Appropriate risk measures, time horizon and valuation principles in economic capital models

Report of the Working group on Economic Capital Models [*]

The Working Group has been set up by *de Raad van Financiële Toezichthouders, de Nederlandse Vereniging van Banken* and *het Verbond van Verzekeraars*

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SUMMARY & CONCLUSIONS

In an earlier study, WECM dealt with the question which risk categories can be distinguished and how risk can be assessed.[1] In order to assess the risk profile of a financial conglomerate, several conceptual issues must be captured.

First, the focus is on the risk measure. Given the aim of using economic capital as a common denominator within the financial conglomerate, is it possible to apply one risk measure that is adequate for all stakeholders? Within a financial conglomerate four distinct stakeholders are identified: management, supervisors, shareholders, and debt/policyholders. Recognizing the distinct interests of the stakeholders and their views on the role of capital, the working group believes that the conglomerate should aim for one economic capital model that is applicable for all stakeholders. The stakeholders face the same statistical distribution function, but are interested in different area of the distribution function. Hence, economic capital can thus be viewed as representing a common ground for the different stakeholders involved. However, to show the distinct interests adequately, the application of several risk measures may help to assess the risks from the perspective of the stakeholders.

The second conceptual issue deals with the time horizon(s) used to determine economic capital. Time horizon is defined as the length of time period during which the behaviour of risk drivers and their impact on the available capital of the enterprise is evaluated. The working group proposes that the time horizon chosen be the time required to orderly cancel out the risk portfolio in question. The appropriate time horizon is not necessarily the same for each risk driver or risk type.

The third conceptual issue deals with the appropriate valuation principle within an economic capital model. The working group is of the opinion that the valuation principle in EC-modelling should be such that an adequate picture of the economic position of the enterprise is given. Appropriate valuation principles are mark-to-market, or as a second-best solution, mark-to-model.
For risk management purposes, mark-to-model is more relevant than accrual value, even with parameter and model uncertainty, because accrual value will certainly result in a larger misspecification of economic capital.

1 The WECM concentrates on the determination of economically (as opposed to regulatory) required capital and abstracts from issues regarding the exact definition and eligibility of different buffers and its interaction with accounting treatment.

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1. INTRODUCTION

1. In the first discussion paper, the Working Group on Economic Capital Models (WECM) presented a common risk language, which is applicable to both insurance companies and banks. The risk types applied in practice by the participating financial conglomerates have been outlined.

2. This second discussion paper considers more fundamental questions, thereby focusing on the use of economic capital as a common denominator within the financial conglomerate. The fundamental questions are placed within an economic framework; we abstain from issues about provisioning.
The amount of economic capital necessary to cover higher than statistically expected losses depends on the financial conglomerate's risk profile. The risk profile is the net result of inherent risks (identified by risk drivers and/or risk types to whom the financial conglomerate is exposed to), and strategic choices made by its management, which may increase or reduce the impact of the risks. Therefore, risks have to be identified, and their potential impact on the financial situation of the conglomerate must be estimated.
The financial conglomerate's risk profile is summarised through appropriate risk measures. Given the risk profile and the applied risk measure(s), the amount of economic capital may be determined.

3. In order to discuss economic capital from a conceptual point of view, we formulated the following questions:
    1. *What risk measure should we take to assess the risk profile of a financial conglomerate? One or several risk measures?* Different stakeholders (shareholders, policyholders, debt holders, management, and supervisors) may have different objectives, possibly resulting in the application of different risk measures. A related question might be if and how we can end up with a common definition of economic capital.
    2. *Which time horizon should be used to determine economic capital?* The duration of assets and liabilities may differ significantly between different business lines within the conglomerate (e.g., trading vs life insurance), and the question arises about how one may cope with that when determining the overall level of economic capital for the conglomerate.
    3. *Does economic capital modeling depend on the valuation principle (accrual, fair value)? If so, what are the implications?* The question is whether economic capital depends on the accounting principles used and how.

