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Search for Excited Leptons in e^+e^- Annihilation at $\sqrt{s} = 130 - 140$ GeV

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Abstract

We report on a search for the excited leptons e^*, μ^*, τ^* and ν^* in e^+e^- collisions at $\sqrt{s} = 130 - 140$ GeV using the L3 detector at LEP. No evidence has been found for their existence. From an analysis of the expected pair produced $\ell^*\ell^*$ in the channels $ee\gamma\gamma$, $\mu\mu\gamma\gamma$, $\tau\tau\gamma\gamma$, eeWW, and $\nu\nu\gamma\gamma$, we determine the lower mass limits at 95% C.L. of 64.7 GeV for e^*, 64.9 GeV for μ^*, 64.2 GeV for τ^*, 57.3 GeV (eW decay mode) and 61.4 GeV ($\nu\gamma$ decay mode) for ν^*. From an analysis of the expected singly produced $\ell\ell^*$ in the channels $ee\gamma$, $\mu\mu\gamma$, $\tau\tau\gamma$, νeW and $\nu\nu\gamma$, we determine upper limits on the couplings λ/m_{ℓ^*} up to $m_{\ell^*} = 130$ GeV.

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1 Introduction

The Standard Model [1] has been successful in describing present experimental data, including all LEP results. However, it has a large number of arbitrary parameters and leaves many fundamental questions unexplained such as the lepton-quark spectrum, mass generation and the origin of the Higgs mechanism. One way to explain the number of families and to make the fermion masses and weak mixing angles calculable, would be to assume that quarks, leptons and gauge bosons are composite [2]. As a consequence of such a model, one would expect the existence of an excited lepton state, ℓ^*, for each known lepton, ℓ.

At e^+e^- colliders, excited leptons would be produced either in pairs ($e^+e^- \longrightarrow \ell^*\ell^*$) or singly ($e^+e^- \longrightarrow \ell\ell^*$). In the first case the maximum ℓ^* mass is limited to the beam energy, whereas in the second case it can reach mass regions close to the center of mass energy. An excited lepton ℓ^* is assumed to have spin $\frac{1}{2}$. It could have both the left and the right-handed component [3] or only a left-handed component as in the Standard Model. Since the production cross section of excited lepton pairs is smaller if only the left-handed component is present, we use this assumption in order to make conservative estimates. An excited lepton, ℓ^*, is expected to decay immediately into its ground state, ℓ, by radiating a photon or a massive vector boson, Z or W.

The excited leptons e^*, μ^*, τ^* and ν^* have been extensively searched for at the LEP e^+e^- collider with $\sqrt{s}$ = 91 GeV and at the HERA ep collider in the last five years [4]. In this paper we describe a new search for excited leptons with L3 at center of mass energies of 130 – 140 GeV.

2 Production and Decays of ℓ^*

The Z and γ are assumed to have the same coupling to an excited lepton pair $\ell^*\ell^*$ as to the standard lepton pair $\ell\ell$. The t-channel contribution for e^* and ν^* is neglected since the couplings $V\ell^*\ell$ are expected to be much smaller than normal couplings $V\ell\ell$ and $V\ell^*\ell^*$, where $V = \gamma$, Z, W. The lowest order pair production cross section can be found in ref. [2].

For single ℓ^* production, the effective Lagrangian [3] can be written as:

$$\mathcal{L}_{\mathrm{eff}} = \sum_{\mathrm{V}=\gamma,\mathrm{Z},\mathrm{W}} \frac{\mathrm{e}}{\Lambda} \bar{\Psi}_{\ell^*} \sigma^{\mu\nu} (\mathrm{C}_{\mathrm{V}\ell^*\ell} - \mathrm{D}_{\mathrm{V}\ell^*\ell}\gamma_5) \Psi_\ell \partial_\mu \mathrm{V}_\nu + \mathrm{h.c.}$$

where Λ is the composite mass scale and $\mathrm{C}_{\mathrm{V}\ell^*\ell}$ and $\mathrm{D}_{\mathrm{V}\ell^*\ell}$ are unknown coupling constants. The precise g-2 measurements impose $|\mathrm{C}_{\mathrm{V}\ell^*\ell}| = |\mathrm{D}_{\mathrm{V}\ell^*\ell}|$. The absence of the electric dipole moment of electrons suggests that both $\mathrm{C}_{\mathrm{V}\ell^*\ell}$ and $\mathrm{D}_{\mathrm{V}\ell^*\ell}$ are real. Therefore we use in the following $\mathrm{C}_{\mathrm{V}\ell^*\ell} = \mathrm{D}_{\mathrm{V}\ell^*\ell}$. The coupling constants can be written as:

