

# Response of two-defect magnetic photonic crystals to oblique incidence of light: Effect of defect layer variation

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The transmission characteristics of a two-defect magnetic photonic crystal (MPC) with respect to oblique incident light are investigated, both for circularly polarized as well as linearly polarized light. It is shown that the transmittivity and Faraday rotation angle are very sensitive to a change of light propagation direction inside the MPC. Possible applications of MPCs as Faraday rotators are discussed. © 2006 American Institute of Physics.

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Magnetic photonic crystals (MPCs) are recently developing into an area in modern photonics, named magnetophotonics.<sup>1,2</sup> Inclusion of magnetic materials in photonic structures leads to peculiarities in the propagation of electromagnetic waves (EMWs), such as nonreciprocity and unidirectionality,<sup>3</sup> that can be tuned by application of an external magnetic field. Another prominent property of MPCs is an enhancement of magneto-optical (MO) effects, such as the Faraday rotation angle  $\theta_F$ , in MPCs with defects.<sup>1,2</sup> Such MPCs can be used as Faraday rotators and isolators or in devices for MO imaging. Steel *et al.*<sup>4</sup> showed that considerable improvement of the transmission characteristics can be achieved in two-defect MPCs. However, they treated the case of normal incidence of light. In our paper<sup>5</sup> it was shown that the positions and widths of photonic band gaps (PBGs) in nondefect MPCs depend on the incidence angle. It can therefore be expected that oblique incident EMWs will also lead to changes in the MO effects in MPCs with defects. Quite recently Vasiliev *et al.* have studied the response of one- and two-defect one-dimensional MPCs for oblique incidence of light.<sup>6</sup> Particularly, it was shown that the widths of the photonic band gaps as well as the position of defect modes (DMs) are changing with the angle of incidence. In this Communication, we show that the transmittivity, Faraday rotation angle, and ellipticity of a two-defect MPC critically depend on deviations of defect layer thickness, especially for oblique light incidence.

Consider a one-dimensional MPC with two magnetic defects constructed as  $(NM)^a D_M (MN)^b M D_M (MN)^a$ , as presented in Fig. 1. Here  $M$  denotes the magnetic layer of thickness  $d_1$  and  $N$  is the nonmagnetic layer of thickness  $d_2$ ;  $a$  and

$b$  are the numbers of  $(NM)$  bilayers. The period of the MPC is  $D=d_1+d_2$ , and  $D_M$  is the defect offset of the thickness  $\Delta$ . The plane EMW of frequency  $\omega$  propagates in the  $yz$  plane with the wave vector  $\mathbf{k}=\{0, k_y, k_z\}$ .

It is well known<sup>7</sup> that such periodic layered structures have PBGs at frequencies approximately defined by the simple Bragg condition<sup>7</sup> for the oblique incidence:

$$(\omega n/c)^2 = (l\pi/D)^2 + k_y^2, \tag{1}$$

where  $l=1, 2, \dots$  denotes the band number and  $n$  is the refractive index. The layer thicknesses are chosen to be  $d_1 = \lambda^{(1)}/(4n_1)$  and  $d_2 = \lambda^{(1)}/(4n_2)$ , where  $\lambda^{(1)}$  is the wavelength of the center of the first PBG. The offset thickness  $\Delta=0$  corresponds to the  $\lambda^{(1)}/(2n_1)$  defect magnetic layer or a defect with a phase shift  $\delta = \pi/2$ , and the transmission peak (or the defect mode) is located at the center of the first PBG, while  $\Delta \neq 0$  corresponds to a phase shift  $\delta \neq \pi/2$ , so the defect mode is shifted from the center of the PBG. Further we shall refer to these cases as the centered and off-centered defect modes.

The case of normal light incidence on one- and two-defect MPCs with the magnetization oriented along the  $z$  axis (polar MO configuration) was investigated in Ref. 4. It is

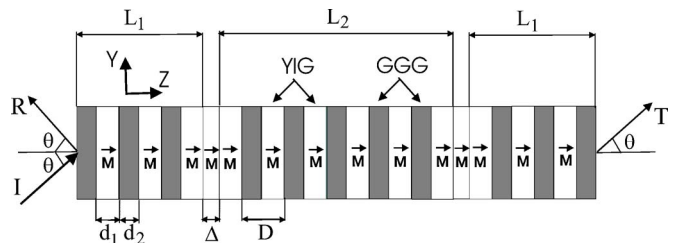


FIG. 1. Schematic figure of the MPC with two magnetic defects. The values  $L_1 = aD$  and  $L_2 = bD + d_1$  are the thicknesses of the MPC segments, separated by the magnetic defect offset  $\Delta$ .

