Measurement of the branching ratios for the decays $\tau \rightarrow \text{hadron } \pi^0\nu$ and $\tau \rightarrow \text{hadron } \pi^0\pi^0\nu$

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The Crystal Ball detector at the DORIS II storage ring at DESY has been used to measure the branching ratios for the decay modes $\tau^+ \rightarrow h^+\pi^0\nu$ and $\tau^- \rightarrow h^-\pi^0\pi^0\nu$, where $h^+$ is any charged hadron. The results are $BR(\tau^+ \rightarrow h^+\pi^0\nu) = (22.0 \pm 0.8 \pm 1.9)\%$, $BR(\tau^- \rightarrow h^-\pi^0\pi^0\nu) = (5.7 \pm 0.5\% \pm 1.3)\%$. The first result is in good agreement with the present world average. The decay mode $h^+\pi^0\pi^0\nu$ is reconstructed in $\tau$ decays for the first time. Its branching ratio, however, is somewhat lower than the corresponding world average, and therefore tends to increase the one-prong problem.
1. Introduction

Since the discovery of the τ lepton [1] in $e^+ e^- \rightarrow \tau^+ \tau^-$, a large number of groups at various laboratories have investigated its properties [2]. All evidence available to date supports the validity of the standard model and the treatment of the τ lepton as the sequential partner of the muon and electron.

The one-prong decays (the decays to a single charged particle) of the τ, however, still present puzzling aspects. The measured exclusive branching ratios agree well with their predictions. However, their sum is smaller than the measured inclusive one-prong branching ratio [3–7]. It is not clear whether this discrepancy indicates as yet undetected decays or is simply due to poor measurements, in particular of the modes involving several neutral particles.

The Crystal Ball detector is an excellent apparatus to detect photons (and $e^\pm$) with high efficiency and to determine their energy and direction with high precision. Therefore it is well suited to investigate τ decay modes involving π0's by explicit reconstruction of the latter. In this paper we present the results of a high statistics analysis of the τ decay modes $\tau^\pm \rightarrow h^\pm \pi^0 \nu$ and $\tau^\pm \rightarrow h^\pm \pi^0\pi^0\nu$, where $h^\pm$ is a charged hadron.

2. Detector

The Crystal Ball detector has been described in detail elsewhere [8,9] and its properties are only briefly summarized here. It is a nonmagnetic calorimeter designed to measure precisely the energies and directions of electromagnetically showering particles. The main part of the detector consists of a spherical shell of 672 NaI(Tl) crystals covering 93% of 4π sr. The length of each crystal corresponds to about 16 radiation lengths and to about 1 nuclear interaction length.

An additional 5% of the solid angle is covered by endcaps, consisting of 40 NaI(Tl) crystals; these endcaps, however, do not allow as accurate a measurement of the energy and direction of a particle and are only used to veto events having particles outside the main detector.

For electromagnetically showering particles the measurement of energy and direction is made using thirteen contiguous crystals in the main detector. This procedure yields an energy resolution given by

$$\sigma_E/E = (2.7\pm0.2)\% /\sqrt{E/\text{GeV}}.$$  

For such particles the angular resolution in the polar angle is $\sigma_\theta = 2^\circ - 3^\circ$, the resolution improving as the energy increases.

Proportional tube chambers surrounding the beam pipe detect charged particles. Depending on the run period, the chambers consisted of three or four double layers of tubes. The inner (outer) layer covers 98% (78%) of 4π sr. Charge division readout allows a determination of the z-position to $\sigma_z = 1.5\%$ of the length of the tube, i.e., from 9.8 mm for the inner layer to 5.5 mm for the outer layer.

Photons, electrons and positrons yield a rather symmetric lateral energy deposition pattern in the NaI(Tl), with typically 70% of the energy in one crystal and about 98% within a group of thirteen contiguous crystals. A charged particle is identified by a

$^\text{11}$ The $z$ axis is in the direction of the $e^+$ beam, the origin is at the center of the overlapping beam bunches, and $\theta$ is the angle with respect to the $z$ axis.
track in the tube chambers that is correlated with an energy deposition in the calorimeter. Muons and charged hadronic particles that do not undergo a strong interaction deposit energy by ionization only. Minimum ionizing particles typically deposit about 200 MeV in one or two crystals. If an energetic charged hadron interacts strongly while traversing the ball, the deposited energy is in general much larger than 200 MeV and the pattern of the hadronic shower is very irregular compared to that of an electromagnetic shower.