$$\mathrm{C}_{\gamma\ell^*\ell} = \frac{1}{2}(\mathrm{t}_3 f + \frac{\mathrm{Y}}{2} f'), \quad \mathrm{C}_{\mathrm{Z}\ell^*\ell} = \frac{1}{2}(\mathrm{t}_3 f \cot\theta_{\mathrm{W}} - \frac{\mathrm{Y}}{2} f' \tan\theta_{\mathrm{W}}), \quad \mathrm{C}_{\mathrm{W}\ell^*\ell} = \frac{f}{2\sqrt{2}\sin\theta_{\mathrm{W}}},$$

where f and f' are respectively the free parameters for SU(2) and U(1), t_3 is the third component of weak isospin of ℓ^*, Y is the hypercharge of ℓ^* and θ_{W} the Weinberg angle. Throughout this paper we assume that t_3 and Y for the ℓ^* are the same as for the standard ℓ. For excited charged leptons, we assume that $f = f'$ so that $f/\Lambda (= \sqrt{2}\lambda/\mathrm{m}_{\ell^*})$ is the only free parameter in the Lagrangian [5]. The differential cross section formulae can be found in ref. [3] and the total cross section is obtained by integration.

An excited lepton is expected to have a narrow width and its mean decay length is less than 1 μm at LEP [6]. An excited charged lepton is expected to decay into a standard lepton and a photon with a 100% branching ratio, if its mass is smaller than that of the W and the Z. At large mass, the decays $\ell^* \rightarrow \mathrm{Z}\ell$ and $\ell^* \rightarrow \mathrm{W}\nu$ become important and the branching ratio of $\ell^* \rightarrow \ell\gamma$ is then a function of m_{ℓ^*} [6]. An excited neutrino, ν^*, can decay by emitting a γ, Z or W, which are either virtual or real depending on the mass of ν^*. We have studied the following two cases:

(1) $f = f'$: In this case the $\gamma\nu\nu^*$ coupling vanishes and $\nu^* \longrightarrow \nu\mathrm{Z}$ and $\nu^* \longrightarrow \mathrm{eW}$ are the only decay modes allowed (the searches are limited to the ν_e^*, which is expected to be the lightest excited neutrino). Since the eW branching ratio is more than 65% [6] for m_{ν^*} = 40 – 140 GeV, and its experimental signatures are much cleaner than those of the Z channel decays, we investigate only the W channel decays. The visible final state is an electron plus two jets if the W decays hadronically or an electron plus another lepton if the W decays leptonically. The coupling parameter in the Lagrangian is the same as for the charged excited leptons: $f/\Lambda (= \sqrt{2}\lambda/m_{\ell^*})$.

(2) $f \neq f'$: In this case the $\gamma\nu\nu^*$ coupling exists. The neutrino has a magnetic moment [7] and the decay $\nu^* \longrightarrow \nu\gamma$ would have a large branching ratio. Hence the W and Z channel decays are neglected in this analysis. The signature of a ν^* is a single energetic photon. There are two coupling parameters f and f' in the Lagrangian. For simplicity we study the two extreme cases: $f = 0$ or $f' = 0$.

All the above processes have been generated by a Monte Carlo program according to the differential cross section of ref. [3] with an angular distribution of $1 + \cos\theta$ assigned to the ℓ^* decay. The relevant branching ratios of ℓ^* decays are taken from ref. [6]. The subsequent τ decays are simulated by the KORALZ Monte Carlo program [8] and the hadronic fragmentation and decays are simulated by the JETSET Monte Carlo program [9]. The effect of initial state radiation is not included in the Monte Carlo generator but is taken into account in our cross section calculations. All generated events have been passed through the L3 detector simulation [10] which includes the effects of energy loss, multiple scattering, interactions and decays in the detector and the beam pipe.

3 The L3 Detector and the Data Sample

The L3 detector is described in detail in ref. [11]. It consists of a silicon microstrip vertex detector, a central tracking chamber (TEC), a high resolution electromagnetic calorimeter composed of bismuth germanium oxide (BGO) crystals, plastic scintillation counters, a uranium hadron calorimeter with proportional wire chamber readout, and a precise muon spectrometer. These detectors are installed in a 12 m diameter solenoid magnet which provides a uniform field of 0.5 T along the beam direction.

