Measurements of $Z^0 \rightarrow b\bar{b}$ decays and the semileptonic branching ratio $BR(b \rightarrow \ell + X)$

L3 Collaboration

We have measured the decay properties of $Z^0 \rightarrow b\bar{b}$ from a study of inclusive muon and electron events. The average ($e$ and $\mu$) $b$-quark semileptonic branching ratio has been determined to be $BR(b \rightarrow \ell + X) = 0.119 \pm 0.003\,_{\text{stat.}} \pm 0.006\,_{\text{syst.}}$, assuming the standard model prediction of $\Gamma_b = 378 \pm 3\,\text{MeV}$. From the ratio of the number of dilepton events to single lepton events, we find $BR(b \rightarrow \ell + X) = 0.113 \pm 0.010\,_{\text{stat.}} \pm 0.006\,_{\text{syst.}}$, without assumptions on $\Gamma_b$. The partial decay width of the $Z^0$ into $b\bar{b}$ has been measured to be $\Gamma_{bb} = 385 \pm 7\,_{\text{stat.}} \pm 11\,_{\text{syst.}}\,\text{MeV}$ with an additional 19 MeV error from the uncertainty in $BR(b \rightarrow \ell + X)$. The average fractional energy of bottom hadrons in $Z^0 \rightarrow b\bar{b}$ events has been determined to be $\langle x_b \rangle = 0.686 \pm 0.006\,_{\text{stat.}} \pm 0.016\,_{\text{syst.}}$.

1. Introduction

In the standard model [1] the partial decay width in $Z^0 \rightarrow q\bar{q}$ depends on the weak isospin of the quark and is expected to be larger for down-type than for up-type quarks. For light quarks the predictions of $\Gamma_q$ have uncertainties of about 10 MeV due to the unknown mass of the top quark [2]. This limits the accuracy of standard model tests in the light quark sector. In contrast, the decay width into $b\bar{b}$ is expected to be insensitive to $m_{t\bar{t}}$. Thus high statistics measurements of $\Gamma_{bb}$ provide a decisive test of the standard model and allow a precise determination of the neutral current couplings to $b$-quarks.

The $e^+e^-$ collider LEP provides an excellent environment in which to study the production and decay...
of bottom quarks and to test their neutral and charged current couplings [2]. A large amount of experimental data on this topic has already been published by LEP and SCL experiments [3–7]. The most precise measurements have been obtained by identifying b-events through their semileptonic decay and determining their properties from measurements of leptons and jets.

This technique has been used recently by the L3 Collaboration in the measurement of the B°–B° mixing parameter [4] and the forward–backward asymmetry of b-quarks [5]. In an earlier L3 measurement [3], based on 1989 data, inclusive muon events were used to determine the partial Z° decay width, Γ_{Bb}, and the b-quark fragmentation parameter, ε_b. The error in Γ_{Bb} included about equal contributions from the measurement error and from the uncertainty in the b-quark semileptonic branching ratio BR(b→ℓ+X).

Measurements of BR(b→ℓ+X) have been performed at CESR and DORIS [8] for low-mass B-mesons, which are produced at the Y(4S). These results are not directly applicable to Z°→bb events, where a large spectrum of B-mesons and b-baryons is produced in the fragmentation process. The semileptonic branching fractions measured at PETRA and PEP can be used in our analysis, but the world average value has an error of about 7% [9,10] and thus limits the accuracy on Γ_{Bb}. An independent measurement of BR(b→ℓ+X) from Z°→bb is needed to improve the precision.

We have performed a measurement of BR(b→ℓ+X) from the ratio of dilepton to single lepton events. Increased statistics, as well as the addition of inclusive electron events into the data sample, has allowed us to reduce the error on the measured values of Γ_{Bb} and ε_b. About 115 000 e^+e^→hadrons events, recorded in 1990 with the L3 detector at √s≈M_Z, are used in this study. This corresponds to an integrated luminosity of 5.5 pb^{-1}, a factor of six increase with respect to our previous measurement.

2. The L3 detector

A detailed description of the L3 detector is given in ref. [11]. The detector consists of a central tracking chamber, a high resolution electromagnetic calorimeter composed of BGO crystals, a ring of scintillation counters, a uranium and brass hadron calorimeter with proportional wire chamber readout, and a precise muon spectrometer. These detectors are installed in a 12 m diameter solenoid which provides a uniform field of 0.5 T along the beam direction.

