We report on the resummation of soft-gluon emissions for squark-antisquark production at next-to-next-to-leading-logarithmic (NNLL) accuracy. We will put particular emphasis on the one loop hard matching coefficients required to perform the resummation. Furthermore we will discuss the numerical effect of the different ingredients in the corrections. We find a significant reduction in the scale uncertainty and a considerable increase in the prediction of the total cross section at the central scale. Compared to the next-to-leading order prediction, the corrections increase the cross section by up to 30% for 1.5 TeV squarks at a centre-of-mass (CM) energy of 7 TeV at the LHC.
1. Introduction

Supersymmetry (SUSY), and in particular the minimal supersymmetric Standard Model [1], can provide a solution to the hierarchy problem, accommodate gauge coupling unification and offer a dark matter candidate. Since these issues are addressed only if the SUSY scale is comparable to the weak scale, one would expect new SUSY particles (sparticles) with masses in the TeV range, which could be measured at the Large Hadron Collider (LHC). Particularly the coloured sparticles, squarks (˜q) and gluinos (˜g), would be pair-produced copiously in hadronic collisions and thus offer the strongest sensitivity. Searches at the LHC with a centre-of-mass (CM) energy of \( \sqrt{S} = 7 \) TeV have placed lower limits on squark and gluino masses around 1 TeV [2, 3].

The leading order (LO) predictions for inclusive squark and gluino hadroproduction depend strongly on the renormalization and factorization scale. This dependence is reduced significantly if higher-order SUSY-QCD corrections are included. If the renormalization and factorization scales are chosen close to the average mass of the produced particles, the corrections generally increase the size of the cross section compared to the LO prediction. Consequently, the SUSY-QCD corrections have a substantial impact on the determination of mass exclusion limits and lead to a significant reduction of the uncertainties on SUSY mass or parameter values in the case of discovery [4, 5].

The squark-antisquark production processes have been known for quite some time at next-to-leading order (NLO) in SUSY-QCD [6, 7]. A significant part of the NLO QCD corrections can be attributed to the threshold region, where the partonic CM energy is close to the kinematic production threshold and the NLO corrections are dominated by soft-gluon emission off the coloured particles in the initial and final state. These soft-gluon corrections can be taken into account to all orders in perturbation theory by means of threshold resummation techniques [8, 9]. This has been done for all MSSM squark and gluino production processes at next-to-leading-logarithmic (NLL) accuracy [10–14].

Recently, resummation at NNLL accuracy for squark-antisquark pair production was presented [15]. Compared to the NLL calculation the new ingredients are the one-loop matching coefficients, which contain the NLO cross section near threshold, and the two-loop soft anomalous dimensions. We will discuss the impact of the corrections and provide an estimate of the theoretical uncertainty due to scale variation.

In section 2 we discuss NNLL resummation for squark-antisquark pair-production. In section 3 we present the calculation of the hard matching coefficients. The numerical results for the LHC with a CM energy of \( \sqrt{S} = 7 \) TeV are presented in section 4. We conclude in section 5.

2. Threshold resummation at NNLL

We first briefly review the formalism of threshold resummation for ˜q ˜q production. The inclusive cross section \( \sigma_{h_1 h_2 \to \bar{q} q} \) can be written in terms of its partonic version \( \sigma_{i j \to \bar{q} q} \) as:

\[
\sigma_{h_1 h_2 \to \bar{q} q}(\rho, \{m^2\}) = \sum_{i,j} \int d\hat{x}_1 d\hat{x}_2 d\hat{p} \delta\left(\hat{p} - \frac{\rho}{x_1 x_2}\right) f_{i/h_1}(x_1, \mu^2) f_{j/h_2}(x_2, \mu^2) \sigma_{i j \to \bar{q} q}(\rho, \{m^2\}, \mu^2),
\]