3. Monte Carlo simulation

We made a Monte Carlo simulation of \( \tau\tau \) events to determine the efficiencies of our selection criteria. The KORALB program [10] was used to generate \( e^+e^-\rightarrow\tau^+\tau^- \) events. This program includes radiative corrections up to \( O(\alpha^3) \) and effects due to spin correlations between the \( \tau \) 's. The \( \tau \) 's are decayed by the package TAUOLA [2], which includes all the standard \( \tau \) decay modes. It has been found that the decay of the \( \tau \) into two (or three) pions proceeds dominantly via \( \rho \) (or \( a_1 \)) resonance production [4]. Resonance production is therefore assumed in the Monte Carlo simulation. Our selection criteria, however, are chosen such that the efficiency does not strongly depend on this assumption.

The generated events are passed through a complete detector simulation including the following features: (a) Electromagnetically showering particles are simulated by the program EGS 3 [15]. (b) Hadronic interactions in the detector are simulated by an improved version of the GHEISHA 6 program [16,17]. (c) Extra energy deposited in the crystals by beam-related background is taken into account by adding special background events to Monte Carlo events. These background events are obtained by triggering on one in every \( 10^7 \) beam crossings, with no other condition imposed. (d) Events are reconstructed using our standard software and subjected to the same cuts as the data.

4. Data sample

The data used in this analysis were collected at the DORIS II storage ring at DESY in the center-of-mass energy (\( E_{CM} \)) range from 9.4 to 10.6 GeV. The data sample corresponds to an integrated luminosity of 256 pb\(^{-1} \) and a corresponding number of produced \( e^+e^-\rightarrow\tau^+\tau^- \) events of \( N_\tau = (267 \pm 11) \times 10^3 \). The luminosity is measured [9] using large-angle Bhabha scattering events with an accuracy better than 3%. The number of produced \( \tau\tau \) pairs is calculated using the measured luminosity and the radiatively corrected QED cross section [10] for \( e^+e^-\rightarrow\tau^+\tau^- \) in the above energy range. For the data taken on the \( Y(1S) \) and \( Y(2S) \) resonances, the additional contribution from \( Y \rightarrow \tau\tau \) is calculated from the number of \( Y \) 's produced and their leptonic branching ratios.

5. Analysis method

To identify \( e^+e^-\rightarrow\tau^+\tau^- \) events we require that one of the \( \tau \) 's decays to one charged particle and nothing else visible: \( \tau \rightarrow ev \), \( \tau \rightarrow \mu v \), \( \tau \rightarrow lv \), where \( h \) is a charged \( \pi \) or \( K \). We refer to the charged particle as the "tag". We are looking for events where the other \( \tau \) decays into \( h^+\pi^-v \) or \( h^+\pi^-h^0 v \). Therefore, we select low multiplicity events with a jet-jet topology: a single charged particle on one side, and a charged particle plus some associated photons on the other side of the event.

In our experiment we do not measure the \( \pi^\pm \) energy; therefore, it is impossible to reconstruct the \( \rho \) and \( a_1 \) resonances. The determination of the branching ratios for \( \tau^\pm \rightarrow h^\pm\pi^0 v \) and \( \tau^\pm \rightarrow h^\pm\pi^0 h^0 v \) is performed by counting the \( \tau \) decays with one or two \( \pi^0 \) 's.

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\(^{42}\) Part of the Monte Carlo data was generated with version 1.4 of TAUOLA, another part with version 1.5. The main difference between these versions is the inclusion of radiative (QED) corrections to the decay products of the \( \tau \). No significant differences, however, were observed in the efficiency determination when using one of the other subsample. For TAUOLA 1.5 see ref. [11].

\(^{43}\) For the \( \rho \) resonance, see ref. [12]; for the \( a_1 \) resonance, see ref. [13], see also ref. [14].
We account for feeddown from other $\tau\tau$ channels into the channels of interest, e.g., feeddown of $\tau\tau\rightarrow p\nu$, $p\bar{\nu}$ into $\tau\tau\rightarrow ta\bar{\nu}$, $a_i\nu$; and of $\tau\tau\rightarrow ta\bar{\nu}$, $a_i\nu$ into $\tau\tau\rightarrow ta\bar{\nu}$, $p\bar{\nu}$. The latter process occurs if one of the $\pi^0$s from the $a_1$ decay misses the main detector or when a $\pi^0$ and a $\pi^0$ from a highly boosted $p$ (from $\bar{a}_1 p \rightarrow \tau^0$) are not resolved by the detector.