The BGO electromagnetic calorimeter covers the polar angle from 11° to 169°. It is divided into a barrel ($42° < \theta < 138°$) and endcaps ($11° < \theta < 38°$, $142° < \theta < 169°$). The energy resolution for photons and electrons is less than 2% for energies above 1 GeV. The angular resolution of electromagnetic clusters is better than 0.5° for energies above 1 GeV. The hadron calorimeter covers the polar angle from 5.5° to 174.5°. It measures the event energy, with the help of TEC and BGO, with a resolution of about 10% at 91 GeV. The muon chambers cover the polar angle from 22° to 158°. They are divided into barrel (air core; $36^\circ < \theta < 144^\circ$), forward (iron core; $22^\circ < \theta < 36^\circ$) and backward (iron core; $144^\circ < \theta < 158^\circ$) regions.

The data used in these searches were taken with the L3 detector at LEP between October and November 1995. The integrated luminosity is 2.75 pb^{-1} at $\sqrt{s} = 130$ GeV, 2.27 pb^{-1} at $\sqrt{s} = 136$ GeV and 0.05 pb^{-1} at $\sqrt{s} = 140$ GeV.

4 Search for Excited Electrons

An electron is identified as an electromagnetic shower with a matched track within 5° in the $r\phi$ projection. If the shower is isolated from all tracks by more than 15° in the $r\phi$ projection, it is identified as a photon.

To remove the background from two-photon collisions, hadrons and taus, we apply the following selection criteria:

i) there is at least one electron in the event;

ii) the number of tracks is at least 1 and at most 4;

iii) the total energy in the electromagnetic calorimeter is more than half the beam energy.

In the following we describe the selection of excited electron candidates in the channels $ee\gamma\gamma$ and $ee\gamma$.

4.1 Pair Production of e^*

Event selection for the reaction $e^+e^- \longrightarrow e^*e^* \longrightarrow ee\gamma\gamma$ requires two electrons and at least two photons, all with energy greater than 5 GeV. Two events pass the selection. The invariant mass is reconstructed for all possible combinations of $e\gamma$, but no structure is evident in the spectrum. All masses are less than 50 GeV. The main background is due to radiative Bhabha events $e^+e^- \longrightarrow ee\gamma\gamma$. Owing to uncertainties of Monte Carlo predictions for hard radiative Bhabha processes, we conservatively make no background subtraction in calculating the upper limit.

The detection efficiency for the signal is estimated from Monte Carlo to be 55%, independent of the mass of the e^*. The decay branching ratio, $e^* \longrightarrow e\gamma$, is about 100% for the mass region concerned. Taking into account the luminosity, the efficiency and the production cross section of e^*, we obtain the number of expected e^* as a function of m_{e^*}. From Poisson statistics, we determine the lower mass limit for excited electrons at 95% Confidence Level (C.L.) to be 64.7 GeV.

4.2 Single Production of e^*

Event selection for the reaction $e^+e^- \longrightarrow ee^* \longrightarrow ee\gamma$ requires at least one electron with energy greater than 5 GeV and exactly one photon with energy greater than 10 GeV. To reduce the background from Bhabha scattering, the photons are required to be in the barrel region. Since the t-channel contribution is large, one electron could be missed in the beam pipe. For events with only one observed electron, we therefore require that the thrust axis of the event should be within the barrel region.

A total of 34 events pass the selection; 23 of them have only one visible electron. Fig. 1a shows the invariant mass, $m_{e\gamma}$, of all combinations. The mass resolution for e^* is about 1 GeV, estimated from Monte Carlo events. No significant structure can be seen. Owing to uncertainties of Monte Carlo predictions for hard radiative Bhabha processes, we conservatively make no background subtraction in calculating the upper limit. For this reason, no background distribution is shown in Fig. 1a.

The detection efficiency is estimated from Monte Carlo to be 44% at $m_{e^*} = 70$ GeV and 66% at $m_{e^*} = 139$ GeV. The decay branching ratio of $e^* \rightarrow e\gamma$ changes from 100% at $m_{e^*} < 80$ GeV to 40% at $m_{e^*} = 140$ GeV. We obtain an upper limit for the number of e^* events in each mass bin at 95% C.L. Taking into account the luminosity, the efficiency and the branching ratio, the upper limit of the coupling constant λ/m_{e^*} at 95% C.L. as a function of m_{e^*} is shown in Fig. 2a. The more stringent limit in the region between 65 – 90 GeV, as determined from a previous analysis of Z decays [12], is combined with the present measurement.

5 Search for Excited Muons

Muons are identified from tracks in the muon chambers with measurements in both the $r\phi$ and rz projections. The transverse and the longitudinal distances of closest approach to the interaction vertex are required to be less than 200 mm.