where \( \{m^2\} \) denotes all masses entering the calculations, \( i, j \) are the initial parton flavours, \( f_{i/h_1} \) and \( f_{j/h_2} \) the parton distribution functions, \( \mu \) is the common factorization and renormalization scale.
and \(x_{1,2}\) are the momentum fractions of the partons in hadrons \(h_1\) and \(h_2\). The hadronic threshold for the inclusive production of two squarks with mass \(m_{\tilde{q}}\) corresponds to a hadronic CM energy squared \(S = 4m_{\tilde{q}}^2\). Therefore we define a threshold variable \(\rho = 4m_{\tilde{q}}^2/S\) and its partonic equivalent \(\tilde{\rho} = 4m_{\tilde{q}}/s\), with \(s = x_1x_2S\) the partonic CM energy squared. The resummation of the soft-gluon contributions is performed after taking a Mellin transform (indicated by a tilde) of the cross section,

\[
\tilde{\sigma}_{h_1h_2 \to \tilde{q}\tilde{q}}(N,\{m^2\}) = \int_0^1 d\rho \rho^{N-1} \sigma_{h_1h_2 \to \tilde{q}\tilde{q}}(\rho,\{m^2\}) = \sum_{i,j} \tilde{f}_{i/h_1}(N+1,\mu^2) \tilde{f}_{j/h_2}(N+1,\mu^2) \tilde{\sigma}_{ij \to \tilde{q}\tilde{q}}(N,\{m^2\},\mu^2).
\]

(2.1)

The threshold limit \(\tilde{\rho} \to 1\) corresponds to \(N \to \infty\). Near threshold, fixed-order perturbation theory does not converge well due to logarithmically enhanced terms. By resumming large logarithmic corrections containing \(L = \log(N)\) to all orders, a new perturbative expansion arises, where first the leading logarithmic (LL) corrections are resummed, followed by the NLL, NNLL, ... contributions.

The relation between this new perturbative expansion and fixed-order perturbation theory is shown schematically in Figure 1.

**Figure 1:** Schematic form of the logarithmically enhanced terms with the LO cross section factored out. The terms enclosed in the solid line occur in the LL approximation, which completes the first column. The new terms in the NLL approximation are enclosed in the dotted line and complete the second column. The shaded region corresponds to new terms coming from the NNLL approximation, which completes the third and fourth column. We resum the logarithms to all orders, so the columns extend downwards to infinity.

The all-order summation of these logarithmic terms is based on the near-threshold factorization of different classes of radiation: hard, (soft)-collinear, and wide-angle soft radiation [8, 9, 16–19]. Near threshold the resummed partonic cross section to NNLL accuracy has the form [8, 9, 20]:

\[
\tilde{\sigma}_{ij \to \tilde{q}\tilde{q}}^{(\text{res})}(N,\{m^2\},\mu^2) = \sum_i \tilde{\sigma}_{ij \to \tilde{q}\tilde{q}}^{(0)}(N,\{m^2\},\mu^2) \exp \left[ Lg_1(\alpha_sL) + g_{2,i}(\alpha_sL) + \alpha_s g_{3,i}(\alpha_sL) \right] \times \left( 1 + \frac{\alpha_s}{\pi} \tilde{C}_{ij \to \tilde{q}\tilde{q}}^{(1)}(N,\{m^2\},\mu^2) \right),
\]

(2.2)

where \(\tilde{\sigma}_{ij \to \tilde{q}\tilde{q}}^{(0)}\) are the colour-decomposed LO cross sections in Mellin-moment space. The colour label \(I\) corresponds to an irreducible representation of the colour structure of the process, which for squark-antisquark production can be either a singlet or an octet [10, 11, 21]. The exponent in the first line of Eq. (2.2) captures all dependence on the large logarithm \(L\). The last line contains the one-loop Coulomb contribution \(\tilde{C}_{ij \to \tilde{q}\tilde{q}}^{(1)}\) and hard matching coefficient \(\tilde{C}^{(1)}\). Setting the hard matching and Coulomb coefficients to 0 and keeping only the \(g_1\) term in Eq. (2.2) constitutes the LL approximation. Including the \(g_2\) term as well corresponds to NLL. For NNLL accuracy also the \(g_3\) term and the one-loop hard matching and Coulomb coefficients need to be taken into account, as can be seen explicitly by expanding Eq. (2.2) and comparing it to
The colour decomposition of the LO matrix element. Due to the orthogonality of the calculation as presented in Ref. [7], we only need corrections to squark-antisquark production, so we cannot take the explicit threshold limit. Contrast to the case of top-pair production in Ref. [30], there is no full analytic result for the real colour basis that we use, the full matrix element squared is then automatically colour-decomposed: 