A $\pi^0$ is reconstructed from its two decay photons. For low-momentum $\pi^0$s the two photons have a large enough opening angle that their energy depositions in the calorimeter are well separated. Such $\pi^0$s are identified by method (1). We account for feeddown from other $xx$ channels into $xx\rightarrow ta\nu$, a,$\nu$; and of $xx\rightarrow ta\nu$, a,$\nu$ into $xx\rightarrow ta\nu$, $p\nu$. The latter process occurs if one of the $\pi^0$s from the $a_1$ decay misses the main detector or when a $\pi^0$ and a $\pi^0$ from a highly boosted $p$ (from $\bar{a}_1 p \rightarrow \tau^0$) are not resolved by the detector.

For $\pi^0$ energies above 500 MeV, it becomes increasingly likely that the photon showers overlap, giving a single elongated energy deposition. Such "merged" $\pi^0$s are identified by method (2), which is an analysis of the shape of the energy deposition, yielding the invariant mass and the direction cosines of the parent $\pi^0$. This method [18] makes use of the second moment, $S$, of the energy distribution, which is calculated using

$$ S = \frac{1}{E} \sum_i E_i (\hat{n}_i - \langle \hat{n} \rangle)^2, \tag{1} $$

where $E_i$ is the energy deposited in crystal $i$ and $\hat{n}_i$ is the unit vector formed by the direction cosines of crystal $i$. The sum runs over all the crystals in the energy cluster under consideration; $E = \sum_i E_i$ is the total energy of the cluster, and $\langle \hat{n} \rangle = \overline{\sum_i E_i \hat{n}_i / E}$ is the direction of its center-of-gravity. The invariant mass $M$ of the shower is given by $M^2 = E^2 - E^2 \langle \hat{n} \rangle^2 = E^2 S$. Using Monte Carlo simulations it was found that an estimate for the real invariant mass (also called shower mass) of particle $X$, decaying into two photons, can be calculated using the expression

$$ M_X^2 = E^2 (S - S_c), \tag{2} $$

where $S_c = 0.004$ is the average second moment of a single photon shower.

In the search for $\tau^\pm\rightarrow h^\pm\pi^0\pi^0\nu$, the highest detection efficiency is obtained by selecting one $\pi^0$ via method (1) and the other via method (2). This is simply a consequence of the energy distribution of the $\pi^0$s in this process. In the search for $\tau^\pm\rightarrow \pi^\pm\pi^0\nu$, the highest efficiency is obtained with method (2). However, use of this method would result in a heavy contamination by events containing a highly energetic photon, e.g., $e^+e^-\rightarrow e^+e^-\gamma$, $\mu^+\mu^-\gamma$. Estimation and subtraction of such background is expected to result in a large systematic error. Therefore, we have chosen to use method (1) for this process.

6. General cuts

We start with a set of general cuts to select the desired $\tau\tau$ sample from a large data sample including radiative Bhabha and $\mu^+\mu^-$ events, continuum hadron events, two-photon events, $Y$ decays and events originating from beam-gas and beam-wall collisions. This first set of cuts is the same for both the $\tau^\pm\rightarrow h^\pm\pi^0\nu$ and the $\tau^\pm\rightarrow h^\pm\pi^0\gamma$ analyses. The cuts below are designed to reduce the above background as much as possible while retaining a maximum acceptance for the channels of interest.

- We require exactly two charged particles in the main detector $(|\cos \theta| < 0.85)$. These particles must have a deposited energy of at least 75 MeV, and at most $0.85 \times E_{beam}$ each. The angle between their directions must be larger than 90°. In addition, at least one of the charged particles must be isolated from neutral particles; that is, there must not be a neutral particle (with an energy of more than 50 MeV) within 60° of the charged particle direction.

- The total energy ($E_{vis}$) measured in the main detector is required to satisfy $2 \text{GeV} < E_{vis} < 0.75 \times E_{CM}$. The lower limit, apart from reducing the machine related background and two photon events, also ensures that the trigger efficiency is 100%. Further, we require the energy deposited in the endcaps to be less than 100 MeV.