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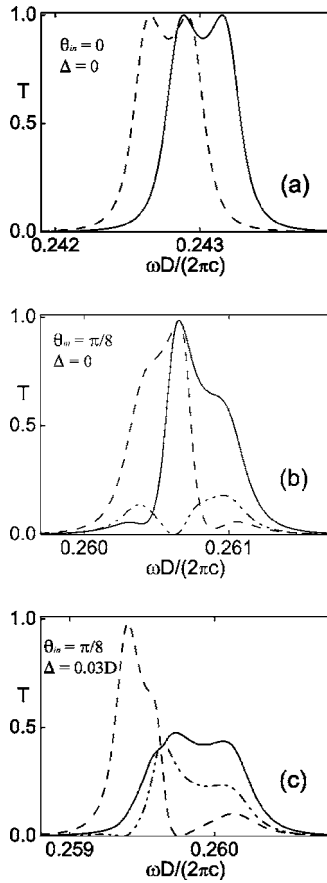


FIG. 2. The transmission coefficients for the MPC with two defects. The solid, dashed, and dash-dot-dotted lines correspond to  $T^+$  (RH mode),  $T^-$  (LH mode), and  $\tilde{T}^+$ , respectively. (a) Normal incidence,  $\theta_m=0$ ,  $\Delta=0$ ; (b) oblique incidence,  $\theta_m \approx \pi/8$ ,  $\Delta=0$ ; and (c) oblique incidence  $\theta_m \approx \pi/8$ ,  $\Delta = 0.03D$ .

well known that in that case, the noninteracting right-handed (RH) and left-handed (LH) circularly polarized waves are the eigenmodes of the MPC. Thus, each defect mode splits into two modes: RH and LH ones. Steel *et al.*<sup>4</sup> explained the response of the MPC to the incident linearly polarized wave, decomposing it into RH and LH modes. The transmission coefficient  $T$ , the angle  $\theta_F$ , and the ellipticity  $e$  of the transmitted linearly polarized light can be presented by means of Fresnel transmission amplitudes  $t^+$  and  $t^-$  of the RH and LH modes, respectively, as<sup>8</sup>

$$T = \frac{1}{2}(|t^+|^2 + |t^-|^2), \quad \theta_F = -\frac{1}{2}(\phi_+ - \phi_-), \quad (2)$$

$$e = \tan(\psi) = \frac{|t^+| - |t^-|}{|t^+| + |t^-|}, \quad (3)$$

where  $\phi_{\pm} = \arg(t^{\pm})$  is the ellipticity angle. For Faraday rotators one requires a large transmittance and a large  $\theta_F$ . As one can deduce from Eqs. (2), one needs to choose the MPC parameters ( $d_{1,2}, D, L_1 = aD, L_2 = bD + d_1$ , and  $\Delta$ ) to keep both  $t^+$  and  $t^-$  as well as the phase difference  $(\phi_+ - \phi_-)$  large enough at this frequency range. This cannot be achieved in a one-defect MPC, but it is possible in a two-defect one.<sup>4</sup> If  $2L_1 > L_2$ , each of the RH and LH defect modes split into two subpeaks and their phases get an increase of about  $\pi$  in the frequency interval of the peak, as shown in Fig. 2(a). So, if

the parameters are chosen to produce a coincidence of the high-frequency subpeak of the LH mode with the low-frequency subpeak of the RH mode, then  $t^+ \approx t^- \approx 1$  and  $(\phi_+ - \phi_-) \sim \pi$ , and the transmission spectrum of the MPC has both a large  $T$  and a large  $\theta_F$ . Moreover, the condition  $t^+ \approx t^-$  provides a low ellipticity [see Eq. (3)].

For numerical calculations of  $T$  and  $\theta_F$  in a two-defect MPC we studied the structure, composed of the materials as in Ref. 9. The magnetic layers  $M$  are assumed to be Ce-doped yttrium iron garnet (YIG), Ce:YIG, with  $n_{\text{YIG}} = 2.21$  at  $\lambda = 1.55 \mu\text{m}$  and the off-diagonal component of the permittivity tensor  $\epsilon' = 0.009$  at  $\lambda = 1.55 \mu\text{m}$ . The nonmagnetic layers  $N$  are considered to be gadolinium gallium garnet (GGG) with  $n_{\text{GGG}} = 1.926$  at  $\lambda = 1.55 \mu\text{m}$ .

