Measurement of the Lifetime Difference in the $B^0_s$ System

We present a study of the decay $B^0 \rightarrow J/\psi \phi$. We obtain the CP-odd fraction in the final state at time zero, $R = 0.16 \pm 0.10 \text{ (stat)} \pm 0.02 \text{ (syst)}$, the average lifetime of the $(B^0, B^\ast_0)$ system, $\tau(B^0) = 1.39 _{-0.02}^{+0.01} \text{ ps}$, and the relative width difference between the heavy and light mass eigenstates, $\Delta \Gamma/\Gamma \equiv \frac{\Gamma_L - \Gamma_H}{\Gamma} = 0.24 ^{+0.26}_{-0.38} \text{ (stat)} ^{+0.03}_{-0.04} \text{ (syst)}$. With the additional constraint from the world average of the $B^0$ lifetime measurements using semileptonic decays, we find $\tau(B^0) = 1.39 \pm 0.06 \text{ ps}$ and $\Delta \Gamma/\Gamma = 0.25 ^{+0.14}_{-0.13}$. For the ratio of the $B^0_s$ and $B^0$ lifetimes we obtain $\frac{\tau(B^0_s)}{\tau(B^0)} = 0.91 \pm 0.09 \text{ (stat)} \pm 0.003 \text{ (syst)}$.

PACS numbers: 13.25.Hw, 11.30.Er

Within the framework of the standard model (SM), the $B^0_s$ mesons are expected to mix in such a way that the
mass and decay width differences between the heavy and light eigenstates, \( \Delta M \equiv M_H - M_L \) and \( \Delta \Gamma \equiv \Gamma_L - \Gamma_H \), are sizeable. The mixing phase \( \delta \phi \) is small and to a good approximation the two mass eigenstates correspond to CP eigenstates. New phenomena may alter \( \delta \phi \), leading to a reduction of the observed \( \Delta \Gamma / \Gamma \) compared to the SM prediction [1].

The decay \( B^0_s \rightarrow J/\psi \phi \), proceeding through the quark process \( b \rightarrow c \bar{c} s \), gives rise to both CP-even and CP-odd final states. It is possible to separate the two CP components of the decay \( B^0_s \rightarrow J/\psi \phi \), and thus to measure the lifetime difference, through a simultaneous study of the time evolution and angular distributions of the decay products of the \( J/\psi \) and \( \phi \) mesons. Angular distribution of \( B^0_s \rightarrow J/\psi(\rightarrow \mu^+\mu^-) \phi(\rightarrow K^+K^-) \) involve three angles. Current statistics are such that the use of all three angles characterizing the final state is not yet beneficial. We use a variable particularly sensitive to separating the \( B^0 \) final states. It is possible to separate the two CP components of the decay \( B^0 \rightarrow J/\psi \phi \), and thus to measure the lifetime difference, through a simultaneous study of the time evolution and angular distributions of the decay products of the \( J/\psi \) and \( \phi \) mesons. Angular distribution of \( B^0 \rightarrow J/\psi(\rightarrow \mu^+\mu^-) \phi(\rightarrow K^+K^-) \) involve three angles. Current statistics are such that the use of all three angles characterizing the final state is not yet beneficial. We use a variable particularly sensitive to separating the \( B^0 \) final states.

The analysis of data collected with the DØ detector [2] at the Fermilab Tevatron collider presented in this Letter is an extension of a recently published study [3] done under the single \( B^0_s \) lifetime hypothesis. We perform an unbinned maximum likelihood fit to the data, including the \( B^0_s \) candidate mass, lifetime, and transversity, in the decay sequence \( B^0_s \rightarrow J/\psi \phi \), \( J/\psi \rightarrow \mu^+\mu^- \), \( \phi \rightarrow K^+K^- \).