To remove the background from cosmic rays, two-photon collisions, hadrons and taus, we apply the following selection criteria:

i) there is at least one identified muon with momentum greater than 10 GeV;

ii) there is at least one scintillator signal in time with the bunch crossing (< 4 ns);

iii) there are at least two tracks in the central tracking chamber;

iv) the number of clusters in the electromagnetic calorimeter is less than 10;

v) the energy deposited in the hadron calorimeter is less than 10 GeV;

vi) the total visible energy is greater than half the beam energy.

In the following we describe the selection of excited muon candidates in the channels $\mu\mu\gamma\gamma$ and $\mu\mu\gamma$.

5.1 Pair Production of μ^*

Event selection for the reaction $e^+e^- \longrightarrow \mu^*\mu^* \longrightarrow \mu\mu\gamma\gamma$ requires at least two photons with energy greater than 5 GeV. Two events pass the selection. The invariant mass is reconstructed for all possible combinations of $\mu\gamma$, but no structure is evident in the spectrum. All masses are less than 52 GeV except one combination which has a mass of 70 GeV. The main background is due to radiative dimuon events $e^+e^- \longrightarrow \mu\mu\gamma\gamma$. A total of 0.8 events is predicted by the Monte Carlo program, KORALZ.

The detection efficiency for μ^* is estimated from Monte Carlo to be 62%, independent of the mass of the μ^*. The decay branching ratio, $\mu^* \longrightarrow \mu\gamma$, is about 100% for the mass region concerned. We determine the lower mass limit for excited muons at 95% C.L. to be 64.9 GeV.

5.2 Single Production of μ^*

Event selection for the reaction $e^+e^- \longrightarrow \mu\mu^* \longrightarrow \mu\mu\gamma$ requires exactly one photon with energy greater than 5 GeV. A total of 15 events pass the selection. The main background is due to radiative dimuon events $e^+e^- \longrightarrow \mu\mu\gamma$, which is predicted by KORALZ to be 12.3 events. Fig. 1b shows the invariant mass, $m_{\mu\gamma}$, of all combinations together with the Monte Carlo prediction for the background. The mass resolution for μ^* is about 3 GeV for muons in the barrel (90% of observed events). No significant structure can be seen from the plot. We conclude that the observed events are compatible with the expected background.

The detection efficiency for singly produced μ^* is estimated to be 67%. The efficiency drops to 58% at μ^* mass close to the center-of-mass energy. The decay branching ratio of $\mu^* \rightarrow \mu\gamma$ changes from 100% at $m_{\mu^*} < 80$ GeV to 40% at $m_{\mu^*} = 140$ GeV. We obtain an upper limit of the coupling constant λ/m_{μ^*} at 95% C.L. as a function of m_{μ^*}, which is combined with the previous analysis [12], as shown in Fig. 2a.

6 Search for Excited Taus

This analysis uses a jet clustering algorithm [13] which groups neighbouring calorimeter energy depositions. The algorithm normally reconstructs one jet for a single isolated electron, photon, muon, high energy tau or hadronic jet. A photon should fulfill the requirements as mentioned in Sec. 4. In addition, it must be isolated from any other calorimetric cluster by at least 15°. A jet is identified as a tau if its energy is greater than 2 GeV and it is not identified as a photon.

To remove background from hadrons, two-photon collisions, muons and Bhabha events, we apply the following selection criteria:

i) the number of tracks is at least two and at most 7;

ii) the number of clusters in the electromagnetic calorimeter is less than 16;

iii) the visible energy is greater than half the beam energy;

iv) there is at least one tau whose energy deposition in the calorimeters is not consistent with that of a muon;

v) the total energy deposition in the electromagnetic calorimeter is less than 85% of the center-of-mass energy;

vi) the polar angle of the missing momentum should satisfy $|\cos\theta| < 0.9$.

In the following we describe the selection of excited tau candidates in the channels $\tau\tau\gamma\gamma$ and $\tau\tau\gamma$.

6.1 Pair Production of τ^*

Event selection for the reaction $e^+e^- \longrightarrow \tau^*\tau^* \longrightarrow \tau\tau\gamma\gamma$ requires at least one tau with energy greater than 2 GeV and two photons each with energy greater than 5 GeV. No events pass the selection. The background from radiative $\tau\tau$ events, $e^+e^- \longrightarrow \tau\tau\gamma\gamma$, is estimated to be 0.3 events by KORALZ.