To satisfy the conditions  $d_1 = \lambda^{(1)}/(4n_1)$  and  $d_2 = \lambda^{(1)}/(4n_2)$ , it is necessary to choose the MPC parameters according to the relation  $d_1 n_1 = d_2 n_2$ . So, for the YIG and GGG, we choose  $d_1 = 0.466D$  and  $d_2 = 0.534D$ . Optimizing the parameters  $a$  and  $b$ , we found that for the MPC under consideration, a coincidence of the RH and LH subpeaks, leading to a high transmission and an angle  $\theta_F$  of about  $45^\circ$  (as required for an optical isolator<sup>4</sup>), takes place for  $a = 16$  and  $b = 29$ . In order to reduce the Fresnel reflection on the MPC surfaces, we surrounded the MPC by a medium with a refractive index, averaged over the period of  $\bar{n} = 2.06$ .

For the calculation of the transmission characteristics of the MPC we used the  $4 \times 4$  transfer matrix method.<sup>10,11</sup> For our MPC the transfer matrix  $\hat{T}$  has the following form:

$$\hat{T} = \hat{A}_3^{-1}(\hat{P})^a \hat{A}_1 \hat{E}_1(d_1 + \Delta) \hat{A}_1^{-1}(\hat{P})^b \hat{A}_1 \hat{E}_1(\Delta) \hat{A}_1^{-1}(\hat{P})^a \hat{A}_3, \quad (4)$$

where

$$\hat{P} = \hat{A}_2 \hat{E}_2(d_2) \hat{A}_2^{-1} \hat{A}_1 \hat{E}_1(d_1) \hat{A}_1^{-1}. \quad (5)$$

In Eqs. (4) and (5) the indices 1, 2, and 3 are related to the YIG, GGG, and surrounding medium, respectively;  $E_i(z)$  is the propagation matrix of the eigenmodes inside the layer,  $\hat{A}_i$  relates the total  $E_{x,y}$  and  $H_{x,y}$  fields at the boundary of the layer to the amplitudes of the eigenmodes. The elements of the matrices  $\hat{A}_i$ , as well as the wave numbers  $k_z$  in YIG and GGG, can be found from the Maxwell equations.<sup>10,11</sup>

In Fig. 2(a) the transmission coefficients  $T^{\pm} = |t^{\pm}|^2$  for the MPC with  $\Delta = 0$  in the case of normal incidence ( $k_y = 0$ ) are plotted against the normalized frequency  $\omega D / (2\pi c)$ . The corresponding figures for  $\Delta \neq 0$  are quite similar, and differ from Fig. 2(a) with a slight shift of the peaks. For oblique incident light, the PBGs and defect modes shift to higher frequencies, following Eq. (1). In this case the RH and LH modes are no longer the eigenmodes, and the ‘‘mixed’’ transmission coefficients  $\tilde{T}^{\pm} = |E_{\text{out}}^{\mp}|^2 / |E_{\text{in}}^{\pm}|^2$  appear. In Figs. 2, 2(b), and 2(c) we present the transmission coefficients for  $k_y D / (2\pi c) = 0.195$  with  $\Delta = 0$  and  $\Delta = 0.03D$ , respectively. As  $\tilde{T}^+ \approx \tilde{T}^-$ , only the coefficient  $\tilde{T}^+$  is shown. The RH and LH modes for  $\Delta \neq 0$  are much more distorted than for  $\Delta = 0$ , and  $\tilde{T}^+$  exceeds the corresponding coefficient for the off-centered defect mode. For the centered defect mode there is a narrow frequency region, where both  $T^{\pm} \approx 1$  and  $\tilde{T}^{\pm} \approx 0$  [see Fig.

