Abstract: This paper reports on a research effort to better understand the act of modelling. In this paper we describe a formal framework by which the process of modelling can be regarded as involving the selection of more and more refined interpretations in terms of the underlying meta-model of the modelling language used. The resulting framework will be used to create a laboratory setup in which we can more closely study (and support) modelling processes.

1 Introduction

Modelling is at the core of information systems engineering. In [Myi98] a distinction is made between usage world, subject world, system world and development world, when producing deliverables during information systems engineering. Understanding each of these worlds require considerable modelling efforts, be it to define the requirements on the system, or be it to produce the design of a system.

The work reported in this paper is part of an ongoing effort to better understand the act of modelling [HPW05a, HPW05b, HPW05c, HPW05d, PVH05, HPR05, PHV05] in the context of information system engineering. One of our longer term goals is to turn the art of modelling into a science of modelling. In the past our focus was mainly on the formal definition of syntax and semantics of modelling languages. We have recently expanded this focus to include the process of modelling and the usage of models in information systems engineering. This expansion was inspired by a desire to better understand the modelling process itself, as well as the requirements on the languages used to express these models by the context in which they are to be used [PVH05]. We remain dedicated to a formal approach to theory development.

The primary concern of this paper is the further elaboration of a hypothesis put forward in [PHW05]. We argue that one can observe how many modelling techniques are in use to model several aspects of domains, such as processes, objects, information being processed, the flow of information, the flow of control, etc. Scholars and practitioners have produced numerous modelling techniques [Bub86, Avi95, BMS98]. The resulting plethora of techniques has, in the past, already been referred to as “a methodology jungle” [Avi95]. Each of these modelling techniques focuses on specific aspects of a domain, and is especially geared towards the representation, study, analysis or design of such aspects. Nevertheless, all of these techniques deal with facts about a domain describing how (from the perspective
of a specific aspect) concepts in the domain relate to each other. Put more operationally, we argue that any activity model, sequence diagram, information model, etc. has an accompanying domain model [PBH04] of the underlying concepts and their relations. This could be done in terms of a general purpose domain modelling language such as ORM [Hal01], but also using ontology modelling languages such as OWL [MH03] and KL-ONE [WS92].

![Diagram](image)

Figure 1: Refinement of models and meta-models

This leads to the situation as depicted in Figure 1. On the right hand side we find the meta-models of the modelling techniques used, while on the left hand side we find the actual models. The ‘XXX’ represents an aspect of the domain that is being modelled. The ‘XXX’ model is a re-interpretation of the original model in terms of the refined ‘XXX’ meta-model. To illustrate this point, consider the example depicted in Figure 2. In this example, we have used the ORM domain modelling technique [Hal01] to represent a general domain model of a small sample domain dealing with involvement of people with a University department. The involvement starts with candidature, then might move on to the coworker level, and will typically end in the alumni status. In the example, we have (partially) re-interpreted the underlying domain model into two directions: an UML class diagram focusing on the core concepts in the domain, and a state-transition diagram focusing on the state changes of the involvement of people with departments.

As another example, consider the compacted version, as depicted in Figure 3, of the case study used in [PHW05]. This example focusses on workflow modelling and shows two interpretation steps. The first step, moving from A) to B), requires modelers to select which object types are really actor and actand types. The second interpretation step, from B) to C), can actually be done automatically given a pre-defined mapping between the meta-models of the modelling techniques involved. The modeller does not need to add additional information to the model. Note that the situation depicted in A) is not a static view on the domain. The arrows from fills in form to examines, etc. show a temporal dependency between states, thus providing a flow of states and activities.

In each of the interpretation steps, modelers need to make a choice of how to re-interpret (if at all) specific concepts in the general domain model in terms of the modelling concepts in the refined meta-model. We argue that modelling can be regarded as a process of (iteratively!) refining ones view on the world in terms of more and more refined modelling concepts (the types in the meta-model). This process is driven by the motivations for producing the model in the first place.

One may argue that in practice modelers will quite often directly produce a UML class diagrams, workflow diagrams, etc. In our view, doing so leaves implicit numerous interpre-
tive decisions about the domain. If one were to first produce a domain model as depicted in Figure 3 A), one could argue that the understanding of the domain being modelled would be deeper, providing a better base from which to then produce model C) via B). Nevertheless, our goal at present is not to cast judgement on how to best model. Some modellers may prefer to start with A) and some will prefer to immediately produce C), where a given C) will still automatically imply an associated B) and A). Our goal is to better understand the actual act of modelling, and as such, we do want to study how modelers implicitly or explicitly move from A) to C). For this purpose, we first need to develop an explicit model of modelling as a process of stepwise interpretation over a hierarchy of meta-models. This model of the modelling process is the focus of attention of the remainder of this paper.