The central tracking chamber is a time expansion chamber which consists of two cylindrical layers of 12 and 24 sectors, with 62 wires measuring the R−φ coordinate. The average single wire resolution is 58 μm over the entire cell. The double-track resolution is 640 μm. The fine segmentation of the BGO detector and the hadron calorimeter allow us to measure the direction of jets with an angular resolution of 2.1°, and to measure the total energy of hadronic events from Z° decay with a resolution of 10.2% [12]. The muon detector consists of three layers of precise drift chambers, which measure a muon trajectory 56 times in the bending plane, and eight times in the non-bending direction.

For the present analysis, we use the data collected in the following ranges of polar angles:
– central chamber: 41°<θ<139°;
– hadron calorimeter: 5°<θ<175°;
– muon chambers: 36°<θ<144°;

3. Event selection

We measure the properties of Z°→b̅b decays from a study of inclusive muon and electron events. Inclusive lepton events are triggered by several independent triggers. The calorimetric trigger requires a total energy of 15 GeV in the BGO and hadron calorimeters. A second trigger for muons requires one of 16 scintillation counter φ sectors in coincidence with a track in the muon chambers. These triggers, combined with an independent charged track trigger and a scintillation counter multiplicity trigger, give a trigger efficiency of greater than 99.9% for hadronic events containing one or more leptons.

Calorimeter clusters are formed by grouping energy deposits in the BGO crystals and in the hadron calorimeter towers. In the electromagnetic calorimeter (BGO), for example, normally one cluster is re-
constructed for an electron or photon and a few clusters for $t^\pm$'s. For each cluster a vector is constructed having as its magnitude the measured calorimeter energy and having the same direction as the line connecting the interaction point to the center of the cluster. Tracks in the muon detector also form clusters. Reconstructed clusters are then grouped together into jets [12].

We select hadronic events using the following criteria:

1. $E_{\text{cal}} > 38 \text{ GeV}$;
2. longitudinal energy imbalance: $|E_L|/E_{\text{vis}} < 0.4$;
3. transverse energy imbalance: $E_T/E_{\text{vis}} < 0.5$.

The energy $E_{\text{cal}}$ is the total energy observed in the calorimeters and $E_{\text{vis}}$ is the sum of the calorimetric cluster energies and the energy of any muons, as measured in the muon chambers, $E_L$ and $E_T$ are the projections of the vector sum of cluster energies onto the axis parallel and transverse to the beam direction. We require that there be at least one jet which has more than 10 GeV in the calorimeters. We reject $t^+t^-$ and $e^+e^-$ events by requiring a minimum of 10 clusters in the BGO, each with an energy greater than 100 MeV.

Muons are identified and measured in the muon chamber system. We require that a muon track consists of track segments in two of the three layers of the muon spectrometer and that the reconstructed track points to the interaction region: the transverse distance of closest approach of the track to the interaction point must be less than $3\sigma$, and the longitudinal distance of closest approach must be less than $4\sigma$. The measurement error, $\sigma$, on the distance of closest approach is dominated by multiple scattering in the calorimeters. A typical value for a 10 GeV muon is $\sigma = 25 \text{ mm}$. These requirements are very effective in reducing backgrounds from hadron punchthrough and from $\pi, K$ decays.

Candidate electrons are found by associating electromagnetic clusters in the BGO calorimeter with charged tracks in the central tracking chamber. We require a cluster in the BGO whose lateral shower shape is consistent with an electromagnetic shower. The centroid of this cluster is required to match to within 5 mrad in azimuthal angle to a track in the central tracking chamber and there should be no other track within 10 mrad of this track. We reject tracks with a measured momentum transverse to the beam direction larger than 35 GeV. Energetic photons and $\pi^0$s accompanying a nearby charged particle are rejected by requiring $E/p < 1.5$, where $E$ is the energy of the cluster measured in the electromagnetic calorimeter and $p$ is the momentum of the matching track as measured in the central tracking chamber. Hadrons are rejected by requiring that the energy in the hadron calorimeter inside a cone around the electron candidate be less than 3 GeV, where the cone is centered on the BGO cluster and has a half opening angle of 7°.