The detection efficiency for τ^* is estimated to be 50%. The efficiency is slightly dependent on the mass of the τ^* and is taken into account. The decay branching ratio of $\tau^* \longrightarrow \tau\gamma$ is about 100% for the mass region concerned. We determine the lower mass limit for excited taus at 95% C.L. to be 64.2 GeV.

6.2 Single Production of τ^*

Event selection for the reaction $e^+e^- \longrightarrow \tau\tau^* \longrightarrow \tau\tau\gamma$ requires at least one tau with energy greater than 2 GeV and exactly one photon with energy greater than 5 GeV. A total of 4 events pass the selection. The main background is due to radiative $\tau\tau$ events, $e^+e^- \longrightarrow \tau\tau\gamma$, and is estimated to be 4.9 events by KORALZ. Due to the large Lorentz boost, most of the tau neutrinos have the same direction as taus. Using this assumption and energy and momentum conservation, we can estimate the total momentum of the taus and reconstruct the invariant mass of $\tau\gamma$. From Monte Carlo events we estimate the mass resolution of τ^*, after applying these kinematic constraints, to be about 2 GeV. Fig. 1c shows the invariant mass of the 4 selected events for all $\tau\gamma$ combinations, together with the Monte Carlo prediction for background. No structure is seen in the plot. We conclude that all events are compatible with the expected background.

The detection efficiency for singly produced τ^* is estimated to be 66%. The efficiency drops to 48% for a τ^* with mass close to the center-of-mass energy. The decay branching ratio of $\tau^* \to \tau\gamma$ changes from 100% at $m_{\tau^*} < 80$ GeV to 40% at $m_{\tau^*} = 140$ GeV. We obtain an upper limit of the coupling constant λ/m_{τ^*} at 95% C.L. as a function of m_{τ^*}, which is combined with the previous analysis [12], as shown in Fig. 2a.

7 Search for Excited Neutrinos in the channel $\nu^* \to \mathrm{eW}$

The jet cluster algorithm used in the tau analysis is also used here. A reconstructed jet corresponds to an isolated high energy lepton, photon or hadronic jet. An electron is identified as an electromagnetic shower with an associated track within 5°. It must be isolated from any other calorimetric cluster by at least 15°.

In the following we describe the selection of excited neutrino candidates in the channels eeWW and νeW.

7.1 Pair Production of ν^*

Event selection for the reaction $e^+e^- \longrightarrow \nu^*\nu^* \longrightarrow$ eeWW requires the following:

i) there must be at least one isolated electron with energy greater than 2 GeV. It should be inconsistent with a converted photon;

ii) the number of tracks must be at least 4;

iii) the energy deposited in the electromagnetic calorimeter must be less than 90% of the center-of-mass energy;

iv) the total visible energy should be greater than 20 GeV.

v) the number of jets should be at least 3;

vi) the number of jets should be at least 5 if there is no identified second electron with energy greater than 1 GeV.

No events pass the selection. The backgrounds from two-photon collisions and hadronic events are estimated from Monte Carlo to be negligible.

The detection efficiency is estimated from Monte Carlo to be 59%. The efficiency is slightly dependent on the mass of the ν^* and is taken into account. The decay branching ratio of $\nu^* \rightarrow$ eW is about 71% in the mass region concerned. The lower mass limit for excited neutrinos at 95% C.L. is determined to be 57.3 GeV.

7.2 Single Production of ν^*

Event selection for the reaction $e^+e^- \longrightarrow \nu\nu^* \longrightarrow \nu eW$ requires the following:

i) there must be exactly one isolated electron with energy greater than 5 GeV. It should be inconsistent with a converted photon and its polar angle should satisfy $|\cos\theta| < 0.9$;

ii) the total visible energy should be greater than 40% of the beam energy;

iii) the polar angle of the missing momentum direction should be greater than 18°;

iv) if the W decays leptonically, i.e. the number of tracks is less than 5, there must be only two jets in the event and their acoplanarity angle should satisfy $\cos\phi < 0.9$;

v) if the W decays hadronically, i.e. the number of tracks is greater than or equal to 5, there must be three jets in the event, each with energy greater than 5 GeV.

No events pass the selection. Backgrounds from $\tau\tau$, two-photon collision and hadrons are estimated from Monte Carlo to be negligible.

The detection efficiency is estimated to be 25% at $m_{\nu^*} = 70$ GeV and 45% at $m_{\nu^*} > 100$ GeV. The decay branching ratio of $\nu^* \rightarrow$ eW is more than 95% at $m_{\nu^*} = 91$ GeV and drops to 67% at $m_{\nu^*} = 140$ GeV. We obtain an upper limit of the coupling constant λ/m_{ν^*} at 95% C.L. as a function of m_{ν^*}, which is combined with the previous analysis [12], as shown in Fig. 2a.