2. RISKS, RISK MEASURES AND ECONOMIC CAPITAL

4. First of all, risks need to be identified and their potential impact should be quantified through appropriate risk measures. Risk is a concept that is given different definitions, depending on the context; an often-applied definition in the context of capital management is 'unexpected loss'. Where the expected loss equals the statistical mean of all possible outcomes, risk relates to the variability of outcomes, or more specifically, the possibility that the actual loss will turn out to be higher than the expected loss. Note that the definition of unexpected loss should cover all potential losses, including those not apparent from the balance sheet (such as a

potential loss arising from a derivatives position, or a guarantee). A risk measure quantifies the potential impact of a risk. Economic capital can be defined as a buffer against all unexpected losses, taking into account the company's desired level of comfort, and incorporating the requirements of all stakeholders (WECM, 2003).

5. Economic capital is often determined by estimating the statistical distribution of potential losses. Given the distribution, a predetermined confidence level, and a selected risk measure, the economic capital can be calculated[2].
Thus, economic capital is not synonymous to a risk measure; a risk measure is required to determine the amount of economic capital. Although risks, risk measures and economic capital are applied interchangeably in practice, from a conceptual point of view they have to be separated.

2.1.Coherent risk measures

6. In order to assess the risk profile of a financial conglomerate adequately, a risk measure is applied. The literature advocates *coherent risk measures* as introduced by Artzner *et al* (1999). A risk measure is coherent if it satisfies the following properties:
    1. *subadditivity* - the value of the risk measure for two risks combined will not be greater than for the risks treated separately;
    2. *monotonicity* - if one risk always leads to equal or greater losses than another risk, the risk measure has the same or a higher value for the first risk;
    3. *positive homogeneity* - the value of the risk measure is independent of scale changes in the unit in which the risk is measured;
    4. *translation invariance* - adding a 'risk-free' asset should not affect the value of the risk measure.

For a mathematical explanation of these properties, we refer to Annex A.

7. We advocate the use of risk measures that satisfy the above-mentioned properties. In practice several risk measures are applied. To what extent do they satisfy the properties of a coherent risk measure? Table 1 outlines the results for some risk measures often applied in practice, or proposed in the literature.

Table 1: Examples of risk measures

Nr.	Risk measure	Coherent	Violation of property
i.	Standard deviation	No	Monotonicity (2)
ii.	Lower semi-standard deviation	No	Subadditivity (1)
iii.	Value-at-Risk (VaR)	No	Subadditivity (1)
iv.	Conditional VaR / Expected Shortfall / TailVaR	Yes	
v.	Wang Transform	Yes	
vi.	Put option premium	No[3]	Translation invariance (4)

[2] Another way to determine economic capital is by means of utility theory (see also Yamai and Yoshiba 2002). Both risk and impact can be described as a utility function. If a distribution function is projected on the shape of the utility function, the amount of economic capital can be determined.

[3] The put option premium does not satisfy all properties of a coherent risk measure. Property (4) of translation invariance is violated. If a relaxation is applied on this property, then the measure satisfies the properties of an insurance risk measure (Jarrow, 2002). See also Annex A.

For each risk measure, the mathematical definition and explanation of those properties which violates a coherent risk measure, are presented in Annex A.

2.2.Relevance of coherent risk measures

8. In the previous section, we presented desirable properties for a risk measure. These may sound rather theoretical, but there may be practical consequences if a risk measure does not satisfy these properties. Some of these consequences are discussed below and may apply to more risk measures. The most-applied risk measure nowadays is the Value-at-Risk. Therefore, we will start to discuss this measure.

9. Artzner (1999) motivates some problems with VaR. Apart from VaR, similar motivations may be given for other measures that do not satisfy the properties of a coherent risk measure. A potential drawback of the value-at-risk measurement is that it may not recognize an undue *concentration* of risks (Artzner, 1999).
Daníelson (2001) presents an example which shows the non-subadditivity of the VaR risk measure, leading to the counterintuitive conclusion that the more-risky portfolio is preferred. For details, see Annex B, which shows a slightly altered version of the example of Daníelson.

10. In conclusion, the Value-at-Risk measure of risks is far from ideal from a theoretical point of view. It has the following shortcomings:
    - VaR may not perform nicely in case of addition of (new) risks, even independent ones. This may create aggregation problems.
    - VaR does not take into account the economic consequences of the events, the probabilities of which it controls. This may neglect diversification benefits.
    - VaR assumes normally distributed returns, which does often not hold in practice. If returns are normal, it has the same qualifications as TailVaR.