Because of the hard fragmentation and large mass of the $b$-quark, leptons from $b$ semileptonic decay have a large momentum and a large transverse momentum with respect to the $b$-quark direction. Therefore by cutting on these quantities we can preferentially select $Z^0 \rightarrow b\bar{b}$ events. Only muons and electrons with momentum larger than 4 GeV and 3 GeV, respectively, are used in this analysis. Also, the transverse momentum $p_{T}$, calculated with respect to the nearest jet axis, must exceed 1 GeV, and be less than 6 GeV. The measured momentum of the lepton is excluded in the calculation of the jet direction. If there is no jet with an energy greater than 6 GeV in the same hemisphere as the lepton, then $p_{T}$ is calculated relative to the thrust axis of the event. The number of selected inclusive lepton and dilepton events is summarized in table 1.

<table>
<thead>
<tr>
<th>Event type</th>
<th>Number of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu+$ hadrons</td>
<td>2621</td>
</tr>
<tr>
<td>$e+$ hadrons</td>
<td>1085</td>
</tr>
<tr>
<td>$\mu\mu$ + hadrons</td>
<td>83</td>
</tr>
<tr>
<td>$ee$ + hadrons</td>
<td>26</td>
</tr>
<tr>
<td>$e\mu$ + hadrons</td>
<td>78</td>
</tr>
</tbody>
</table>

4. Analysis method

We classify inclusive lepton events into the following categories (and their charge conjugate reactions):

1. prompt $b \rightarrow \ell$;
2. cascade $b \rightarrow c \rightarrow \ell$ and $b \rightarrow c + c + s$ where $c \rightarrow \ell$;
3. cascade $b \rightarrow \tau \rightarrow \ell$;
4. prompt $c \rightarrow \ell$.
(5) background from non b-quark events and misidentified leptons.

The expected contributions from each category have been given earlier [4,5]. The prompt b-decay (1) dominates at large transverse momenta as shown in fig. 1, where the measured momentum and transverse momentum spectra for electrons and muons are shown after the above cuts. The expected contributions from prompt b-decay (1), b-quarks only (2 and 3) and backgrounds (4 and 5) is indicated. By requiring 1 GeV < p_x < 6 GeV, we estimate that the muon and the electron samples consist of 62.7% and 76.7% prompt b-decays, respectively. The purity is higher for the electron sample because of the implicit isolation requirements in the electron selection.

We obtain the results of our measurement with an unbinned maximum likelihood fit to the momentum and transverse momentum spectra of single lepton and dilepton events. The likelihood function is determined from the number and type of Monte Carlo generated leptons found within a rectangular region, centered on each data point in the p-p_L plane. The expected number of events is evaluated for each event category (1)-(5). The size of the region is increased until it contains at least 20 Monte Carlo events. The Monte Carlo events have been normalized using our measured value of the Z° hadronic partial width, \( \Gamma_{\text{had}} = 1742 \pm 19 \text{ MeV} \) [13]. A similar fitting method was used in our measurements of B°-B° mixing [4] and b-quark forward–backward asymmetry [5].

For each event category we generate Monte Carlo events with the parton shower program JETSET 7.2

![Graphs](image-url)
The events are passed through the L3 detector simulation which includes the effects of energy loss, multiple scattering, interactions and decays in the detector materials. The events were generated with $A_{\text{LL}} = 290$ MeV and string fragmentation. The Peterson et al. fragmentation function [17],

$$f(z) = \frac{N}{z} \left( 1 - \frac{1}{z} - \frac{\epsilon_b}{1-z} \right)^{-2}$$

was used in the Monte Carlo to describe the fragmentation of c- and b-quarks, where $N$ is a normalization constant and $z = (E + p_t)/E + p_t$ quaternion. The input fragmentation parameters used in the generator are $\epsilon_c = 0.07$ for charm quarks and $\epsilon_b = 0.008$ for bottom quarks. These values agree with our earlier measurement [3] and with extrapolations from measurements at PETRA and PEP [3].

In the fits for $\Gamma_{bs}$, $BR(b \rightarrow \ell + X)$ and $\epsilon_b$, we weight the Monte Carlo generated events such that the $x_{\text{hadron}} = 2E_{\text{hadron}}/\sqrt{s}$ distributions for b- and c-hadrons follow a Peterson et al. function with the parameters $\epsilon_c$ and $\epsilon_b$. The result of the $\epsilon_b$ fit ($\epsilon_b = 0.050 \pm 0.004$) is used in the determination of $\Gamma_{bs}$ and $BR(b \rightarrow \ell + X)$. For the c-quark fragmentation we use $\epsilon_c = 0.5$ in agreement with extrapolations from PETRA and PEP [18].