\[
\beta \propto \frac{\text{counterterm}}{\text{section}} = \int \text{d}\sigma^\text{NLO} \frac{1}{2\pi i} \rho^{-N} \tilde{f}_{i/h_1} (N + 1, \mu^2) \tilde{f}_{j/h_2} (N + 1, \mu^2) \]  

We adopt the “minimal prescription” of Ref. [27] for the counter CT of the inverse Mellin transform in Eq. (2.3). In order to use standard parametrizations of parton distribution functions in x-space we employ the method introduced in Ref. [28].

### 3. Calculation of the hard matching coefficients

We now discuss the calculation of the one-loop hard matching coefficients \( \varphi^{(1)} \). The calculations were done using FORM [29]. After performing an expansion of the NLO cross section in \( \beta \), the hard matching coefficients \( \varphi^{(1)} \) are determined by terms in the NLO cross section that are proportional to \( \beta, \beta \log(\beta) \) and \( \beta \log^2(\beta) \). Terms that contain higher powers of \( \beta \) are suppressed by powers of \( 1/N \) in Mellin-moment space and do not contribute to the matching coefficient. In contrast to the case of top-pair production in Ref. [30], there is no full analytic result for the real corrections to squark-antisquark production, so we cannot take the explicit threshold limit.

The virtual corrections for squark-antisquark production can be obtained from the full analytic calculation as presented in Ref. [7]. First we need to colour-decompose the result. We only need the colour decomposition of the LO matrix element. Due to the orthogonality of the s-channel colour basis that we use, the full matrix element squared is then automatically colour-decomposed:

\[
|\mathcal{M}|^2_{\text{NLO},I} = 2 \text{Re}(\mathcal{M}_{\text{NLO}} \mathcal{M}^*_\text{LO,I}).
\]

Then we are left with an expression in terms of masses, Mandelstam variables and scalar integrals. Since we need the cross section to \( \theta(\beta) \), we have to expand \( |\mathcal{M}|^2 \) to zeroth order in \( \beta \) to obtain the virtual part of the hard matching coefficients.

The integrated real corrections at threshold are formally phase-space suppressed near threshold unless the integrand compensates this suppression. Therefore we can construct the real corrections at threshold from the singular behaviour of the matrix element squared, which can be obtained using dipole subtraction [31, 32]. First we recall that the cross section can be split into three parts: a part with three-particle kinematics \( \sigma^{(3)} \), one with two-particle kinematics \( \sigma^{(2)} \), and a collinear counterterm \( \sigma^{C} \) to remove the initial-state collinear singularities. These parts are well-defined in \( n = 4 - 2\varepsilon \) dimensions, but their constituents diverge for \( \varepsilon \to 0 \). With the aid of an auxiliary cross section \( \sigma^A \), which captures all singular behaviour, all parts are made finite and integrable in four space-time dimensions. This auxiliary cross section is subtracted from the real corrections \( \sigma^R \) at the integrand level to obtain \( \sigma^{(3)} \) and added to the virtual corrections \( \sigma^V \), which defines \( \sigma^{(2)} \):