- To obtain events having a large energy flow perpendicular to the beam directions, we require that the transverse energy $E_T$ be greater than 1500 MeV. The transverse energy is defined as $E_T = \sum_i E_i \sin \theta_i$, where $E_i$ and $\theta_i$ are the energy deposited in, and the polar angle of, crystal $i$, respectively.

- The events must have a jet-jet topology; this is achieved by requiring the second Fox–Wolfram moment [19] to be greater than 0.4. This moment is defined as $H_2 = \sum_{ij} E_i E_j (3 \cos^2 \alpha_{ij} - 1)/2 (\sum_i E_i)^2$, where $E_i$ is the energy deposition in crystal $i$ and $\alpha_{ij}$ is the angle between crystals $i$ and $j$.

Further selection criteria depend on the specific $\tau$ decay mode under study.
7. $\tau$ decays with one $\pi^0$

We first discuss the selection of $\tau\tau$ events where one of the $\tau$'s decays into $h^+\pi^0\nu$. Recall (section 5) that we reconstruct such a $\pi^0$ using two well-separated photons. Therefore, we require two neutral particles that must fulfill the following conditions:

- For their energy ($E_{\gamma_1}$) and polar angle ($\theta_{\gamma_1}$), we require $E_{\gamma_1} > 50$ MeV and $|\cos \theta_{\gamma_1}| < 0.85$, respectively. Their lateral energy deposition patterns must be consistent with that of electromagnetically showering particles. Furthermore, the sum of their energies, $E_{\gamma_1} + E_{\gamma_2}$, is required to be smaller than 1200 MeV, because it is more likely that a $\pi^0$ above 1200 MeV produces a "merged" shower in the calorimeter. In addition, we require the angle between the directions of $\gamma_1$ and $\gamma_2$ to be smaller than $120^\circ$.

- The direction of their total momentum, $p_{\gamma_1} + p_{\gamma_2}$, must be within $90^\circ$ of the direction of one of the charged particles. The $\pi^0$ and the charged particle $h^*$ whose direction is closest to that of the $\pi^0$ are assumed to originate from the same $\tau$. The other charged particle is the tagging particle.

In addition to these two photons, we allow at most one neutral particle with an energy less than 50 MeV in the main detector. We thus allow at most one soft radiative photon. Further, we require:

- The pseudo-invariant mass of the $h^+\gamma\gamma$ system to be smaller than 1400 MeV (i.e., well below the $\tau$ mass). The term "pseudo" refers to the fact that we do not measure the real $h^+$ energy. In the calculation of the invariant mass, the energy deposited by the charged hadron is used instead.

- The tagging particle to be a highly energetic electron or positron (as expected from $\tau\rightarrow e\nu\bar{\nu}$); that is, (a) the lateral energy deposition must be consistent with that of an electromagnetically showering particle, and (b) its energy must be greater than 1000 MeV.

Tagging on an $e^\pm$ results in events with a typical $\tau\tau$ signature: a lepton on one side, hadrons on the other. We do not lose much efficiency by this cut because most of the events where the tag is a $\mu^\pm$, $\pi^\pm$ or $K^\pm$ were already (unavoidably) rejected by the lower limit on the visible energy $E_{\text{vis}}$. By requiring an energy deposition of at least 1000 MeV for the tagging particle, we avoid the problem that most of the energy needed to pass the lower limit on $E_{\text{vis}}$ comes from the $h^+\pi^0$ system. Since the selected photons are relatively soft, the absence of this cut would have biased the events such that the charged pion deposits a lot of energy in the calorimeter. This would have led to a large systematic error because the uncertainty in the Monte Carlo simulation for the $h^+$ interaction in the NaI increases with the deposited $h^+$ energy.

Fig. 1 shows the distribution of the invariant mass of the two photons for the events that passed the above cuts. To obtain the number of $\tau\tau$ events with one $\pi^0$, the signal plus background is fitted with a gaussian on top of a third order polynomial for the background. The $\pi^0$ mass thus obtained was consistent with the known value of the $\pi^0$ mass, and the width of the distribution with the resolution of our detector. From the fit we find $800 \pm 28$ events. From a Monte Carlo simulation of continuum hadron (qq) events [20] we expect the continuum to contribute $21 \pm 14$ events to the peak. No evidence for other background not originating from $\tau\tau$ events was found. The number of $\tau\tau$ events with one $\pi^0$ thus becomes $779 \pm 31$.