11. Although these criticisms seem very severe at first sight, they appear to be less problematic in practice. VaR is still the most widely applied single risk measure and for good reasons. VaR provides a very workable compromise between practical applicability and theoretical robustness. Although some other measures are theoretically more appealing, their application may be problematic in practice. More advanced measures may be difficult to implement due to data limitations and limited IT-support. Probably even more important is the consideration that users (including management) meanwhile have become familiar with VaR-measures, which consequently have evolved into the lingua franca of risk measurement. For most users VaR offers a rather convenient means of communicating about risk. As long as users are aware of the shortcomings of VaR and compensate for them through other aspects of their risk management—like stress testing and the employment of internal limits, the shortcomings associated with using a VaR rather than a more advanced measure seem to be fairly manageable.

3. ONE RISK MEASURE FOR ALL STAKEHOLDERS?

12. In the previous sections we outlined the properties a risk measure should satisfy. In this section we will discuss the application of appropriate risk measures for the different stakeholders involved.

13. In an ideal world according to Modigliani and Miller (proposition 1 and 2), the stakeholders within the financial conglomerate have no preferences with respect to the amount of capital held in the company. In reality, the existence of taxes, transaction costs, no infinite division of assets, etc., imply that stakeholders care about the amount of capital held in the company.

3.1.Identification of stakeholders

14. With respect to the stakeholders that are involved within the financial conglomerate, we consider four groups[4]:
    - stakeholders with a primary focus on risk-return, i.e. shareholders,
    - stakeholders with delegated responsibility from shareholders, i.e. management
    - stakeholders with a primary focus on solvency, i.e. debt holders, and policyholders,
    - stakeholders with delegated responsibility from debt holders and policy holders, i.e. supervisors

The perspectives of these four groups could be characterized as follows.

Shareholders

15. Shareholders evaluate return vis-à-vis the risk involved and compare it to the risk-return trade-off level in the market. Shareholders are the providers of capital and hence they are the ones that actually run the risk, although not to a greater extent than the capital provided. From the point of view of the shareholders there is a need for transparency in the trade-off between the risk taken and the return promised. Shareholders compare the actual returns on their capital invested against those required, i.e. the hurdle rate, given the overall risk profile. The required returns can be considered as a reward on the volatility of the returns. The demand of the shareholders is often translated into a maximization of the value of the conglomerate (e.g. by finding the best trade-off between growth, margins and risks).

Management

16. Shareholders appoint management to run the business. Their primary goal is to optimize the value of the conglomerate under the constraint that they maintain a required capital level, which would include both regulatory and economic solvency criteria.
Although management is accountable to the shareholders, its mandate is generally so large that it can be considered a distinct stakeholder.

Policy and debt holders

17. Policyholders reduce their individual risks by insuring these risks at an insurer. The insurer can pool these risks and therefore the overall risk is lower than the sum of the risks faced by individuals. Policy holders require that the insurer maintains a certain level of ongoing solvency, such that the company is likely to be able to honor the obligations over the full term

[4] Of course, other stakeholders within the financial conglomerate are involved, like personnel, suppliers, etc. In the discussion paper we omit these types of stakeholders.

of the contract. Debt holders do not pool their risks, and they also require a certain level of ongoing solvency. In that sense, for debt holders similar arguments can be provided. They require a minimum amount of capital to reduce the likelihood of insolvency to a desired target level.

