Search for Neutral MSSM Higgs Bosons at LEP

ALEPH, DELPHI, L3 and OPAL Collaborations
The LEP Working Group for Higgs Boson Searches

Abstract

The four LEP collaborations, ALEPH, DELPHI, L3 and OPAL, have searched for the neutral Higgs bosons which are predicted by the Minimal Supersymmetric Standard Model (MSSM). The data of the four collaborations are statistically combined and examined for their consistency with the background hypothesis and with a possible Higgs boson signal. The combined LEP data show no significant excess of events which would indicate the production of Higgs bosons. The search results are used to set upper bounds on the cross-sections of various Higgs-like event topologies. The results are interpreted within the MSSM in a number of “benchmark” models, including CP-conserving and CP-violating scenarios. These interpretations lead in all cases to large exclusions in the MSSM parameter space. Absolute limits are set on the parameter tan β and, in some scenarios, on the masses of neutral Higgs bosons.

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1See Appendix C for the list of authors
1 Introduction

One of the outstanding questions in particle physics is that of electroweak symmetry breaking and the origin of mass. The leading candidate for an answer is the Higgs mechanism [1] whereby fundamental scalar Higgs fields acquire nonzero vacuum expectation values and spontaneously break the electroweak symmetry. Gauge bosons and fermions obtain their masses by interacting with the resulting vacuum Higgs fields. Associated with this description is the existence of massive scalar particles, the Higgs bosons.

The Standard Model [2] requires one complex Higgs field doublet and predicts a single neutral Higgs boson of unknown mass. After extensive searches at LEP, a lower bound of 114.4 GeV/c² has been established for the mass of the Standard Model Higgs boson, at the 95% confidence level (CL) [3].

Supersymmetric (SUSY) [4] extensions of the Standard Model are of interest since they provide a consistent framework for the unification of the gauge interactions at a high energy scale and for the stability of the electroweak scale. Moreover, their predictions are compatible with existing high-precision data [5]. The Minimal Supersymmetric Standard Model (MSSM) (reviewed, e.g., in [6]) is the SUSY extension with minimal new particle content. It requires two Higgs field doublets and predicts the existence of three neutral and two charged Higgs bosons. The lightest of the neutral Higgs bosons is predicted to have a mass less than about 140 GeV/c² including radiative corrections [7]. This prediction provided a strong motivation for the searches at LEP energies.

Most of the experimental investigations carried out in the past at LEP and elsewhere were interpreted in MSSM scenarios where CP conservation in the Higgs sector was assumed. In such scenarios the neutral Higgs bosons are CP eigenstates. However, CP violation in the Higgs sector cannot be a priori excluded [8]. Scenarios with CP violation are theoretically appealing since they provide one of the ingredients needed to explain the observed cosmic matter-antimatter asymmetry. The observed size of CP violation in B and K meson systems is not sufficient to drive this asymmetry. In the MSSM, however, substantial CP violation can be induced by complex phases in the soft SUSY-breaking sector, through radiative corrections, especially from third-generation scalar quarks [9]. In such scenarios the three neutral Higgs mass eigenstates are mixtures of CP-even and CP-odd fields, with production and decay properties different from those in the CP-conserving scenarios. Hence, the experimental exclusions published so far for the CP-conserving MSSM scenarios may be weakened by CP-violating effects. There is currently one publication on searches interpreted in CP-violating scenarios [10].

In this paper we describe the results of a statistical combination based on the searches of the four LEP collaborations [10–13], which was carried out by the LEP Working Group for Higgs Boson Searches. These searches include all LEP2 data up to the highest energy, 209 GeV; in the case of Refs. [10,12] they also include the LEP1 data collected at energies in the vicinity of 91 GeV (the Z boson resonance). The combined LEP data show no significant signal for Higgs boson production. The search results are used to set upper bounds on topological cross-sections for a number of Higgs-like final states. Furthermore, they are interpreted in a set of
representative MSSM “benchmark” models, with and without CP-violating effects in the Higgs sector.

2 The MSSM framework

The LEP searches and their statistical combination presented in this paper are interpreted in a constrained MSSM model. At tree level, two parameters are sufficient (besides the known parameters of the Standard Model fermion and gauge sectors) to fully describe the Higgs sector. A convenient choice is one Higgs boson mass ($m_A$ is chosen in CP-conserving scenarios and $m_{H^\pm}$ in CP-violating scenarios), and the ratio $\tan \beta = v_2/v_1$ of the vacuum expectation values of the two Higgs fields ($v_2$ and $v_1$ refer to the fields which couple to the up- and down-type fermions). Additional parameters, $M_{\text{Susy}}$, $M_2$, $\mu$, $A$ and $m_{\tilde{g}}$, enter at the level of radiative corrections. $M_{\text{Susy}}$ is a soft SUSY-breaking mass parameter and represents a common mass for all scalar fermions (sfermions) at the electroweak scale. Similarly, $M_2$ represents a common SU(2) gaugino mass at the electroweak scale. The “Higgs mass parameter” $\mu$ is the strength of the supersymmetric Higgs mixing; $A = A_t = A_b$ is a common trilinear Higgs-squark coupling at the electroweak scale and $m_{\tilde{g}}$ the gluino mass. Three of these parameters define the stop and sbottom mixing parameters $X_t = A - \mu \cot \beta$ and $X_b = A - \mu \tan \beta$. In CP-violating scenarios, the complex phases related to $A$ and $m_{\tilde{g}}$, $\text{arg}(A)$ and $\text{arg}(m_{\tilde{g}})$, are supplementary parameters.

In addition to all these MSSM parameters, the top quark mass also has a strong impact on the predictions through radiative corrections. In this paper, four fixed values are used in the calculations: $m_t = 169.3, 174.3, 179.3$ and $183.0$ GeV/$c^2$. For the purposes of illustration, $m_t = 174.3$ GeV/$c^2$ is used in producing the figures (unless explicitly specified otherwise), which is a previous world-average value [14] and which is within the current experimental range of $172.7 \pm 2.9$ GeV/$c^2$ [15]. The influence of the top quark mass on the exclusion limits is discussed in Sections 5 and 6 along with the other results.

The combined LEP data are compared to the predictions of a number of MSSM “benchmark” models [16]. Within each of these models, the two tree-level parameters, $\tan \beta$ and $m_A$ (in the CP-conserving scenarios) or $m_{H^\pm}$ (in the CP-violating scenarios) are scanned while the other parameters are set to fixed values. Each scan point thus represents a specific MSSM model. The ranges of the scanned parameters and the values of the fixed parameters are listed in Table 1 for the main scenarios studied. The first five models represent the main benchmarks for CP-conserving scenarios while the last model, labelled $\text{CPX}$, is a benchmark model for CP-violating scenarios. Some variants of these benchmark scenarios, which are also investigated, are presented in the text below.

The scan range of $\tan \beta$ is limited by the following considerations. For values of $\tan \beta$ below the indicated lower bounds, the calculations of the observables in the Higgs sector (masses, cross-sections and decay branching ratios) become uncertain; for values above the upper bounds, the decay width of the Higgs bosons may become larger than the experimental mass resolution (typically a few GeV/$c^2$) and the modelling of the kinematic distributions of the signal becomes
Table 1: Parameters of the main benchmark scenarios investigated in this paper. The values of $\tan \beta$ and the mass parameters $m_A$ (in the CP-conserving scenarios) or $m_{H^\pm}$ (in the CP-violating scenarios) are scanned within the indicated ranges. For the definitions of $A$ and $X_t$, the Feynman-diagrammatic on-shell renormalisation scheme is used in the CP-conserving scenarios and the $\overline{\text{MS}}$ renormalisation scheme in the CP-violating scenarios.

Inaccurate. The scan range of $m_A$ is limited in most cases to less than 1000 GeV/c$^2$; at higher values the Higgs phenomenology is insensitive to the choice of $m_A$.

For a given scan point, the observables in the Higgs sector are calculated using two theoretical approaches, both including one- and two-loop corrections. The FeynHiggs2.0 code [17] is based on a Feynman-diagrammatic approach and uses the on-shell renormalization scheme. The SUBHPOLE calculation and its CP-violating variant CPH [18] are based on a renormalization-group improved effective potential calculation [19] and use the $\overline{\text{MS}}$ scheme.

In the CP-conserving case, the FeynHiggs calculation is retained for the presentation of the results since it yields slightly more conservative results (the theoretically allowed parameter space is wider) than SUBHPOLE does. Also, FeynHiggs is preferred on theoretical grounds since its radiative corrections are more detailed than those of SUBHPOLE.

In the CP-violating case, neither of the two calculations is preferred on theoretical grounds. While FeynHiggs contains more advanced one-loop corrections, the CPH code has a more precise phase dependence at the two-loop level. We opted therefore for a solution where, in each scan point, the CPH and FeynHiggs calculations are compared and the calculation yielding the weaker

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<table>
<thead>
<tr>
<th>Benchmark parameters</th>
<th>(1) $m_h$-max</th>
<th>(2) no-mixing</th>
<th>(3) large-$\mu$</th>
<th>(4) gluophobic</th>
<th>(5) small-$\alpha_{eff}$</th>
<th>(6) CPX</th>
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<tr>
<td>$\tan \beta$</td>
<td>0.4–40</td>
<td>0.4–40</td>
<td>0.7–50</td>
<td>0.4–40</td>
<td>0.4–40</td>
<td>0.6–40</td>
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<tr>
<td>$m_A$ (GeV/c$^2$)</td>
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<td>0.1–1000</td>
<td>0.1–400</td>
<td>0.1–1000</td>
<td>0.1–1000</td>
<td>–</td>
</tr>
<tr>
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<td>–</td>
</tr>
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<td>Fixed parameters</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{\text{SUSY}}$ (GeV)</td>
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<td>350</td>
<td>800</td>
<td>500</td>
</tr>
<tr>
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<td>200</td>
<td>400</td>
<td>300</td>
<td>500</td>
<td>200</td>
</tr>
<tr>
<td>$\mu$ (GeV)</td>
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<td>–200</td>
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<td>300</td>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>$m_{\tilde{g}}$ (GeV/c$^2$)</td>
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<td>200</td>
<td>500</td>
<td>500</td>
<td>1000</td>
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<td>$X_t$ (GeV)</td>
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<td>–750</td>
<td>–1100</td>
<td>$A - \mu \cot \beta$</td>
</tr>
<tr>
<td>$A$ (GeV)</td>
<td>$X_t + \mu \cot \beta$</td>
<td>$X_t + \mu \cot \beta$</td>
<td>$X_t + \mu \cot \beta$</td>
<td>$X_t + \mu \cot \beta$</td>
<td>$X_t + \mu \cot \beta$</td>
<td>1000</td>
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<tr>
<td>$\arg(A) = \arg(m_{\tilde{g}})$</td>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>90°</td>
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exclusion (more conservative) is retained. However, we also discuss in Section 6 the effect of using separately either one or the other of the two calculations. Rather large discrepancies between the two codes are found in calculating the partial width for the Higgs boson cascade decay $\Gamma(\mathcal{H}_2 \to \mathcal{H}_1 \mathcal{H}_1)$ ($\mathcal{H}_1$ and $\mathcal{H}_2$ are the lightest and the second-lightest neutral MSSM Higgs bosons). Aiming at conservative exclusion limits, therefore, the CPH formula for this decay was also used within the FeynHiggs code.

All codes are implemented in a modified version of the \textsc{HZHA} program package [21], which takes into account initial-state radiation and the interference between identical final states from Higgsstrahlung and boson fusion processes.

### 2.1 CP-conserving scenarios

Assuming CP conservation, the spectrum of MSSM Higgs bosons consists of two CP-even neutral scalars, $h$ and $H$ ($h$ is defined to be the lighter of the two), one CP-odd neutral scalar, $A$, and one pair of charged Higgs bosons, $H^\pm$. The following ordering of masses is valid at tree level: $m_h < (M_Z, m_A) < m_H$ and $m_{W^\pm} < m_{H^\pm}$. This ordering may be substantially modified by radiative corrections [7] where the largest contribution arises from the incomplete cancellation between top and scalar top (stop) loops. The corrections affect mainly the neutral Higgs boson masses and decay branching ratios.

In $e^+e^-$ collisions at LEP energies, the main production processes of $h$, $H$ and $A$ are the Higgsstrahlung processes $e^+e^- \to hZ$ and $HZ$ and the pair production processes $e^+e^- \to hA$ and $HA$ (in most of the MSSM parameter space only the $hZ$ and $hA$ processes are possible by kinematics). The fusion processes $e^+e^- \to (WW \to h)\nu_e\bar{\nu}_e$ and $e^+e^- \to (ZZ \to h)e^+e^-$ play a marginal role at LEP energies but they are also taken into account in the derivation of the results.

The cross-sections for Higgsstrahlung and pair production can be expressed in terms of the Standard Model Higgs boson production cross-section $\sigma_{hZ}^{SM}$. The following expressions hold for the processes involving the lightest scalar boson $h$:

$$\sigma_{hZ} = \sin^2(\beta - \alpha) \sigma_{hZ}^{SM}$$

$$\sigma_{hA} = \cos^2(\beta - \alpha) \lambda \sigma_{hZ}^{SM}.$$  \hspace{1cm} (1, 2)

Here $\alpha$ is the mixing angle which diagonalises the CP-even Higgs mass matrix (at lowest order it can be expressed in terms of $m_A$, $M_Z$ and $\tan\beta$) and $\lambda$ is a kinematic factor:

$$\lambda = \lambda_A^{3/2}/[(12M_Z^2/s + \lambda_{Zh})]$$

$$\lambda_{ij} = [1 - (m_i + m_j)^2/s][1 - (m_i - m_j)^2/s],$$

with $s$ is the square of the centre-of-mass energy. The cross-sections for the processes involving the heavy scalar boson $H$ are obtained by interchanging the MSSM suppression factors.
sin^2(\beta - \alpha) and \cos^2(\beta - \alpha) in Eqs. 1 and 2 and replacing the index h by H in Eqs. 1, 2 and 3. The Higgsstrahlung and pair production cross-sections are complementary, as seen from Eqs. 1 and 2. At LEP energies, the process e^+e^-\rightarrow hZ is typically more abundant at small tan \beta and e^+e^-\rightarrow hA at large tan \beta, but the latter process can be suppressed also by the kinematic factor \lambda.

The following decay features are relevant to the neutral MSSM Higgs bosons. The h boson decays mainly to fermion pairs, with only a small fraction of WW^* and ZZ^* decays, since its mass is below the threshold of the on-shell processes h\rightarrow WW and h\rightarrow ZZ. However, for particular choices of the parameters, the fermionic final states may be strongly suppressed. The A boson also decays predominantly to fermion pairs, independently of its mass, since its coupling to vector bosons is zero at leading order. For tan \beta > 1, decays of h and A to bb and \tau^+\tau^- pairs are preferred while the decays to cc and gluon pairs are suppressed. Decays to cc may become important for tan \beta < 1. The decay h\rightarrow AA may be dominant if allowed by kinematics [22]. Higgs boson decays into SUSY particles, such as sfermions, charginos or invisible neutralinos, are suppressed due to the high values of the SUSY-breaking scale MSUSY which have been chosen.

In the following we describe the CP-conserving benchmark scenarios [16] which are examined in this paper. The corresponding parameters are listed in Table 1.

