Period Changes in Ultra-compact Double White Dwarfs

T. R. Marsh¹, G. Nelemans²
1. Department of Physics, University of Warwick, Coventry CV4 7AL, UK
2. Department of Astrophysics, Radboud University Nijmegen, Nijmegen, The Netherlands

Accepted ; Received ; in original form

ABSTRACT
In recent years there has been much interest in the nature of two stars, V407 Vul and RX J0806+1527, which are widely thought to be binary white dwarfs of very short orbital period, 570 and 321 seconds respectively. As such they should be strong sources of gravitational waves and possible ancestors of the accreting AM CVn stars. Monitoring at X-ray and optical wavelengths has established that the period of each star is decreasing, at rates compatible with that expected from gravitational radiation. This has been taken to support the “unipolar inductor” model in which the white dwarfs are detached and the X-rays produced by the dissipation of magnetically-induced electric currents. In this paper we show that this interpretation is incorrect because it ignores associated torques which transfer angular momentum between the spin of the magnetic white dwarf and the orbit. We show that this torque is \( \sim 10^5 \) times larger than the GR term in the case of V407 Vul and \( \sim 10 \) times larger for RX J0806+1527. For V407 Vul, the unipolar inductor model can only survive if the white dwarf spins \( \sim 100 \) times faster than the orbit. Since this could only come about through accretion, the validity of the unipolar inductor appears questionable for this star. We also consider whether accretion models can fit the observed spin-up, concluding that they can, provided that a mechanism exists for driving the mass transfer rate away from its equilibrium value.

Key words: binaries: close — accretion, accretion discs — gravitational waves — white dwarfs — novae, cataclysmic variables

1 INTRODUCTION
Over the past decade, observations have established the existence of a population of some 100-200 million double white dwarfs within our Galaxy (Marsh et al. 1995; Napiwotzki et al. 2003). These and their descendants are thought likely to be a dominant source of low frequency gravitational waves in the Galaxy (e.g. Hils et al. 1990; Nelemans et al. 2001b), and are a possible progenitor population of Type Ia supernovae. A significant fraction of these binary stars are close enough that gravitational wave losses will cause them to undergo mass transfer within a Hubble time. Most will merge to form single stars, variously suggested to be Type Ia supernovae, or in the majority of cases, sdB, sdO or R CrB stars (e.g. Webbink 1984; Iben 1990; Saio & Jeffery 2002). If any systems survive the onset of mass transfer as binary stars, then they would become semi-detached accreting binary stars with white dwarf donors, properties matched by the AM CVn stars, which feature helium-dominated spectra and orbital periods which range from 10 to 65 minutes.

The question of survival as a binary is key to whether double white dwarfs are the ancestors of AM CVn stars and is important to the prediction of gravitational waves. The nearest we can get to a proof that the start of mass transfer can be survived is to identify accreting pairs of double white dwarfs with periods short of 10 minutes since there are alternative routes for systems with periods longer than this (Nelemans et al. 2001a; Podsiadlowski et al. 2003). In this context, two stars, V407 Vul and RX J0806+1527 have generated much interest in the past few years because they show periods of 570 and 321 seconds respectively, and there are several reasons to think that these may be orbital (Cropper et al. 1998), rather than, for example, the spin of a magnetic accreting white dwarf as seen in “intermediate polars” (but see Norton et al. 2004). The key features of these systems are (i) X-ray light curves which are off for about half the period (Cropper et al. 1998; Israel et al. 1999), suggestive of a spot of emission on a spinning star, (ii) optical and infrared light curves that show the same period and no other (Ramsay et al. 2000, 2002a; Israel et al. 2002), (iii) weak optical line emission (Ramsay et al. 2002b; Israel et al. 2002), and (iv) very soft X-ray spectra. Items ii–iv in partic-
ular count against Norton et al. (2004)’s intermediate polar model.