The JETSET 7.2 Monte Carlo, as used in our acceptance calculation, does not include the production of high-mass D-mesons (1P and 2S states) in its simulation of b-semileptonic decays. This can affect our results since the addition of heavy mesons changes the predicted shape of the prompt-lepton momentum spectrum. Any non-resonant production of D- or D* -mesons with extra pions in the b-semileptonic decay would have a similar effect, and is also not included in the standard Monte Carlo simulation. A theoretical estimate [19] for the relative fraction of high-mass D-mesons in B-meson semileptonic decay gives $\approx 0.1$. However, experimental results from the $Y(4S)$ [20] give values in the range 0.2–0.3, albeit with large errors. Of course, the fraction could be different for b-hadrons coming from $Z^0$ decay.

We have studied the effect of higher mass states in b-semileptonic decay by simulating the decay $b \rightarrow D^*_s(2460) + \ell \nu$. Varying the relative branching ratio $BR(b \rightarrow D^*_s(2460) + \ell \nu)/BR(b \rightarrow X + \ell \nu)$ in the Monte Carlo from 0 to 0.15 increases our measured values of $\Gamma_{bs}$ by $7$ MeV, $BR(b \rightarrow X + \ell \nu)$ by $0.002$ and $\epsilon_b$ by less than $0.001$. We have corrected our fitted values by these amounts. Thus, all of the results quoted in this paper correspond to the relative fraction $BR(b \rightarrow D^*_s(2460) + \ell \nu)/BR(b \rightarrow X + \ell \nu) = 0.15$.

As discussed below, we have varied this fraction by $\pm 0.15$ in order to estimate the systematic effects due to its uncertainty.

Similarly, we have investigated the effect of higher mass B-meson production. Since the measured mass difference between $B^*$ and $B$ mesons is small ($52 \pm 2 \pm 4$ MeV [21]), the relative contribution of $B^*$ and $B$ mesons has an insignificant effect on our results. The ratio of the number of $B^*$ to $B$ mesons is about three in our Monte Carlo. Varying this ratio from 2 to 4 changes our measured values of $\Gamma_{bs}$, $BR(b \rightarrow \ell + X)$ and $\epsilon_b$ by less than $0.3\%$. Our standard Monte Carlo does not include $B^*$ production in the fragmentation process. We have investigated its effect on our results by generating in the Monte Carlo $B^*$'s, with $m_{B^-} - m_{B^+} = 500$ MeV. Varying the fraction of the number of $B^*$ to all b-hadrons between 0 and 10% changes our results by less than $0.5\%$.

We use the average branching ratio, $BR(c \rightarrow \ell + X) = 0.096 \pm 0.006$, from measurements at PETRA and PEP [9,10]. This value represents an average over the production and semileptonic decay of c-hadrons produced in the c-quark fragmentation. Other parameters used in our analysis are $M_2 = 91.181 \pm 0.010 \pm 0.02$ GeV, $M_{Higgs} = 100$ GeV, $M_{top} = 150$ GeV and $\alpha_s(M_Z) = 0.115 \pm 0.009$. The $M_2$ and $\alpha_s$ values are taken from recent L3 measurements [13,22]. With these parameters, the expected partial $Z^0$ decay width into $b\bar{b}$ is $\Gamma_{bs} = 378$ MeV in the standard model [22]. This prediction changes less than 3 MeV, if $M_Z$ and $\alpha_s$ vary within errors and the top quark mass and the Higgs boson mass vary in the ranges $90$ GeV $< m_{top} < 250$ GeV and $50$ GeV $< m_{Higgs} < 1000$ GeV.
5. Determination of BR(b→l+X)