2.1.1 The \(m_h\)-max scenario

In the \(m_h\)-max scenario the stop mixing parameter is set to a large value, \(X_t = 2M_{SUSY}\). This model is designed to maximise the theoretical upper bound on \(m_h\) for a given tan \beta and fixed \(m_t\) and \(M_{SUSY}\) (uncertainties due to unknown higher-order corrections are ignored). This model thus provides the largest parameter space in the \(m_h\) direction and conservative exclusion limits for tan \beta.

We also examine a variant of this scenario where the sign of \(\mu\) is changed to positive, since this is favoured by presently available results on \((g - 2)_\mu\) [23, 24]. This variant is labelled \(m_h\)-max (a) below. Furthermore, we examine the case where, besides changing the sign of \(\mu\) to positive, the sign of the mixing parameter \(X_t\) is changed to negative. This choice of parameters gives better agreement with measurements of the branching ratios and of the CP- and isospin-asymmetries for the process b\rightarrow s\gamma [16, 25]. This variant is labelled \(m_h\)-max (b) below.

2.1.2 The no-mixing scenario

In the no-mixing scenario the stop mixing parameter \(X_t\) is set to zero, giving rise to a relatively restricted MSSM parameter space. We also examine a variant of this scenario where the sign of \(\mu\) is changed to positive, for a better agreement with recent measurements of \((g - 2)_\mu\) [23, 24], and \(M_{SUSY}\) is raised to 2 TeV in order to enlarge the parameter space of the standard no-mixing
scenario [16]. This variant is labelled no-mixing (a) below. In this case, $\tan \beta$ is scanned only from 0.7 upward due to numerical instabilities at lower values in the diagonalisation of the mass matrix.

### 2.1.3 Special scenarios

Some scenarios were designed to illustrate choices of the MSSM parameters for which the detection of Higgs bosons at LEP, at the Tevatron and at the LHC is expected to be difficult a priori due to the suppression of some main discovery channels [16].

- **The large-$\mu$ scenario** is constructed in such a way that, while the $h$ boson is accessible by kinematics at LEP for all scan points, the decay $h \to b\bar{b}$, on which most of the searches at LEP and at the Tevatron are based, is typically strongly suppressed. For many of the scan points the decay $h \to \tau^+\tau^-$ is also suppressed, such that the dominant decay modes are $h \to c\bar{c}$, $gg$ and $WW^*$. The detection of Higgs bosons thus relies mainly on flavour- and decay-mode-independent searches. Moreover, for some of the scan points, the $e^+e^- \to hZ$ process is suppressed altogether by a small value of $\sin^2(\beta - \alpha)$. In such cases, however, the heavy neutral scalar $H$ is within reach ($m_H < 111$ GeV/$c^2$) and the cross-section for $e^+e^- \to HZ$, proportional to $\cos^2(\beta - \alpha)$, is large; the search may thus proceed via the heavy Higgs boson $H$.

- **The gluophobic scenario** is constructed in such a way that the Higgs boson coupling to gluons is suppressed due to a cancellation between the top and the stop loops at the $hgg$ vertex. Since at the LHC the searches will rely heavily on producing the Higgs boson in gluon-gluon fusion, and since the mass determination will rely in part on the decays into gluon pairs, such a scenario may present experimental difficulties.

- **In the small-$\alpha_{\text{eff}}$ scenario** the couplings governing the decays $h \to b\bar{b}$ and $h \to \tau^+\tau^-$ are suppressed with respect to their Standard Model values by a factor $-\sin \alpha_{\text{eff}} / \cos \beta$ ($\alpha_{\text{eff}}$ is the effective mixing angle of the neutral CP-even Higgs sector including radiative corrections). The suppression occurs mainly for large $\tan \beta$ and moderate $m_A$.

### 2.2 CP-violating scenarios

In CP-violating MSSM scenarios the three neutral Higgs mass eigenstates $\mathcal{H}_i$ ($i = 1, 2, 3$) do not have well defined CP quantum numbers. Each of them can thus be produced by Higgsstrahlung ($e^+e^- \to \mathcal{H}_iZ$) via the CP-even field component and in pairs ($e^+e^- \to \mathcal{H}_i\mathcal{H}_j$ ($i \neq j$)). The relative rates depend on the choice of the parameters describing the CP-even/odd mixing.

Experimentally, the CP-violating scenarios are more challenging than the CP-conserving scenarios. For a wide range of model parameters, the coupling of the lightest Higgs boson $\mathcal{H}_1$ to the $Z$ boson may be suppressed. Furthermore, the second- and third-lightest $\mathcal{H}_2$ and
\( \mathcal{H}_3 \) bosons may both have masses close to or beyond the kinematic reach of LEP. Also, in CP-violating scenarios, the decays to the main “discovery channels”, \( \mathcal{H}_1 \to bb \), \( \mathcal{H}_2 \to bb \) and \( \mathcal{H}_2 \to \mathcal{H}_1 \mathcal{H}_1 \to b \bar{b} b \bar{b} \), may have lower branching ratios. One therefore anticipates less search sensitivity in the CP-violating scenarios than in the CP-conserving scenarios. An example illustrating this situation is given in Table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>FeynHiggs</th>
<th>CPH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{H}^+ ) (GeV/c²)</td>
<td>129.0</td>
<td>129.0</td>
</tr>
<tr>
<td>( \tan \beta )</td>
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<td>5.0</td>
</tr>
<tr>
<td>( m_{\mathcal{H}_1} ) (GeV/c²)</td>
<td>38.1</td>
<td>33.4</td>
</tr>
<tr>
<td>( m_{\mathcal{H}_2} ) (GeV/c²)</td>
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<td>102.4</td>
</tr>
<tr>
<td>( \sigma(\mathcal{H}_1 \to bbZ) ) (pb)</td>
<td>0.0051</td>
<td>0.0019</td>
</tr>
<tr>
<td>( \sigma(\mathcal{H}_2 \to b \bar{b}Z) ) (pb)</td>
<td>0.0156</td>
<td>0.0197</td>
</tr>
<tr>
<td>( \sigma(\mathcal{H}_2 \to \mathcal{H}_1 \mathcal{H}_1 \to b \bar{b} b \bar{b} Z) ) (pb)</td>
<td>0.0866</td>
<td>0.0978</td>
</tr>
<tr>
<td>( \sigma(\mathcal{H}_1 \mathcal{H}_2 \to b \bar{b} b \bar{b}) ) (pb)</td>
<td>0.0066</td>
<td>0.0094</td>
</tr>
</tbody>
</table>

Table 2: A typical parameter set which is difficult to address by the present searches. The results of the two calculations, FeynHiggs and CPH, are given for a centre-of-mass energy of 206 GeV. The main input parameters are listed in the first two lines; all other input parameters correspond to the CPX benchmark scenario and are listed in the last column of Table 1. The output masses \( m_{\mathcal{H}_1}, m_{\mathcal{H}_3} \) and the relevant topological cross-sections are listed below the second horizontal line.

The cross-sections for Higgsstrahlung and pair production are given by [9]

\[
\sigma_{\nu \tau Z} = g_{\nu \tau ZZ}^2 \sigma_{\mathcal{H}Z}^{SM} \tag{5}
\]

\[
\sigma_{\nu \nu hj} = g_{\nu \nu hjZ}^2 \bar{\lambda} \sigma_{\mathcal{H}Z}^{SM} \tag{6}
\]

(in the expression for \( \bar{\lambda} \), Eq. 3, the indices \( h \) and \( A \) have to be replaced by \( \mathcal{H}_i \) and \( \mathcal{H}_j \)). The couplings

\[
g_{\nu \tau ZZ} = \cos \beta \mathcal{O}_{1i} + \sin \beta \mathcal{O}_{2i} \tag{7}
\]

\[
g_{\nu \nu hjZ} = \mathcal{O}_{3i}(\cos \beta \mathcal{O}_{2j} - \sin \beta \mathcal{O}_{1j}) - \mathcal{O}_{3j}(\cos \beta \mathcal{O}_{2i} - \sin \beta \mathcal{O}_{1i}) \tag{8}
\]

obey the complementarity relation

\[
\sum_{i=1}^{3} g_{\nu \nu ZZ}^2 = 1 \tag{9}
\]

\(^4\)Regarding the decay properties, the CP-violating scenarios maintain a certain similarity to the CP-conserving scenarios although the branching ratios are, in general, different. The lightest mass eigenstate \( \mathcal{H}_1 \) predominantly decays to \( bb \) if allowed by kinematics, with a small fraction decaying to \( \tau^+ \tau^- \) and \( cc \). The second-lightest Higgs boson \( \mathcal{H}_2 \) may decay to \( \mathcal{H}_1 \mathcal{H}_1 \) when allowed by kinematics; otherwise it decays preferentially to \( bb \).
\[ g_{\nu_2\nu_2} = \varepsilon_{ijk} g_{\nu_i\nu_j}\nu \]  

(10)

where \( \varepsilon_{ijk} \) is the usual Levi-Civita symbol.

In CP-violating scenarios, the orthogonal matrix \( O_{ij} \) \((i, j = 1, 2, 3)\) relating the weak CP eigenstates to the mass eigenstates has non-vanishing off-diagonal elements. These elements, giving rise to CP-even/odd mixing, are proportional to

\[ \frac{m_i^4 \text{Im}(\mu A)}{v^2 M^2_{\text{SUSY}}} \]  

(11)

with \( v = \sqrt{v_1^2 + v_2^2} \). Substantial deviations from the CP-conserving scenarios are thus expected for small \( M_{\text{SUSY}} \) and large \( \text{Im}(\mu A) \), which are obtained if the CP-violating phase \( \text{arg}(A) \) takes values close to 90°. Furthermore, the effects from CP violation strongly depend on the precise value of the top quark mass [15].

The parameters of the benchmark model \( \text{CPX} \) have been chosen [18] to maximise the phenomenological differences with respect to the CP-conserving scenarios. Constraints from measurements of the electron and neutron electric dipole moments [26] were also taken into account. The basic set of parameters is listed in the last column of Table 1. Note that the scan of \( m_{H^\pm} \) started at 4 GeV/c\(^2\) but values less than about 100 GeV/c\(^2\) give unphysical results and are thus considered as theoretically inaccessible.

The parameters which follow have been varied one-by-one while all the other parameters were kept at their standard \( \text{CPX} \) value.

- Top quark mass: \( m_t = 169.3, 174.3, 179.3 \) and 183.0 GeV/c\(^2\), embracing the current experimental value, \( m_t = 172.7 \pm 2.9 \) GeV/c\(^2\) [15].
- The CP-violating phases: \( \text{arg}(A) = \text{arg}(m_{\tilde{g}}) = 0°, 30°, 60°, 90° \) (\( \text{CPX} \) value), 135° and 180° (the values 0° and 180° correspond to CP-conserving limits).
- The Higgs mass parameter: \( \mu = 0.5, 1.0, 2.0 \) (\( \text{CPX} \) value) and 4.0 TeV.
- The SUSY-breaking scale: \( M_{\text{SUSY}} = 0.5 \) TeV (\( \text{CPX} \) value) and 1.0 TeV. The proposal of the \( \text{CPX} \) scenario [18] predicts a weak dependence on \( M_{\text{SUSY}} \) if the relations \( |A| = |m_{\tilde{g}}| = \mu/2 = 2M_{\text{SUSY}} \) are preserved. This behaviour is examined by studying a model where \( M_{\text{SUSY}} \) is increased from 0.5 TeV to 1 TeV and the values of \( A, m_{\tilde{g}} \) and \( \mu \) are scaled to 2000 GeV, 2000 GeV and 4000 GeV, respectively.

3 Experimental searches

The searches carried out by the four LEP collaborations are based on \( e^+e^- \) collision data which span a large range of centre-of-mass energies, from 91 GeV to 209 GeV. The searches include the Higgsstrahlung and pair production processes, ensuring by their complementarity
a high sensitivity over the accessible MSSM parameter space. It is important to note that the kinematic properties of the signal processes are to a large extent independent of the CP composition of the Higgs bosons. This implies that the same topological searches can be applied to study the CP-conserving and CP-violating scenarios. For Higgsstrahlung this is natural since only the CP-even components of the Higgs fields couple to the Z boson. In pair production involving CP-even and CP-odd field components, the similarity of the kinematic properties (e.g., angular distributions) arises from the scalar nature of the Higgs bosons. Small differences may occur from spin-spin correlations between final-state particles but these were found to have no noticeable effect on the signal detection efficiencies. We therefore adopt in the following a common notation for the CP-conserving and CP-violating processes in which \( H_i \) \( (i = 1, 2, 3) \) designate three generic neutral Higgs bosons of increasing mass, with undefined CP properties; in the CP-conserving limit \( \arg(A) = \arg(m_\Phi) = 0^\circ \), these become the CP eigenstates \( h, A, H \) (the correspondence depends on the mass hierarchy).

In each of the four LEP experiments, the data analysis is done in several steps. A pres­election is applied to reduce some of the largest backgrounds, in particular, from two-photon processes. The remaining background, mainly from production of fermion pairs and WW or ZZ (possibly accompanied by photon or gluon radiation), is further reduced by more selective cuts or by applying multivariate techniques such as likelihood analyses and neural networks. The identification of b-quarks in the decay of the Higgs bosons plays an important role in the discrimination between signal and background, as does the kinematic reconstruction of the Higgs boson masses. The detailed implementation of these analyses, as well as the data samples used by the four collaborations, are described in the individual publications. A full catalog of the searches provided by the four LEP collaborations for this combination, with corresponding references to the detailed descriptions, is given in Appendix A.

### 3.1 Search topologies

Searches have been carried out for the two main signal processes, the Higgsstrahlung process \( e^+e^- \rightarrow H_1Z \) (which also apply in some cases to \( e^+e^- \rightarrow H_2Z \)) and the pair production process \( e^+e^- \rightarrow H_2H_1 \).

(a) Considering first the Higgsstrahlung process \( e^+e^- \rightarrow H_1Z \), the principal signal topologies are those used in the search for the Standard Model Higgs boson at LEP [3], namely:

- the four-jet topology, \( (H_1 \rightarrow b\bar{b})(Z \rightarrow q\bar{q}) \), in which the invariant mass of two jets is close to the Z boson mass \( M_Z \) while the other two jets contain b-flavour;

- the missing energy topology, \( (H_1 \rightarrow b\bar{b}, \tau^+\tau^-)(Z \rightarrow \nu\bar{\nu}) \), in which the event consists of two b-jets or identified tau decays and substantial missing momentum and missing mass, compatible with \( M_Z \);

- the leptonic final states, \( (H_1 \rightarrow b\bar{b})(Z \rightarrow e^+e^-, \mu^+\mu^-) \), in which the invariant mass of the two leptons is close to \( M_Z \);
• the final states with tau-leptons, \((H_1 \to \tau^+\tau^-)(Z \to q\bar{q})\) and \((H_1 \to b\bar{b}, \tau^+\tau^-)(Z \to \tau^+\tau^-)\), in which either the \(\tau^+\tau^-\) or the \(q\bar{q}\) pair has an invariant mass close to \(M_Z\).