Even on the assumption that we are seeing orbital periods, questions remain over the nature of these stars. There are currently three popular models which are (a) the polar model in which we see X-rays from the magnetic poles of a white dwarf locked to its companion (Cropper et al. 1998), (b) the unipolar inductor (UI) model in which X-rays come from the dissipation of electric currents generated from a slight asynchronism between the spin of a magnetic white dwarf and its companion (Wu et al. 2002), and (c) the direct impact model in which X-rays come from the point at which the mass transfer stream impacts the white dwarf, which can happen directly in these very compact systems (Nelemans et al. 2001a; Marsh & Steeghs 2002; Ramsay et al. 2002b). The polar and direct impact models rely on accretion and so imply that the systems have survived contact; the system in the UI model is not in contact and therefore has not necessarily had to survive it. The polar model has fallen somewhat out of favour because of the weak optical line emission, an absence of, or, at best, weak polarisation (Ramsay et al. 2002b; Reinsch et al. 2004; Israel et al. 2004) and the X-ray spectra, which are unusually soft, even for polars (but see Cropper et al. 2004). The two most promising models left are the UI and direct impact models. It was soon realised that period changes might help distinguish between these models since on the detached UI model it was predicted that the period would decrease while in accreting models the period should increase as the donor expands. In a series of papers, the periods of both V407 Vul and RX J0806+1527 have been definitively established to be decreasing (Strohmayer 2002; Hakala et al. 2003; Strohmayer 2003; Israel et al. 2004; Hakala et al. 2004; Ramsay et al. 2005; Strohmayer 2005), which, in spite of some problems which face the UI model (Marsh & Steeghs 2002; Barros et al. 2005), has been taken to be strong evidence in its favour (Cropper et al. 2004).

In much of this work it has been assumed, almost by default, that, since the systems in the unipolar inductor (UI) model are detached, their orbital periods change at a rate governed by gravitational radiation alone (see e.g. Israel et al. 2003; Cropper et al. 2004; Hakala et al. 2004; Willes & Wu 2005). In this paper we show that this is very far from being the case, and that on the contrary the observed period changes rule out Wu et al. (2002)’s original model once-and-for-all. We further show that it can only survive in a very different form which necessarily requires accretion to have occurred, raising the question of why it ever should have ceased. We begin by discussing the period change expected for the UI model.

2 PERIOD CHANGES FROM INDUCTION TORQUES

The key point about the UI model which has not always been appreciated is that although the two white dwarfs are detached, their period evolution reflects the combination of angular momentum loss through gravitational radiation and induction-driven angular momentum interchange within the system. Although this was stated explicitly by Wu et al. (2002), who also present equations which correctly account for the effect, its importance seems not to have been generally recognised. As we will show, in Wu et al. (2002)’s model, the angular momentum interchange term dwarfs the GR loss term, and therefore the idea that the UI model evolves in the same way as a pair of detached stars is completely wrong.

In the UI model the X-rays come from the dissipation of electric currents generated by magnetic induction. This implies that a torque $T$ exists which transfers angular momentum between the spin of the magnetic primary star and the binary orbit. If the spin angular frequency of the primary star is $\Omega_s$, then the rate of work done by the torque is $T\Omega_s$ (we define the sign of $T$ such that a positive value spins the primary star up). An equal but opposite torque acts to extract angular momentum from the orbit, removing energy from it at rate $T\Omega_o$ where $\Omega_o$ is the orbital angular frequency. The difference between the two is dissipated, at least in part, in the form of X-rays so that

$$L_N \leq T(\Omega_o - \Omega_s).$$

(1)

Therefore the rate of change of orbital angular momentum $J$ is given by

$$J = J_{GR} - \frac{L_N}{(1 - \alpha)\Omega_o},$$

(2)

where we assume from now on that equation 1 is an equality and where we have introduced $\alpha = \Omega_s/\Omega_o$, following Wu et al. (2002). In this equation $J_{GR}$ is the rate of angular momentum lost due to gravitational waves alone. For two detached stars, the rate of angular momentum loss immediately leads to the period change through $P/P = 3J/J$, therefore whether the period change occurs at the GR rate depends upon the relative magnitudes of the two terms in Eq. 2. If we multiply through by the orbital angular frequency, then we obtain an illuminating version:

$$E = -L_{GR} - \frac{L_N}{(1 - \alpha)}.$$

(3)

where $L_{GR}$ is the luminosity in gravitational waves and $E$ is the rate of change of orbital energy. Equation 3 is identical in physical content to Equation 9 of Wu et al. (2002), except that for simplicity we have ignored the moment of inertia of the secondary star as it makes only a small difference to the results. Equation 3 shows that the simple GR formula for the period derivative can only be used if

$$L_N \ll |1 - \alpha|L_{GR}.$$

(4)

We now show that, on Wu et al. (2002)’s model, this is not at all the case for either V407 Vul or RX J0806+1527.