Supervisors

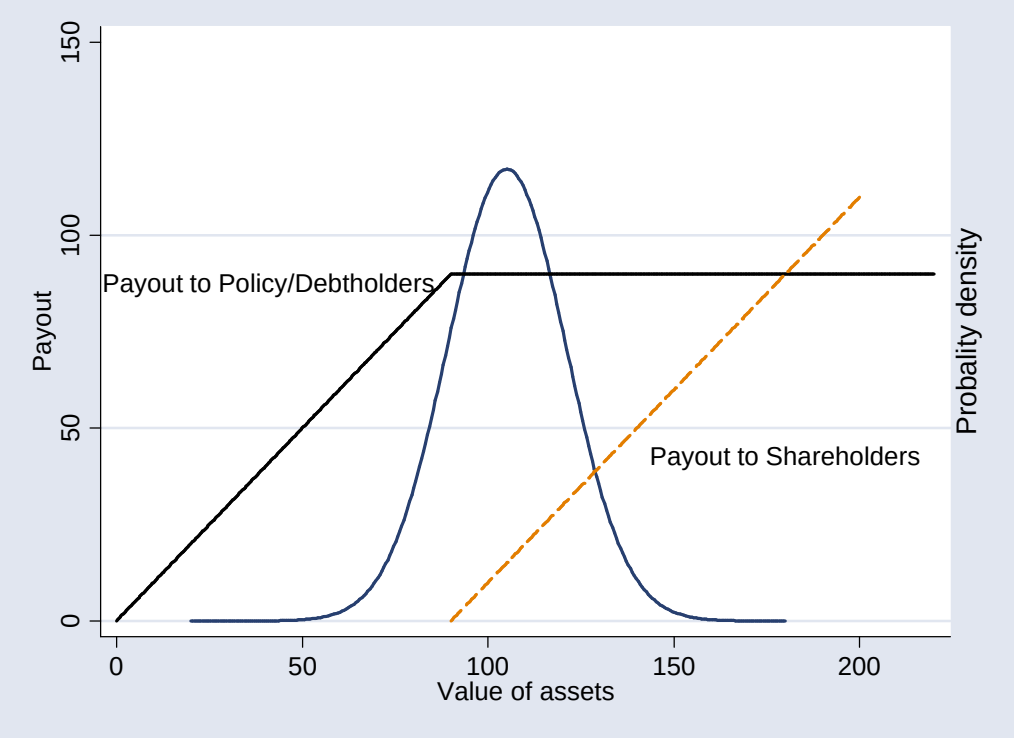
18. Supervisors have the legislative powers to protect the policyholders and debt holders against defaults. Although they are not direct stakeholders, their power makes it necessary to consider them as a distinct stakeholder.

3.2.Appropriate risk measures for the stakeholders involved

19. Although one could restrict the consideration to shareholders and policyholders/debt holders as the two primary stakeholders in a company, we explicitly mention supervisors on one side and management on the other, because they have (legislative) powers to affect the conglomerate's policy. Apart from the question whose risks are considered, a principal-agent problem is identified with respect to the stakeholders. The objectives of management may deviate from the interest of shareholders. The supervisor faces a similar principal-agent problem. Thus, the principal is the shareholder, debt holder, or policyholder; the agent is either the management or the supervisor.
Recognizing the distinct interests of the stakeholders and their views on the role of capital can help in building the overall risk, capital and value framework within which economic capital is incorporated.

20. To further clarify the differences between the different stakeholders concerning the risks they face, and the choice of an appropriate risk measure that follows from that, suppose that the management of an institution has chosen as its primary goal to provide an attractive risk-return proposition to the shareholders. Then it has to find a balance between the desire to provide a high expected return by having a small capital base, and the desire to limit the risk of very low returns (or even bankruptcy, driving equity prices to zero) by having a large capital base. The actual balance will depend on the risk attitude (or utility function) of the (targeted) equity investors. Whatever balance is struck, equity investors will only care about the probability distribution of asset returns (and therefore equity returns) to the extent that losses are not so large as to deplete capital. An equity investor will not care about the potential size of losses beyond this level as this has no effect on the return on equity (which will be -100% irrespectively). This is in line with the interpretation that equity investors effectively own a call option on the assets of the institution with strike price equal to the amount of debt ("limited liability"). For the owner of a call option, only the probability of values of the underlying asset in excess of the strike price is relevant as only then will the option be worth something.

21. As an illustration, consider a simple balance sheet with liabilities of 90 and assets with a value of 100, thereby implying an equity value of 10. Then, the strike price of the option of both policy or debt holder and shareholder equals 90. The point of interest is where the value of the assets of the firm would drop to 90. At that point the equity value becomes zero while policy and debt holders could still be paid back in full.



22. All stakeholders obviously face the same probability distribution function, and although the probability with which the available capital could be exhausted by extreme losses is relevant to all, beyond that they differ in their emphasis on different parts of the probability distribution. Shareholders are interested in the part of the probability distribution that lies to the right of the asset value of 90, while policy and debt holders are interested in the part to the left of an asset value of 90.