Most of these signatures are relevant for Higgs boson masses above the \(b\bar{b}\) threshold and rely on the identification of \(b\)-quarks in the final state. Searches for lighter Higgs bosons, listed in Appendix A, use signatures which are described in the specific publications. In some regions of the MSSM parameter space, the \(H_1 \to b\bar{b}\) decay may be suppressed while decays into other quark flavours or gluon pairs are favoured. The above searches are therefore complemented or replaced\(^5\) by flavour-independent searches for \((H_1 \to q\bar{q})Z\) in which there is no requirement on the quark-flavour of the jets. Finally, the searches for Higgsstrahlung also include the Higgs cascade decay \(e^+e^- \to H_2Z \to (H_1H_1)Z\), giving rise to a new class of event topologies. These processes may play an important role in those regions of the parameter space where they are allowed by kinematics.

(b) In the case of the pair production process, \(e^+e^- \to H_2H_1\), the principal signal topologies at LEP are:

• the four-\(b\) final state \((H_2 \to b\bar{b})(H_1 \to b\bar{b})\);
• the mixed final states \((H_2 \to \tau^+\tau^-)(H_1 \to b\bar{b})\) and \((H_2 \to b\bar{b})(H_1 \to \tau^+\tau^-)\);
• the four-tau final state \((H_2 \to \tau^+\tau^-)(H_1 \to \tau^+\tau^-)\).

The Higgs cascade decay, \(e^+e^- \to H_2H_1 \to (H_1H_1)H_1\), gives rise to event topologies ranging from six \(b\)-jets to six tau-leptons. Most of these searches are relevant for Higgs boson masses above the \(\tau^+\tau^-\) threshold. Similarly to the Higgsstrahlung case, the above searches for pair production are complemented or replaced, whenever more efficient, by flavour-independent searches.

### 3.2 Additional experimental constraints

If the combination of the above searches is not sufficiently sensitive for excluding a given model point, additional constraints are applied; these are listed below.

• Constraint from the measured decay width of the \(Z\) boson, \(\Gamma_Z\), and its possible deviation, \(\Delta\Gamma_Z\), from the Standard Model prediction. The model point is regarded as excluded if the following relation between the relevant cross-sections is found to be true:

\[
\sum_i \sigma_{\eta_iZ}(m_Z) + \sum_{i,j} \sigma_{\eta_i\eta_j}(m_Z) > \frac{\Delta\Gamma_Z}{\Gamma_Z} \cdot \sigma^{\text{tot}}_Z(m_Z),
\]

\(^5\)The replacement is necessary whenever the overlap in terms of selected events is important, in order to avoid double-counting.
where $\Delta \Gamma_Z = 2.0$ MeV [27] stands for the 95% CL upper bound on the possible additional decay width of the $Z$ boson, beyond the Standard Model prediction, and $\sigma^\text{tot}_Z$ is the $Z$ pole cross-section.

- Constraint from a decay mode independent search for $e^+e^- \rightarrow H_1Z$ [28]. The model point is regarded as excluded if the condition
  \[ \sigma_{\eta_1Z} > k(m_{\eta_1}) \cdot \sigma_{HZ}^\text{SM} \]  
  (13)
  is fulfilled, where $k(m_{\eta_1})$ is a mass-dependent factor which scales the Standard Model Higgs production cross-section to the value that is excluded at the 95% CL.

- Constraint from a search for light Higgs bosons produced by the Yukawa process\(^6\). The model point is regarded as excluded if the predicted Yukawa enhancement factor $\xi(m_{\eta_1})$, defined in [29], is excluded by this search. To be conservative, the weaker of the two enhancement factors, for CP-even and CP-odd couplings, is used.

These additional constraints are particularly useful at small $m_{H_1}$ and $m_{H_2}$, below the $b\bar{b}$ threshold.

### 3.3 Statistical combination of search channels

The statistical method by which the topological searches are combined is described in Refs. [3, 30].

After selection, the combined data configuration (distribution of all selected events in several discriminating variables) is compared in a frequentist approach to a large number of simulated configurations generated separately for two hypotheses: the background ($b$) hypothesis and the signal-plus-background ($s+b$) hypothesis. The ratio

\[ Q = \frac{\mathcal{L}_{s+b}}{\mathcal{L}_b} \]  
(14)

of the corresponding likelihoods is used as the test statistic. The predicted, normalised, distributions of $Q$ (probability density functions) are integrated to obtain the $p$-values [31]

\[ 1 - CL_b = 1 - \mathcal{P}_b(Q \leq Q_{\text{observed}}) \]  
and

\[ CL_{s+b} = \mathcal{P}_{s+b}(Q \leq Q_{\text{observed}}); \] 

these measure the compatibility of the observed data configuration with the two hypotheses. Here $\mathcal{P}_b$ and $\mathcal{P}_{s+b}$ are the probabilities for a single experiment to obtain a value of $Q$ smaller than or equal to the observed value, given the background or the signal-plus-background hypothesis. More details can be found in Ref. [3].

Systematic errors are incorporated in the calculation of the likelihoods by randomly varying the signal and background estimates in each channel\(^7\) according to Gaussian error distributions.

---

\(^6\)Note that, in the case of DELPHI, the Yukawa channels are not used as external constraints but are combined with the other search channels.

\(^7\)The word “channel” designates any subset of the data in which a search has been carried out. These subsets may correspond to specific final-state topologies, to data sets collected at different centre-of-mass energies or to the subsets of data collected by different experiments.
and widths corresponding to the systematic errors. For a given source of uncertainty, correlations are addressed by applying these random variations simultaneously to all those channels for which the source of uncertainty is relevant. Errors which are correlated among the experiments arise mainly from using the same Monte Carlo generators and cross-section calculations for the signal and background processes. The uncorrelated errors arise mainly from the limited statistics of the simulated background event samples.

In a purely frequentist approach, the exclusion limit is computed from the confidence $CL_{s+b}$ for the signal-plus-background hypothesis: a signal is regarded as excluded at the 95% CL, for example, if an observation is made such that $CL_{s+b}$ is lower than 0.05. However, this procedure may lead to the undesired situation in which a large downward fluctuation of the background would exclude a signal hypothesis for which the experiment has no sensitivity since the expected signal rate is too small. This problem is avoided by using the ratio

$$CL_s = CL_{s+b}/CL_b$$

instead of $CL_{s+b}$. We adopt this quantity for setting exclusion limits and consider a given model to be excluded at the 95% CL if the corresponding value of $CL_s$ is less than 0.05. Since $CL_b$ is a positive number less than one, $CL_s$ is always larger than $CL_{s+b}$ and the limits obtained in this way are therefore conservative.

### 3.4 Comparisons of the data with the expected background

The distribution of the $p$-value $1 - CL_b$ over the parameter space covered by the searches provides a convenient way of studying the agreement between the data and the expected background and of discussing the statistical significance of any local excess in the data. While a purely background-like behaviour\(^8\) would yield $p$-values close to 0.5, much smaller values are expected in the case of a signal-like excess. For example, a local excess of three or five standard deviations would give rise to a $p$-value $1 - CL_b$ of $2.7 \times 10^{-3}$ or $5.7 \times 10^{-7}$, respectively.

One has to be careful, however, when interpreting these numbers as probabilities for local excesses occurring over the extended domains covered by the searches. For example, the probability for a fluctuation of three standard deviations to occur anywhere in the parameter space is much larger than the number $2.7 \times 10^{-3}$ just quoted. A multiplication factor has to be applied to the probability $1 - CL_b$ which reflects the number of independent “bins” of the parameter space; this factor can be estimated from the total size of the parameter space and the experimental resolutions. For example, the searches for the Higgsstrahlung process $e^+e^- \rightarrow H_1Z$, covering the range $0 < m_{H_1} < 120$ GeV/$c^2$ with a mass resolution $\Delta m_{H_1}$ of about 3 GeV/$c^2$, would yield about twenty fairly independent mass-bins of width $2\Delta m_{H_1}$; hence, a multiplication factor of about twenty. Much bigger multiplication factors are expected in the searches for the pair production process $e^+e^- \rightarrow H_2 H_1$ with two independent search parameters (masses).

\(^8\)Single, background-like, experiments have values of $1 - CL_b$ uniformly distributed between zero and one.
These simple considerations do not take into account, for example, possible correlations from resolution tails extending over several adjacent bins or correlations between different searches sharing candidate events. A more elaborate evaluation of the multiplication factor has therefore been performed. A large number of background experiments was simulated, covering the whole parameter space, using realistic resolution functions and taking correlations into account. From these random experiments, the probability to obtain $1 - CL_b$ smaller than a given value, anywhere in the parameter space of a given scenario, has been determined (the $m_{h\text{-max}}$ scenario was taken for this study). A scale factor of at least 60 was obtained in this manner. According to this estimate, the probability of observing a background fluctuation of three standard deviations anywhere in the parameter space of a given scenario (e.g., $m_{h\text{-max}}$) can be 16% or more. Also, to observe two fluctuations with two standard deviations turns out to be more likely than to observe only one.

Figure 1 shows the distribution of the $p$-value $1 - CL_b$, determined from the present combined searches, for the CP-conserving benchmark scenario $m_{h\text{-max}}$ and the CP-violating scenario $CPX$. Over the largest part of the parameter space, the local excesses are smaller than two standard deviations. In the $m_{h\text{-max}}$ scenario, the lowest value, $1 - CL_b = 1.3 \times 10^{-2}$, lies within the vertical band at $m_h$ around 100 GeV/$c^2$ and corresponds to 2.5 standard deviations. This excess, and a less significant excess at about 115 GeV/$c^2$, come from the Higgsstrahlung search; both are discussed in Ref. [3] in the context of the search for the Standard Model Higgs boson. In the $CPX$ scenario, one observes two small regions at $m_{H_1} \approx 35-40$ GeV/$c^2$, $m_{H_2} \approx 105$ GeV/$c^2$ and $\tan \beta \approx 10$, where the significance exceeds three standard deviations; they arise from the search for the pair production process.

The exact position and size of these fluctuations may vary from one scenario to the other. In Tables 3 and 4 we list the parameters of the most significant excesses for all CP-conserving and CP-violating benchmark scenarios considered in this paper. The largest fluctuation of all has a significance of 3.5 standard deviations; its probability is estimated as 3.6% at least, when the scale factor of 60 or more is applied.

From these studies one can conclude that there is a reasonable agreement between the data and the simulated background, with no compelling evidence for a Higgs boson signal, and that the excesses observed are compatible with random fluctuations of the background.

4 Limits on topological cross-sections

In this section we present upper bounds on the cross-sections for the most important final-state topologies expected from the Higgsstrahlung process $e^+e^- \rightarrow HZZ$ and the pair production process $e^+e^- \rightarrow HZH_1$. These can be used to test a wide range of specific models.

We define the scaling factor

$$S_{95} = \sigma_{\text{max}}/\sigma_{\text{ref}}, \quad (16)$$

where $\sigma_{\text{max}}$ is the largest cross-section compatible with the data, at the 95% CL, and $\sigma_{\text{ref}}$
Table 3: The most significant excesses with respect to the predicted background, for each of the CP-conserving benchmark scenarios. Columns 2 to 6 show the mass parameters (in GeV/c^2) and tan β at which the excess occurs. Column 7 gives the corresponding p-values 1 − CL_b. In the last column, the significances of the excesses, in standard deviations, are listed.

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>m_h</th>
<th>m_H</th>
<th>m_A</th>
<th>m_{H^±}</th>
<th>tan β</th>
<th>1 − CL_b</th>
<th>σ (st.dev.)</th>
</tr>
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<tbody>
<tr>
<td>m_h-max</td>
<td>99</td>
<td>253</td>
<td>169</td>
<td>184</td>
<td>0.7</td>
<td>1.3×10^{-2}</td>
<td>2.5</td>
</tr>
<tr>
<td>m_h-max (a)</td>
<td>99</td>
<td>277</td>
<td>156</td>
<td>171</td>
<td>0.6</td>
<td>1.4×10^{-2}</td>
<td>2.5</td>
</tr>
<tr>
<td>m_h-max (b)</td>
<td>99</td>
<td>345</td>
<td>300</td>
<td>319</td>
<td>0.9</td>
<td>1.6×10^{-2}</td>
<td>2.4</td>
</tr>
<tr>
<td>no-mixing</td>
<td>99</td>
<td>165</td>
<td>152</td>
<td>171</td>
<td>3.7</td>
<td>1.4×10^{-2}</td>
<td>2.5</td>
</tr>
<tr>
<td>no-mixing (a)</td>
<td>99</td>
<td>134</td>
<td>114</td>
<td>138</td>
<td>5.4</td>
<td>1.1×10^{-2}</td>
<td>2.5</td>
</tr>
<tr>
<td>large-μ</td>
<td>59</td>
<td>108</td>
<td>67</td>
<td>104</td>
<td>3.1</td>
<td>1.0×10^{-2}</td>
<td>2.6</td>
</tr>
<tr>
<td>gluophobic</td>
<td>56</td>
<td>124</td>
<td>69</td>
<td>105</td>
<td>4.1</td>
<td>5.5×10^{-3}</td>
<td>2.8</td>
</tr>
<tr>
<td>small-α_{eff}</td>
<td>60</td>
<td>121</td>
<td>75</td>
<td>109</td>
<td>5.5</td>
<td>2.4×10^{-3}</td>
<td>3.0</td>
</tr>
</tbody>
</table>

is a reference cross-section. For the topologies motivated by Higgsstrahlung, σ_{ref} is taken to be the Standard Model Higgs production cross-section; for final states motivated by the pair production process, σ_{ref} is taken to be the MSSM Higgs production cross-section of Eq. 2 with the MSSM suppression factor set to 1. Numerical values for the cross-section limits are listed in Appendix B.

Figure 2 shows the upper bound $S_{95}$ for final states motivated by the Higgsstrahlung process $e^+e^− \rightarrow \mathcal{H}_1Z$ (the figure is reproduced from Ref. [3]). In part (a), the Higgs boson is assumed to decay into fermions and bosons with branching ratios as given by the Standard Model. Contributions from the fusion processes $WW \rightarrow \mathcal{H}_1$ and $ZZ \rightarrow \mathcal{H}_1$, according to the Standard Model, corrected for initial-state radiation, are assumed to scale with energy like the Higgsstrahlung process. In part (b) it is assumed that the Higgs boson decays exclusively to bb and in part (c) exclusively to $\tau^+\tau^-$. Besides representing bounds on topological cross-sections, this figure also illustrates the overall agreement between the data and the expected background from Standard Model processes. The largest deviations observed barely exceed two standard deviations.