The selected data sample does, however, contain events originating from $\tau$ decays involving an $a_1$ as described in section 5; their contribution is also determined by a Monte Carlo simulation. Significant feeddown comes from the following decay channels: $\tau\tau\rightarrow e\nu\bar{\nu}a_1, \pi^0(K)\nu\bar{\nu},$ and $p\pi^0\nu$, where the $a_1$ subsequently decays into $p\pi^0$. The fractions of such events that pass our cuts for $\tau^\pm\rightarrow p^\pm\nu\rightarrow \pi^\pm\pi^0\nu$ are: $f_{\text{eal}} =$
(0.34 ± 0.04)%, \( f_{\pi^0(K\pi)} = (0.07 ± 0.03)\% \) and \( f_{\pi^0} = (0.04 ± 0.02)\% \), respectively. Because the actual feeddown is dependent on the branching ratio for \( \tau^- \rightarrow a^+ \gamma \rightarrow \pi^+ \pi^0 \nu \), the former can only be determined after (or simultaneously with) the determination of the latter.

8. \( \tau \) decays with two \( \pi^0 \)s

We now proceed with the selection of \( \tau \) events where one of the \( \tau \)'s decays to two neutral pions. Recall (section 5) that we reconstruct one of the \( \pi^0 \)s using the shower-mass technique and the other \( \pi^0 \) using two well-separated photons. In addition to the set of general cuts described in section 6, we make the following cuts:

- We require exactly three neutral energy clusters, at least one with \( E > 500 \) MeV and the other(s) with \( E > 50 \) MeV, in the main ball. The cluster with the highest shower mass \( M \) and with \( E > 500 \) MeV is the "merged"-\( \pi^0 \) candidate. The other two clusters are the single-photon candidates.
- In addition to these energy clusters, we allow at most one neutral particle with an energy less than 50 MeV in the main detector (to allow for a radiative photon).
- The single-photon candidates must have lateral energy deposition patterns consistent with being a single photon. Just as in the selection of events containing only one \( \pi^0 \), we require that \( E_{\gamma_1} + E_{\gamma_2} < 1200 \) MeV and that the angle between the directions of \( \gamma_1 \) and \( \gamma_2 \) be smaller than 120°.

We now introduce some cuts to ensure to a high degree that the \( \pi^0 \) and both photons originate from the decay of the same \( \tau \). These cuts mainly select against feeddown of events of the type \( \tau \rightarrow p\bar{p}\nu\bar{\nu} \).

- The direction of the total momentum of the neutral particles, \( p_{\text{neut}} = p_{\pi^0} + p_{\gamma_1} + p_{\gamma_2} \), must be close to one of the charged particles, i.e., if \( n_{\text{ch}} \) is the direction of a charged particle, then for at least one of the charged particles one must have \( (p_{\text{neut}} - n_{\text{ch}}) / |p_{\text{neut}}| > 0.2 \). In addition, the other charged particle should be at least 120° away from the direction of \( p_{\text{neut}} \).
- The pseudo-invariant mass of the \( h^\pm \pi^0 \gamma \gamma \) system must be smaller than 1400 MeV.

We do not make any additional cuts on the tagging particle such as those used in the \( \tau^- \rightarrow h^\pm \pi^0 \nu \) analysis. Such cuts are unnecessary here. The bias of the selected events towards high energy depositions from charged pions (as described in the last item of the previous section) does not occur here, because we now have two neutral pions that deposit their total energy in the calorimeter.

In fig. 2 we plot the invariant mass of the two photons versus the shower mass of the "merged"-\( \pi^0 \) candidate. Clear clustering of \( \pi^0 \) events is observed. After a cut on the shower mass \( (90 < M < 180 \) MeV), \( M(\gamma, \gamma) \) axis (fig. 3). This \( \gamma \gamma \) mass distribution is fitted with a gaussian \( \pi^0 \) signal plus a linear polynomial for the background. The \( \pi^0 \) mass from the fit was consistent with the known value and the width with the resolution of our detector. The number of \( \tau \) events with two \( \pi^0 \)s thus found is \( 133 ± 12 \) events. This number contains a small contribution from the process \( \tau^- \rightarrow p\bar{p}\nu\bar{\nu} \). From a Monte Carlo sample of 11 000 events only two events survived the selection criteria for \( \tau^- \rightarrow h^\pm \pi^0 \gamma \gamma \). Again, the actual feeddown can only be given after (or simultaneously with) the determination of BR(\( \tau^- \rightarrow h^\pm \pi^0 \nu \)). No evidence of other background to the peak was found.
Fig. 3. The invariant mass of the two photons in $\tau\tau$ events after one $n^0$ has already been identified by the shower mass technique.