The resulting framework will be integrated with the logbook perspective [BFW96, BW04, HPW05a] on the modelling process to create a system that will allow us to conduct modelling experiments in a laboratory setting.

We have structured the remainder of this paper as follows. In Section 2 we briefly explore the notion of subjectivity in relation to modelling. Section 3 then focuses on hierarchies of modelling languages, i.e. meta-model hierarchies. Given such a hierarchy, Section 4 shows how hierarchies of models as depicted in Figure 3 can be represented formally. The final step is made in Section 5 where we cater for the possible evolution of a hierarchy of models during a modelling process.
Figure 3: Activity modelling
2 Subjectivity in Modelling

The aim of this section is to define more precisely what we mean by the modelling of a domain, in other words, our fundamental way of thinking about modelling. In doing so, we will start by introducing a framework describing the essential processes that take place when an observer observes a domain.

It is our assumption, based on the work of C.S. Peirce [Pei69], that observers perceive a universe and then produce a conception of that part they deem relevant. The conceptions harboured by an observer are impossible to communicate and discuss with other observers unless they are articulated somehow (the need for this ability in the context of information systems engineering is evident). In other words, a conception needs to be represented. Peirce argues that both the perception and conception of an observer are strongly influenced by their interest in the observed universe. This leads to the following set of definitions (also inspired by the ones provided in [FVV+98], which are based on the work by Peirce as well):

**Universe** – the ‘world’ around the observer.

**Observer** – an actor perceiving and conceiving the universe, using their senses.

**Perception** – that what results, in the mind of an observer, when they observe the universe, using their senses.

**Conception** – that what results, in the mind of an observer, when they interpret a perception of the universe.

An observer may zoom in on a particular part of the universe they observe, or to state it more precisely, they may zoom in on a particular part of their conception of the universe.

**Domain of interest** – any ‘part’ or ‘aspect’ of a conception of the universe, a observer may zoom in on.

Note that when an observer zooms in on a domain of interest, they produce yet another conception.

In the context of information systems engineering, observers may have different domains of interest depending on their concern with regards to the information system being engineered. For example, the operators who will be required to maintain a planned information system, will regard this system in terms of costs of keeping the system up and running, costs and efforts involved in implementing the system, etc. Future users of the same planned system, however, will be more interested in the impact/support the system is likely to have on their work related tasks. In our effort to obtain a fundamental understanding of the act of modelling, we initially focus on situations where we only have one specific concern and associated domain of interest.

In line with [FVV+98] we define a model to be a specific kind of conception:

**Model** – a purposely abstracted and unambiguous conception of a domain of interest.
Conceptions that are harboured by an observer are impossible to communicate and discuss with other observers, unless they are articulated somehow. In other words, the conception needs to be represented:

**Representation** – the result of an observer representing a conception, using some language to express themselves.

The resulting situation is illustrated in Figure 4 showing how an observer in observing the universe has a conception, which may be represented in terms of a representation.

![Figure 4: An observer observing a universe](image)

We are now also in a position to define more precisely what we mean by modelling:

**Modelling** – The act of purposely abstracting a model from (what is conceived to be) a part of the universe, and representing the resulting model by means of some language and medium.

The same domain of interest may be regarded by different observers, which is bound to lead to different conceptions, depending on the specific observers. The fact that when referring to the same universe, people are likely to refer to different models is, as reported in e.g. [FVV+98], one serious cause for the current confusion in the development of information systems. People, tend to think about a system as something that can be objectively determined [FVV+98]. An assumption that is bound to lead to serious ‘accidents’. However, at present our focus is on better understanding the act of modelling when only one observer is involved, which is difficult enough as even one observer is not likely to behave like a monotonic function when modelling.

In the context of information systems engineering, observers will approach a domain with the aim of expressing the domain in terms of some set of modelling constructs, such as classes, activity (types), event (types), constraints, etc. The set of modelling constructs a observer is used to using (or trained to use) when modelling a domain, will strongly influence their conceptions. For example, when viewing a domain of interest from the perspective of UML class diagrams, this is bound to lead to a different model than when the same domain is viewed from the perspective of UML sequence diagrams. To make this explicit, we therefore presume that when observers model a domain, they do so from a certain perspective; their Weltanschauung [WAA85]. In Figure 5 we have depicted a
situation where an observer observes (a domain of interest within) a universe from the perspective of different meta-models \((M_1, \ldots, M_n)\), leading to even so many models \((m_1, \ldots, m_n)\) and model representations \((r_1, \ldots, r_n)\).