23. If we return to the risk measures discussed in section 2, which risk measure is most appropriate for which stakeholder? From an institution's viewpoint that focuses on shareholder value, expected shortfall does not seem an appropriate risk measure. Although VaR is typically used as a measure of economic capital in financial institutions, it clearly is somewhat awkward as well, not only because it is not coherent (under normality VaR is generally coherent), but also because it implies a very awkward (assumed) utility function for the equity investors.
On the other hand, for the regulator expected shortfall could be an appropriate risk measure. The regulator's objective is to protect debt and policyholders. Effectively debt and policyholders have written a put option on the assets of the firm, with strike equal to the amount of debt. This implies that they are primarily concerned about the probability that the asset value will drop below the value of debt (capital <0), as well as the probability distribution of asset values below that level which could determine the amount paid back to them in case of default.

4. EXPERIENCES OF FINANCIAL CONGLOMERATES

24. In practice, economic capital is generally defined as the level of capital that is sufficient to cover potential losses with a pre-specified probability. In the definition of losses, all sources

of risk to which a conglomerate is exposed and which can therefore lead to losses, should be accounted for. In fact, one should not only consider losses in the determining the level of (economic) capital, but also take into account the earnings capacity of the conglomerate, as net earnings form the first buffer in covering losses. It is thus more accurate to define economic capital as the level of capital that is sufficient to cover potential declines in value of the conglomerate with a pre-specified probability.

25. The probability in the definition of economic capital is typically linked to the desired rating for the conglomerate. As this desired rating may differ between the different stakeholders (e.g., shareholders and management may strive for a higher rating than the minimum rating that the regulator requires), the corresponding level of capital may differ, although it follows from the same economic capital model.

26. In light of the previous discussion, it is clear that economic capital cannot capture the risk profile of any stakeholders in full. In fact, the discussion is aimed to clarify that no single risk measure can. However, economic capital is relevant to all stakeholders. Hence, economic capital defined in this manner seems most suitable to present the common ground for the different stakeholders involved with the conglomerate.

27. Each stakeholder may obviously be interested in additional risk measures. For example, shareholders will be interested in the volatility of earnings. Regulators, on the other hand, may want to look at the expected shortfall. In fact, the Canadian supervisor requires reports using an expected shortfall risk measure, which is more or less similar to a TailVaR risk measure.

Capital requirements in Canada for life insurers

The Canadian insurance supervisor assesses the minimum capital requirements for an insurer based on a formula-driven approach, where each relevant risk is multiplied by a factor. The factors themselves are the result of a stochastic process and represent the main risk drivers. In case of the default method, the supervisor provides default values, which have been refined over the years. Should the insurer be evaluating a product type that is materially different from those presented in the tables, or where a company needs to evaluate a complex reinsurance or hedging arrangement, it will be necessary to use stochastic modelling to calculate factors for their particular product.
The factors should then be calculated by the insurer applying a Tail Conditional Expection (TCE) risk measure at a confidence level of 95%. The TCE is a variant of TailVaR. The supervisor assesses their adequacy; approved factors apply until new factors are determined or an internal model is approved by the Canadian supervisor.

28. Economic capital as defined above obviously uses a value-at-risk risk measure. As shown earlier, this risk measure may violate the desirable property of sub-additivity, and hence may not recognise diversification effects adequately. Although violation of this property can easily be shown through stylised examples, it is not thought of as a problem in practical situations. Moreover, no institution manages its risk solely on the basis of value-at-risk related risk measures. A whole host of other limitations are in place, ranging from simple bounds on notional amounts to sophisticated limit structures on the ‘Greeks’ of trading positions. This mitigates the risk that an incomplete or incorrect picture of the complete risk profile is obtained by using only one single risk measure.
However, the fact that value-at-risk risk measures have gained and maintained such a prominent position within the financial community shows that it is experienced as providing a useful summary representation of risk in many situations. That the value-at-risk measure is

relatively easy to understand and to communicate has certainly supported its acceptance and popularity, but this alone would not have allowed it to survive for so long.