8 Search for Excited Neutrinos in the channel $\nu^* \rightarrow \nu\gamma$

Photons are identified as electromagnetic showers. To remove background from charged final states and cosmic rays, we apply the following selection criteria:

i) there are no tracks in the central tracking chamber;

ii) the energy deposited in the electromagnetic calorimeter is larger than 10 GeV;

iii) the energy deposited in the hadron calorimeter is less than 5 GeV;

iv) the thrust axis has a polar angle $|\cos\theta| < 0.91$;

v) there is at least one scintillator signal in time with the beam crossing ($<$ 4 ns) caused by leakage of the electromagnetic showers.

In the following we describe the selection of excited neutrino candidates in the channels $\nu\nu\gamma\gamma$ and $\nu\nu\gamma$.

8.1 Pair Production of ν^*

Event selection for the reaction $e^+e^- \longrightarrow \nu^*\nu^* \longrightarrow \nu\nu\gamma\gamma$ requires at least two photons, each with energy greater than 10 GeV. The directions of the missing momentum and of both photon candidates are required to be greater than 20° with respect to the beam direction. This cut removes most of the Bhabha and $e^+e^- \rightarrow \gamma\gamma$ events. To further remove $e^+e^- \rightarrow \gamma\gamma$ events, we require that the energy sum of two photons is less than 75% of the center-of-mass energy.

No events pass the selection. The main background is due to events of the type $e^+e^- \longrightarrow \gamma\gamma$. A total of 0.6 events are predicted by GGG [14].

The detection efficiency is estimated to be 52%. It is slightly dependent on the mass of the ν^* and is taken into account. The decay branching ratio of $\nu^* \rightarrow \nu\gamma$ is 100% in the mass region concerned. We determine the lower mass limit for excited neutrinos at 95% C.L. to be 61.4 GeV.

8.2 Single Production of ν^*

Event selection for the reaction $e^+e^- \longrightarrow \nu\nu^* \longrightarrow \nu\nu\gamma$ requires exactly one photon with energy greater than 10 GeV. This photon is required to be in the barrel part of BGO in order to reject background from radiative Bhabha and $e^+e^- \longrightarrow Z\gamma \longrightarrow \nu\nu\gamma$ events. A total of 20 events pass the selection. The background is mainly due to a) radiative Bhabha events $e^+e^- \longrightarrow (e)(e)\gamma$ with two electrons missing in the beam pipe, b) radiative neutrino events $e^+e^- \longrightarrow \nu\nu\gamma$, and c) $e^+e^- \longrightarrow \gamma\gamma$ with one γ missing in the beam pipe. For the present luminosity, no events are predicted by TEE [15] for radiative Bhabha events, 14.9 events are predicted by both NNGSTR [16] and KORALZ [8] for radiative neutrino events and 1.2 events are predicted by GGG [14] for $e^+e^- \longrightarrow \gamma\gamma$. Fig. 1d shows the recoil mass of the 20 observed events compared with Monte Carlo predictions for the background. The peak at 91 GeV shows that most of the events are due to $e^+e^- \rightarrow Z\gamma$, where the γ is from initial state radiation, and the Z decays to $\nu\nu$. We conclude that all events are compatible with the expected background.

The detection efficiency is estimated to be about 56%. It is slightly dependent on the mass of the ν^* and is the same for both $f = 0$ and $f' = 0$. For $f = 0$, the decay branching ratio of $\nu^* \rightarrow \nu\gamma$ is 100% at $m_{\nu^*} < 91$ GeV and drops to about 90% at $m_{\nu^*} = 140$ GeV. For $f' = 0$, the decay branching ratio is 100% at $m_{\nu^*} < 80$ GeV and drops to 15% at $m_{\nu^*} = 140$ GeV. Since it is not possible to reconstruct the $\nu\gamma$ invariant mass, we derive the upper limit on the basis of 20 observed events with 16.1 expected background events. An upper limit is obtained for the coupling constant λ/m_{ν^*} at 95% C.L. as a function of m_{ν^*} for both $f = 0$ and $f' = 0$. The result, combined with the previous analysis, is shown in Fig. 2b, in which $\lambda = f$ if $f' = 0$, and $\lambda = f'$ if $f = 0$.