Figure 3 shows contours of $S_{95}$ for the cascade process $e^+e^− \rightarrow \mathcal{H}_2Z \rightarrow (\mathcal{H}_1\mathcal{H}_1)Z$, projected onto the $(m_{\mathcal{H}_2}, m_{\mathcal{H}_1})$ plane, assuming that the $\mathcal{H}_2$ boson decays exclusively to $\mathcal{H}_1\mathcal{H}_1$. In part (a) it is assumed that the $\mathcal{H}_1$ boson decays exclusively to bb and in part (b) exclusively to $\tau^+\tau^-$. In part (c), as an example, an equal mixture of $\mathcal{H}_1 \rightarrow bb$ and $\mathcal{H}_1 \rightarrow \tau^+\tau^-$ is assumed, which implies 25% bbbbZ, 25% $\tau^+\tau^-\tau^+\tau^-$ and 50% bb$\tau^+\tau^-Z$ final states. The sensitivity of the bbZ channel starts at the bb threshold and extends almost to the kinematic limit. In the $\tau^+\tau^-\tau^+\tau^-Z$ channel the sensitivity is altogether weaker (the discontinuities reveal the limited and inhomogeneous mass coverage of the four experiments in this channel).
Table 4: The most significant excesses with respect to the predicted background in the CP-violating benchmark scenario CPX and its variants. The first column indicates either the CPX scenario or the parameter value which differs from the standard CPX set listed in the last column of Table 1. Columns 2 to 6 show the mass parameters (in GeV/c^2) and tan β at which the excesses occur (the more conservative of the CPH and FeynHiggs calculations is used). Columns 7 and 8 give the corresponding p-values, 1 − CLb, using in turn the CPH and FeynHiggs codes (note the overall agreement of the two calculations in this respect). In the last column, the significances of the excesses, in standard deviations, are listed.

Figure 4 shows S95 for final states motivated by the pair-production process e^+e^- → H_2H_1, for the particular case where the masses m_{H_2} and m_{H_1} are approximately equal. Such is the case, for example, in the CP-conserving MSSM scenario m_{h-max} for tan β larger than about 10 and small m_{H_2} (≡ m_A). In part (a), the H_2 and H_1 decay branching ratios correspond to the m_h-max benchmark scenario with tan β = 10 (see the caption for the exact values); in part (b), both H_2 and H_1 are assumed to decay exclusively to b̅b; in part (c), one Higgs boson is assumed to decay exclusively to b̅b while the other exclusively to τ^+τ^-; in part (d), H_2 and H_1 are both assumed to decay exclusively to τ^+τ^- at low masses, the exclusion limits are completed using the constraint from the measured decay width of the Z boson (see Section 3.2). This figure also illustrates the overall agreement between the data and the expected background from Standard Model processes since the largest deviations are within two standard deviations.

Figure 5 shows contours of S_95 for final states motivated by the process e^+e^- → H_2H_1, projected onto the (m_{H_2}, m_{H_1}) plane. In part (a), both Higgs bosons are assumed to decay exclusively to b̅b and in part (b) exclusively to τ^+τ^- In parts (c) / (d), the H_2 / H_1 boson is assumed to decay exclusively to b̅b while the other boson is assumed to decay exclusively to
Figure 6 shows contours of $S_{95}$ for the cascade process $e^+e^- \rightarrow H_2H_1 \rightarrow (H_1H_1)H_1$, projected onto the $(m_{H_2}, m_{H_1})$ plane, assuming that the $H_2$ boson decays exclusively to $H_1H_1$. In part (a), the $H_1$ boson is assumed to decay exclusively to $b\bar{b}$ and in part (b) exclusively to $\tau^+\tau^-$. In part (c), as an example, an equal mixture of $H_1 \rightarrow b\bar{b}$ and $H_1 \rightarrow \tau^+\tau^-$ is assumed, which implies 12.5% $b\bar{b}b\bar{b}bb$, 37.5% $b\bar{b}b\bar{b}\tau^+\tau^-$, 37.5% $b\tau^+\tau^-\tau^+\tau^-$ and 12.5% $\tau^+\tau^-\tau^+\tau^-\tau^+\tau^-$ final states.

A word of caution is in place concerning the correlations which exist between some of the above cross-section limits which arise from overlapping candidates in the corresponding selections. Such correlations are present, for example, between b-tagged and flavour-independent searches of a given experiment or between searches addressing direct decays (e.g., $H_1Z \rightarrow b\bar{b}b$) and cascade decays (e.g., $H_2 \rightarrow H_1H_1Z \rightarrow b\bar{b}b$); they may be a source of problems if several of the cross-section limits are used in conjunction to test a given model. Note, however, that these correlations are properly taken into account in the model interpretations which follow.

5 Results interpreted in CP-conserving MSSM scenarios

In this section, the search results are interpreted in the CP-conserving benchmark scenarios presented in Section 2.1. The exclusion limits, which are shown in the figures below at the 95% CL and the 99.7% CL, are obtained from the values of $CL_s$ (see Eq. 15), for an assumed top quark mass of $m_t = 174.3$ GeV/$c^2$. The exclusion limits are presented in four projections of the MSSM parameter space. The limits expected on the basis of Monte Carlo simulations with no signal, at the 95% CL, are also indicated. The exact mass bounds and exclusions for $\tan \beta$ are listed in Table 5, for four values of $m_t$.

The exclusions for the $m_h$-max benchmark scenario are shown in Figure 7. In the region with $\tan \beta$ less than about five, the exclusion is provided mainly by the Higgsstrahlung process, giving a lower bound of about 114 GeV/$c^2$ for $m_h$. At high $\tan \beta$, the pair production process is most useful, providing limits in the vicinity of 93 GeV/$c^2$ for both $m_h$ and $m_A$. For $m_h$ in the vicinity of 100 GeV/$c^2$, one observes a deviation between the expected and the experimental exclusions. This deviation, which is also present in other CP-conserving scenarios, is due to the excess in the Higgsstrahlung channel which was discussed in Ref. [3] and gives rise to the vertical bands in Figures 1 (a) and (b). Note that the mass bounds obtained are largely insensitive to the top quark mass.

The data also exclude certain domains of $\tan \beta$. This is best illustrated in the ($m_h$, $\tan \beta$) projection (plot (b)) where the upper boundary of the parameter space along $m_h$ is indicated for four values of $m_t$; the intersections of these boundaries with the experimental exclusions define the regions of $\tan \beta$ which are excluded. The exclusion in $\tan \beta$, as a function of the assumed top quark mass, is summarised in Figure 8; for $m_t$ larger than about 181.5 GeV/$c^2$, no 95% CL limit on $\tan \beta$ can be set in this scenario.
One should be aware that the upper boundary of the parameter space along $m_h$ also depends moderately on the choice of $M_{\text{SUSY}}$. For example, changing $M_{\text{SUSY}}$ from 1 TeV to 2 TeV would broaden the parameter space by about 2 GeV/$c^2$ along $m_h$, with corresponding effects on the exclusions in $\tan \beta$. This observation holds for all CP-conserving scenarios which follow.

Figures 9 and 10 show the same set of plots for the two variants, (a) and (b), of the $m_h$-max scenario introduced in Section 2.1.1. The change of the sign of the Higgs mass parameter $\mu$ or of the mixing parameter $X_t$ barely affect the mass limits; however, sizable differences occur in the exclusions of $\tan \beta$ (see Table 5). For example, in variant (b), a small domain of $\tan \beta$ is excluded even for $m_t = 183.0$ GeV/$c^2$, which is not the case in the standard $m_h$-max scenario and its variant (a). Note, in Figure 9, the small domains at $m_h$ between 60 and 75 GeV/$c^2$, small $m_A$ and $\tan \beta < 0.9$ which are excluded at the 95% CL but not at the 99.7% CL.

The exclusions for the CP-conserving no-mixing benchmark scenario are shown in Figure 11. In this scenario, the theoretical boundaries of the parameter space are more restricted than in the $m_h$-max scenario. As a consequence, large domains of $\tan \beta$ are excluded for all the top quark masses considered. Note the relatively strong variation of the exclusion limits with $m_t$ in this scenario (see Table 5), which is caused by the proximity of the experimental lower bound of $m_h$ from the Higgsstrahlung searches and the theoretical upper bound of $m_h$.

An interesting feature of this scenario is that, for $m_h$ larger than about 100 GeV/$c^2$ and large $\tan \beta$, the heavy scalar boson $H$ is within kinematic reach. Moreover, the cross-section for the process $e^+e^- \rightarrow HZ$ is increasing with $\tan \beta$, resulting in an improved search sensitivity; this explains the nearly circular shape of the expected limit in Figure 11 (b).

Note the small domain at $m_h$ between 75 and 80 GeV/$c^2$, small $m_A$ and $\tan \beta < 0.7$, barely perceptible in the plots, which is not excluded in this scenario at 95% CL (this domain is excluded for $m_t = 169.3$ GeV/$c^2$). The branching ratio for $h \rightarrow bb$ is small and the decay $h \rightarrow AA$ is dominant in this region. The A boson, with mass below the $\tau^+\tau^-$ threshold, may decay to final states which are not sufficiently covered by the present searches. For this reason, the mass limits given in Table 5 for this scenario and for $m_t$ larger than 169.3 GeV/$c^2$ are valid only for $\tan \beta \geq 0.7$. Conversely, for $m_t$ larger than 169.3 GeV/$c^2$, the quoted exclusion of $\tan \beta$ is valid only for $m_A$ larger than about 3 GeV/$c^2$.

Figure 12 shows the exclusion plots for the (a) variant of the no-mixing scenario introduced in Section 2.1.2. The change of sign of the Higgs mass parameter $\mu$ and the increase of the weak SUSY-breaking scale from 1 TeV to 2 TeV affect only the theoretical bounds of the parameter space but barely change the mass limits, except for $m_t=169.3$ GeV/$c^2$. There are moderate changes though in the exclusions of $\tan \beta$. In the hatched domain ($\tan \beta<0.7$), the contributions from top and stop quark loops to the radiative corrections are large and uncertain; hence, no exclusions can be claimed there.

The exclusions for the large-$\mu$ benchmark scenario are shown in Figure 13. As mentioned in Section 2.3, this scenario was constructed to test the sensitivity of LEP to MSSM scenarios which may be a priori difficult to handle experimentally since the Higgs boson decays to $bb$ are largely suppressed. It turns out that the flavour-independent and decay-mode-independent
searches are sufficiently powerful to exclude all such situations at 95% CL, for top quark masses up to 174.3 GeV/c². There remains a thin strip at \( \tan \beta \) larger than about 10 and running from \( m_A \) of about 100 to about 200 GeV/c², which is excluded at the 95% CL but not at 99.7% CL because the suppression of the \( bb \) channel is particularly strong in that region. This strip is found to grow with increasing \( m_A \) and becomes gradually non-excluded at the 95% CL. Other small, weakly excluded, regions are located at \( m_H \approx 60 \text{ GeV/c}^2 \) and small \( m_A \), and along the \( m_h \approx m_A \) “diagonal” of plot (a).

Similar plots are shown in Figures 14 and 15 for the gluophobic and small-\( \alpha_{eff} \) scenarios defined in Section 2.1.3. These scenarios were designed to test situations which can be problematic at the Tevatron and LHC colliders. In both cases, large domains of the parameter space are excluded by the LEP searches.

6 Results interpreted in CP-violating MSSM scenarios

In this section, the search results are interpreted in the CP-violating benchmark scenario \( \text{CPX} \) presented in Section 2.2, and in some variants of \( \text{CPX} \) where the basic model parameters are varied one-by-one. Note that in these scenarios \( m_{H_3} \) is always larger than 120 GeV/c², except where the CP-violating phases \( \arg(A) = \arg(m_{H_3}) \) are put to 0° or 180°.

The experimental exclusions for the \( \text{CPX} \) benchmark scenario are shown in Figure 16, in four projections. For large \( m_{H_3} \), the \( H_1 \) is almost completely CP-even; in this case the limit on \( m_{H_1} \) is close to 114 GeV/c², the limit obtained for the Standard Model Higgs boson [3]. For example, for \( m_{H_3} \) larger than 133 GeV/c², one can quote a lower bound of 113 GeV/c² for \( m_{H_1} \). Large CP-odd admixtures to \( H_1 \) occur, however, for smaller \( m_{H_3} \), giving rise to domains at lower \( m_{H_1} \) which are not excluded.

The exclusion is particularly weak for \( \tan \beta \) between about 3.5 and 10. Here, the signal is spread over several channels arising from the Higgsstrahlung and pair-production processes, including the \( H_2 \to H_1 H_1 \) cascade decays, which give rise to complex final states with six jets. The parameter set of Table 2 is a typical example of this situation. This is illustrated in Figure 17 where the main final-state cross-sections are plotted as a function of \( \tan \beta \) (the \text{FeynHiggs} calculation is used). In general, these signal contributions cannot be added up statistically because of a large overlap in the selected events; hence, a relatively low overall detection efficiency is expected. Moreover, one of the experiments presents a local excess of about two standard deviations in this domain of \( \tan \beta \) and for \( m_{H_1} \) of about 45 GeV/c² [10], which lowers the exclusion power below the expectation. Nonetheless, the region defined by \( m_{H_1} < 114 \text{ GeV/c}^2 \) and \( \tan \beta < 3.0 \) is excluded by the data (see Figure 16 (b)) and a 95% CL lower bound of 2.9 can be set on \( \tan \beta \) in this scenario.

Figure 18 illustrates the exclusions in the \((m_{H_1}, \tan \beta)\) projection, using the CPH calculation (part (a)) and the \text{FeynHiggs} calculation (part (b)). Differences occur mainly at large \( \tan \beta \) where the \text{FeynHiggs} calculation predicts a larger Higgsstrahlung cross-section and hence a bet-
ter search sensitivity than the CPH calculation. In parts (a) and (b) of the figure, one observes two distinct domains at moderate $\tan \beta$, with $m_{\tilde{t}_1} < 15 \text{ GeV/c}^2$ and $30 \text{ GeV/c}^2 < m_{\tilde{t}_1} < 55 \text{ GeV/c}^2$, which are not excluded at the 95% CL. The values of $1 - CL_b$ indicate that these domains are excluded, respectively, at the 55% CL and 77% CL using the CPH calculation, and at the 50% CL and 66% CL, respectively, using the FeynHiggs calculation. A third domain appears in part (b) at higher $m_{\tilde{t}_1}$ (where the CPH calculation indicates no exclusion power at all); this domain is excluded at the 42% CL using FeynHiggs.

As explained in Section 2, neither of the two approaches, CPH or FeynHiggs, are preferred on theoretical grounds. For this reason, part (c) of this figure was obtained by choosing in each scan point of the parameter space the more conservative of the two approaches, i.e., the one for which the less significant exclusion is observed. The same procedure was adopted in Figure 16 and in all the figures which follow.

The significant impact of the top mass on the CP-violating effects, indicated by Eq. 11, is illustrated in Figure 19 where the $(m_{\tilde{t}_1}, \tan \beta)$ projection is shown for four values of $m_t$. With increasing $m_t$, one observes a reduction of the exclusion power, especially in the region of $\tan \beta$ between 3.5 and 10. No lower bound on $m_{\tilde{t}_1}$ can be quoted in this domain. In plot (a) (for $m_t = 169.3 \text{ GeV/c}^2$), the two domains with $m_{\tilde{t}_1} < 15 \text{ GeV/c}^2$ and $30 \text{ GeV/c}^2 < m_{\tilde{t}_1} < 55 \text{ GeV/c}^2$ are excluded at the 60% CL and 88% CL, respectively.

Figure 20 illustrates the exclusion in the $(m_{\tilde{t}_1}, \tan \beta)$ plane as a function of the CP-violating phases, $\arg(A) = \arg(m_{\tilde{g}})$, which are varied together. For phase angles close to 0°, the experimental exclusions are similar to those in the CP-conserving scenarios (see, for example, Figure 7 but note the differences in the allowed parameter space). Sizable differences are observed for larger phase angles, especially for $\arg(A) = \arg(m_{\tilde{g}}) = 90°$ (the CPX value). At $\arg(A) = \arg(m_{\tilde{g}}) = 180°$ (another CP-conserving scenario), the allowed parameter space is excluded almost completely. Note however that in the hatched region, with $\tan \beta$ greater than about 12, the calculation of the bottom-Yukawa coupling has large theoretical uncertainties; hence no exclusion can be claimed in this domain.