5. APPROPRIATE TIME HORIZONS IN ECONOMIC CAPITAL

29. The next question to be addressed is the question as to which time horizon should be used in the modeling of economic capital. In practice, we often see that a time horizon of 1 year is used in EC models. By time horizon is meant the length of the time period during which the behaviour of risk drivers and their impact on the available capital of the enterprise is evaluated.
Theoretically, the appropriate time horizon should be set as the time required to orderly cancel out the risk profile of the portfolio in question.[5]

The following example may illustrate the given definition.

Example 1

Current banking regulation for the calculation of market risk in the trading book assumes a ten days' holding period with no management intervention during these ten days, and closing out all positions in the trading book at market prices on the tenth day. It is implicitly assumed that it could take ten days before all positions are closed out at market prices.

30. Sometimes the time horizon in ALM models is set at the maturity of the contracts. This approach is called a "run off" approach. This means that a run-off of the company is assumed in the absence of supervisory intervention. In that case, the risk that is measured relates to the situation when the money has run out before the last deposit holder or policy holder has been paid out. Assuming that interim positions of the balance sheet, i.e. before maturity, do not matter to the behaviour of management or deposit holders and policyholders, the valuation principle does not play a role. Compared to the aforementioned approach, this approach can however be regarded as highly hypothetical to most companies that operate in a competitive environment. Only institutions that have an 'eternal time horizon' due to government regulation or other monopolistic entitlements might convincingly argue that interim financial positions do not matter. Even then, however, a supervisor (if no one else, such as shareholders) will require appropriate actions be taken.

31. The appropriate time horizon is not necessarily the same for each risk driver or risk type. It can be argued that the time horizon for illiquid instruments, such as bank loans, should be longer than for liquid instruments, such as the trading book of a bank. The longer the time horizon, the larger the model uncertainty and the impact of the assumptions regarding management intervention. Therefore in order to calculate EC numbers, most practitioners hesitate to use very long time horizons. The possible difference between time horizons for different instruments leads to the question as to how these time horizons can be compared. Should different time horizons be scaled up to the lowest common multiple? In principle, we do not think so. Using different time horizons for different instruments can be a perfectly consistent way of modeling the possible behavior of the company's available capital in a fully

[5] Paraphrased from the IAA (2003) "[…] a holding period (time horizon) for risk assessment which corresponds to the longest period of time required for an orderly disposition of the portfolio in question in order to unwind the positions."

integrated stochastic EC model which covers all risk drivers and instruments. However, in most practical applications of EC models different risk types are modeled separately, so one has to aggregate over different risk types and therefore one has to use a common denominator of the possibly different time horizons.

32. In practice both companies and regulators take a more or less pragmatic stance regarding the appropriate time horizon. In many economic capital models a one year period is chosen it being the time period corresponding to the annual planning and control cycle of the company and also the time period between the subsequent annual accounts. Furthermore, as most economic capital models use Value-at-Risk as the risk measure for a one year period, these models are relatively easily understood and communicated even when the audience is relatively inexperienced in the field of risk modelling.

6. VALUATION PRINCIPLES

33. The valuation principles used in Economic Capital modelling should be chosen in such a way that an adequate picture of the economic position of the company is given. Assuming that the company can carry out any (financial) transaction at market prices, the appropriate valuation principle in economic capital modelling is fair value.
Sometimes it is argued that the valuation principle for insurance liabilities does not play any role. This is true for the run off approach, but it is not true for the other situations described. In order to define what is meant by ‘zero available capital’ with a time horizon shorter than the maturity of the insurance liabilities, one has to value the balance sheet items at this time horizon.

34. Notice however that we face an intriguing paradox here, as mentioned by Froot and Stein (1998): the core of the business (of banks and insurance companies) is difficult to value on fair value basis due to a lack of a liquid secondary market in insurance liabilities and loan portfolios. We can conclude that the basis for valuation in EC models should be mark-to-market, or as a second-best solution, mark-to-model. The consequence of ‘mark-to-model’ valuation is that parameter uncertainty and model uncertainty are introduced. But, from the viewpoint of economic capital determination, marking-to-model still is preferable above accrual value. With mark-to-model you may either overestimate or underestimate the true potential change in value, but that is still better than assuming no potential change in value. Accrual valuation may therefore lead to larger misspecifications of economic capital.