9. Results

If $N(\pi^0)$ is the number of observed $\tau\tau$ events with one $\pi^0$, the branching ratio for $\tau^\pm \rightarrow h^\pm \pi^0\nu$ is given by

$$\text{BR}(\tau^\pm \rightarrow h^\pm \pi^0\nu) = \frac{N(\pi^0) - N_{\text{bg}}(\pi^0)}{2\epsilon_{\text{tag}} + \epsilon_{b}} \times 100\%,$$

where $N_{\text{bg}}(\pi^0)$ is the feeddown from $\pi\pi\rightarrow\text{tag}$ events (section 7), i.e.,

$$N_{\text{bg}}(\pi^0) = 2 \left[ \text{BR}(\tau^\pm \rightarrow \pi^0\nu) \epsilon_{\text{tag},\pi} + \text{BR}(\tau^\pm \rightarrow h^\pm \pi^0\nu) \epsilon_{\text{tag},h} \right] N_t,$$

and

$$\epsilon_{\text{tag},\pi} = \text{BR}(\tau^\pm \rightarrow \pi^0\nu) \epsilon_{\text{sp}} + \text{BR}(\tau^\pm \rightarrow H\nu) \epsilon_{\text{sp}}$$

$$+ \text{BR}(\tau^\pm \rightarrow \pi^0\nu) \epsilon_{\text{sp},\pi}$$

$$= 0.62 \pm 0.02\%.$$

$N_t = (267 \pm 11) \times 10^3$ is the total number of $\tau\tau$ events in our full data sample. For the branching ratios of the tagging particles we use the Particle Data Group values \cite{7}, $\text{BR}(\tau^\pm \rightarrow \pi^0\nu) = (17.7 \pm 0.4)\%$, $\text{BR}(\tau^\pm \rightarrow H\nu) = (17.8 \pm 0.4)\%$ and $\text{BR}(\tau^\pm \rightarrow \pi^0\nu) = (11.7 \pm 0.5)\%$. The selection efficiencies $\epsilon_{\text{tag},\pi}$ and $\epsilon_{\text{tag},h}$ for the $\tau\tau$ channels contributing to our selected data samples are shown in table 1. The substracts $p$ and $a_1$ of the efficiencies $\epsilon$ above are used to remind the reader that $p$ and $a_1$ dominance is assumed in the simulated $\tau$ decays. The efficiencies are defined in the following way: e.g., $\epsilon_{\text{sp}}$ is the ratio of $xx$ Monte Carlo events (one $x$ decaying into $e\nu\nu$ and the other into $\mu\nu\nu$) accepted by our selection criteria to the total number of such events.

The branching ratio for $\tau^\pm \rightarrow h^\pm \pi^0\pi^0\nu$ is given by

$$\text{BR}(\tau^\pm \rightarrow h^\pm \pi^0\pi^0\nu) = \frac{N(\pi^0\pi^0) - N_{\text{bg}}(\pi^0\pi^0)}{2\epsilon_{\text{tag},\pi} + \epsilon_{\text{sp}}}$$.\footnote{Such a test is not possible for the $\tau^\pm \rightarrow h^\pm \pi^0\nu$ analysis because there the tagging particle is required to be an $e\nu\nu$ (cf. section 7).}

$$\epsilon_{\text{tag},\pi} = 3.36 \pm 0.09 \%, \quad \epsilon_{\text{sp}} = 0.01 \%, \quad \epsilon_{\text{sp},\pi} = 0.19 \pm 0.03 \%, \quad \epsilon_{\text{sp},h} = 0.25 \pm 0.05 \%.$$

where $N(\pi^0\pi^0)$ is the number of $xx$ events that passed our selection criteria, $N_{\text{bg}}(\pi^0\pi^0)$ is the feeddown from $\tau\tau\rightarrow\text{pp\nu}$ (section 8),

$$N_{\text{bg}}(\pi^0\pi^0) = \frac{1}{\epsilon_{\text{tag},\pi} + \epsilon_{\text{sp}} + \epsilon_{\text{sp},\pi}} N_t,$$

and

$$\epsilon_{\text{tag},\pi} = \text{BR}(\tau^\pm \rightarrow \pi^0\pi^0) \epsilon_{\text{tag},\pi} + \text{BR}(\tau^\pm \rightarrow \mu\nu\nu) \epsilon_{\text{sp}}$$

$$+ \text{BR}(\tau^\pm \rightarrow \pi^0\nu) \epsilon_{\text{tag},\pi} + \text{BR}(\tau^\pm \rightarrow \mu\nu\nu) \epsilon_{\text{sp}}.$$

