The subject of event-by-event fluctuations has currently drawn a lot of attention in both theoretical and experimental studies of relativistic heavy ion collisions [1, 2]. It is argued that information on the QCD phase transition—the formation of Quark Gluon Plasma (QGP)—can be inferred from measurements [3] among which event-by-event charge fluctuations are considered as a promising signature [2, 4]. Due to the fractional electric charges of quarks, the charges spread more evenly throughout the QGP volume than in a hadronic gas and, therefore, the fluctuations are expected to suffer an observable suppression in a QGP [4].

Recently, it has been demonstrated that event-by-event charge fluctuation can be directly related to a thermodynamic signature—the anomalous proton-number fluctuation at the critical point [5], which is supposed to enhance the charge fluctuations. The observed enhancement of charge fluctuations at RHIC and SPS [2] seems to be a good support for these arguments, though the effects of limited detector acceptance and other corrections need to be further investigated.

The charge fluctuations are also sensitive to other effects, as the number of resonances at chemical freeze-out [6, 7] and fluctuations occurring in the initial stage [8]. The corresponding analyses are interesting by their own beyond the QGP hypothesis [9].

There are mainly two kinds of measures for the event-by-event charge fluctuations on the market at present, others being related to these under reasonable assumptions [2]. One is that of net charge fluctuations, the other that of charge ratio fluctuations. The direct measure of net charge fluctuations is the variance of net charge $Q$, where $n^+$ and $n^-$ are the numbers of positive and negative particles observed in a particular phase space window under consideration. The average is over all events in the sample. If charge is randomly assigned to each particle, $\delta Q^2 = \langle n_{ch} \rangle$, where $n_{ch} = n^+ - n^-$. So the measure for net charge fluctuations is defined as

$$D(Q) = 4 \frac{\delta Q^2}{\langle n_{ch} \rangle},$$

i.e., equal to 4 for independent emission.

In order to reduce the fluctuations of $n^+$ and $n^-$ due to the variation of impact parameter, charge ratio $R = n^+/n^-$ fluctuations are recommended in [4] and the corresponding measure is

$$D(R) = \langle n_{ch} \rangle \cdot \delta R^2,$$

where $\delta R^2 = \langle R^2 \rangle - \langle R \rangle^2$.

In the high multiplicity limit, the above two measures are approximately equal, with the leading order correction being $\sim 1/\langle n_{ch} \rangle$.

In accounting for the charge conservation in a large rapidity window and a non-zero net charge due to non-negligible baryon stopping, two correction factors [10],

$$C_Y = 1 - \frac{\langle n_{ch} \rangle_{\Delta y}}{\langle n_{ch} \rangle_{\text{total}}}, \quad C_{\mu} = \frac{\langle n^+ \Delta y \rangle^2}{\langle n^+ \Delta y \rangle^2},$$

are applied to the $D$-measures of Eq’s. (2) and (3):

$$\tilde{D} = \frac{D}{C_Y C_{\mu}}.$$
The theoretical prediction [2, 4] for the $D$-measure in a QGP is $D = 1$. It is 2.9 for a resonance gas [2, 4] and 3.26 in a quark coalescence model [2, 11].

Before comparing the data from different experiments with the above expectations, one must know how the measurements depend on the size of the rapidity window. This dependence has been estimated in various models [4, 12], but the results depend strongly on the assumptions for the rapidity correlator and the width of acceptance. Therefore, a model-independent study of the dependence of the fluctuations on the size of the rapidity window in the full rapidity domain is called for.

In addition, one should test whether the correction factors given by Eq. (4) are valid. If correct, a rapidity window size scaling should be observed in large rapidity windows due to the global charge conservation. Moreover, the rapidity size for the onset of the scaling will offer us a valuable scale for the relaxation time of long-range correlation caused by charge conservation [5, 13].

Due to the limited acceptance in current heavy ion experiments [14, 15, 16], this kind of study can only be performed in hadron-hadron experiments, such as NA22, which is equipped with a rapid cycling bubble chamber as an active vertex detector and has excellent momentum resolution over its full $4\pi$ acceptance.

