge spillover literature whether stock or flow data of FDI should be used. Those in favor of the former argue that knowledge embodied in FDI accumulates over time, so that stocks more accurately reflect the total amount of accumulated knowledge embodied in FDI (usually the perpetual inventory method is applied). Those in favor of flow data argue that stock data suffer from serious stationarity problems so that they will bias the estimated coefficients. Hence the former view is mainly based on conceptual considerations whereas the latter is more methodologically founded.
spillover potential, no such distinction is made in the empirical spillover literature.\textsuperscript{29} I.e. the empirical literature has largely disregarded the level of control-effect. Hence, regardless of the sign and magnitude of $\beta$, we cannot infer anything about the spillover effects of specific types of FDI. Note that this also impedes proper policy recommendations beyond very general ones about whether or not FDI is beneficial in terms of knowledge spillovers \textit{de facto}.

A second point of concern is that the coefficient $\beta$ not only very crudely measures \textit{net} spillover effects from total FDI, it also does not allow us to infer anything about specific spillover transfer channels. I.e. the empirical estimation strategy as given by \cite{11} simply \textit{assumes} that the theoretically distinguished spillover transfer channels function as such. Consequently, any measured spillover effect must be going through any or all of these transfer channels. The question of the validity of these transfer channels and whether or not there are more that we have overlooked thus remains unanswered.\textsuperscript{30} This implies that the channel variety-effect has not been addressed by the empirical literature either. Again this impedes proper policy recommendations as we are not able to identify the exact workings of knowledge spillovers.

Although these objections may seem valid based on the discussion in section 3, this discussion itself is to some extent subjective as it is based on personal (and thus maybe arbitrary) observations. We thus may ask how the results of the empirical research efforts hold up compared to each other. The answer is: not too well.\textsuperscript{31} A large number of empirical studies have set-out to measure spillovers from FDI and the evidence ranges from positive spillovers (e.g. Keller and Yeaple, 2003; Kinoshita, 2000; Barba Navaretti and Castellani, 2003), to no spillovers (e.g. Braconier et al., 2001; Veugelers and Cassiman, 2004; Cornish, 1997) to even negative spillovers (Aitken and Harrison, 1999). According to Görg and Strobl (2001), this large variety in results has three main causes: (1) different levels of analysis, (2) cross-section versus panel data analysis, and (3) different measures of spillovers. They even argue that the number of studies finding 'evidence' of spillovers is approximately as high as cases where no significant results are obtained. Narula and Zanfei (2005) implicitly argue that most evidence in favor of the existence of spillovers is found in developed-county studies, whereas studies

\textsuperscript{29} Two notable exceptions are Belderbos et al. (2001) and Javorcik (2004). These authors indeed distinguish between different types of FDI in terms of MNE-ownership and find that there is a relationship between type of FDI and knowledge spillovers.

\textsuperscript{30} Again, one notable exception here is Belderbos et al. (2001). These authors specifically investigate the extent of knowledge spillovers through \textit{vertical linkages}.

\textsuperscript{31} Barba Navaretti and Venables (2004, Ch. 7) present an elegant discussion of the ambiguity in empirical results on FDI-related knowledge spillovers. Specifically Table 7.5 (p.177) is illuminating. Also see Table 2 (p.177-178) in Görg and Greenaway (2004) for a more elaborate exposition.
for developing countries do not point to significant spillovers. Explanations for the lack of consistency in empirical results have thus been claimed to be mainly of an empirically methodological nature.

Based on the above however, we would like to argue that the problem that is plaguing empirical modeling efforts of FDI-related knowledge spillovers is more fundamental. Our main objection is that the empirical literature disregards the fact that FDI comprises a large variety of different types of investments. Consequently, the empirical literature goes past the specific spillover transfer channels. Referring back to sections 2 and 3 we can summarize these objections by noting that the empirical literature fails to take proper account of the theoretical validation of the existence of FDI-related knowledge spillovers. What has been lacking thus far is a coherent general framework in which empirical modeling efforts are embedded. As we have argued, a consequence of this current state of affairs has been an inability to make proper policy recommendations. Not only because of the lack in consistent empirical results, but also because the results have been far too general to warrant policy inference. Moreover, the missing underlying framework has largely impeded proper comparison of different strands of research.

We are thus in need of a general and coherent framework that enables proper empirical modeling of knowledge spillovers by being firmly established in theoretical and conceptual notions. In our view, the framework proposed in sections 3 and 4 provides a first step in fulfilling this need. From a theoretical point of view, our model will be useful for generating testable hypothesis concerning the spillover potential of different types of FDI through specific spillover transfer channels. Empirically, it may serve as a general construct that provides a guide-line as to what relationships and hypotheses are relevant for testing.