2.1 V407 Vul and RX J0806+1527

In order to estimate the gravitational wave luminosities, let’s assume that we are seeing the pure GR rate, in which case it can be shown that

$$L_{GR} = \frac{5\tilde{P}^2}{1152G\pi^2},$$

(5)

independent of the masses. The observed rates of period change of V407 Vul and RX J0806+1527 are $\dot{P} = 3 \times 10^{-12}$/s (Strohmayer 2004; Ramsay et al. 2005) and $3.7 \times 10^{-11}$/s (Israel et al. 2004; Strohmayer 2005) respectively. These give $L_{GR} = 1.4 \times 10^{33}$ and $2.1 \times 10^{35}$ ergs/s for the two systems.
The X-ray luminosity of V407 Vul is \( \sim 10^{35} (d/1 \text{ kpc})^2 \text{ ergs/s} \) (Ramsay et al. 2005). From Eq. 3, and assuming Wu et al. (2002)'s value of \( 1 - \alpha = 0.001 \), one would therefore predict that the induction torque term is \( \sim 10^2 (d/1 \text{ kpc})^2 \) times larger than the GR term. In other words the observed rate of period change in V407 Vul is far too small compared to a prediction based upon Wu et al. (2002)'s model. The model is nearer the mark in the case of RX J0806+1527 for which \( L_X \sim 2 \times 10^{33} (d/1 \text{ kpc})^2 \) ergs/s (Strohmayer 2005), and the predicted induction torque term is "only" \( \sim 10 \) times the GR term. However, this still shows that it is incorrect to assume that period changes in the UI model run at a rate given solely by GR.

A problem for the UI model has always been its short lifetime before synchronisation occurs, estimated to be \( \sim 1000 \) years by Wu et al. (2002). In order to have (at least) two such systems within a kiloparsec of the Sun, this suggests a formation rate of order 1 per year within the Galaxy, unless the asynchronism can be regenerated by some as-yet-undetermined mechanism (Cropper et al. 2004). We will now show that the problem is actually much worse than even this estimate suggests. During synchronisation, the spin rate of the primary will change much faster than that of the orbit, so we can estimate the timescale by neglecting the change of orbital period. The rate of change of the spin energy is then given by the power injected into the spin = \( TQ_s \) so using Eq. 1

\[
I_0 \dot{\Omega}_s = \frac{\Omega_s L_X}{\Omega_o - \Omega_s},
\]

where \( I \sim (1/5) M_1 R_1^2 \) is the moment-of-inertia of the primary star. Dividing through by \( \Omega_o \) gives

\[
\dot{\alpha} = \frac{L_X}{I \Omega_o^2} \frac{1}{1 - \alpha},
\]

This equation is identical to equation 8 of Wu et al. (2002) who also find that \( L_X \propto (1 - \alpha)^2 \), so we can write

\[
L_X = \frac{(L_X)_o}{I(1 - \alpha)} (1 - \alpha)^2,
\]

where the subscript zeroes indicate initial values. Therefore

\[
\dot{\alpha} = \frac{(L_X)_o}{I(1 - \alpha)_o} (1 - \alpha),
\]

which can be integrated to give

\[
1 - \alpha = (1 - \alpha)_o e^{-t/\tau_s},
\]

with the synchronisation timescale \( \tau_s \) given by

\[
\tau_s = \frac{I \Omega_o^2}{I(1 - \alpha)_o^2}.
\]

Since \( L_X \propto (1 - \alpha)^2 \), the X-ray luminosity decays with a time constant of \( \tau_s/2 \). Taking \( M_1 = 0.6 M_\odot \), \( R_1 = 0.01 R_\odot \), we find \( \tau_s \sim 4400 (1 - \alpha)^2 (d/1 \text{ kpc})^2 \) years for V407 Vul, and \( 7 \times 10^2 (1 - \alpha)^2 (d/1 \text{ kpc})^2 \) years for RX J0806+1527. Putting in Wu et al. (2002)'s value of \( 1 - \alpha = 0.001 \) leads to X-ray flux decay constants of 0.8 days for V407 Vul and 4.2 months for RX J0806+1527. Putting in Wu et al. (2002)'s value of \( 1 - \alpha = 0.001 \) leads to X-ray flux decay constants of 0.8 days for V407 Vul and 4.2 months for RX J0806+1527. Note that the horizontal axes of the two panels differ from each other.