In this letter, the dependence of the event-by-event net charge and charge ratio fluctuations on the size of the rapidity window is presented for $\pi^+p$ and $K^+p$ collisions at 250 GeV/c. Since no statistically significant differences are seen between the results for $\pi^+$ and $K^+$ induced reactions, the two data samples are combined for the purpose of this analysis. A total of 44 524 non-single-diffractive events is obtained after all necessary selections as described in detail in [17]. Secondary interactions are suppressed by a visual scan and the requirement of charge balance, $\gamma$ conversion near the vertex by electron identification.

The $D(Q)$-measure in central rapidity windows $|y| < \delta_y/2$ with $0.001 \text{GeV/c} < p_t < 0.1 \text{GeV/c}$ (open circles), $p_t > 0.1 \text{GeV/c}$ (open squares), $p_t > 0.2 \text{GeV/c}$ (crosses), and $p_t > 0.2 \text{GeV/c}$, $\Delta \phi = \pi/2$ (open triangles), and $p_t > 0.2 \text{GeV/c}$, $A_0 = n/2$ (open triangles), the open circles correspond to random charge assigned to each particle and open diamonds are the results from PYTHIA.

The dependence of $D(Q)$ on the size of the central rapidity window $|y| < \delta_y/2$ with $0.001 \text{GeV/c} < p_t < 10 \text{GeV/c}$ (open circles), $p_t > 0.1 \text{GeV/c}$ (open squares), $p_t > 0.2 \text{GeV/c}$ (crosses), and $p_t > 0.2 \text{GeV/c}$, $\Delta \phi = \pi/2$ (open triangles), and $p_t > 0.2 \text{GeV/c}$, $A_0 = n/2$ (open triangles), the open circles correspond to random charge assigned to each particle and open diamonds are the results from PYTHIA.

The loss of small-$p_t$ particles and the cut in azimuthal angle both enhance the fluctuations. The $D(Q)$ obtained under the same cuts as PHENIX [14] and STAR [15] are also presented.

The solid circles correspond to random charge assigned to each particle, which indeed gives the value of 4 as expected, no matter how small the multiplicity is in very narrow rapidity intervals. This shows that the accuracy of event-by-event analysis hardly depends on event multiplicity and thus can be useful even for low multiplicity cases [19]. So, the dependence of the data on centrality is not caused by an insufficient number of particles [20].

From Fig. 1(a), it can be seen that with the widening of the rapidity window, the NA22 data keep decreasing from the value close to 4 (as expected for independent emission) to 1 (as expected for a QGP) and even below. The loss of small-$p_t$ particles and the cut in azimuthal angle both enhance the fluctuations. The $D(Q)$ obtained under the same cuts as PHENIX [14] and STAR [15] are listed in Tab. I. Their values are consistent with ours.

In this letter, the dependence of the event-by-event net charge and charge ratio fluctuations on the size of the rapidity window is presented for $\pi^+p$ and $K^+p$ collisions at 250 GeV/c. Since no statistically significant differences are seen between the results for $\pi^+$ and $K^+$ induced reactions, the two data samples are combined for the purpose of this analysis. A total of 44 524 non-single-diffractive events is obtained after all necessary selections as described in detail in [17]. Secondary interactions are suppressed by a visual scan and the requirement of charge balance, $\gamma$ conversion near the vertex by electron identification.

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The $D(Q)$-measure in central rapidity windows $|y| < \delta_y/2$ with $0.001 \text{GeV/c} < p_t < 0.1 \text{GeV/c}$ (open circles), $p_t > 0.1 \text{GeV/c}$ (open squares), $p_t > 0.2 \text{GeV/c}$ (crosses), and $p_t > 0.2 \text{GeV/c}$, $\Delta \phi = \pi/2$ (open triangles), and $p_t > 0.2 \text{GeV/c}$, $A_0 = n/2$ (open triangles), the open circles correspond to random charge assigned to each particle and open diamonds are the results from PYTHIA.
vation is about 4 rapidity units. The corrections reduce the measure in small rapidity windows and enhance it in large ones. Since the influence of global charge conservation always enhances the fluctuations, i.e., $C_y < 1$ in Eq. (4), the suppression in small rapidity windows shows that the leading-particle stopping is non-negligible. If only the effect of global charge conservation is taken into account, as in STAR [15], this will always enhance the fluctuations and the scaling behavior disappears. The results for such a correction are presented in Fig. 1(c). So, the data from both STAR and PHENIX exaggerate the fluctuations, the former considering only one correction and the latter without corrections at all.