6. Conclusion and research agenda

This paper has reviewed the main conceptual and theoretical issues surrounding knowledge spillovers, specifically with respect to FDI. Based on these issues it has established a coherent framework of analyzing knowledge spillovers related to FDI. An important observation in this respect has been that a coherent theory of FDI-related knowledge spillovers should recognize that there are many different types of FDI, which we have termed the level of control-effect. Different types of FDI in turn may yield different degrees of spillover potential. This process runs mainly through the channel variety-effect, which states that different types of FDI are related to a different number and nature of spillover transfer channels. We have then argued
that the combination of these two effects lies at the heart of the OLI-paradox as they
determine the total spillover potential of different types of FDI. With respect to actual
knowledge spillovers, we have argued that these are determined both by spillover potential
and by technological distance. We define technological distance as the amount of knowledge
transferred to the MNE's subsidiary, relative to that of recipient (partner) firms. Subsequently
we have presented a preliminary model for modeling knowledge spillover potential of FDI.
This model shows, *inter alia*, that the investment choice of the MNE is determined by profits
and spillover-costs simultaneously and that the optimal degree of ownership is positively
related to potential local partner firms and sunk costs. Finally, it has been shown that the
empirical literature thus far fails to tackle the key issues that we address in our framework. In
our opinion this implies a lack of theoretical foundation which we believe is the main cause
for the lack of consistency in results. An important consequence of this state of affairs is that
it has impeded proper and specific policy inference, as well as valid cross-comparison of the
empirical research.

The eventual aim of our research agenda is to properly assess the extent and potential
of FDI-related knowledge spillovers empirically. The first step in this process is the model we
introduced in section 4. However, as we have stated this is far from complete. For instance, it
does not incorporate weighted channel variety-effects. But also, and more importantly, we
would like to embed our framework in a full-fledged theory of the MNE. That is, including
the why, how and where decision of foreign operations. The geographical economics
literature may provide useful modeling tools in this respect.

In order to test the hypotheses that can be derived from the model however, we
conjecture that the data that are presently used in empirical research are not sufficient. More
specifically, we believe that we are in need of survey-data from MNEs.\(^{32}\) Such data should be
able to specifically address the relationship between different types of FDI and specific
spillover transfer channels. In light of this, not only should MNEs and their foreign
subsidiaries be surveyed, but also matched surveys might be conducted among competitors or
supplier- and customer-networks of MNEs. Although innovation surveys are already
conducted at regular intervals (e.g. the Community Innovation Survey), these surveys do not
allow for specific spillover analysis as they focus mainly on input and output of the
innovation process. Moreover, there is no explicit focus on MNEs. Accordingly, a next step in

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\(^{32}\) Economists are in general not very fond of survey-data since mostly they do not allow for the much desired
generalizations of results - unless the survey can be conducted at some very large critical scale. However, using
MNE survey data to measure technology transfer is certainly not novel (e.g. Teece, 1977), nor is the suggestion
of doing so (e.g. Sharma, 1998, p.897).
our modeling efforts of FDI-related spillovers is to conduct such a survey. Currently we are aiming to establish contacts with (Dutch) MNEs in order to secure the possibility of an empirical test in an early stadium of our research.

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Appendix\textsuperscript{33}

We take the case as denoted in Figure 3, where \( n = 6 \). From this it follows that [7] becomes:

\[
S = -3x^2 + \frac{3x}{12} \tag{[7]'}
\]

[7]' thus gives us the function of spillover potential. Applying [9b] yields a function of MNE profits. In order to specify [9b] however, we need a value for \( c \). In order to preserve the proper scale, let us set \( c \) to \( 1/500 \). This yields the MNE profit function:

\[
\pi_{MNE} = 2x^2 - \frac{1}{500} \tag{[9b]'}
\]

The graphs belonging to these functions for the specific parameter values are shown below in Figure A1. The \( SS \)-curve denotes the spillover potential function, whereas the \( PP \)-curve denotes the MNE profit function.

\[\text{Figure A1: Spillover Potential (n=6)}\]

\textsuperscript{33} The numerical values that result from the equations for \( \pi \) and \( S \) may seem unrealistic. Note that this is due to the fact that we normalize total technological distance to the interval \([0,1]\). The reader may verify that by normalizing the distance between two local firms \((1/n)\) to 1, similar results are obtained albeit the numerical values become somewhat larger. However, since the parameter \( n \) disappears from the equations we prefer our original specification as this enables us to analyze the effects of an change in \( n \) on the variables.
From this figure and from [8] and [10] we can derive that spillover potential is largest at \( I = 0.5 \), SME profits are smallest at \( I = 0 \) and the critical value \( I' \) after which the MNE finds it profitable to invest abroad is \( +/- 0.67 \). Also note that due to fixed costs, MNE profits only become positive as \( I \approx 0.37 \). It thus follows that due to knowledge spillovers, positive profits are not a sufficient condition for investing abroad. Also note that if the government of the foreign economy wants to maximize spillovers, it should subsidize the MNE's investment by an amount of \( c + 1/16\alpha^2 \) (footnote 24) which in our example thus amounts to \( +/- 0.004 \).