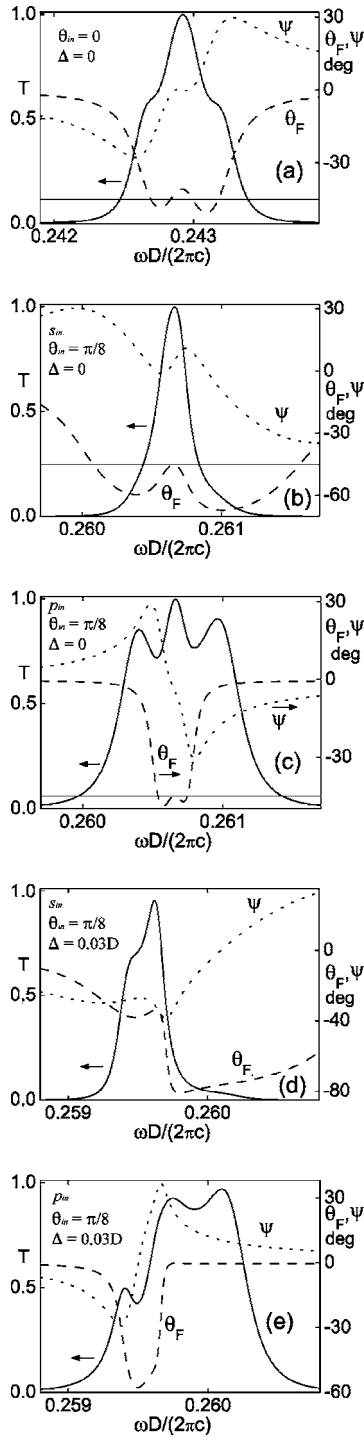


FIG. 3. The response to the incident linearly polarized light: The transmission coefficient  $T$  (solid line),  $\theta_F$  (dashed line), and ellipticity (dotted line) vs the normalized frequency. The thin horizontal line corresponds to  $\theta_F = -\pi/4$ . (a) Normal incidence,  $\theta_{in} = 0$ ,  $\Delta = 0$ ; (b) oblique incidence with  $\theta_{in} \approx \pi/8$ ,  $\Delta = 0$ ,  $s$ -polarized input light; (c) the same as for (b), but with  $p$ -polarized input; (d) oblique incidence with  $\theta_{in} \approx \pi/8$ ,  $\Delta = 0.03D$ ,  $s$ -polarized input light; and (e) the same as for (d), but with  $p$ -polarized input.

2(b)]. This feature is not typical for the off-centered modes, where only  $T^-$  has a maximum near unity, while  $T^+$  does not exceed 0.5, as shown in Fig. 2(c).

Figure 3(a) shows the response of the MPC to an incident linearly polarized EMW for normal incidence and  $\Delta$

$= 0$ . The response to an oblique incident linearly polarized EMW in the case of a centered defect mode is shown in Figs. 3(b) and 3(c) for  $s$ - and  $p$ -polarized inputs, respectively, whereas the ellipticity grows to  $|e| = 0.09$  at the transmission peak.

For a  $p$ -polarized input light the transmission peak is wider than for an  $s$ -polarized one. This fact corresponds to an increase of the Fresnel transmission coefficient through MPC layers with incident angle (up to the Brewster angle) for  $p$  polarization. On the contrary, the frequency region with high  $\theta_F$  is wider for an  $s$ -polarized input, whereas  $\theta_F$  also reaches larger absolute values in this case.

One should also realize that for oblique incidence,  $\theta_F$  is several degrees larger than for normal incidence, and it is exactly equal to  $-45^\circ$  for  $k_y D / (2\pi) = 0.195$ . Thus,  $\theta_F$  can be fine-tuned with the parameter  $k_y$ , while this is impossible with the discrete parameters  $a$  and  $b$ .

The response of the MPCs with the off-centered defect modes for a linearly polarized EMW is shown in Figs. 3(d) and 3(e) for  $s$ - and  $p$ -polarized inputs, respectively. As one can see, the transmission peaks are considerably distorted in comparison with those for the MPC with the centered defect modes, although the offset  $|\Delta|$  is quite small. In particular, the maximum in transmission does not correspond anymore to the maximum in the Faraday rotation  $\theta_F$ . This fact means that MPCs with an off-centered defect mode cannot be used as Faraday rotators in the regime of oblique incident light. For example, for the MPC considered above, at the incidence angle  $\theta_{in} \approx \pi/8$  a defect thickness deviation of 3% leads to a complete destruction of the DM and, consequently, to the impossibility of performance as a Faraday rotator.

In conclusion, the transmission characteristics of a MPC with two magnetic defects with respect to the oblique incidence of light are investigated. We found that a MPC with centered defect modes retains good transmission characteristics even for quite large incidence angles. Thus, oblique incidence can be used for the fine-tuning of  $\theta_F$  in Faraday rotators. The above analysis is also valid for an incident light with a wavelength different from  $\lambda^{(1)}$ , if the incidence angle is properly selected, according to Eq. (1). On the other hand, as the off-centered defect modes are distorted considerably in the case of oblique incident light, the thicknesses of the defect layers should be selected with great accuracy.

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