2.2 Revision of the Unipolar Inductor model

We have shown that Wu et al. (2002)'s model is ruled out by the measured rates of period change in V407 Vul and by its very short lifetime for both V407 Vul and RX J0806+1527. We now consider whether it can be adjusted to match them.
The problem stems from the inefficient X-ray generation as only 1 part in 1000 of the power transferred from the orbit to the spin is dissipated. This can be avoided if very different values of \( \alpha = \Omega_i / \Omega_o \) are considered. For V407 Vul, a value of \( |1 - \alpha| \gtrsim 100 \) would reduce the induction torque term to a value comparable to or smaller than the gravitational wave term. Since we expect prograde rotation (\( \alpha > 0 \)), this means that \( \alpha \gtrsim 100 \), i.e. that the magnetic white dwarf must spin of order 100 times faster than the orbit so that \( P_{\text{spin}} \lesssim 6 \text{sec} \), which is or order the break-up spin rate of typical white dwarfs. Since \( \alpha > 1 \), the induction torque acts in the reverse sense to GR and therefore the large value of \( \alpha \) is needed not only to obtain the correct order-of-magnitude for \( \dot{P} \), but also to give the correct sign.

Such a large value of \( \alpha \) can only be attained through accretion-driven spin-up. The increased value of \( 1 - \alpha \) also means that the magnetic moment of the white dwarf must be much less than supposed by Wu et al. (2002). According to their model, the power dissipated in electric currents scales as \( \propto \mu^2(1-\alpha)^2 \), where \( \mu \) is the magnetic moment of the white dwarf. Since in V407 Vul we require \( 1 - \alpha \) to be \( 10^5 \) times larger than Wu et al. (2002) assume, the magnetic moment of the white dwarf must correspondingly drop by \( 10^5 \), which would therefore end up at about \( 10^{27} \) to \( 10^{28} \text{Gcm}^3 \), corresponding to a surface field of only 1 to 10 G. We would then have a case of accretion onto a very weakly magnetic white dwarf (non-magnetic as far as one could tell from observations), which begs the question of why the accretion should ever have stopped to allow the unipolar inductor to get going in V407 Vul but not in similar systems, such as the AM CVn stars and cataclysmic variable stars. Occam’s razor suggests that if accretion has to be invoked to allow the UI model to work at all, then one should favour accretion-only models, which we will look at in the next section.

The much lower X-ray luminosity of RX J0806+1527 leads to less stringent requirements. For \( 1 - \alpha = 0.001 \) the induction term is only 10 times the GR term, and so a modest increase leads to an acceptable match given the uncertainties. A tougher constraint comes from the X-ray decay timescale, which at 4 months for \( 1 - \alpha = 0.001 \) is much too short. However, an increase to \( 1 - \alpha = 0.01 \) gives a timescale of \( \sim 30 \) years which is probably long enough that it could not be ruled out by observations.

The larger values of \( 1 - \alpha \) needed to match V407 Vul and RX J0806+1527 have one further consequence: they make it easier to see the phase shifts between optical and X-ray pulses predicted under the UI model, but not observed, which were used by Barros et al. (2005) to rule out the dipolar field geometry used by Wu et al. (2002). For small values of \( 1 - \alpha \), it was always possible that an unlucky distribution of observations had lead to our missing these phase shifts. The raised values of \( 1 - \alpha \) and Fig. 6 of Barros et al. (2005) show that this is no longer the case.