Eqs. (3)–(6) are solved. The result for the feeddown is $N_{\text{bg}}(\pi^0\pi^0) = 24 \pm 5$ and $N_{\text{bg}}(\pi^0\pi^0) = 23 \pm 1.5$. For the branching ratios we obtain $\text{BR}(\tau^\pm \rightarrow h^\pm \pi^0\nu) = (22.0 \pm 0.8 \pm 1.2)\%$ and $\text{BR}(\tau^\pm \rightarrow h^\pm \pi^0\nu) = (5.7 \pm 0.5 \pm 0.5)\%$. The systematic error includes the error due to limited Monte Carlo statistics, the uncertainty on the total number of $\tau\tau$ events ($N_t$), and the errors on the branching ratios for the tagging particles $e$, $\mu$, $\pi$ or $K$.

A nice check on the $\tau^\pm \rightarrow h^\pm \pi^0\pi^0\nu$ analysis can be performed by making subsamples according to the nature of the tagging particle $S_5$. Restricting the tagging particle to $S_5$ (e.g., to $e^\pm$) the selection efficiency $\epsilon_{\text{tag},\pi}$ is increased by a factor of $S_5$. Such a test is not possible for the $\tau^\pm \rightarrow h^\pm \pi^0\nu$ analysis because there the tagging particle is required to be an $e\nu\nu$ (cf. section 7).
giving particle to a highly energetic $e^\pm$ (originating from $\tau \to e\nu\bar{\nu}$), we obtain $\text{BR}(\tau^\pm \to h^\pm \pi^0\pi^0\nu) = (5.0 \pm 0.8 \pm 0.5)\%$. By instead requiring the tag to be a minimum ionizing particle (i.e., originating from $\tau \to \mu\nu\bar{\nu}$ or $\tau \to \pi(J/\psi)\nu$), we find a consistent value, $\text{BR}(\tau^\pm \to h^\pm \pi^0\pi^0\nu) = (5.5 \pm 0.8 \pm 0.7)\%$. The systematic error only includes the sources mentioned in the previous paragraph. These consistent results give us confidence that there is indeed no significant background in our data sample.

The most important additional systematic errors for the $\tau^\pm \to h^\pm \pi^0\pi^0\nu$ analysis are: (i) An uncertainty in the charged tagging efficiency (i.e., the efficiency that a charged particle is identified as charged by the tube chamber system): 5%; (ii) An uncertainty in the estimation of the background from continuum hadron events (see fig. 1): 2.5%; (iii) An uncertainty arising from variation of the fit interval and the background function (within reasonable limits) of the selection criteria: 3%. Adding these systematic errors quadratically, the total error for $\text{BR}(\tau^\pm \to h^\pm \pi^0\pi^0\nu)$, including the ones mentioned above, becomes 8.4%.

The most important additional systematic errors for the $\tau^\pm \to h^\pm \pi^0\pi^0\nu$ analysis are: Uncertainties of 5%, another 5%, and 3% as described by the items (i)–(iii) above, and 12.6% due to variation of the selection criteria. The latter error is dominated by a small discrepancy between the experimental data and the Monte Carlo simulation of the distribution $p_{\text{true}}: p_{\text{ch}} / |p_{\text{true}}|$ (defined in section 8), i.e., an angular distribution of the $\pi^\pm \pi^0\pi^0$ system. The cuts on this quantity were made rather loose in order not to become too dependent on the specific Monte Carlo model (which assumes $a_1$ dominance in the $\pi^\pm \pi^0\pi^0$ system). A maximum difference of 10% was found by using a Monte Carlo sample in which the $\tau$ decays directly into $h^\pm \pi^0\pi^0\nu$. Adding all the systematic errors in quadrature gives a total systematic error for $\text{BR}(\tau^\pm \to h^\pm \pi^0\pi^0\nu)$ of 17.1% or an absolute systematic error of 1.0%.