We extract three parameters characterizing the \( B^0_s \) system and its decay: \( \tau(B^0_s) = 1/\bar{\Gamma} \), where \( \bar{\Gamma} \equiv (\Gamma_H + \Gamma_L)/2 \); \( \Delta \Gamma / \Gamma \); and \( R_{1,2} \), the relative rate of the decay to the CP-odd states at time zero. The average lifetimes of \( B^0_s \) and \( B^0 \), as defined above, are expected to be equal to within 1\% [4], and their ratio is determined by measuring the lifetime of \( B^0 \) in the similar decay topology of \( J/\psi K^* \).

The data were collected between June 2002 and August 2004. The sample is selected by requiring two reconstructed muons with a transverse momentum \( p_T > 1.5 \) GeV. Each muon is required to be detected as a track segment in at least one layer of the muon system and to be matched to a central track. One muon is required to have segments both inside and outside the toroid. We require the events to satisfy a muon trigger that does not have segments both inside and outside the toroid. We select the one with the highest transverse momentum.

To reconstruct the primary vertex, we select tracks with \( p_T > 0.3 \) GeV that are not used as decay products of the \( B_s \) candidate and apply a constraint to the average beam spot position. The proper decay length, \( c t \), is defined by the relation \( c t = L_{xy}^0 M_{B_s}/p_T \) where \( M_{B_s} \) is the world average mass of the \( B_s \) meson [5]. The distribution of the proper decay length uncertainty \( \sigma( c t ) \) of \( B_s \) mesons peaks around 25 \( \mu m \). We accept events with \( \sigma( c t ) < 60 \) \( \mu m \). There are 9699 events satisfying the above cuts.

The resulting invariant mass distribution of the \( \{ J/\psi, \phi \} \) system is shown in Fig. 1. The curves are projections of the maximum likelihood fit, described below. The fit assigns 513\pm33 events to \( B^0_s \) decay.

Using the same data sample and analogous kinematic and quality cuts, we select a sample of 1913 events of the decay sequence \( B^0_s \rightarrow J/\psi K^* \), \( J/\psi \rightarrow \mu^+\mu^- \), \( K^+ \rightarrow K^+ \pi^+ \), and update the \( B^0 \) lifetime measurement reported in Ref. [3] with larger statistics.

We perform a simultaneous unbinned maximum likelihood fit to the proper decay length, transversity, and mass. The likelihood function \( \mathcal{L} \) is given by:

\[
\mathcal{L} = \prod_{i=1}^{N} \left[ f_{\text{sig}} F_{\text{sig}}^i + (1 - f_{\text{sig}}) F_{\text{back}}^i \right],
\]

where \( N \) is the total number of events, \( F_{\text{sig}}^i \) (\( F_{\text{back}}^i \)) is the product of the mass, proper decay length, and the
transversity probability density functions for the signal (background), and $f_{\text{sig}}$ is the fraction of signal in the sample. Background is divided into two categories, based on their origin and lifetime characteristics. “Prompt” background is due to directly produced $J/\psi$ mesons accompanied by random tracks arising from hadronization. This background is distinguished from “non-prompt” background, where the $J/\psi$ meson is a product of a B-hadron decay while the tracks forming the $\phi$ candidate emanate from a multibody decay of the same B hadron or from the underlying event. We allow for independent parameters for the two background components in mass, lifetime, and transversity. There are nineteen free parameters in the fit.

For the signal mass distribution, we use a sum of two Gaussian functions with a fixed ratio of widths and normalizations, obtained in a fit to the signal-dominated subset satisfying $ct/\sigma(ct) > 5$. We allow for two free parameters, the common mean value and the width of the narrow component. The lifetime distribution of the signal is parametrized by an exponential convoluted with a Gaussian function with the width taken from the event-by-event estimate of $\sigma(ct)$. To allow for the possibility of the lifetime uncertainty to be systematically underestimated, we introduce a free scale factor.