In Figure 21, the value of the Higgs mass parameter $\mu$ is varied from 500 GeV through 1000 GeV and 2000 GeV (the CPX value) to 4000 GeV. At small values, the CP-violating effects are small (see Eq. 11) and the exclusion power is strong (as in the CP-conserving case). For $\mu$ larger than 2000 GeV and large $\tan \beta$, the FeynHiggs and CPH calculations both provide bottom-Yukawa coupling in the non-perturbative regime, giving rise to negative values for the square of $m_{\tilde{t}_1}$ and to other unphysical results. For $\mu \leq 2000 \text{ GeV}$ this regime sets in only at $\tan \beta$ larger than 40 whereas for $\mu = 4000 \text{ GeV}$ this situation already occurs at $\tan \beta$ above 20. Hence, in Figure 21 (d), the hatched domain should not be considered as being integrally part of the allowed parameter space.

Figure 22 illustrates the dependence on the soft SUSY-breaking scale parameter, $M_{\text{SUSY}}$, which is increased from the CPX value of 500 GeV in part (a) to 1000 GeV in part (b). This decreases the CP-violating effects (see Eq. 11) and leads to a larger exclusion. The “scaling” behaviour mentioned in Section 2.3, namely the relative insensitivity of the exclusions to changes in $M_{\text{SUSY}}$ as long as the relations $|A_{t,b}| = |m_{\tilde{g}}| = \mu/2 = 2M_{\text{SUSY}}$ are preserved, is qualitatively
confirmed by comparing parts (a) and (c) of the figure.

7 Summary

The searches for neutral Higgs bosons described in this paper are based on the data collected by the four LEP collaborations, ALEPH, DELPHI, L3 and OPAL, which were statistically combined by the LEP Working Group for Higgs Boson Searches. The data samples include those collected during the LEP 2 phase at $e^+e^-$ centre-of-mass energies up to 209 GeV; two experiments also provided LEP 1 data, at energies in the vicinity of the $Z$ boson resonance. The searches address a large number of final-state topologies arising from the Higgsstrahlung process $e^+e^- \rightarrow H_iZ$ and from the pair production process $e^+e^- \rightarrow H_2H_1$. The combined LEP data do not reveal any excess of events which would indicate the production of Higgs bosons. The differences with respect to the background predictions are compatible with statistical fluctuations of the background.

From these results, upper bounds are derived for the cross-sections of a number of Higgs-like event topologies. These upper bounds cover a wide range of Higgs boson masses and are typically well below the cross-sections predicted within the MSSM framework; these limits can be used to constrain a large number of theoretical models.

The combined search results are used to test several MSSM scenarios which include CP-conserving and CP-violating benchmark models. These models are motivated mainly by physics arguments but some of them are constructed to test specific situations where the detection of Higgs bosons at the Tevatron and LHC colliders might present experimental difficulties. It is found that in all these scenarios the searches conducted at LEP exclude sizable domains of the theoretically allowed parameter space.

In the CP-conserving case, lower bounds can be set on the masses of neutral Higgs bosons and the value of $\tan \beta$ can be restricted. Taking, for example, the CP-conserving scenario $m_h$-$max$ and a top quark mass of 174.3 GeV/$c^2$, values of $m_h$ and $m_A$ less than 92.8 GeV/$c^2$ and 93.4 GeV/$c^2$, respectively, are excluded at the 95% CL. In the same scenario, values of $\tan \beta$ between 0.7 and 2.0 are excluded, but this range depends considerably on the assumed top quark mass and may also depend on $M_{SUSY}$.

In the CP-violating benchmark scenario $CPX$ and the variants which have been studied, the combined LEP data show large domains which are not excluded, down to the lowest mass values; hence, no absolute limits can be set for the Higgs boson masses. The excluded domains vary considerably with the precise value of the top quark mass and the MSSM model parameters. For example, in the $CPX$ scenario with standard parameters and $m_t = 174.3$ GeV/$c^2$, $\tan \beta$ can be restricted to values larger than 2.9 at the 95% CL.
Acknowledgements

We congratulate the LEP Accelerator Division for the successful running of LEP over twelve years, up to the highest energies. We also would like to express our thanks to the engineers and technicians in all our institutions for their contributions to the excellent performance of the four LEP experiments.
Table 5: Lower mass bounds and exclusions in tan β, at 95% CL, obtained in the case of the CP-conserving MSSM benchmark scenarios, for various values of the top quark mass. In each case, the observed limit is followed, between parentheses, by the value expected on the basis of Monte Carlo simulations with no signal. In the \( m_h\)-max scenario and its variant (a), there is no exclusion in tan β for \( m_t = 183.0 \text{ GeV/c}^2 \) or larger. The no-mixing scenario is entirely excluded for \( m_t = 169.3 \text{ GeV/c}^2 \) or smaller. In the no-mixing scenario and for \( m_t \) larger than 169.3 GeV/c\(^2\), the quoted mass limits are only valid for tan β > 0.7 and the exclusion in tan β is only valid for \( m_A \) larger than about 3 GeV/c\(^2\). The large-μ scenario is entirely excluded for \( m_t = 174.3 \text{ GeV/c}^2 \) or smaller.
References


S. Heinemeyer, hep-ph/0407244;
The code is accessible via http://www.feynhiggs.de.


M. Carena, H. E. Haber, S. Heinemeyer, W. Hollik, C. E. M. Wagner and G. Weiglein,


Appendix A: Catalog of searches

The searches of the four LEP collaborations which contribute to this combined analysis are listed in Tables 6 to 13. The list is structured into two tables per experiment, one for the Higgsstrahlung process $e^+e^- \rightarrow \mathcal{H}_1Z$ and one for the pair production process $e^+e^- \rightarrow \mathcal{H}_2\mathcal{H}_1$. In each of these tables, the upper part contains the final states of the direct process and the lower part contains, where it applies, those of the cascade process $\mathcal{H}_2 \rightarrow \mathcal{H}_1\mathcal{H}_1$.

The final-state topologies are listed in the first column. In the notation adopted, $\mathcal{H}_1$ represents the lightest and $\mathcal{H}_2$ the second-lightest neutral Higgs boson. In the CP-conserving case, $\mathcal{H}_1$ is identified with the CP-even eigenstate $h$. The $\mathcal{H}_2$ is identified in most cases with the CP-odd eigenstate $A$ (the cascade process $\mathcal{H}_2 \rightarrow \mathcal{H}_1\mathcal{H}_1$ is identified with $h \rightarrow AA$).

The symbol $q$ indicates an arbitrary quark flavour, $u, d, s, c$ or $b$. “Hadrons” include quarks and gluons. In the missing energy channel, in addition to the $\mathcal{H}_1Z \rightarrow \mathcal{H}_1\nu\bar{\nu}$ process, the $W$ fusion process $\mathcal{H}_1\nu\bar{\nu}$ (including interference) is also considered; similarly, in the leptonic channel, in addition to the $\mathcal{H}_1Z \rightarrow \mathcal{H}_1\ell^+\ell^-$ process, the $Z$ fusion process $\mathcal{H}_1e^+e^-$ (including interference) is also considered.

The contributions based on LEP1 data (from two experiments only) can be identified by their value “91” in the second column which indicates the $e^+e^-$ collision energy, $\sqrt{s}$ (GeV); the LEP1 data used in this combination represent an integrated luminosity $L$ of about 125 pb$^{-1}$. The LEP2 data span an energy range between 133 GeV and 209 GeV; they represent an integrated luminosity of about 2400 pb$^{-1}$. The integrated luminosities for the individual searches are listed in the third column.

Responding to the increasing data samples and $e^+e^-$ energies, the searches were gradually upgraded or replaced so as to become more efficient in detecting Higgs bosons of higher masses. The mass ranges where the searches are relevant are listed in the next column(s). In the last column, references are given to the publications where the details of the searches can be found.
| $\mathcal{H}_1 Z \rightarrow (...) (...) \big| \begin{array}{c|c|c|c|c} \sqrt{s} \text{ (GeV)} & \mathcal{L} \text{ (pb}^{-1}) & \text{Mass range} \text{ (GeV}/c^2) & \text{Ref.} \\

| (bb)(qq), (bb,cc,\tau\tau,gg)(\nu\nu) & 189 & 176.2 & 75 - 110 & [32] \\
| (any)(e^+e^-,\mu^+\mu^-) & 189 & 176.2 & 75 - 110 & [32] \\
| (bb)(\tau^+\tau^-), (\tau^+\tau^-)(qq) & 189 & 176.2 & 65 - 110 & [32] \\
| (bb)(qq,\nu\nu) & 192 - 202 & 236.7 & 60 - 120 & [33] \\
| (bb,\tau^+\tau^-,cc,gg)(e^+e^-,\mu^+\mu^-) & 192 - 202 & 236.7 & 60 - 120 & [33] \\
| (bb,\tau^+\tau^-,cc,gg)(\tau^+\tau^-),(\tau^+\tau^-)(qq) & 192 - 202 & 236.7 & 60 - 120 & [33] \\
| (bb)(qq) & 199 - 209 & 217.2 & 75 - 120 & [11,34] \\
| (bb,\tau^+\tau^-,cc,gg,WW)(\tau^+\tau^-,\nu\nu) & 199 - 209 & 217.2 & 75 - 120 & [11,34] \\
| (bb,\tau^+\tau^-,cc,gg)(e^+e^-,\mu^+\mu^-) & 199 - 209 & 217.2 & 70 - 120 & [11,34] \\
| (bb,cc,ss,gg)(qq) & 189 & 176.2 & 40 - 100 & [35] \\
| (bb,cc,ss,gg)(\nu\nu) & 189 & 176.2 & 60 - 100 & [35] \\
| (bb,cc,ss,gg)(e^+e^-,\mu^+\mu^-) & 189 & 176.2 & 60 - 115 & [32,35] \\
| (\tau^+\tau^-)(qq) & 189 & 176.2 & 65 - 110 & [32] \\
| (bb,cc,ss,gg)(qq) & 192 - 202 & 236.7 & 40 - 110 & [35] \\
| (bb,cc,ss,gg)(\nu\nu) & 192 - 202 & 236.7 & 60 - 116 & [35] \\
| (bb,cc,ss,gg)(e^+e^-,\mu^+\mu^-) & 192 - 202 & 236.7 & 60 - 115 & [33,35] \\
| (\tau^+\tau^-)(qq) & 192 - 202 & 236.7 & 60 - 120 & [33] \\
| (bb,cc,ss,gg)(\nu\nu) & 199 - 209 & 217.2 & 75 - 120 & [35] \\
| (bb,cc,ss,gg)(e^+e^-,\mu^+\mu^-) & 199 - 209 & 217.2 & 70 - 120 & [11,34,35] \\
| (\tau^+\tau^-)(qq) & 199 - 209 & 217.2 & 60 - 120 & [11,34] \\

Table 6: Summary of the ALEPH searches for the Higgstrahlung process $e^+e^- \rightarrow \mathcal{H}_1 Z$. The top part of the table lists the searches originally developed for the Standard Model Higgs boson. The bottom part lists flavour-independent searches where the decays of the Higgs boson into a quark pair of any flavour, a gluon pair or a tau pair were considered; the signal efficiencies were evaluated for all indicated hadronic decays of the Higgs boson. In the cases of the $(\tau^+\tau^-)(qq)$ and leptonic channels listed in the flavour-independent part, the event selections of the Standard Model Higgs boson searches were used.
<table>
<thead>
<tr>
<th>( \mathcal{H}_2 \mathcal{H}_1 \to (...)(...) )</th>
<th>( \sqrt{s} ) (GeV)</th>
<th>( \mathcal{L} ) (pb(^{-1}))</th>
<th>Mass range (GeV/c(^2))</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(bb)(bb), ((\tau^+\tau^-))(bb), (bb)((\tau^+\tau^-))</td>
<td>189</td>
<td>176.2</td>
<td>65 - 95</td>
<td>[32]</td>
</tr>
<tr>
<td>(bb)(bb), (bb, (\tau^+\tau^-), c(\bar{c}), gg)((\tau^+\tau^-)), ((\tau^+\tau^-))(bb, (\tau^+\tau^-), c(\bar{c}), gg)</td>
<td>192 - 202</td>
<td>236.7</td>
<td>60 - (\sqrt{s}/2)</td>
<td>[33]</td>
</tr>
<tr>
<td>(bb)(bb), (bb, (\tau^+\tau^-), c(\bar{c}), gg)((\tau^+\tau^-)), ((\tau^+\tau^-))(bb, (\tau^+\tau^-), c(\bar{c}), gg)</td>
<td>199 - 209</td>
<td>217.2</td>
<td>75 - (\sqrt{s}/2)</td>
<td>[11,34]</td>
</tr>
</tbody>
</table>