The remainder of this paper is primarily concerned with the development of a precise understanding of the relationships between these meta-models, the corresponding models (or rather their representations), as well as their evolution during a modelling process. Here we will operate under the hypothesis that modelling can be viewed as an iterative process of:

- Defining an (unspecific) model of a domain using some suitable (suitable; not necessarily the) generic meta-model, focusing on domain concepts and their relationships in a general sense.
  
  In the examples of the previous section, we used ORM (with temporal extensions) as an example of such a generic meta-model.

- Selecting more specific interpretations of the concepts identified in the initial model, using more refined meta-models.
  
  In the previous section we showed examples of interpretations in terms of a UML class diagram, a state-transition diagram, and a workflow model.

The latter step, selection of interpretation, is the essential aspect of the way of thinking as put forward in this paper, and is therefore also used as the title for this paper.

3 Meta-model Hierarchies

The foundation of our modelling framework is formed by a hierarchy of meta-models. The concept of a \textit{meta-model hierarchy} is not new. It was already introduced in [FO94] as a way of comparing modelling techniques, and to some extend refined further in [FVV+98].

Our goals of viewing the act of modelling as a process of stepwise selection of interpretations over a hierarchy of meta-models is a way to operationalise the ‘old’ notion of a meta-model hierarchy.
3.1 Meta-models

A meta-model is seen as a formal system [Men87]. Such a system consists of (1) a signature that specifies its concepts, providing a base for the definition of well-formed formulae, and (2) a set of such well-formed formulae (also referred to as axioms) that are assumed/required to hold for concrete systems that realize the formal system. In this context we shall refer to the concepts of the formal system as the (modeling) types of the meta-model. We will denote a meta-model by its signature and its axioms. We will use \((T, A)\) to denote the system with signature \(T\) and axioms \(A\).

Let \(\mathcal{MT}\) be the set of all meta-types from some class of modelling techniques, \(\mathcal{MA}\) be the set of all axioms, and \(\mathcal{MM} \subseteq \mathcal{MT} \times \mathcal{MA}\) the set of all meta-models. We focus on meta-models that satisfy the following rules.

Each meta-model is consistent, meaning that the axioms are not contradictory.

[M1] If \((T, A) \in \mathcal{MM}\), then \(A\) is a consistent set of well-formed formulae over \(T\).

Each meta-model is required to have different modeling types.

[M2] If \(M_1 = (T_1, A_1)\) and \(M_2 = (T_2, A_2)\), such that \(M_1, M_2 \in \mathcal{MM}\), then:
\[
M_1 \neq M_2 \Rightarrow T_1 \cap T_2 = \emptyset
\]

This latter requirement is added to allow us to study relations between modeling concepts in more depth.

3.2 Models

An actual model is an instantiation of a formal system; the associated meta-model. This model thus contains instantiations of the meta-types contained in that meta-model. Let \(\mathcal{EM}\) be the set of all those instantiations, which are referred to as model elements. We define the possible interpretation of these elements in terms of the meta-types: \(\mathcal{IN} = \mathcal{EM} \times \mathcal{MT}\). In other words, an interpretation is the combination of a model element and a meta-type. Since meta-models may contain sub-types, elements may be associated to multiple meta-types.

If \(m\) is a model with associated meta-model \(M\), we will also say that \(m\) is an \(M\)-model. An \(M\)-model \(m\) can be regarded as a set of interpretations \(m \subseteq \mathcal{IN}\) that meet the axioms of meta-model \(M\). The set of valid \(M\)-models for a given meta-model \(M = (T, A)\) is therefore defined as:
\[
\mathcal{M}(M) \triangleq \{ m \subseteq \mathcal{EM} \times T \mid m \models A \}
\]

The set of interpretations fitting a meta-model is defined as:
\[
\mathcal{I}(M) \triangleq \bigcup \mathcal{M}(M)
\]
3.3 Hierarchies of meta-models

The next step is to introduce hierarchies of meta-models. Such a hierarchy is composed of refinement relations between meta-models. Let $RF$ be the set of possible refinement relations for the considered class of meta-models and let $From, To : RF \rightarrow \mathcal{MM}$ be functions returning the start and destination meta-model of a refinement respectively. Then $RF, From$ and $To$ together span a space in which we will be able to identify meta-model hierarchies to be used in modelling. We do require $RF$ to be acyclic:

[M3] The graph spanned over $\mathcal{MM}$ by $From$ and $To$ is acyclic.