The transversity distribution of the signal is determined in the following way. The time-dependent three-angle distribution for the decay of $\phi$ mesons, i.e., summed over $B^0_s$ and $\bar{B}^0_s$, expressed in terms of the linear polarization amplitudes $|A_x(t)|$ and their relative phases $\delta_i$ is [6]:

$$
\frac{d^3\Gamma(t)}{d\cos \theta \, d\phi \, d\cos \psi} \propto 2|A_0(0)|^2 e^{-\Gamma t} \cos^2 \psi (1 - \sin^2 \theta \cos^2 \varphi) + \sin^2 \psi |A_1(0)|^2 e^{-\Gamma t} (1 - \sin^2 \theta \sin^2 \varphi)
$$

$$
+ |A_\perp(0)|^2 e^{-\Gamma t} \sin^2 \theta + \frac{1}{\sqrt{2}} \sin 2\varphi |A_0(0)||A_1(0)| \cos (\delta_2 - \delta_1) e^{-\Gamma t} \sin^2 \theta \sin 2\varphi
$$

$$
+ \left( \frac{1}{\sqrt{2}} |A_0(0)||A_\perp(0)| \cos \delta_2 \sin 2\psi \sin 2\theta \sin \varphi - |A_1(0)||A_\perp(0)| \cos \delta_1 \sin^2 \psi \sin 2\theta \sin \varphi \right) \frac{1}{2} (e^{-\Gamma t} - e^{-\Gamma t}) \sin \delta . \quad (2)
$$

In the coordinate system of the $J/\psi$ rest frame (where the $\phi$ meson moves in the $x$ direction, the $z$ axis is perpendicular to the decay plane of $\phi \rightarrow K^+ K^-$, and $p_y(K^+) \geq 0$), the transversity polar and azimuthal angles ($\theta, \varphi$) describe the direction of the $\mu^+$, and $\psi$ is the angle between $\vec{p}(K^+)$ and $-\vec{p}(J/\psi)$ in the $\phi$ rest frame.

We model the acceptance in the three angles by polynomials, with parameters determined using Monte Carlo simulations. We have used the SVV-HELAMP model in the EvtGen generator [7], interfaced to the PYTHIA program [8]. Reconstructed events were reweighted to match the kinematic distributions with the data.

To obtain the one angle (transversity) distribution, we integrate the three-angle distribution over the angles $\psi$ and $\varphi$. The resulting distribution depends on one free parameter, $R_\perp = |A_\perp(0)|^2$. There is a small correction term due to the nonuniformity of the acceptance in the angle $\varphi$, which is proportional to $|A_0(0)|^2 - |A_1(0)|^2$. We use the CDF Collaboration measurement [9] of this difference, 0.355±0.066.

The lifetime shape of the background is described as a sum of a prompt component, simulated as a Gaussian function centered at zero, and a non-prompt component, simulated as a superposition of one exponential for the negative $ct$ region and two exponentials for the positive $ct$ region, with free slopes and normalizations. The mass distributions of the backgrounds are parametrized by first-order polynomials. The transversity distributions of backgrounds are parametrized as $(1 + a_2 \cos^2 \theta + a_4 \cos^4 \theta)$.

Results of the fit are presented in Figs. 1 - 4. The proper decay length distribution, and the transversity distribution, both with the fit results overlaid are shown in Figs. 2 and 3. Figure 4 shows the one standard deviation (one-$\sigma$) contour for $c\tau(B_s^0)$ versus $\Delta t/\Gamma$. It provides the best display of the uncertainty range for these correlated parameters. Our best fit returns $R_\perp = 0.16\pm0.10$ and $\Delta t/\Gamma = 0.24^{+0.28}_{-0.38}$ at $\bar{B}^0_s = 3.39^{+0.13}_{-0.16}$ ps.