Table 7: Summary of the ALEPH searches for the pair production process \(e^+e^- \to \mathcal{H}_2 \mathcal{H}_1\). The searches are restricted to \(|m_{\mathcal{H}_2} - m_{\mathcal{H}_1}| \) less than about 20 GeV/c\(^2\).
<table>
<thead>
<tr>
<th>Process</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$L$ (pb$^{-1}$)</th>
<th>Mass ranges (GeV/c$^2$)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow H_1Z \rightarrow (...)(...)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(any)($e^+e^-, \mu^+\mu^-$), ($\nu\bar{\nu}$)(any)</td>
<td>91</td>
<td>2.5</td>
<td>&lt; 0.21</td>
<td>[36]</td>
</tr>
<tr>
<td>(jet)($e^+e^-, \mu^+\mu^-$)</td>
<td>91</td>
<td>0.5</td>
<td>0.21 - 2</td>
<td>[37]</td>
</tr>
<tr>
<td>(jet jet)($\ell^+\ell^-, \nu\bar{\nu}$)</td>
<td>91</td>
<td>3.6</td>
<td>12 - 50</td>
<td>[38]</td>
</tr>
<tr>
<td>(jet jet)($e^+e^-, \mu^+\mu^-, \nu\bar{\nu}$)</td>
<td>91</td>
<td>33.4</td>
<td>35 - 70</td>
<td>[39]</td>
</tr>
<tr>
<td>(bb)(any), ($\tau^+\tau^-$(qq)</td>
<td>161,172</td>
<td>19.9</td>
<td>40 - 80</td>
<td>[40]</td>
</tr>
<tr>
<td>(bb)(any), ($\tau^+\tau^-$(qq)</td>
<td>183</td>
<td>52.0</td>
<td>45 - 95</td>
<td>[41]</td>
</tr>
<tr>
<td>(bb)(any), ($\tau^+\tau^-$(qq)</td>
<td>189</td>
<td>158.0</td>
<td>65 - 100</td>
<td>[42]</td>
</tr>
<tr>
<td>(bb)(any)</td>
<td>192-209</td>
<td>452.4</td>
<td>45 - 120</td>
<td>[43, 44]</td>
</tr>
<tr>
<td>($\tau^+\tau^-$(qq)</td>
<td>192-209</td>
<td>452.4</td>
<td>45 - 120</td>
<td>[43, 44]</td>
</tr>
<tr>
<td>(qq, $gg$)(qq, $\nu\bar{\nu}$, $e^+e^-, \mu^+\mu^-$)</td>
<td>189-209</td>
<td>610.4</td>
<td>4 - 116</td>
<td>[45]</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow H_2Z \rightarrow (H_1H_1)Z \rightarrow (...)(...)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(any)($q\bar{q}$)</td>
<td>91</td>
<td>16.2</td>
<td>12 - 70</td>
<td>&lt; 0.21</td>
</tr>
<tr>
<td>($\nu\bar{\nu}$)(any but $\tau^+\tau^-$)</td>
<td>91</td>
<td>9.7</td>
<td>0.5 - 55</td>
<td>&lt; 0.21</td>
</tr>
<tr>
<td>($\gamma\gamma$)(any)</td>
<td>91</td>
<td>12.5</td>
<td>0.5 - 60</td>
<td>&lt; 0.21</td>
</tr>
<tr>
<td>(4 prongs)(any)</td>
<td>91</td>
<td>12.9</td>
<td>0.5 - 60</td>
<td>0.21 - 10</td>
</tr>
<tr>
<td>(hadrons)($\nu\bar{\nu}$)</td>
<td>91</td>
<td>15.1</td>
<td>1 - 60</td>
<td>0.21 - 30</td>
</tr>
<tr>
<td>($\tau^+\tau^-\tau^+\tau^-$)($\nu\bar{\nu}$)</td>
<td>91</td>
<td>15.1</td>
<td>9 - 73</td>
<td>3.5 - 12</td>
</tr>
<tr>
<td>(any)($q\bar{q}$, $\nu\bar{\nu}$)</td>
<td>161,172</td>
<td>20.0</td>
<td>40 - 70</td>
<td>20 - 35</td>
</tr>
<tr>
<td>(bbbb)$($qq$)</td>
<td>183</td>
<td>54.0</td>
<td>45 - 85</td>
<td>12 - 40</td>
</tr>
<tr>
<td>(bbbb, bbcc, cccc)$($qq$)</td>
<td>192-208</td>
<td>452.4</td>
<td>30 - 105</td>
<td>12 - 50</td>
</tr>
<tr>
<td>(cccc)$($qq$)</td>
<td>192-208</td>
<td>452.4</td>
<td>10 - 105</td>
<td>4 - 12</td>
</tr>
</tbody>
</table>

Table 8: List of the DELPHI searches for the Higgsstrahlung processes $e^+e^- \rightarrow H_1Z$ and $H_2Z$. 
Table 9: List of the DELPHI searches for the pair production process $e^+e^- \rightarrow \mathcal{H}_2\mathcal{H}_1$. 

<table>
<thead>
<tr>
<th>Process</th>
<th>$\sqrt{s}$(GeV)</th>
<th>$\mathcal{L}$ (pb$^{-1}$)</th>
<th>Mass ranges (GeV/$c^2$)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow \mathcal{H}_2\mathcal{H}_1 \rightarrow (...)(...)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 prongs</td>
<td>91</td>
<td>5.3</td>
<td>0.2 − 10, 0.2 − 10</td>
<td>[39]</td>
</tr>
<tr>
<td>$(\tau^+\tau^-)$(hadrons)</td>
<td>91</td>
<td>0.5</td>
<td>4 − 35, 4 − 35</td>
<td>[48]</td>
</tr>
<tr>
<td>$(\tau^+\tau^-)$(jet jet)</td>
<td>91</td>
<td>3.6</td>
<td>25 − 42, 25 − 42</td>
<td>[49]</td>
</tr>
<tr>
<td>(bb)(bb), (bb)(cc)</td>
<td>91</td>
<td>33.4</td>
<td>15 − 46, 15 − 46</td>
<td>[38]</td>
</tr>
<tr>
<td>$\tau^+\tau^-bb$</td>
<td>91</td>
<td>79.4</td>
<td>4 − 70, 4 − 70</td>
<td>[47]</td>
</tr>
<tr>
<td>bbbb</td>
<td>91</td>
<td>79.4</td>
<td>12 − 40, 20 − 70</td>
<td>[50]</td>
</tr>
<tr>
<td>bbbb, $\tau^+\tau^-bb$</td>
<td>133</td>
<td>6.0</td>
<td>40 − 68, 35 − 73</td>
<td>[51]</td>
</tr>
<tr>
<td>bbbb, $\tau^+\tau^-bb$</td>
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<td>20.0</td>
<td>40 − 70, 35 − 75</td>
<td>[40]</td>
</tr>
<tr>
<td>bbbb, $\tau^+\tau^-bb$</td>
<td>183</td>
<td>54.0</td>
<td>50 − 80, 25 − 105</td>
<td>[41]</td>
</tr>
<tr>
<td>bbbb, $\tau^+\tau^-bb$</td>
<td>189</td>
<td>158.0</td>
<td>65 − 90, 40 − 115</td>
<td>[42]</td>
</tr>
<tr>
<td>$\tau^+\tau^-bb$</td>
<td>192-208</td>
<td>452.4</td>
<td>50 − 100, 60 − 150</td>
<td>[43,44]</td>
</tr>
<tr>
<td>bbbb</td>
<td>192-208</td>
<td>452.4</td>
<td>12 − 100, 40 − 190</td>
<td>[43,44]</td>
</tr>
<tr>
<td>$\tau^+\tau^-\tau^+\tau^-$</td>
<td>189-208</td>
<td>570.9</td>
<td>4 − 90, 4 − 170</td>
<td>[50]</td>
</tr>
<tr>
<td>bbbb</td>
<td>189-208</td>
<td>610.2</td>
<td>12 − 70, 30 − 170</td>
<td>[50]</td>
</tr>
<tr>
<td>quarks or gluons</td>
<td>189-208</td>
<td>610.4</td>
<td>4 − 170, 4 − 170</td>
<td>[45]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process</th>
<th>$\sqrt{s}$(GeV)</th>
<th>$\mathcal{L}$ (pb$^{-1}$)</th>
<th>Mass ranges (GeV/$c^2$)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow \mathcal{H}_2\mathcal{H}_1 \rightarrow (\mathcal{H}_1\mathcal{H}_1)\mathcal{H}_1 \rightarrow (...)(...)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\gamma\gamma)$(\gamma\gamma)</td>
<td>91</td>
<td>12.5</td>
<td>0.5 − 60, &lt; 0.21</td>
<td>[46]</td>
</tr>
<tr>
<td>(4 prongs)(2 prongs)</td>
<td>91</td>
<td>12.9</td>
<td>0.5 − 60, 0.21 − 10</td>
<td>[46]</td>
</tr>
<tr>
<td>(hadrons)(hadrons)</td>
<td>91</td>
<td>15.1</td>
<td>1 − 60, 0.21 − 30</td>
<td>[46]</td>
</tr>
<tr>
<td>$(\tau^+\tau^-\tau^+\tau^-)(\tau^+\tau^-)$</td>
<td>91</td>
<td>15.1</td>
<td>9 − 60, 3.5 − 12</td>
<td>[46]</td>
</tr>
<tr>
<td>(any)(any)</td>
<td>161,172</td>
<td>20.0</td>
<td>40 − 70, 20 − 35</td>
<td>[40]</td>
</tr>
<tr>
<td>Mass ranges (GeV/c²)</td>
<td>Ref.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------------</td>
<td>------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 – 100</td>
<td>[52]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 – 110</td>
<td>[53]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 – 120</td>
<td>[54]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 – 85</td>
<td>[55]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 – 42</td>
<td>[56]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10: List of the L3 searches for the Higgsstrahlung processes $e^+e^- \rightarrow \mathcal{H}_1Z \rightarrow (...)(...)$. 

\[
\begin{array}{cccc}
\sqrt{s} \text{ (GeV)} & L \text{ (pb}^{-1} \text{)} & m_{\mathcal{H}_1} & m_{\mathcal{H}_2} \\
(\text{bb})(\text{any}), (\tau^+\tau^-)(\text{q\bar{q}}) & 189 & 176.4 & 60 – 100 \\
(\text{bb})(\text{any}), (\tau^+\tau^-)(\text{q\bar{q}}) & 192 – 202 & 233.2 & 60 – 110 \\
(\text{bb})(\text{any}), (\tau^+\tau^-)(\text{q\bar{q}}) & 203 – 209 & 217.3 & 60 – 120 \\
(\text{bb}, \text{cc}, \text{gg})(\text{any}) & 189 & 176.4 & 60 – 100 \\
(\text{bb}, \text{cc}, \text{gg})(\text{any}) & 192 – 202 & 233.2 & 60 – 110 \\
(\text{bb}, \text{cc}, \text{gg})(\text{any}) & 204 – 209 & 214.5 & 60 – 120 \\
(\text{bb}, \text{cc gg})(\text{qq}) & 189 – 209 & 626.9 & 30 – 85 \\
(\mathcal{H}_1 \rightarrow \text{bb,cc,gg})(\text{qq}) & & & 10 – 42 \\
\end{array}
\]

Table 11: List of the L3 searches for the pair production process $e^+e^- \rightarrow \mathcal{H}_2\mathcal{H}_1$. 

\[
\begin{array}{cccc}
\sqrt{s} \text{ (GeV)} & L \text{ (pb}^{-1} \text{)} & m_{\mathcal{H}_2} & m_{\mathcal{H}_1} \\
(\text{bb})(\text{bb}), (\text{bb})(\tau^+\tau^-), (\tau^+\tau^-)(\text{bb}) & 189 & 176.4 & 50 – 95 \\
(\text{bb})(\text{bb}), (\text{bb})(\tau^+\tau^-), (\tau^+\tau^-)(\text{bb}) & 192 – 202 & 233.2 & 50 – 105 \\
(\text{bb})(\text{bb}), (\text{bb})(\tau^+\tau^-), (\tau^+\tau^-)(\text{bb}) & 204 – 209 & 216.6 & 50 – 110 \\
\end{array}
\]
<table>
<thead>
<tr>
<th>$\mathcal{H}_1 Z \rightarrow (...) (\ldots)$</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\mathcal{L}$ (pb$^{-1}$)</th>
<th>Mass ranges (GeV/c$^2$)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(bb)(qq)$</td>
<td>161–172</td>
<td>20.4</td>
<td>40 – 80</td>
<td>[59,60]</td>
</tr>
<tr>
<td>$(bb)(qq)$</td>
<td>183</td>
<td>54.1</td>
<td>40 – 95</td>
<td>[61]</td>
</tr>
<tr>
<td>$(bb)(qq)$</td>
<td>189</td>
<td>172.1</td>
<td>40 – 100</td>
<td>[62]</td>
</tr>
<tr>
<td>$(bb)(qq)$</td>
<td>192–209</td>
<td>421.2</td>
<td>80 – 120</td>
<td>[63]</td>
</tr>
<tr>
<td>$(bb)(\nu\nu)$</td>
<td>161–172</td>
<td>20.4</td>
<td>50 – 70</td>
<td>[59,60]</td>
</tr>
<tr>
<td>$(bb)(\nu\nu)$</td>
<td>183</td>
<td>53.9</td>
<td>50 – 95</td>
<td>[61]</td>
</tr>
<tr>
<td>$(bb)(\nu\nu)$</td>
<td>189</td>
<td>171.4</td>
<td>50 – 100</td>
<td>[62]</td>
</tr>
<tr>
<td>$(bb)(\nu\nu)$</td>
<td>192–209</td>
<td>419.9</td>
<td>30 – 120</td>
<td>[63]</td>
</tr>
<tr>
<td>$(bb)(\tau^+\tau^-)$, $(\tau^+\tau^-)(qq)$</td>
<td>161–172</td>
<td>20.4</td>
<td>30 – 95</td>
<td>[59,60]</td>
</tr>
<tr>
<td>$(bb)(\tau^+\tau^-)$, $(\tau^+\tau^-)(qq)$</td>
<td>183</td>
<td>53.7</td>
<td>30 – 100</td>
<td>[61]</td>
</tr>
<tr>
<td>$(bb)(\tau^+\tau^-)$, $(\tau^+\tau^-)(qq)$</td>
<td>189</td>
<td>168.7</td>
<td>30 – 100</td>
<td>[62]</td>
</tr>
<tr>
<td>$(bb)(\tau^+\tau^-)$, $(\tau^+\tau^-)(qq)$</td>
<td>192–209</td>
<td>417.4</td>
<td>80 – 120</td>
<td>[63]</td>
</tr>
<tr>
<td>$(bb)(e^+e^-)$, $(bb)(\mu^+\mu^-)$</td>
<td>183</td>
<td>55.9</td>
<td>60 – 100</td>
<td>[61]</td>
</tr>
<tr>
<td>$(bb)(e^+e^-)$, $(bb)(\mu^+\mu^-)$</td>
<td>189</td>
<td>170.0</td>
<td>70 – 100</td>
<td>[62]</td>
</tr>
<tr>
<td>$(bb)(e^+e^-)$, $(bb)(\mu^+\mu^-)$</td>
<td>192–209</td>
<td>418.3</td>
<td>40 – 120</td>
<td>[63]</td>
</tr>
<tr>
<td>$(qq, gg)(\tau^+\tau^-, \nu\nu)$, $(\tau^+\tau^-)(qq)$</td>
<td>91</td>
<td>46.3</td>
<td>0 – 70</td>
<td>[64,65]</td>
</tr>
<tr>
<td>$(qq, gg)(e^+e^-, \mu^+\mu^-)$</td>
<td>91</td>
<td>46.3</td>
<td>20 – 70</td>
<td>[64,65]</td>
</tr>
<tr>
<td>$(any)(e^+e^-, \mu^+\mu^-)$</td>
<td>161–172</td>
<td>20.4</td>
<td>35 – 80</td>
<td>[59,60]</td>
</tr>
<tr>
<td>$(qq, gg)(qq)$</td>
<td>183</td>
<td>174.1</td>
<td>60 – 100</td>
<td>[66]</td>
</tr>
<tr>
<td>$(qq, gg)(qq)$</td>
<td>189</td>
<td>171.8</td>
<td>30 – 100</td>
<td>[66]</td>
</tr>
<tr>
<td>$(qq, gg)(\nu\nu)$</td>
<td>189</td>
<td>171.8</td>
<td>30 – 100</td>
<td>[66]</td>
</tr>
<tr>
<td>$(qq, gg)(\nu\nu)$</td>
<td>192–209</td>
<td>414.5</td>
<td>30 – 110</td>
<td>[67]</td>
</tr>
<tr>
<td>$(qq, gg)(\tau^+\tau^-, \tau^+\tau^-)(qq)$</td>
<td>183</td>
<td>168.7</td>
<td>30 – 100</td>
<td>[66]</td>
</tr>
<tr>
<td>$(qq, gg)(\tau^+\tau^-, \tau^+\tau^-)(qq)$</td>
<td>189</td>
<td>168.7</td>
<td>30 – 100</td>
<td>[66]</td>
</tr>
<tr>
<td>$(qq, gg)(e^+e^-, \mu^+\mu^-)$</td>
<td>183</td>
<td>174.1</td>
<td>60 – 100</td>
<td>[66]</td>
</tr>
<tr>
<td>$(qq, gg)(e^+e^-, \mu^+\mu^-)$</td>
<td>189</td>
<td>171.8</td>
<td>30 – 100</td>
<td>[66]</td>
</tr>
<tr>
<td>$(qq, gg)(e^+e^-, \mu^+\mu^-)$</td>
<td>192–209</td>
<td>418.3</td>
<td>40 – 120</td>
<td>[63]</td>
</tr>
<tr>
<td>$(e^+e^- \rightarrow \mathcal{H}_2 Z \rightarrow (\mathcal{H}_1 \mathcal{H}_1)Z \rightarrow (...) (\ldots))$</td>
<td>$m_{\mathcal{H}_2}$</td>
<td>$m_{\mathcal{H}_1}$</td>
<td>$m_{\mathcal{H}_2}$</td>
<td>$m_{\mathcal{H}_1}$</td>
</tr>
<tr>
<td>$(qqqq)(\nu\nu)$</td>
<td>91</td>
<td>46.3</td>
<td>10 – 75</td>
<td>0 – 35</td>
</tr>
<tr>
<td>$(bbbbb)(qq)$</td>
<td>183</td>
<td>54.1</td>
<td>40 – 80</td>
<td>10.5 – 38</td>
</tr>
<tr>
<td>$(bbbbb)(qq)$</td>
<td>189</td>
<td>172.1</td>
<td>40 – 100</td>
<td>10.5 – 48</td>
</tr>
<tr>
<td>$(bbbbb)(qq)$</td>
<td>192–209</td>
<td>421.2</td>
<td>80 – 120</td>
<td>12 – $m_{\mathcal{H}_2}/2$</td>
</tr>
<tr>
<td>$(bbbbb)(\nu\nu)$</td>
<td>183</td>
<td>53.9</td>
<td>50 – 95</td>
<td>10.5 – $m_{\mathcal{H}_2}/2$</td>
</tr>
<tr>
<td>$(qqqq)(\nu\nu)$</td>
<td>189</td>
<td>171.4</td>
<td>50 – 100</td>
<td>10.5 – $m_{\mathcal{H}_2}/2$</td>
</tr>
<tr>
<td>$(bbbbb)(\nu\nu)$</td>
<td>199–209</td>
<td>207.2</td>
<td>100 – 110</td>
<td>12 – $m_{\mathcal{H}_2}/2$</td>
</tr>
<tr>
<td>$(bbbbb)(\tau^+\tau^-)$</td>
<td>183</td>
<td>53.7</td>
<td>30 – 100</td>
<td>10.5 – $m_{\mathcal{H}_2}/2$</td>
</tr>
<tr>
<td>$(bbbbb)(\tau^+\tau^-)$</td>
<td>189</td>
<td>168.7</td>
<td>30 – 100</td>
<td>10.5 – $m_{\mathcal{H}_2}/2$</td>
</tr>
<tr>
<td>$(bbbbb, bbr\tau^+\tau^-, \tau^+\tau^-\tau^+\tau^-)$ $(\nu\nu, e^+e^-, \mu^+\mu^-)$</td>
<td>189–209</td>
<td>598.5</td>
<td>45 – 90</td>
<td>2 – 10.5</td>
</tr>
</tbody>
</table>