We use two methods to determine the b-semileptonic branching ratio. In the first method, we measure the ratio of the number of selected dilepton events to the number of single lepton events. This ratio is, in first order, proportional to the semileptonic branching ratio and independent of $\Gamma_{b\bar{b}}$. We have determined the contributions to the dilepton and single lepton event sample for all event categories (1)-(5) with Monte Carlo simulations. For dilepton events, in addition to the standard cuts, we require that the opening angle between both leptons be larger than 60°. We find the following results from this measurement:

$$BR(b\rightarrow \mu + X) = 0.113 \pm 0.012,$$

$$BR(b\rightarrow e + X) = 0.138 \pm 0.032,$$

where the errors are statistical only. The systematic uncertainty has been estimated by varying the cuts and the input parameters. Various sources of systematic errors and their sizes are listed in table 2. The variations correspond to at least one standard deviation changes using the known or estimated uncertainties. The error in the selection efficiency is larger for electrons due to uncertainties in the separation of electrons and hadrons. The systematic error accounts for uncertainties in the simulation of hadronic showers and in the matching efficiency of calorimeter clusters with tracks in the central tracking chamber. The effect of the $p_\bot$ cut has been studied by varying the cut by ±0.25 GeV. From these studies we estimate the systematic error to be ±0.006 for muons and ±0.008 for electrons.

If additionally we include the $e\mu$ events and perform a combined fit of the muon and electron events, we obtain the average b-semileptonic branching ratio

$$BR(b\rightarrow l + X) = 0.113 \pm 0.010\,(\text{stat.}) \pm 0.006\,(\text{syst.}).$$

Our measurements agree with the average b-semileptonic branching ratios measured at PETRA and PEP [9,10]: $BR(b\rightarrow \mu + X) = 0.117 \pm 0.010$ and $BR(b\rightarrow e + X) = 0.121 \pm 0.010$.

Averaging our measurement with the values from PETRA and PEP, we obtain

$$BR(b\rightarrow l + X) = 0.117 \pm 0.006,$$

where the statistical and systematic errors have been added in quadrature. This average is nearly independent of assumptions about the neutral current couplings to b-quarks. Our result depends only weakly on $\Gamma_{b\bar{b}}$. The PETRA and PEP results have been obtained at lower center-of-mass energies where electroweak effects contribute less than 3% to the $e^+e^-\rightarrow b\bar{b}$ cross section. We use this combined average branching ratio in our determination of $\Gamma_{b\bar{b}}$.

In the second method, we perform a one-parameter fit to determine the branching ratio $BR(b\rightarrow l + X)$ with $\Gamma_{b\bar{b}}$ set to the standard model value of 378 MeV. This method mainly relies on the number of single lepton events, in contrast to the first method, where the statistical error is dominated by the number of dilepton events.

The result of the fit is

The table shows contributions to the systematic error in the measurement of $BR(b\rightarrow e + X)$ and $BR(b\rightarrow \mu + X)$ from the ratio of the number of dilepton and single lepton events.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variation</th>
<th>$\Delta Br(b\rightarrow e + X)$</th>
<th>$\Delta Br(b\rightarrow \mu + X)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{b\bar{b}} = 378,\text{MeV}$</td>
<td>±40,MeV</td>
<td>±0.002</td>
<td>±0.002</td>
</tr>
<tr>
<td>$BR(c\rightarrow l + X)=0.096$</td>
<td>±0.006</td>
<td>±0.001</td>
<td>±0.002</td>
</tr>
<tr>
<td>$s_\mu=0.050$</td>
<td>±0.004</td>
<td>±0.002</td>
<td>±0.002</td>
</tr>
<tr>
<td>$c_\mu=0.5$</td>
<td>±0.1</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>background</td>
<td>±10%</td>
<td>±0.001</td>
<td>±0.001</td>
</tr>
<tr>
<td>selection efficiencies ($\mu$)</td>
<td>±0.5%</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>selection efficiencies ($e$)</td>
<td>±3%</td>
<td>±0.006</td>
<td>×</td>
</tr>
<tr>
<td>$p_\bot$ cut = 1,GeV</td>
<td>±0.25,GeV</td>
<td>±0.003</td>
<td>±0.002</td>
</tr>
<tr>
<td>$D(2460)$ fraction = 0.15</td>
<td>±0.15</td>
<td>±0.002</td>
<td>±0.002</td>
</tr>
</tbody>
</table>
Table 3
Contributions to the systematic error in the $\Gamma_{bs}$ measurement.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variation</th>
<th>$\Delta \Gamma_{bs}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>electrons</td>
</tr>
<tr>
<td>BR($b\rightarrow\mu +X$) =0.117</td>
<td>±0.006</td>
<td>±19</td>
</tr>
<tr>
<td>BR($c\rightarrow\mu +X$) =0.096</td>
<td>±0.006</td>
<td>±2</td>
</tr>
<tr>
<td>$e_b$=0.5</td>
<td>±0.004</td>
<td>±2</td>
</tr>
<tr>
<td>background</td>
<td>±0.1</td>
<td>±1</td>
</tr>
<tr>
<td>selection efficiencies (\mu)</td>
<td>±0.1</td>
<td>±5</td>
</tr>
<tr>
<td>selection efficiencies (\epsilon)</td>
<td>±0.5%</td>
<td>±7</td>
</tr>
<tr>
<td>$\rho_x$ cut = 1 GeV</td>
<td>±0.25 GeV</td>
<td>±1</td>
</tr>
<tr>
<td>$D_{T}$ (2460) fraction = 0.15</td>
<td>±0.15</td>
<td>±7</td>
</tr>
</tbody>
</table>