We do a series of alternative fits, at discrete values of $\bar{B}^0_s$. The results for $\Delta t/\Gamma$, its one standard deviation range, and the corresponding value of the likelihood, are listed in Table I. We verify the procedure by performing fits on a sample of approximately 50,000 MC events passed through the full chain of detector simulation, event reconstruction, and maximum likelihood fitting. We see no bias in the event reconstruction or
in the fitting procedure. The fits reproduce the inputs
($\Delta t = 439$ µm, $\Delta t/\Delta t = 0$, and ranges of $R_L$ between 0 and 1) correctly within the statistical precision of 2 µm
for $\Delta t$, 0.01 for $R_L$, and 0.025 for $\Delta t/\Delta t$. We test the
sensitivity of the results to the parametrization of the
signal and background mass distributions by varying the
parameters of the two-Gaussian function. To test the
sensitivity of the results to the background model, we
add a quadratic term in the background mass distribu­
tion. We find a non-negligible effect from the extra term
in the non-prompt background in $\Delta t$ and $\Delta t/\Delta t$. We have
also tested the sensitivity of the results to the assump­
tion that the lifetime and the transversity distributions
of background are independent of mass. The effect of
the uncertainty of the detector alignment on the lifetime
measurement was estimated in Ref. [3]. The effects of
systematic uncertainties are listed in Table II.

We also conduct a test with an ensemble of 1000
pseudo-experiments with similar statistical sensitivity,
chosen from the distribution described by Eq. 1, with
the same parameters as obtained in this analysis. Both
the spread of uncertainties and of the central values of
the fit parameters are in good agreement with the results
reported here. Our results are consistent with the CDF
Collaboration results [9], also shown in Fig. 4.

$B_s^0$ lifetime measurements from semileptonic (flavor-
specific) data provide an independent constraint on the
average lifetime and lifetime difference in the $B_s^0$ sys­
tem. The world average [5] $B_s^0$ lifetime is
$\tau_{fs} = 1/\Gamma_{fs} = 1.442 ± 0.066$ ps. This result is based on single­
exponential fits in the flavor-specific decay channels,
which determine the following relation [10] (shown in Fig.
4) of $\Gamma_{fs}$ and $\Delta t/\Delta t$: $\Gamma_{fs} = \Gamma - (\Delta t/\Delta t)^2/2\Gamma + O(\Delta t/\Delta t)^3/\Delta t$.

Applying the above constraint to our measurement, we
obtain $\tau(B_s^0) = 1.39 ± 0.06$ ps and $\Delta t/\Delta t = 0.29_{-0.15}^{+0.14}$. This result is consistent with the SM expectation [11] of

<table>
<thead>
<tr>
<th>$\tau(B_s^0)$ (ps)</th>
<th>$\Delta t/\Delta t$</th>
<th>$\Delta \ln(L)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.23</td>
<td>-0.13 ± 0.15</td>
<td>0.51</td>
</tr>
<tr>
<td>1.27</td>
<td>-0.03 ± 0.17</td>
<td>0.32</td>
</tr>
<tr>
<td>1.31</td>
<td>0.07 ± 0.19</td>
<td>0.17</td>
</tr>
<tr>
<td>1.35</td>
<td>0.16 ± 0.21</td>
<td>0.04</td>
</tr>
<tr>
<td>1.39</td>
<td>0.24 ± 0.20</td>
<td>0.0</td>
</tr>
<tr>
<td>1.43</td>
<td>0.31 ± 0.19</td>
<td>0.06</td>
</tr>
<tr>
<td>1.47</td>
<td>0.37 ± 0.18</td>
<td>0.20</td>
</tr>
<tr>
<td>1.51</td>
<td>0.43 ± 0.18</td>
<td>0.42</td>
</tr>
<tr>
<td>1.55</td>
<td>0.48 ± 0.18</td>
<td>0.69</td>
</tr>
</tbody>
</table>
TABLE II: Sources of systematic uncertainty. The numbers reflect the variation of the fitted central values associated with the one-σ variation of the corresponding external input parameters. The second item includes contributions from the variation of the acceptance as a function of θ and ψ, as well as from a one-σ variation of the quantity \(|A_0(0)|^2 - |A_1(0)|^2\).