\[
\sigma^{\text{NLO}} = \int_3 \left[ d\sigma^R - d\sigma^A \right]_{\varepsilon=0} + \int_2 \left[ d\sigma^V + \int_1 d\sigma^A \right]_{\varepsilon=0} + \sigma^C \equiv \sigma^{(3)} + \sigma^{(2)} + \sigma^C
\]
Compared to the case of two-parton kinematics, the phase space of $\sigma^{(3)}$ is limited by the energy of the third, radiated massless particle. Near the two-particle threshold, the maximum energy of the radiated particle, and thus the available phase space, equals $E_{\text{max}} = \sqrt{3} - 2m_{q} \propto \beta^{2}$. Since after subtracting $\sigma^{4}$ no divergences are left in the integrand of $\sigma^{(3)}$, the leading contribution of $\sigma^{(3)}$ is at most proportional to $\beta^{2}$ and can thus be neglected. So at threshold the real radiation is completely specified by the singular behaviour contained in $\sigma^{4}$ and can therefore be determined by summing over dipoles that correspond to pairs of ordered partons [32] and taking the threshold limit.

After combining the real and the virtual corrections the hard matching coefficients can be obtained by taking the Mellin transform and omitting the Coulomb corrections and the $\log(N)$ terms. The complete expressions for the hard matching coefficients of the squark-antisquark production processes can be found in Ref. [15]. Their behaviour for varying gluino mass is shown in Fig. 2.

![Figure 2: Gluino-mass dependence of the colour-decomposed NLO hard matching coefficients for the $q\bar{q}$ initiated channel (a) and the $gg$ initiated channel (b). We have set $\mu = m_{q} = 1.2$ TeV and $m_{t} = 172.9$ GeV.](image)

### 4. Numerical results

We now present numerical results for NNLL-resummed squark-antisquark pair-production at the LHC for a CM energy of 7 TeV. We use the 2008 NLO MSTW parton distribution functions [33] with the corresponding $\alpha_{s}(M_{Z}^{2}) = 0.120$. We have used a top quark mass of $m_{t} = 172.9$ GeV [34]. In order to study the effects from the hard matching coefficients and the Coulomb corrections separately, we compare the NLL matched cross section $\sigma^{\text{NLO+NLL}}$, which is based on the calculations presented in [10–12], the NNLL matched cross section without the Coulomb contributions to the resummation $\sigma^{\text{NLO+NNLL w/o Coulomb}}$, which has $C^{\text{Coul,1}} = 0$ in Eq. (2.2), and the NNLL matched cross section $\sigma^{\text{NLO+NNLL}}$, which does include the Coulomb correction $C^{\text{Coul,1}}$. All cross sections are matched to the NLO result calculated with PROSPINO [7,35] using Eq. (2.3).

Figure 3(a) shows the mass dependence of the scale uncertainty for the squark-antisquark cross section. The squark and gluino mass have been taken equal and the scale has been varied in the range $m_{q}/2 \leq \mu \leq 2m_{q}$. We see that the scale uncertainty reduces when higher-order corrections are included. In this range of squark masses, the NNLL resummation without the Coulomb corrections $C^{\text{Coul,1}}$ already reduces the scale uncertainty to at most 10%. The inclusion of the Coulomb term $C^{\text{Coul,1}}$ in the resummed NNLL prediction results in a scale uncertainty of only a few percent.
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5. Conclusions

We have discussed the NNLL resummation of threshold corrections for squark-antisquark hadroproduction and presented numerical results for the NLO+NNLL resummed cross section for squark-antisquark production at the LHC with a CM energy of 7 TeV. For a squark mass of 2 TeV, the NLO+NNLL squark-antisquark cross section is up to 45% larger than the corresponding NLO cross section. The correction is reduced to 25% if the resummation contributions due to Coulombic interactions are omitted. The scale dependence is reduced significantly, particularly after inclusion of the Coulomb corrections. However, the observed reduction in the scale dependence might be modified somewhat by the inclusion of the width of the particles or by matching to the full NNLO result, which is not available. A very conservative estimate of the scale uncertainty is provided by the NLO+NNLL w/o Coulomb results, which do not include the Coulomb corrections. This information could be used to improve current limits on SUSY masses or, in the case that SUSY is found, to more accurately determine the masses of the sparticles.
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References


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