Table 12: List of the OPAL searches for the Higgsstrahlung processes $e^+e^- \rightarrow \mathcal{H}_1 Z$ and $\mathcal{H}_2 Z$. 
<table>
<thead>
<tr>
<th>Process</th>
<th>( \sqrt{s} ) (GeV)</th>
<th>( L ) (pb(^{-1}))</th>
<th>Mass ranges (GeV/c(^2))</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(bb)(bb)</td>
<td>130–136</td>
<td>5.2</td>
<td>( \Sigma = 80 - 130 ) ( \Delta = 0 - 50 )</td>
<td>[60]</td>
</tr>
<tr>
<td>(bb)(bb)</td>
<td>161</td>
<td>10.0</td>
<td>( \Sigma = 80 - 130 ) ( \Delta = 0 - 60 )</td>
<td>[59,60]</td>
</tr>
<tr>
<td>(bb)(bb)</td>
<td>172</td>
<td>10.4</td>
<td>( \Sigma = 80 - 130 ) ( \Delta = 0 - 60 )</td>
<td>[59,60]</td>
</tr>
<tr>
<td>(bb)(bb)</td>
<td>183</td>
<td>54.1</td>
<td>( \Sigma = 80 - 150 ) ( \Delta = 0 - 60 )</td>
<td>[61]</td>
</tr>
<tr>
<td>(bb)(bb)</td>
<td>189</td>
<td>172.1</td>
<td>( \Sigma = 80 - 180 ) ( \Delta = 0 - 70 )</td>
<td>[62]</td>
</tr>
<tr>
<td>(bb)(bb)</td>
<td>192</td>
<td>28.9</td>
<td>( \Sigma = 83 - 183 ) ( \Delta = 0 - 70 )</td>
<td>[10]</td>
</tr>
<tr>
<td>(bb)(bb)</td>
<td>196</td>
<td>74.8</td>
<td>( \Sigma = 80 - 187 ) ( \Delta = 0 - 70 )</td>
<td>[10]</td>
</tr>
<tr>
<td>(bb)(bb)</td>
<td>200</td>
<td>77.2</td>
<td>( \Sigma = 80 - 191 ) ( \Delta = 0 - 70 )</td>
<td>[10]</td>
</tr>
<tr>
<td>(bb)(bb)</td>
<td>202</td>
<td>36.1</td>
<td>( \Sigma = 80 - 193 ) ( \Delta = 0 - 70 )</td>
<td>[10]</td>
</tr>
<tr>
<td>(bb)(bb)</td>
<td>199–209</td>
<td>207.3</td>
<td>( \Sigma = 120 - 190 ) ( \Delta = 0 - 70 )</td>
<td>[10]</td>
</tr>
<tr>
<td>(bb)(bb)</td>
<td>199–209</td>
<td>207.3</td>
<td>( \Sigma = 100 - 140 ) ( \Delta = 60 - 100 )</td>
<td>[10]</td>
</tr>
<tr>
<td>(bb)((\tau^+\tau^-)), ((\tau^+\tau^-))(bb)</td>
<td>161</td>
<td>10.0</td>
<td>40 – 160 ( \Delta = 52 - 160 )</td>
<td>[59,60]</td>
</tr>
<tr>
<td>(bb)((\tau^+\tau^-)), ((\tau^+\tau^-))(bb)</td>
<td>172</td>
<td>10.4</td>
<td>37 – 160 ( \Delta = 28 - 160 )</td>
<td>[59,60]</td>
</tr>
<tr>
<td>(bb)((\tau^+\tau^-)), ((\tau^+\tau^-))(bb)</td>
<td>183</td>
<td>53.7</td>
<td>( \Sigma = 70 - 170 ) ( \Delta = 0 - 70 )</td>
<td>[61]</td>
</tr>
<tr>
<td>(bb)((\tau^+\tau^-)), ((\tau^+\tau^-))(bb)</td>
<td>189</td>
<td>168.7</td>
<td>( \Sigma = 70 - 190 ) ( \Delta = 0 - 90 )</td>
<td>[62]</td>
</tr>
<tr>
<td>(bb)((\tau^+\tau^-)), ((\tau^+\tau^-))(bb)</td>
<td>192</td>
<td>28.7</td>
<td>( \Sigma = 10 - 174 ) ( \Delta = 0 - 182 )</td>
<td>[10]</td>
</tr>
<tr>
<td>(bb)((\tau^+\tau^-)), ((\tau^+\tau^-))(bb)</td>
<td>196</td>
<td>74.7</td>
<td>( \Sigma = 10 - 182 ) ( \Delta = 0 - 191 )</td>
<td>[10]</td>
</tr>
<tr>
<td>(bb)((\tau^+\tau^-)), ((\tau^+\tau^-))(bb)</td>
<td>200</td>
<td>74.8</td>
<td>( \Sigma = 10 - 182 ) ( \Delta = 0 - 191 )</td>
<td>[10]</td>
</tr>
<tr>
<td>(bb)((\tau^+\tau^-)), ((\tau^+\tau^-))(bb)</td>
<td>202</td>
<td>35.4</td>
<td>( \Sigma = 10 - 174 ) ( \Delta = 0 - 182 )</td>
<td>[10]</td>
</tr>
<tr>
<td>(bb)((\tau^+\tau^-)), ((\tau^+\tau^-))(bb)</td>
<td>199–209</td>
<td>203.6</td>
<td>( \Sigma = 70 - 190 ) ( \Delta = 0 - 90 )</td>
<td>[10]</td>
</tr>
<tr>
<td>(qq)((\tau^+\tau^-)), ((\tau^+\tau^-))(qq)</td>
<td>91</td>
<td>46.3</td>
<td>12 – 75 ( \Delta = 10 - 78 )</td>
<td>[64,65]</td>
</tr>
</tbody>
</table>

Table 13: List of the OPAL searches for the pair production process \( e^+e^- \rightarrow \mathcal{H}_2\mathcal{H}_1 \rightarrow (\mathcal{H}_1\mathcal{H}_1) \rightarrow (...)(...) \). The symbols \( \Sigma \) and \( \Delta \) stand for the mass sum \( m_{\mathcal{H}_2} + m_{\mathcal{H}_1} \) and mass difference \( |m_{\mathcal{H}_2} - m_{\mathcal{H}_1}| \).
Appendix B: Limits on topological cross-sections

The tables presented below summarise the 95% CL upper bounds, as a function of the Higgs boson masses, of the scaling factor $S_{95}$ defined in the text (see Eq. 16). Tables 14, 15 and 16 refer to final-state topologies arising from the Higgsstrahlung processes $e^+e^- \rightarrow H_1Z$ and $e^+e^- \rightarrow (H_2 \rightarrow H_1H_1)Z$; Tables 18 to 21 refer to those arising from the pair production processes $e^+e^- \rightarrow H_2H_1$ and $e^+e^- \rightarrow (H_2 \rightarrow H_1H_1)H_1$. The corresponding figures, showing the same results, are mentioned in the table captions.
Table 14: The 95% CL upper bound, \( S_{95} \), obtained for the normalised cross-section (see text) of the Higgsstrahlung process \( e^+e^- \rightarrow H_1Z \), as a function of the Higgs boson mass. The numbers listed in this table correspond to the observed limit (full line) in Figure 2, which is reproduced from Ref. [3]. In the columns labelled (a) the Higgs boson is assumed to decay as in the Standard Model; in columns (b) it is assumed to decay exclusively to \( bb \) and in columns (c) exclusively to \( \tau^+\tau^- \).

<table>
<thead>
<tr>
<th>( m_{H_1} ) (GeV/c(^2))</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>( m_{H_1} ) (GeV/c(^2))</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.0204</td>
<td>0.0154</td>
<td>0.0925</td>
<td>62</td>
<td>0.0273</td>
<td>0.0218</td>
<td>0.0287</td>
</tr>
<tr>
<td>14</td>
<td>0.0176</td>
<td>0.0143</td>
<td>0.0899</td>
<td>64</td>
<td>0.0276</td>
<td>0.0218</td>
<td>0.0287</td>
</tr>
<tr>
<td>16</td>
<td>0.0158</td>
<td>0.0134</td>
<td>0.0923</td>
<td>66</td>
<td>0.0271</td>
<td>0.0246</td>
<td>0.0287</td>
</tr>
<tr>
<td>18</td>
<td>0.0150</td>
<td>0.0131</td>
<td>0.0933</td>
<td>68</td>
<td>0.0291</td>
<td>0.0274</td>
<td>0.0271</td>
</tr>
<tr>
<td>20</td>
<td>0.0156</td>
<td>0.0139</td>
<td>0.1060</td>
<td>70</td>
<td>0.0320</td>
<td>0.0301</td>
<td>0.0297</td>
</tr>
<tr>
<td>22</td>
<td>0.0177</td>
<td>0.0156</td>
<td>0.1080</td>
<td>72</td>
<td>0.0421</td>
<td>0.0380</td>
<td>0.0351</td>
</tr>
<tr>
<td>24</td>
<td>0.0194</td>
<td>0.0174</td>
<td>0.1110</td>
<td>74</td>
<td>0.0469</td>
<td>0.0424</td>
<td>0.0350</td>
</tr>
<tr>
<td>26</td>
<td>0.0207</td>
<td>0.0186</td>
<td>0.1140</td>
<td>76</td>
<td>0.0435</td>
<td>0.0441</td>
<td>0.0316</td>
</tr>
<tr>
<td>28</td>
<td>0.0223</td>
<td>0.0195</td>
<td>0.1110</td>
<td>78</td>
<td>0.0467</td>
<td>0.0475</td>
<td>0.0281</td>
</tr>
<tr>
<td>30</td>
<td>0.0203</td>
<td>0.0181</td>
<td>0.0893</td>
<td>80</td>
<td>0.0539</td>
<td>0.0585</td>
<td>0.0222</td>
</tr>
<tr>
<td>32</td>
<td>0.0193</td>
<td>0.0173</td>
<td>0.0796</td>
<td>82</td>
<td>0.0762</td>
<td>0.0816</td>
<td>0.0257</td>
</tr>
<tr>
<td>34</td>
<td>0.0191</td>
<td>0.0172</td>
<td>0.0682</td>
<td>84</td>
<td>0.112</td>
<td>0.118</td>
<td>0.0296</td>
</tr>
<tr>
<td>36</td>
<td>0.0241</td>
<td>0.0187</td>
<td>0.0653</td>
<td>86</td>
<td>0.153</td>
<td>0.152</td>
<td>0.0331</td>
</tr>
<tr>
<td>38</td>
<td>0.0299</td>
<td>0.0235</td>
<td>0.0634</td>
<td>88</td>
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Table 15: The 95% CL upper bound, $S_{95}$, obtained for the normalised cross-section (see text) of the Higgsstrahlung cascade process $e^+e^- \rightarrow (H_2 \rightarrow H_1H_1)Z \rightarrow (b\bar{b}b\bar{b})Z$, as a function of the Higgs boson masses $m_{H_1}$ and $m_{H_2}$. The numbers correspond to the contours shown in Figure 3 (a).
Table 16: The 95% CL upper bound, \( S_{95} \), obtained for the normalised cross-section (see text) of the Higgsstrahlung cascade process \( e^+e^- \rightarrow (H_2 \rightarrow H_1H_1)Z \rightarrow (\tau^+\tau^-\tau^+\tau^-)Z \), as a function of the Higgs boson masses \( m_{H_1} \) and \( m_{H_2} \). The numbers correspond to the contours shown in Figure 3 (b).
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<th>(c)</th>
<th>(d)</th>
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Table 17: The 95% CL upper bound, $S_{95}$, obtained for the normalised cross-section (see text) of the pair production process $e^+e^-$ $\rightarrow H_2H_1$, as a function of the Higgs boson mass sum $m_{H_1} + m_{H_2}$. The bounds are given for the particular case where $m_{H_2}$ and $m_{H_1}$ are approximately equal. This occurs, for example, in the CP-conserving MSSM scenario $m_{h_{\text{max}}}$ for $\tan \beta$ greater than 10 and small $m_{H_2} = m_{H_1}$. The numbers listed in this table correspond to the four plots in Figure 4 (see the corresponding labels). For $m_{H_1} + m_{H_2}$ less than 30 GeV/c², the bounds are derived from the measured decay width of the Z boson, see Section 3.2. Columns labelled (a): the Higgs boson decay branching ratios correspond to the $m_{h_{\text{max}}}$ benchmark scenario with $\tan \beta = 10$, giving 94% for $H_1 \rightarrow b\bar{b}$, 6% for $H_1 \rightarrow \tau^+\tau^-$, 92% for $H_2 \rightarrow b\bar{b}$ and 8% for $H_2 \rightarrow \tau^+\tau^-$. Columns (b): both Higgs bosons are assumed to decay exclusively to $b\bar{b}$; columns (c): one Higgs boson is assumed to decay exclusively to $b\bar{b}$ only and the other exclusively to $\tau^+\tau^-$; columns (d): both Higgs bosons are assumed to decay exclusively to $\tau^+\tau^-$. 