35. One has to bear in mind that the choice for accrual value instead of fair value as a valuation principle can have consequences for determining the amount of economic capital. For example, consider a bank that has provided a 5-year loan of 100 to a company, and funded itself with 95 of floating-rate debt and 5 equity capital. Now suppose that because of interest and/or credit spread changes, the fair value of the loan drops to 94. Although the bank does not get in direct financial problems as long as the company does not default and thus it keeps paying coupons on the loan (and assuming that the loan income exceeds the funding cost of debt), the value of equity capital has become negative (the floating-rate debt is still worth 95). Hence, on an accrual accounting basis, the bank would still look healthy, while on a fair-value basis, the bank would be in trouble. The choice for accrual value can thus lead to large deviations from the actual financial position. For the purpose of economic capital modelling the financial situation should therefore preferably be represented by using fair value.

7. CONCLUDING REMARKS

36. In this discussion paper we have answered three fundamental questions, thereby focusing on the use of economic capital as a common denominator within the financial conglomerate. Here, we summarize the conclusions on the main discussion topics.

1. *What risk measure should we take to assess the risk profile of a financial conglomerate? One or several risk measures?*

37. The paper has discussed the meaning of risk, risk measure and economic capital. In practice these elements are applied interchangeably. However, from a conceptual point of view, we have separated risk, risk measure and economic capital.
In order to assess the appropriateness of a particular risk measure, the desired properties of a risk measure we have to be clear. In the literature, the use of coherent risk measures is advocated. Often-applied risk measures (*e.g.* VaR) may violate the properties of a coherent risk measure. Although violation of coherence can easily be shown with stylised examples, it is not thought of as a severe problem in practical situations.
Furthermore, the purpose of the measuring of risk within a financial conglomerate may depend on the perspective of the stakeholder involved. Four distinct stakeholders were considered within a financial conglomerate: management, supervisors, shareholders, and debt/policyholders. Recognizing the distinct interests of the stakeholders and their views on the role of capital, the working group believes that the conglomerate should aim for one economic capital model that is applicable for all stakeholders. The stakeholders face the same statistical distribution function of possible outcomes with regard to the available capital, but are interested in different areas of the distribution function. Hence, economic capital can thus be viewed as representing a common ground for the different stakeholders involved with the conglomerate. However, to show the distinct interests adequately, several risk measures should be applied in assessing the risks of a financial conglomerate from the perspective of the stakeholders.

2. *Which time horizon should be used to determine economic capital?*

38. By time horizon is meant the length of the time period during which the behaviour of risk drivers and their impact on the available capital of the enterprise is evaluated. The Working Group proposes that the time horizon chosen be the time required to orderly cancel out the risk profile of the portfolio in question. The appropriate time horizon is not necessarily the same for each risk driver or risk type. Many EC-models use a time horizon of 1 year.
In most practical applications, aggregation over different risk types is performed using a common denominator for the time horizon. We abstained from these aspects here; it will be part of the third discussion paper.

3. *Does economic capital modeling depend on the valuation principle (accrual, fair value)? If so, what are the implications?*

39. The valuation principles in EC-modelling should be such that an adequate picture of the economic position of the company is given. Appropriate valuation principles are mark-to-market, or as a second-best solution, mark-to-model.

Mark-to-model is more relevant than accrual value, even with parameter and model uncertainty because accrual valuation will certainly result in a larger misspecification of economic capital. Therefore, the financial situation should preferably be represented by using fair value.

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9. ANNEX A COHERENT AND INSURANCE RISK MEASURES

40. A coherent risk measure Artzner *et al*. (1999) satisfies the following set of consistency rules:
    1. *Subadditivity*: For all random losses X and $Y, r(X+Y) \le r(X) + r(Y)$
    2. *Monotonicity*: If $X \le Y$ for each outcome, then $r(X) \ge r(Y)$
    3. *Positive homogeneity*: For all positive constant $b, r(bX) = br(X)$
    4. *Translation invariance*: For all constant a, $r(X+ar) = r(X) - a$, with *r* the return of a riskless asset

41. Insurance risk measures represent a relaxation of the coherent risk measure, where the translation invariance property is replaced by two other properties (Jarrow, 2002):
    4a. *Translation monotonicity*: For all $a > 0$, $r(X+ar) < r(X) < r(X-ar)$.
    4b. *Boundary relevance*: For all X, with $X < 0, X$ nonempty, $r(X) > 0$.