for inclusive muons and electrons respectively. The errors are statistical only. We estimate the systematic error by varying several parameters (see table 3) by at least one standard deviation of their known or estimated uncertainties.

We estimate the systematic error by varying the parameters shown in table 2, obtaining similar systematic errors as for the first method. If the value of $\Gamma_{bs}$ is altered by ±3 MeV (the uncertainty in the standard model prediction) a change in the semileptonic branching ratio of ±0.001 is observed. From these studies we estimate a combined systematic error of ±0.006 for muons and ±0.008 for electrons. From a combined fit to electron and muon data we obtain

BR($b\rightarrow\mu +X$) =0.123±0.003(stat.)±0.006(syst.)

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BR($b\rightarrow\epsilon +X$)

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BR($b\rightarrow\epsilon +X$)

The third error gives the uncertainty from the semileptonic branching ratios. Combining all errors in quadrature we obtain

$\Gamma_{bs} = 385\pm7$ (stat.) ±11 (syst.) ±19 (BR) MeV.

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$\Gamma_{bs} = 385\pm23$ MeV,

in good agreement with the standard model prediction of 378±3 MeV.

6. Determination of $\Gamma_{bs}$

In the determination of $\Gamma_{bs}$, we perform a one-parameter fit to the data, with $\Gamma_{bs}$ as free parameter and all other parameters fixed to their nominal values. We obtain

$\Gamma_{bs} = 394\pm9$ MeV ($\mu +X$),

$\Gamma_{bs} = 370\pm12$ MeV ($\epsilon +X$)

for inclusive muons and electrons respectively. The errors are statistical only. We estimate the systematic error by varying several parameters (see table 3) by at least one standard deviation of their known or estimated uncertainties.

From these studies we assign systematic errors of ±19 MeV from the uncertainty in the branching ratios, BR($b\rightarrow\mu +X$) and BR($c\rightarrow\epsilon +X$), and $\Delta \Gamma_{bs} = 11$ MeV for muons and $\Delta \Gamma_{bs} = 15$ MeV for electrons from other sources. If we perform a combined fit using the electron and muon samples, we obtain

$\Gamma_{bs} = 385\pm7$ (stat.) ±11 (syst.) ±19 (BR) MeV.

The third error gives the uncertainty from the semileptonic branching ratios. Combining all errors in quadrature we obtain

$\Gamma_{bs} = 385\pm23$ MeV,

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7. b-quark fragmentation

We use the same fitting method to determine the fragmentation function of b-quarks from $Z^0$ decays.