9 Conclusion

We see no evidence for excited electrons, muons, taus or neutrinos; the observed events are consistent with Standard Model expectations. From pair production searches the lower mass limits are found to be 64.7 GeV for e^*, 64.9 GeV for μ^*, 64.2 GeV for τ^*, 57.3 GeV (eW decay mode) and 61.4 GeV ($\nu\gamma$ decay mode) for ν^*. From single production searches, we derive upper limits on the couplings λ/m_{ℓ^*} in the range of $(10^{-4} - 1)$ GeV^{-1} for ℓ^* masses up to 130 GeV.

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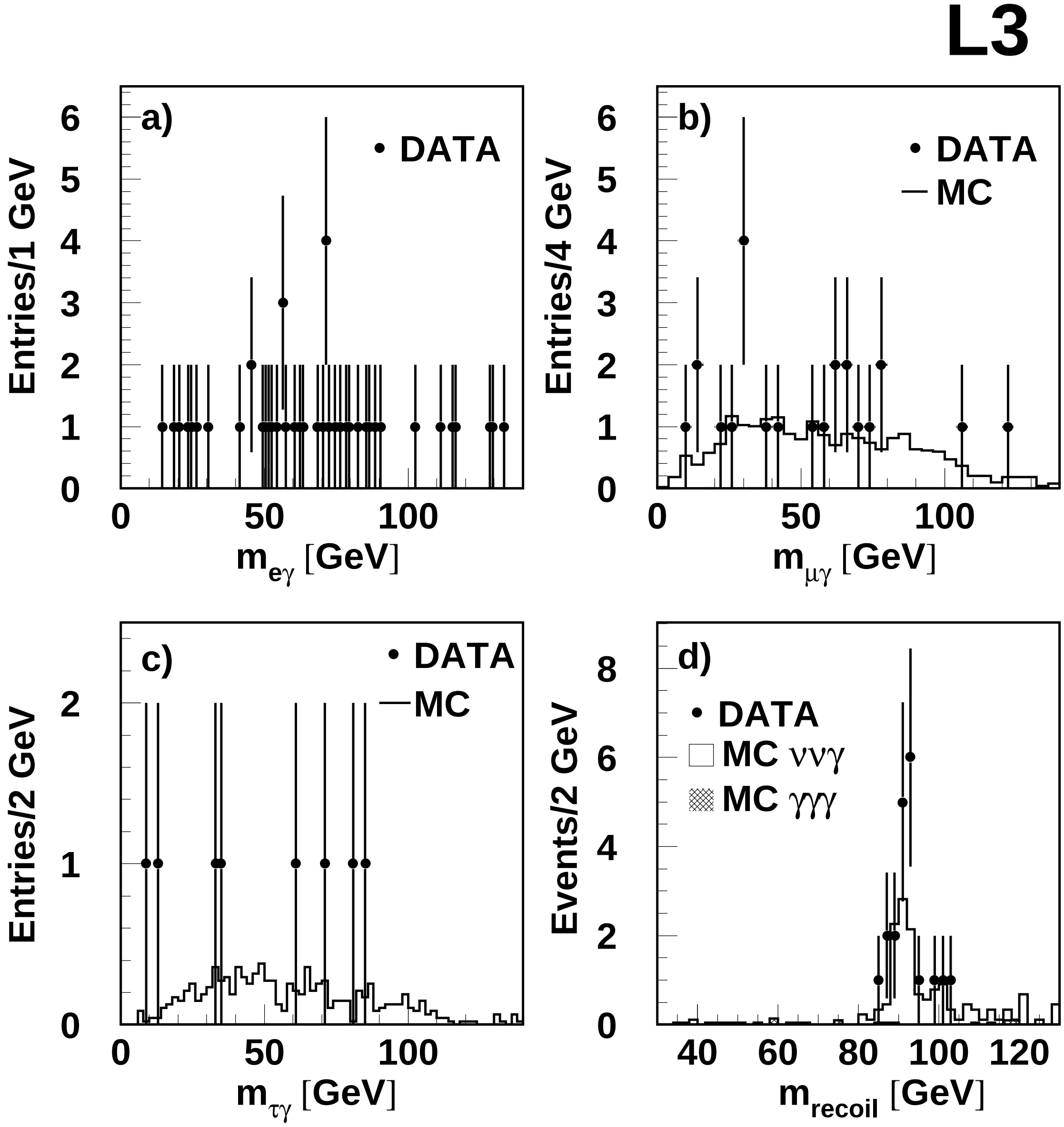

Figure 1: Selected events in single production searches: a) invariant mass of all $e\gamma$ combinations; b) invariant mass of all $\mu\gamma$ combinations; c) invariant mass of all $\tau\gamma$ combinations (after applying kinematic constraints); d) recoil mass of single photons. For each plot, the bin size is comparable to the expected mass resolution.