Table 2: Some examples of applied risk measures

i.	Standard deviation	$r_{st}(X) = s(X)$
ii.	Lower semi-standard deviation	$X^- = \begin{cases} -X, & X < 0 \\ 0, & X \ge 0 \end{cases}$, $r_{\text{semi}}(X) = -\mathrm{E}(X) + s\left([X - \mathrm{E}(X)]^-\right)$
iii.	Value-at-Risk (VaR) at level $a \in (0,1)$	$r_{\text{VaR}}(X) = -\inf\{x \mid P[(X) \le x \cdot r] > a\}$
iv.	Conditional VaR / Expected shortfall / TailVaR at level $a \in (0,1)$	$r_{\text{CVaR}}(X) = -\mathrm{E}\left[X/r \mid X/r \le -r_{\text{VAR}}(X)\right]$
v.	Wang Transform (Wang, 2001)	$r_{\text{WT}}(X) = \mathrm{E}^*(X) = \int_{-\infty}^{0} g(F(x))dx + \int_{0}^{\infty}[1 - g(F(x))]dx$, with $g(u) = \Phi\left[\Phi^{-1}(u) - l\right]$ and Φ the standard normal cumulative distribution function
vi.	Put option premium (Jarrow, 2002)	$r_{PO}(X) = \dfrac{\mathrm{E}(\max[-X, 0])}{r}$

42. With respect to the required properties of a coherent risk measure, the standard deviation (i) risk measure does not satisfy the monotonicity property; the lower semi-standard deviation (ii) and VaR (iii) do not satisfy the subadditivity property.
The put option premium (vi) risk measure is not coherent, but satisfies the properties of the insurance measure. Theoretically, it has been proven that the insurance risk measure refers to the same so-called acceptance set as a coherent risk measure, which means that decisions are taken with the same amount of information.

43. The Wang Transform (Wang, 2001) is developed to pay attention to low-frequency and high severity losses. For most lines of business in banking the shape of the distribution function should not have this form, although the exposures in e.g. telecom business may represent a low-frequency/high severity loss shape; in insurance it is a relevant item. However, in its

current stage, the Wang Transform is more a theoretical risk measure with nice properties, but the intuitive meaning of the risk measure has not been set very clear.

44. Thus, only the TailVaR (iv) and the Wang Transform (v) risk measures are coherent; the put option premium risk measure satisfies the properties of the insurance risk measure. Note that, if a risk measure is coherent, it also belongs to the insurance risk measure; the other way around does not hold.

10. ANNEX B: EXAMPLE OF VIOLATION OF SUBADDITIVITY PROPERTY OF THE VAR-MEASURE

Suppose, we have 100 different bonds $X(i), i = 1,\ldots,100$, iid distributed, with probability of default equal to 10% and a reward of $20 of the period end coupon when it does not default. Suppose each bond has face value $100. The P&L of each bond is then:

$$X(i) = \begin{cases} -100 & \text{with probability } 0.10 \\ +20 & \text{with probability } 0.90 \end{cases} \quad i = 1,\ldots,100$$

Consider two distinct portfolios $P_j, j = 1,2$:

$$P_1 : X(1) + \ldots + X(100)$$

$$P_2 : 100 \times X(1)$$

Consider the following set of 10 scenarios and a VaR-measure at a confidence level of 95%. Then

Scenario	P_1	P_2
1	800	2000
2	800	2000
3	800	2000
4	800	2000
5	800	2000
6	800	2000
7	800	2000
8	800	2000
9	800	2000
10	800	-10000
VaR@95%	800	-10000

It is easy to check that at a confidence interval α, the $VaR(P_1) > VaR(P_2) = 100 \times VaR(X(1))$, hence subadditivity is not satisfied. On the basis of the Value-at-Risk measure, the non-diversified portfolio is preferred, which intuitively is more risky; *diversification* of the portfolio has *increased* the measure of risk, while the ‘‘piling up’’ of risky bonds issued by the same company has remained undetected.