Table 18: The 95% CL upper bound, $S_{95}$, obtained for the normalised cross-section (see text) of the pair production process $e^+e^- \rightarrow \mathcal{H}_1 \mathcal{H}_1 \rightarrow b\bar{b}b\bar{b}$, as a function of the Higgs boson masses $m_{\mathcal{H}_1}$ and $m_{\mathcal{H}_2}$. The numbers correspond to the contours shown in Figure 5 (a).
The numbers correspond to the contours shown in Figure 5 (b).

Table 19: The 95% C.L. upper bound $\sigma^{\text{max}}$ obtained for the normalized cross-section $\sigma^b_L$ as a function of the Higgs boson mass and $m_H$. $H^\pm_1 \rightarrow \gamma \gamma$ for the partial production process $e^- e^+ \rightarrow H^\pm_1 \rightarrow \gamma \gamma$ (see text).

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\[
\left( \frac{\sigma^b_L}{\Lambda_{\text{QCD}}} \right)^{\text{max}}^{\text{H}}
\]

\[
\left( \frac{\sigma^b_L}{\Lambda_{\text{QCD}}} \right)^{\text{max}}^{\text{H}}
\]
Table 20: The 95% CL upper bound, $S_{95}$, obtained for the normalised cross-section (see text) of the pair production cascade process $e^+e^- \rightarrow (H_2 \rightarrow H_1 H_1) H_1 \rightarrow (bbbb)bb$, as a function of the Higgs boson masses $m_{H_1}$ and $m_{H_2}$. The numbers correspond to the contours shown in Figure 6 (a).
### Table 21

The 95% CL upper bound, $S_{95}$, obtained for the normalised cross-section (see text) of the pair production cascade process $e^+e^- \rightarrow (H_2 \rightarrow H_1 H_1)H_1 \rightarrow (\tau^+\tau^-\tau^+\tau^-)\tau^+\tau^-$, as a function of the Higgs boson masses $m_{H_1}$ and $m_{H_2}$. The numbers correspond to the contours shown in Figure 6 (b).

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Appendix C: List of authors

The ALEPH, DELPHI, L3 and OPAL Collaborations have provided the inputs for the combined results presented in this paper. The LEP Working Group for Higgs Boson Searches has performed the combinations. The Working Group consists of members of the four collaborations and of theorists among whom S. Heinemeyer\(^9\), A. Pilaftsis\(^{10}\) and G. Weiglein\(^{11}\) are authors of this paper. The lists of authors from the collaborations follow.

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\(^{11}\)Institute for Particle Physics Phenomenology, University of Durham, Durham DH1 3LE, UK
Appendix C1: The ALEPH Collaboration


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Appendix C4: The OPAL Collaboration

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Figure 1: Contours of the observed p-values, $1 - CL_b$, indicating the statistical significances of local excesses in the data. Plots (a) and (b) refer to the CP-conserving MSSM benchmark scenario $m_h$-max and plots (c) and (d) to the CP-violating scenario CPX. For each scenario, the parameter space is shown in two projections. Regions which are not part of the parameter space (labelled “Theoretically Inaccessible”) are shown in light-grey or yellow. In the medium-grey or light-green regions the data show an excess of less than one standard deviation above the expected background. Similarly, in the dark-grey or dark-green regions the excess is between one and two standard deviations while in the darkest-grey or blue regions it is between two and three standard deviations. In plots (c) and (d), two small regions with excesses larger than three standard deviations are shown in white. The dashed lines show the expected exclusion limit at 95% CL. The hatched areas represent regions where the median expected value of $CL_s$ in the background hypothesis is larger than 0.4; apparent excesses in these regions would not be significant.
Figure 2: The 95% CL upper bounds, $S_{95}$ (see text), for various topological cross-sections motivated by the Higgsstrahlung process $e^+e^- \to H_1Z$, as a function of the Higgs boson mass (the figure is reproduced from Ref. [3]). The full lines represent the observed limits. The dark (green) and light (yellow) shaded bands around the median expectations (dashed lines) correspond to the 68% and 95% probability bands. The horizontal lines correspond to the Standard Model cross-sections. In part (a) the Higgs boson decay branching ratios are assumed to be those predicted by the Standard Model; in part (b) the Higgs boson is assumed to decay exclusively to $b\bar{b}$ and in part (c) exclusively to $\tau^+\tau^-$. 
Figure 3: Contours of the 95% CL upper bound, $S_{95}$ (see text), for various topological cross-sections motivated by the Higgsstrahlung cascade process $e^+e^- \rightarrow (H_2 \rightarrow H_1 H_1)Z$, projected onto the $(m_{H_2}, m_{H_1})$ plane. The scales for the shadings are given on the right-hand side of each plot. In plot (a) the $H_1$ boson is assumed to decay exclusively to $b\bar{b}$ and in plot (b) exclusively to $\tau^+\tau^-$; in plot (c) it is assumed to decay with equal probabilities to $b\bar{b}$ and to $\tau^+\tau^-$. 

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Figure 4: The 95% CL upper bounds, $S_{95}$ (see text), for various topological cross-sections motivated by the pair production process $e^+e^- \rightarrow \mathcal{H}_2\mathcal{H}_1$. The bounds are obtained for the particular case where $m_{\mathcal{H}_2}$ and $m_{\mathcal{H}_1}$ are approximately equal. Such is the case, for example, in the CP-conserving MSSM scenario $m_{\mathcal{H}}$-max for $\tan \beta$ greater than 10 and small $m_{\mathcal{H}_0}$ ($\equiv m_A$). The abscissa is the sum of the two Higgs boson masses. The full lines represent the observed limits. The dark (green) and light (yellow) shaded bands around the median expectations (dashed lines) correspond to the 68% and 95% probability bands. The curves which complete the exclusions at low masses are obtained using the constraint from the measured decay width of the Z boson, see Section 3.2. Plot (a): the Higgs boson decay branching ratios correspond to the $m_{\mathcal{H}}$-max benchmark scenario with $\tan \beta=10$, namely 94% $\mathcal{H}_1 \rightarrow b\bar{b}$, 6% $\mathcal{H}_1 \rightarrow \tau^+\tau^-$, 92% $\mathcal{H}_2 \rightarrow b\bar{b}$ and 8% $\mathcal{H}_2 \rightarrow \tau^+\tau^-$; plot (b): both Higgs bosons are assumed to decay exclusively to $b\bar{b}$; plot (c): one of the Higgs bosons is assumed to decay exclusively to $b\bar{b}$ and the other exclusively to $\tau^+\tau^-$; plot (d): both Higgs bosons are assumed to decay exclusively to $\tau^+\tau^-$. 
Figure 5: Contours of the 95% CL upper bound, $S_{95}$ (see text), for various topological cross-sections motivated by the pair production process $e^+e^- \rightarrow \mathcal{H}_2 \mathcal{H}_1$, projected onto the $(m_{\mathcal{H}_2}, m_{\mathcal{H}_1})$ plane. The scales in terms of the shadings are given on the right-hand side of each plot. In plot (a) both Higgs bosons are assumed to decay exclusively to $b\bar{b}$ and in plot (b) exclusively to $\tau^+\tau^-$. In plot (c) the $\mathcal{H}_2$ boson is assumed to decay exclusively to $b\bar{b}$ and the $\mathcal{H}_1$ boson exclusively to $\tau^+\tau^-$ and in plot (d) the $\mathcal{H}_1$ boson is assumed to decay exclusively to $b\bar{b}$ and the $\mathcal{H}_2$ boson exclusively to $\tau^+\tau^-$. The dashed lines represent the approximate kinematic limits of the processes.
Figure 6: Contours of the 95% CL upper bound, $S_{95}$ (see text), for various topological cross-sections motivated by the pair production cascade process $e^+e^- \rightarrow (H_2 \to H_1 H_1) H_1$, projected onto the ($m_{H_2}$, $m_{H_1}$) plane. The scales in terms of the shadings are given on the right-hand side of each plot. In plot (a) the $H_1$ boson is assumed to decay exclusively to $bb$ and in plot (b) exclusively to $\tau^+\tau^-$. In plot (c) the $H_1$ boson is assumed to decay with equal probability to $bb$ and to $\tau^+\tau^-$. The dashed line in part (a) represents the approximate kinematic limit of the process.
Figure 7: Exclusions, at 95% CL (medium-grey or light-green) and the 99.7% CL (dark-grey or dark-green), in the case of the CP-conserving $m_h$-max benchmark scenario, for $m_t = 174.3$ GeV/c$^2$. The figure shows the theoretically inaccessible domains (light-grey or yellow) and the regions excluded by this search, in four projections of the MSSM parameters: (a): ($m_h$, $m_A$); (b): ($m_h$, $\tan\beta$); (c): ($m_A$, $\tan\beta$); (d): ($m_{H^\pm}$, $\tan\beta$). The dashed lines indicate the boundaries of the regions which are expected to be excluded, at 95% CL, on the basis of Monte Carlo simulations with no signal. In the ($m_h$, $\tan\beta$) projection (plot (b)), the upper boundary of the parameter space is indicated for four values of the top quark mass; from left to right: $m_t = 169.3$, 174.3, 179.3 and 183.0 GeV/c$^2$. 
Figure 8: Domains of $\tan \beta$ which are excluded at the 95% CL (light-grey or light-green) and the 99.7% CL (dark-grey or dark-green), in the case of the CP-conserving $m_h$-max benchmark scenario, as a function of the assumed top quark mass.
Figure 9: Exclusions in the case of the CP-conserving $m_h$-max benchmark scenario, variant (a) (see Section 2.1.1.). See the caption of Figure 7 for the legend. Note the small domains at $m_h$ between 60 and 75 $\text{GeV}/c^2$, small $m_A$ and $\tan \beta < 0.9$ which, although excluded at the 95% CL, are not excluded at the 99.7% CL.
Figure 10: Exclusions in the case of the CP-conserving $m_h$-max benchmark scenario, variant (b) (see Section 2.1.1.). See the caption of Figure 7 for the legend.
Figure 11: Exclusions in the case of the CP-conserving no-mixing benchmark scenario. See the caption of Figure 7 for the legend. Note the small domain at $m_h$ between 75 and 80 GeV/c$^2$, small $m_A$ and $\tan\beta < 0.7$ which is not excluded at the 95% CL.
Figure 12: Exclusions in the case of the CP-conserving no-mixing benchmark scenario, variant (a) (see Section 2.1.2). See the caption of Figure 7 for the legend. In the hatched domain ($\tan \beta < 0.7$), the contributions from top and stop quark loops to the radiative corrections are large and uncertain. Note the small domain at $m_h$ between 56 and 72 GeV/c$^2$, small $m_A$, and $\tan \beta < 1$ which, although excluded at the 95% CL, is not excluded at the 99.7% CL.
Figure 13: Exclusions in the case of the CP-conserving large-$\mu$ benchmark scenario (see Section 2.1.3). See the caption of Figure 7 for the legend. In the hatched domain ($\tan\beta < 0.7$), the contributions from top and stop quark loops to the radiative corrections become large and uncertain; hence, no exclusions can be claimed there.
Figure 14: Exclusions in the case of the gluophobic benchmark scenario (see Section 2.1.3). See the caption of Figure 7 for the legend.
Figure 15: Exclusions in the case of the CP-conserving small-$\alpha_{\text{eff}}$ benchmark scenario (see Section 2.1.3). See the caption of Figure 7 for the legend.
Figure 16: Exclusions, at 95% CL (medium-grey or light-green) and the 99.7% CL (dark-grey or dark-green), for the CP-violating CPX scenario with $m_t = 174.3$ GeV/c$^2$. The figure shows the theoretically inaccessible domains (light-grey or yellow) and the regions excluded by the present search, in four projections of the MSSM parameter space: $(m_{H_2}, m_{H_2})$, $(m_{H_1}, \tan \beta)$, $(m_{H_2}, \tan \beta)$ and $(m_{H^+}, \tan \beta)$. The dashed lines indicate the boundaries of the regions expected to be excluded, at the 95% CL, on the basis of Monte Carlo simulations with no signal. In each scan point, the more conservative of the two theoretical calculations, FeynHiggs or CPH, is used.
Figure 17: Cross-sections, as a function of $\tan \beta$, for some of the dominant signal processes, in the CP-violating scenario CPX, using the FeynHiggs calculation, with a centre-of-mass energy of 202 GeV, $m_t = 175$ GeV/$c^2$, and $m_{H_1}$ between 35 and 45 GeV/$c^2$. 

\[ \sqrt{s} = 202 \text{ GeV} \]
Figure 18: Exclusions, in the case of the CP-violating CPX scenario, for the two theoretical approaches, CPH and FeynHiggs. See the caption of Figure 16 for the legend. In part (a) the CPH calculation is used and in part (b) the FeynHiggs calculation. In part (c) the procedure is adopted where, in each scan point of the parameter space, the more conservative of the two calculations is used.
Figure 19: Exclusions, in the case of the CP-violating CPX scenario, for four top quark masses: $m_t = 169.3$ GeV/$c^2$, $174.3$ GeV/$c^2$, $179.3$ GeV/$c^2$ and $183.0$ GeV/$c^2$. See the caption of Figure 16 for the legend.
Figure 20: Exclusions, in the case of the CPX scenario with various CP-violating phases, \( \arg(A) = \arg(m_g) \): 0°, 30°, 60°, 90° (the CPX value), 135° and 180°. See the caption of Figure 16 for the legend. In the hatched region in part (f) the calculations are uncertain (see text).
Figure 21: Exclusions, for the CP-violating CPX scenario with various values of the Higgs mass parameter $\mu$: 500 GeV, 1000 GeV, 2000 GeV (the standard CPX value) and 4000 GeV. See the caption of Figure 16 for the legend. In the hatched region in part (d) the calculations are uncertain (see text).
$M_{\text{SUSY}} = 500\,\text{GeV}$

Figure 22: Exclusions, for the CP-violating CPX scenario with various values of the soft SUSY-breaking scale $M_{\text{SUSY}}$. (a): $M_{\text{SUSY}} = 500\,\text{GeV}$ (the standard CPX value); (b): $M_{\text{SUSY}} = 1000\,\text{GeV}$ while all other parameters are kept at their standard CPX values; (c): $M_{\text{SUSY}} = 1000\,\text{GeV}$ while $A$, $m_g$ and $\mu$ are “scaled” to 2000 GeV, 2000 GeV and 4000 GeV, respectively. See the caption of Figure 16 for the legend.