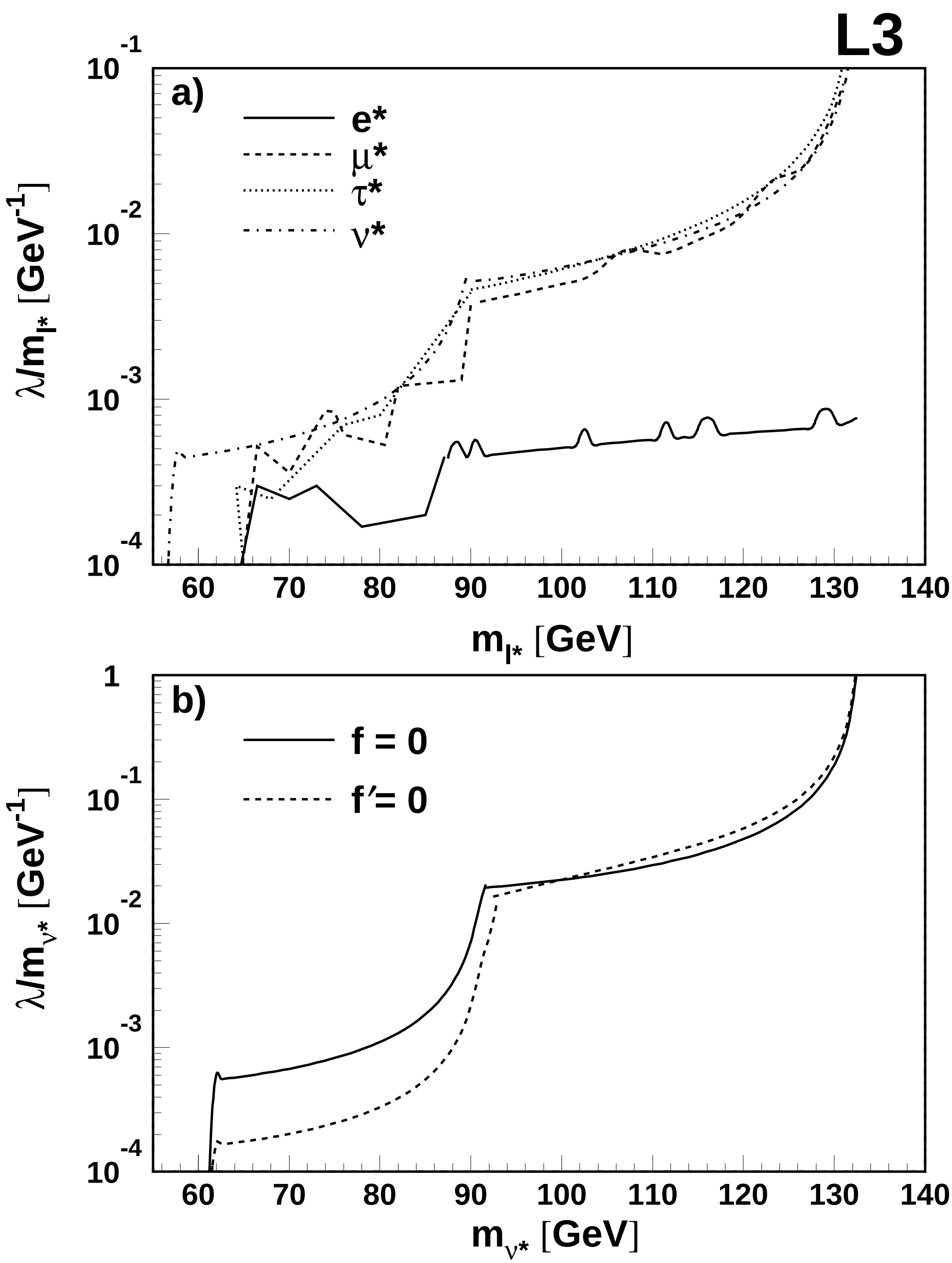

Figure 2: The upper limit of the coupling constant λ/m_{ℓ^*} at 95% C.L. as a function of m_{ℓ^*}: a) $f = f'$ for all excited leptons; b)$f \neq f'$ for excited neutrinos with $\lambda = f$ if $f' = 0$, and $\lambda = f'$ if $f = 0$. The excluded region is above and to the left of the curves.