A specific meta-model hierarchy is a set of refinements, so we can define the set of possible meta-model hierarchies as $\mathcal{MH} \subseteq \wp(RF)$, where we do require:

[M4] If $R \in \mathcal{MH}$ then $R$ is a tree.

Let $Top(R)$ denote the top of such a tree. We will write $R_{\mathcal{MM}}$ as an abbreviation for the set of meta-models involved in $R$.

3.4 Grounding of models

To really capture the notion of refinement between meta-models, we must be able to map models upward in the hierarchy. We therefore need a function that is able to ground models stated in a refined meta-model in terms of the more general meta-model:

$$\text{Ground}_{r} : RF \times (\wp(\mathcal{IN}) \times \wp(\mathcal{IN}'))$$

In terms of the example shown in Figure 3 the grounding function would have to map any actor type and actand type in a workflow model onto an object type in an ORM model, and each activity type onto an ORM relationship type. The working of the grounding function is illustrated in Figure 6. Models are grounded by grounding the interpretations they are made of. Multiple models conform a refined meta-model may be grounded onto the same generalized model. For example, in Figure 3 we might have selected a person being examined to be an actand (i.e. passive) in the examination, rather than considering it to be an actor as well (as is currently shown in B). In either case, the grounding of model B would still be the model shown in A).

For a given refinement $r$, the grounding function should limit itself to interpretations associated to the meta-models involved in the refinement:

[M5] $x \in \text{dom}(\text{Ground}_{r}) \Rightarrow x \subseteq \mathbb{I}(To(r))$

[M6] $y \in \text{ran}(\text{Ground}_{r}) \Rightarrow y \subseteq \mathbb{I}(From(r))$

Empty models have an empty grounding:
Figure 6: Grounding of models and interpretations

[M7] \text{Ground}_r(\emptyset) = \emptyset

Even more, the grounding function should behave strict monotonous in terms of inclusion of sets of interpretations:

[M8] m_1 \subseteq m_2 \subseteq \mathcal{I} \Rightarrow \text{Ground}_r(m_1) \subseteq \text{Ground}_r(m_2)

where \( \subseteq \) is used as a proper subset. This allows us to ground any non-empty fragment of a re-interpreted model back to (a non-empty) fragment at the more generic level:

Corollary 3.1
\[ m \neq \emptyset \Rightarrow \text{Ground}_r(m) \neq \emptyset \]

4 Model Hierarchies

In this section we extend the meta-model hierarchy of the previous section to a hierarchy of models over such meta-model hierarchies. First we follow the interpretation of a single model element in a hierarchy. When modelling, decisions are made pertaining to interpretations of the domain. These modeling decisions are almost as important as the resulting model. Let \( \mathcal{M}^\mathcal{V} \) be a carrier set for motivations of such decisions, then we can define an interpretation hierarchy as a \emph{partial} function: \( h : \mathcal{M}^\mathcal{M} \rightarrow \mathcal{P}(\mathcal{I} \mathcal{N}) \times \mathcal{M}^\mathcal{V} \). Let \( \mathcal{I} \mathcal{H} \) be the set of all such interpretation hierarchies. If we are only interested in the set of interpretations, we will use: \( \mathcal{H}(M) \triangleq \{ I \text{ such that } h(M) = (I, v) \} \).

An interpretation hierarchy should follow a meta-model hierarchy. This is laid down in three rules. We consider \( h \) to be an interpretation hierarchy fitting a meta-model hierarchy.
$R$, written as $h \in I(R)$, iff:

1. The first condition requires that an interpretation hierarchy can only contain interpretations for the meta-models present in $R$. Formally: $\text{dom}(h) \subseteq R_{\text{MM}}$.

2. The second condition requires the top of the interpretation hierarchy to contain one interpretation only; the root. Formally: $|h!(\text{Top}(R))| = 1$.

3. The third condition requires the interpretation hierarchy to obey the grounding function. Formally this is enforced by: $\forall r \in R [\text{Ground}_r (h!(\text{To}(r))) \subseteq h!(\text{From}(r))]$.

Two interpretation hierarchies are disjoint iff they do not overlap for any meta-model:

$h \otimes i \triangleq \forall M \in \text{MM} [h!(M) \cap i!(M) = \emptyset]$

A model hierarchy is a set $H$ of interpretation hierarchies. If $H$ is a model hierarchy, then for any meta-model $M$, the complete model is defined as the union of the interpretations in the interpretation hierarchies:

$H!(M) \triangleq \bigcup_{h \in H} h!(M)$

This is illustrated in Figure 7.