We assume that the $x_f = 2E_{b\text{hadron}}/\sqrt{s}$ distribution can be approximated by a Peterson et al. function and we fit the parameter $e_b$ to our measured lepton momentum and transverse momentum spectra. All other parameters are fixed to their nominal values. In the fit we vary $e_b$ and weight the $x_f$ distribution of the
Table 4
Contributions to the systematic error in the $e_b$ and $\langle x_E \rangle$ measurement.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variation</th>
<th>$\Delta e_b$</th>
<th>$\Delta \langle x_E \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Γ_{b^+} = 378$ MeV</td>
<td>$\pm 10$ MeV</td>
<td>$\pm 0.002$</td>
<td>$\pm 0.003$</td>
</tr>
<tr>
<td>BR($b\to e+X$) = 0.117</td>
<td>$\pm 0.006$</td>
<td>$\pm 0.006$</td>
<td>$\pm 0.010$</td>
</tr>
<tr>
<td>BR($e\to e+X$) = 0.096</td>
<td>$\pm 0.006$</td>
<td>$\pm 0.002$</td>
<td>$\pm 0.003$</td>
</tr>
<tr>
<td>background</td>
<td>$\pm 10%$</td>
<td>$\pm 0.001$</td>
<td>$\pm 0.015$</td>
</tr>
<tr>
<td>selection efficiencies ($\mu$)</td>
<td>$\pm 0.5%$</td>
<td>$&lt; 0.001$</td>
<td>$&lt; 0.001$</td>
</tr>
<tr>
<td>selection efficiencies ($e$)</td>
<td>$\pm 3%$</td>
<td>$\pm 0.002$</td>
<td>$\pm 0.003$</td>
</tr>
<tr>
<td>$p_\perp$ cut = 1 GeV</td>
<td>$\pm 0.25$ GeV</td>
<td>$\pm 0.001$</td>
<td>$\pm 0.0015$</td>
</tr>
<tr>
<td>$D^0$ (2460) fraction = 0.15</td>
<td>$\pm 0.15$</td>
<td>$&lt; 0.001$</td>
<td>$&lt; 0.001$</td>
</tr>
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</table>

Monte Carlo events accordingly. The result of the fit is

$e_b = 0.047 \pm 0.005$ for $\mu + X$ events,

$e_b = 0.056 \pm 0.008$ for $e + X$ events,

where the errors are statistical. We perform the tests shown in table 4 to check the results of the fit and to estimate the systematic errors.

From these studies we estimate a systematic error of 0.01 in $e_b$ and 0.016 in $\langle x_E \rangle$, for both the electron and the muon measurement. We perform a combined fit using electron and muon events and obtain as our final result

$e_b = 0.050 \pm 0.004$ (stat.) $\pm 0.010$ (syst.),

which gives a precise determination of the average energy fraction of $b$-hadrons:

$\langle x_E \rangle = 0.686 \pm 0.006$ (stat.) $\pm 0.016$ (syst.).

We have performed a study to check whether the Peterson et al. function gives an adequate description of the $x_E$ distribution. We determine the $b$-quark fragmentation function from the data without assumptions on a functional form and compare the result with the Peterson et al. function. For this test we use only the inclusive muon sample. The $c_E$ distribution is approximated by a histogram with seven bins, and the value for each bin is allowed to vary freely in the fit. The fit is constrained to enforce overall normalization of the fragmentation function. We perform a six-parameter fit in the allowed range $x_E \geq 2m_b/\sqrt{s} \approx 0.11$. The points with error bars (statistical only) in fig. 2 give the result of the fit. From this fit we obtain $\langle x_E \rangle = 0.680 \pm 0.011$, where the error is statistical only and includes correlations between all the points. Fig. 2 also shows, for comparison, the Peterson et al. function for $e_b = 0.05$. The measured fragmentation function agrees within errors with a Peterson et al. form.

8. Summary

We have measured the decay properties of $Z^0 \to b\bar{b}$ from a study of inclusive muon and electron events. The average (electron and muon) $b$-semileptonic branching ratios has been determined to be
\[ \text{BR}(b \rightarrow \ell + X) = 0.119 \pm 0.003 \text{(stat.)} \pm 0.006 \text{(syst.)} , \]

assuming the standard model prediction \( \Gamma_{b\ell} = 378 \pm 3 \) MeV. From the ratio of dilepton events to single lepton events, we find

\[ \text{BR}(b \rightarrow \ell + X) = 0.113 \pm 0.010 \text{(stat.)} \pm 0.006 \text{(syst.)} , \]

without any assumptions on \( \Gamma_{b\ell} \). The partial decay width of the \( Z^0 \) into \( b \bar{b} \) is measured to be

\[ \Gamma_{b\bar{b}} = 385 \pm 7 \text{(stat.)} \pm 11 \text{(syst.)} \pm 19 \text{(BR)} \text{ MeV} . \]

The average fractional energy of bottom hadrons in \( Z^0 \rightarrow b\bar{b} \) decays is determined to be

\[ \langle x_b \rangle = 0.686 \pm 0.006 \text{(stat.)} \pm 0.016 \text{(syst.)} . \]

These measurements improve previous LEP and SLC results [3,6] by more than a factor of two.

Acknowledgement

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