In Fig. 1(d), $D(Q)$ is presented for different positions of a unit-width rapidity window. It is almost a constant near that of a resonance gas, showing that the charge fluctuations are insensitive to the position of the rapidity window and that the local charge is non-equilibrium, as pointed out in [21].

We now turn to a similar study of the charge ratio fluctuations. Due to the positive charge of the initial-state particles, the average number of positively charged particles is higher than that of negatively charged ones. Therefore, we present the $D$-measures in terms of the charge ratios $R^+ = n^+/n^-$ and $R^- = n^-/n^+$ in Fig's. 2(a) and (b) separately, where events with $n^- = 0$ and $n^+ = 0$ have been excluded from the analysis of $R^+$ and $R^-$, respectively. It can be seen from the figures that $D(R^+)$ have much larger values than $D(R^-)$. Both of them have behavior very different from that of net charge fluctuations.

The charge ratio measures corrected according to Eq. (4) is given in Fig's. 2(c) and (d). All points are above independent emission and increase rapidly with the widening of the central rapidity window, in analogy with the model calculation [22] for A-A collisions. These results show that the corrections proposed for net charge in the observed window as given by Eq. (4) are invalid for charge ratio fluctuations.

It is further interesting to check how these measures do in recording the change of charge fluctuations with multiplicity in different rapidity windows. This is important, in particular, because the even- and odd-multiplicity distributions coincide in small rapidity windows, e.g., $|y| < 2$, while separation of them appears in large windows, e.g., $|y| < 3$, [23].

The dependence of $D(Q)$, $D(R^+)$ and $D(R^-)$ on multiplicity in two rapidity windows is presented in Fig's. 3. The following can be observed: (1) First of all, all plots show clear multiplicity dependence, while the results from PHENIX [14] in a small central rapidity window show that only $D(R)$ depends on multiplicity, while $D(Q)$ is independent of it. (2) For $|y| < 2$, the fluctuations of even and odd multiplicities in terms of net charge and charge ratios coincide within the error bars, consistent with the coincidence of even- and odd-multiplicity distributions in small rapidity windows. (3) For $|y| < 3$, the $D(Q)$ separate for even- and odd-multiplicities, consistent with the separation of even- and odd-multiplicity distributions in large rapidity windows. The $D(Q)$ have almost equal separation distance for all multiplicities. While the $D(R^+)$ and $D(R^-)$ are separated differently for different multiplicities, with very big errors for odd multiplicities, as they could be the combinations of very different $n^+$ and $n^-$. These observations show that $D(Q)$ is better in recording the change of charge fluctuations with multiplicity in different size of central rapidity windows.

The above results can be summarized as follows: (1) $D(Q)$, $D(R^+)$ and $D(R^-)$ depend strongly on the size of the central rapidity window. (2) $D(Q)$ eliminates the influence of global charge conservation and leading-particle stopping. Its scaling behavior is observed when the central rapidity window is wider than 4 rapidity units.
The same corrections are invalid for charge ratio fluctuations. (3) \( D(Q) \) is insensitive to the position of the rapidity bin. (4) \( D(Q), D(R^+) \) and \( D(R^-) \) all have clear multiplicity dependence. \( D(Q) \) has a better record in distinguishing the charge fluctuations of even and odd multiplicities than \( D(R^+) \) and \( D(R^-) \). (5) PYTHIA can reproduce almost all the data for charge fluctuations, while it fails to describe the transverse momentum fluctuations in different central rapidity windows [24].

In summary, the dependence of charge fluctuations on the size of the rapidity window is presented for the first time in the full rapidity domain. The correction factors for net charge fluctuations given by Eq. (4) eliminate the influence of global charge conservation and leading-particle stopping. The latter is non-negligible in small rapidity windows. Due to the incomplete consideration on these two corrections, both STAR and PHENIX exaggerate the fluctuations. The scale of long-range correlations caused by charge conservation is about 4 rapidity units at \( \sqrt{s} = 22 \text{ GeV/c} \). The measure in terms of net charge fluctuations is better than that of charge ratio ones.

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