In all the above, the X-ray luminosity, and therefore the distance, is crucial. Smaller distances reduce \( L_X \) and therefore alleviate the problems discussed above. We have assumed \( d = 1 \text{kpc} \) in each case. For V407 Vul this comes from assuming that the variable is at the same distance as the G star which dominates its spectrum (Steehgs et al., in prep.). For RX J0806+1527 on the other hand, the estimate comes from its blue colour, magnitude and adopting \( M_V = 11 \) as the absolute magnitude of a white dwarf of comparable colour. Neither constraint is secure, and lower distances are certainly possible. A crude lower limit comes from the absence of detectable proper motion in either star, which suggests that \( d > 100 \text{pc} \). However, it is hard to see that RX J0806+1527 can be near this limit at the same time as being blue and faint (\( U - B = -1.1, V = 21.1 \) Israel et al. 2002), unless the emitting area is much less than that of an average white dwarf. A similar argument applies to V407 Vul, because although it is brighter than RX J0806+1527, much of its flux in V comes from the G star, leaving the variable comparable to RX J0806+1527 in brightness. Thus \( d > 300 \text{pc} \), is probably a better lower limit, so the X-ray fluxes could perhaps be lowered by a factor of 10. This would leave RX J0806+1527 compatible with Wu et al. (2002)’s model, but V407 Vul still a good way from it. Better constraints on the distances to these systems are needed.

We now look at the whether accreting models can do any better in matching the observed period decreases in RX J0806+1527 and V407 Vul.

### 3 Period Changes in Accreting Binary Stars

Assuming conservative mass transfer one can show that

\[
\frac{\dot{P}}{P} = 3 \left( \frac{J_{\text{eff}}}{J} - (1 - q) \frac{M_2}{M_2} \right),
\]

(12)

where \( q = M_2/M_1 \) and \( M_2 \) is the mass of the donor. Since for stability we require that \( q < 1 \), and since \( J < 0 \) and \( M_2 < 0 \), mass transfer offsets the angular momentum loss term. The angular momentum term depends upon whether the accretor’s spin is strongly coupled to the orbit or not. If it is we have simply \( J = J_{GR} \); if not then Eq. 1 from Marsh et al. (2004) gives

\[
J = \bar{J}_{GR} + \sqrt{G M_1 R_k M_2},
\]

(13)

where \( R_k \) is the circularisation radius, which effectively increases the rate of loss of orbital angular momentum.

An illuminating comparison with the section 2 can be made by calculating the “effective” angular momentum loss rate \( J_{\text{eff}} \) such that the detached formula can still be applied as \( \dot{P}/P = 3J_{\text{eff}}/J \) because this can then be compared directly to Eq. 2. Assuming that \( L_X = -GM_1 M_2/R_1 \) where \( M_1 \) and \( R_1 \) are the mass and radius of the accretor, and using Eq. 12 (strong coupling), one can show that

\[
J_{\text{eff}} = J_{GR} + (1 - q) \frac{R_1}{a} \frac{L_X}{\Omega_o},
\]

(14)

in the strongly coupled case where \( a \) is the orbital separation. Comparing with Eq. 2, we see that the factor \( 1/(1 - \alpha) \sim 1000 \), which causes Wu et al. (2002)’s model so much trouble, is replaced by \( (1 - q)R_1/a \sim 0.1 \). This is a considerable improvement, and given the uncertainties in \( L_X \) and \( L_{GR} \) discussed in section 2.2, could be made consistent with the observed \( \dot{P} \) values for both systems.

For RX J0806+1527, the X-ray luminosity is already lower than the gravitational wave luminosity, as pointed out by Strohmayer (2005), and so Eq. 14 predicts that the observed period change must be close to the pure GR one. Weak coupling is better still since it increases the loss of

© 2005 RAS, MNRAS 000, 1–6
angular momentum from the orbit, but since it is otherwise qualitatively the same, we won’t discuss it further.

This discussion hides the ugly truth about accreting models which is that the equilibrium value of \( M_2 \) leads to an increasing period. Put differently, the current mass transfer rates in V407 Vul and RX J0806+1527 must be \( \lesssim 60\% \) of their equilibrium values in order for their periods to decrease at all (Marsh et al. 2004) (with some uncertainty over V407 Vul since some of its apparent period change could be caused by light-travel time effects if it is truly associated with the G star mentioned earlier). This is the price one must pay to accept accreting models. We regard it as a small one because there are already examples of systems which do not have mass transfer rates that match expectations, such as long period dwarf novae, and there are ways to make the mass transfer rate deviate from its equilibrium value as we will discuss below.