With an efficiency for the $\tau^\pm \to \pi^\pm \pi^0\pi^0\nu$ analysis at the level of 1%, a very good understanding of the acceptance of the detector is needed to achieve an accurate final result. Indeed, uncertainties in the Monte Carlo determination of the acceptance dominate the systematic error.

Earlier preliminary Crystal Ball analyses [21,22], using essentially the same data as in the present analysis, found $\text{BR}(\tau^\pm \to h^\pm \pi^0\pi^0\nu) = (7.0 \pm 0.7 \pm 1.4)\%$ and $(7.4 \pm 0.6 \pm 1.3)\%$. The sensitivity of the result to the Monte Carlo details is the main reason that these previous preliminary results have not been published. In the interim, the analysis presented here was performed in conjunction with an extensive reexamination of the Monte Carlo used to calculate the acceptance. These studies resulted in a number of improvements [17] to the Monte Carlo, accounting in part for the noticeable difference in the value of the branching ratio between the previous preliminary analyses and the present one. It must be noted that all three analyses are in good agreement with each other for $\text{BR}(\tau^\pm \to h^\pm \pi^0\pi^0\nu)$.

As described above, an estimate of the absolute systematic error yields $\pm 1\%$. This error does not cover the central values of the previous preliminary analyses. We believe that the best current estimate of the branching ratio from our data is the new value presented in this paper. However, we choose to increase the size of the systematic error to cover the central values of the previous preliminary analyses. This leads to a final absolute systematic error of $\pm 1.3\%$.

10. Conclusions

We have measured the branching ratios for the $\tau$ decay modes $\tau^\pm \to h^\pm \pi^0\nu$ and $\tau^\pm \to h^\pm \pi^0\pi^0\nu$, where $h^\pm$ is a charged hadron. The results are

$\text{BR}(\tau^\pm \to h^\pm \pi^0\nu) = (22.0 \pm 0.8 \pm 1.9)\%$,

$\text{BR}(\tau^\pm \to h^\pm \pi^0\pi^0\nu) = (5.7 \pm 0.5 \pm 1.3)\%$.

As discussed in section 5, the charged hadron may be a $\pi$ or $K$. Subtracting a contribution [7] of $(0.7 \pm 0.1)\%$ for the branching ratio of the decay $\tau^\pm \to K^{\pm} \pi^0 \nu$ and assuming $p$ dominance in the $\pi^0$ system, we obtain $\text{BR}(\tau^\pm \to p^\pm \nu \to \pi^\pm \pi^0\nu) = (21.3 \pm 0.8 \pm 1.9)\%$. For the two-$\pi^0$ mode there is a kaon contribution [7] of 0.1% due to the decay chain $\tau^\pm \to K^{\ast \pm} \nu \to K^0\pi^\pm \nu \to \pi^\pm \pi^0\pi^0\nu$. Thus, assuming $a_1$ dominance, $\text{BR}(\pi^\pm \to a_1^\pm \nu \to \pi^\pm \pi^0\pi^0\nu) = (5.6 \pm 0.5 \pm 1.2)\%$. It should be noted that, although we assumed $p$ and $a_1$ dominance in the efficiency determination, our selection criteria are chosen such that the result
does not critically depend on this hypothesis.

Our experiment is the first to measure the branching ratio of \( \pi^{\pm} \rightarrow \pi^0\pi^0\pi^0\pi^0\nu \) by reconstruction of both neutral pions from their decay photons.

Our value for \( BR(\pi^{\pm} \rightarrow h^0K^0\nu) \) (and \( BR(\pi^- \rightarrow p^+\nu) \)) is in good agreement with other measurements. The result for \( BR(\pi^{\pm} \rightarrow \pi^0\pi^0\pi^0\pi^0\nu) \) is somewhat lower than the present world average \( (7.5 \pm 0.9)\% \) of \( (7.5 \pm 0.9)\% \). Furthermore, it is about 1.7 standard deviations below the recent CELLO result \( [23] \) of \( (10.0 \pm 1.5 \pm 1.1)\% \) which was among the results essential to the observed absence of the one-prong problem in their data. The discrepancy between their and our result for this mode suggests that the problem(s) with \( \tau \) decay modes are not yet solved. In particular, our result tends to increase the one-prong problem \( [3-7] \).

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