<table>
<thead>
<tr>
<th>Source</th>
<th>(σ(B^0_s)), pm</th>
<th>(ΔΓ/Γ)</th>
<th>(R_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceptance vs. cos θ</td>
<td>±0.6</td>
<td>±0.001</td>
<td>±0.005</td>
</tr>
<tr>
<td>Integration over φ, ψ</td>
<td>±0.2</td>
<td>±0.001</td>
<td>±0.02</td>
</tr>
<tr>
<td>Procedure test</td>
<td>±2.0</td>
<td>±0.025</td>
<td>±0.01</td>
</tr>
<tr>
<td>Momentum scale</td>
<td>-3.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Signal mass model</td>
<td>±1.0</td>
<td>+0.009,-0.017</td>
<td>±0.007</td>
</tr>
<tr>
<td>Background mass</td>
<td>-3.5</td>
<td>+0.02</td>
<td>-0.002</td>
</tr>
<tr>
<td>Detector alignment</td>
<td>±2.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Background model</td>
<td>±0.5</td>
<td>±0.016</td>
<td>±0.005</td>
</tr>
<tr>
<td>Total</td>
<td>-5.6,+3.1</td>
<td>-0.04,+0.03</td>
<td>±0.02</td>
</tr>
</tbody>
</table>

0.12 ± 0.05.

All the results presented above are obtained under a tacit assumption that the CP-violating phase is negligible, as predicted by the SM (\(δφ = φ_{CKM} = -0.03\)). Future improvements on the measurement of \(ΔΓ/Γ\) may exclude models predicting large deviations of \(δφ\) from the SM value.

In summary, we have measured the CP-odd fraction for the decay \(B^0_s \rightarrow J/ψ φ\), and the correlated parameters of the average lifetime of the \((B^0_s, B^0_{s0})\) system \(\tau(B^0_s) = 1/Γ\), and the relative width difference \(ΔΓ/Γ\), or, equivalently, the mean lifetimes of the light and heavy \(B^0_s\) eigenstates, respectively. We obtain

\[
R_1 = 0.16 ± 0.10 \text{ (stat)} ± 0.02 \text{ (syst)},
\]

\[
ΔΓ/Γ = 0.24^{+0.28}_{-0.38} \text{ (stat)} ± 0.03 \text{ (syst)},
\]

\[
\tau(B^0_s) = 1.39^{+0.13}_{-0.16} \text{ (stat)} ± 0.01 \text{ (syst)} \text{ ps},
\]

\[
τ_{L} = 1.24^{+0.14}_{-0.11} \text{ (stat)} ± 0.01 \text{ (syst)} \text{ ps},
\]

\[
τ_{H} = 1.58^{+0.39}_{-0.42} \text{ (stat)} ± 0.02 \text{ (syst)} \text{ ps}.
\]

We have updated the measurement of the mean lifetime of the \(B^0\) meson with doubled statistics. With the systematic uncertainty estimated in Ref. [3], the updated measurement is

\[
τ(B^0) = 1.530 ± 0.043 \text{ (stat)} ± 0.023 \text{ (syst)} \text{ ps}.
\]

For the ratio of the average \(B^0_s\) lifetime to the \(B^0\) lifetime, we obtain

\[
\frac{τ(B^0_s)}{τ(B^0)} = 0.91 ± 0.09 \text{ (stat)} ± 0.003 \text{ (syst)}.
\]

We thank the staffs at Fermilab and collaborating institutions, and acknowledge support from the DOE and NSF (USA); CEA and CNRS/IN2P3 (France); FASI, Rosatom and RFBR (Russia); CAPES, CNPq, FAPERJ, FAPESP and FUNDUNESP (Brazil); DAE and DST (India); Colciencias (Colombia); CONACyT (Mexico); KRF (Korea); CONICET and UBACyT (Argentina); FOM (The Netherlands); PPARC (United Kingdom); MSMT (Czech Republic); CRC Program, CFI, NSERC and WestGrid Project (Canada); BMBF and DFG (Germany); SFI (Ireland); Research Corporation, Alexander von Humboldt Foundation, and the Marie Curie Program.