For a given meta-model hierarchy $R$, the set of valid model hierarchies consists of those interpretation hierarchies $H$ such that:

$\forall M \in \text{dom}(H) [H!(M) \in M(M)] \land \forall h, i \in H [h \neq i \Rightarrow h \otimes i]$
5 Evolution of Model Hierarchies

During a modelling process, a model hierarchy is likely to evolve. The final step in the definition of our framework is therefore the introduction of a temporal dimension. This essentially boils down to the extension of the framework so-far with a schema evolution mechanism as defined before in e.g. [PW94, Pro97]. Within the confines of this paper, we can only provide a brief overview of how evolution of model hierarchies can be formalized.

Let \( T \) be a set of points in time with a total ordering \(<\). Let furthermore \( \triangleright \) be a function returning the next point in time, such that:

\[
\text{T1} \quad \forall t \in T, [t < \triangleright t] \\
\text{T2} \quad \neg \exists s \ [t < s < \triangleright t]
\]

The evolution of a model hierarchy is modelled by tracing the evolution of the underlying interpretation hierarchies. The latter evolutions are modelled as a function \( e : T \rightarrow \mathcal{H} \).

Let \( \mathcal{I} = T \times \mathcal{H} \) be the set of possible evolutions of interpretation hierarchies. The evolution of a model hierarchy can then be regarded as a subset \( E \subseteq \mathcal{I} \). At any point in time, the current version of the model hierarchy can be derived as:

\[ E(t) \triangleq e(t) \]

The set of valid model hierarchy evolutions for a given meta-model hierarchy \( R \) is defined as:

\[
E(R) \triangleq \left\{ E \subseteq \mathcal{I} \mid \forall t \in T, [E(t) \in \mathcal{I}(R)] \wedge \forall e \in E, [e(t) \subseteq e(\triangleright t) \vee e(t) \supseteq e(\triangleright t)] \right\}
\]

The first condition requires evolutions to obey the meta-model hierarchy. The second condition is used to ensure that the steps in the modelling process are not too large, retracting and extending a hierarchy cannot be done in one step.

In line with [Pro97], information providing motivation for may be added to each of the evolution steps. To be able to do so, we enrich the hierarchy evolutions with a partial function:

\[ \text{Motivation} : \wp(\mathcal{I}) \times T \times \wp(\mathcal{I}) \rightarrow \wp \]

where the intuition is that if \( \text{Motivation}(E, t, C) = o \), then a change occurring within a hierarchy evolution \( E \) at time \( t \), involving the interpretation evolutions \( C \) is motivated by \( o \).

We require the motivations for changes are limited to those interpretation evolutions that are part of the given hierarchy evolution:

\[ \text{Motivation} \left( E, t, C \right) \in \text{dom}(\text{Motivation}) \Rightarrow C \subseteq E \]

The reason for \( C \) to be a set of evolutions has to do with the fact that during the modelling process a modeler may decide to merge and/or split existing evolutions. In [Pro97] a
terminology based on particle physics was introduced to identify such situations as fusion, fission, emission and absorption.

We require all changes to be motivated:

\[ \text{[M10]} \text{ If } E \in E(R) \text{ then:} \]
\[ \forall e \in E \left[ e(t) \neq e(\triangleright t) \Rightarrow \exists C \left[ e \in C \land \langle E, \triangleright t, C \rangle \in \text{dom(Motivation)} \right] \right] \]

Only changes can be motivated:

\[ \text{[M11]} \langle E, t, C \rangle \in \text{dom(Motivation)} \Rightarrow \forall e \in C \left[ e(t) \neq e(\triangleright t) \right] \]

6 Conclusion

In this paper we have discussed a framework to study the act of modelling, where a modelling process is regarded as involving the selection of more and more refined interpretations in terms of the underlying meta-model of the modelling language used. The resulting framework will be used, in conjunction with the logbook system, to create a laboratory environment in which modelling experiments can be conducted.

The logbook system [BFW96, BW04, HPW05a] takes the view that a modelling process is a (controlled) dialogue between a domain expert, a modelling mediator and a model builder. This process is regarded as a questioning & answering process involving these three roles. When combined with the theory as presented in this paper, the goal of such a questioning & answering process can be made explicit as the creation of a model hierarchy on top of a pre-determined (dictated by the modelling goals at hand [PVH05, PHV05]) meta-model hierarchy.

In future versions of our framework we intend to refine it such that we are able to deal with multiple views and concerns, as well as multiple (contradicting?) observers. In the latter case we would like to be able to even log the negotiation that may have to take place in reconciling different views held by different observers of the same domain.

References