If the mass transfer rate is below its equilibrium value, how long will it stay like this and would we expect to see a noticeable increase over the course of a few years? The mass transfer rate in the two systems is expected to be in the adiabatic regime (Webbink 1984; Marsh et al. 2004) for which \( \dot{M} \propto \Delta^2 \) where \( \Delta = R_O - R_L \) is the amount by which the donor overfills its Roche lobe (Marsh et al. 2004). Therefore the rate of change of mass transfer rate is given by

\[
\dot{M} = \frac{3}{2} \frac{\dot{M}}{\Delta},
\]

and so the timescale for significant change of the mass transfer rate is given by

\[
\tau_e = \frac{\dot{M}}{\dot{M}_e} = \frac{1}{3} \frac{\Delta}{\dot{\Delta}}.
\]

Using Equation 19 of (Marsh et al. 2004), assuming strong coupling and neglecting the \( M \) dependent term in order to calculate the maximum rate of change of \( M \) gives

\[
\tau_e \approx \frac{1}{6} \frac{\Delta}{R_G} \left( \frac{J_{orb}}{-J_G} \right).
\]

Taking \( M_1 = 0.6 M_\odot \), and \( M_2 = 0.07 \) and \( 0.12 M_\odot \) for V407 Vul and RX J0806+1527 respectively, and using equations 10-12 of (Marsh et al. 2004) to calculate \( \Delta \), we find timescales of \( \sim 1000 \) and 100 years for significant alterations in the mass transfer rate to occur in V407 Vul and RX J0806+1527 respectively. These numbers suggest that below-equilibrium mass transfer rates can be sustained for many years, as observed. We must appeal to some unknown mechanism to force the departure from equilibrium in the first place, but there is no shortage of candidates. For example, a star-spot moving over the inner Lagrangian point is one possibility (Livio & Pringle 1994; Hessman et al. 2000; Hessman 2004), irradiation-induced cycles another (Ritter et al. 2000), widening of the orbit caused by the mass ejected in nova explosions a third (Shara et al. 1986) and synchronisation-induced detachment a fourth (Lamb & Melia 1987). One can even circumvent the relation between mass transfer rate and period derivative altogether through alterations in the structures of the stars (Applegate 1992), a mechanism which is thought to be responsible for cyclical variations in the periods of many eclipsing cataclysmic variables (Baptista et al. 2003). This mechanism can make the period decrease even if mass transfer occurs at the equilibrium rate, although the low X-ray luminosity of RX J0806+1527 means that there remains a need for below-equilibrium mass transfer whatever the cause of the period decrease. Which of these mechanisms, if any, can be applied to double white dwarfs is not clear, but neither is it clear that any of them are not applicable.

4 CONCLUSIONS

We have shown that, contrary to widespread assumption, the unipolar inductor model (Wu et al. 2002) does a poor job at predicting the magnitude of the period changes observed in the two candidate ultra-compact binary stars V407 Vul and RX J0806+1527 in the sense that the predicted changes are much larger than those observed. This removes the main piece of evidence supporting the model. The reason for this is that if there is only a small asynchronism between the spin and orbital periods, then much more energy is transferred between the orbit and spin than is dissipated in X-rays. The problem is particularly acute in the case of V407 Vul for which the unipolar inductor model predicts a rate of period change some 100,000 times greater than is observed. The unipolar inductor model can only apply to V407 Vul if its primary star rotates much faster than the 570 second putative orbital period; a larger asynchronism is also required to allow the unipolar inductor model to last more than a few years. This suggests that for the unipolar inductor model to work, accretion is a necessary precursor, without there being any obvious reason for it to have ceased. The problem with accretion-only models remains that the equilibrium mass transfer rates should lead to increasing orbital periods (as opposed to the observed decreases), and so the two systems must currently be transferring mass at below equilibrium rates or other mechanisms must be affecting the orbital periods if accretion is to work. Given that there are many examples of systems where this is the case, this difficulty seems less significant than those facing the unipolar inductor model. Nevertheless, the nature of these systems remains far from clear, and observational efforts to pin them down must continue to be pursued.

ACKNOWLEDGEMENTS

TRM acknowledges the financial support of a PPARC SRF. GN is supported by NWO-VENI grant 639.041.405. This research has made use of NASA’s Astrophysics Data System Bibliographic Services and the SIMBAD database, operated at CDS, Strasbourg, France.

REFERENCES


© 2005 RAS, MNRAS 000, 1–6
Israel, G. L., et al., 2004, Memorie della Societa Astronomica Italiana Supplement, 5, 148