INTERTIMMENTY AT INTERMEDIATE ENERGIES

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Abstract
Apart from some misunderstandings of this summer, recent developments have led to considerable progress in this controversial but challenging field. Intermittency is now established as an effect in at least two dimensions, where it has turned out to be stronger and cleaner than in one. The effect is strongest in \( \eta \) and \( e^+e^- \) collisions. In \( h\bar{h} \) collisions, where the energy range covered is the largest, the effect is stronger at intermediate then at high \( \sqrt{s} \). Next to parton showering, intermittency is very sensitive to the soft phase, which will have to be studied with large statistics at intermediate energies, where it is not overshadowed by hard effects. In terms of the space-time development of a scattering process, the intermittency phenomenon and the determination of its parameters are shown to give a genuine information on the nature of hadronization at large distances compared to the hard interaction region. As such, it is expected to be sentitive to the nuclear environment of a deep-inelastic electron collision.

In recent years events have been observed containing high particle density “spikes” in rapidity space. Fig.1a shows the JACEE event [1] at a pseudo-rapidity resolution of \( \delta\eta = 0.1 \) unit, with local fluctuations up to \( dn/\delta\eta \approx 300 \) and with a signal to background ratio of about 1:1. The NA22 event [2] of Fig.1b contains a rapidity “spike” of \( dn/dy = 100 \) at a rapidity resolution \( \delta y = 0.1 \). This corresponds to 60 times the average density in this experiment. Also UA5 [3] has reported “spikes” of \( dn/d\eta \) up to 30 (10 times average) as early as JACEE, but found these to be in agreement with a short range cluster Monte Carlo.

No doubt, local density fluctuations exist. The question is whether they are of statistical or dynamical origin, whether the underlying probability density is continuous or intermittent.

To study these, Bialas and Peschanski [4] have suggested to study the dependence of scaled factorial multiplicity moments

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Fig. 1a. The JACEE event [1]  

Fig. 1b. The NA22 event [2]

\[ \langle F_i \rangle = \frac{1}{M} \sum_{m=1}^{M} \frac{(n_m(n_m-1)\cdots(n_m-i+1))}{(n_m)^i} \]  

as a function of the resolution \( \delta y \). Here, \( n_m \) is the (charged) particle multiplicity in bin \( m \) \((m=1\ldots M)\) of size \( \delta y=\Delta y/M \), with \( M \) the number of bins into which an original region \( \Delta y \) is divided. The averages under the sum are over the events in the sample. High order moments resolve the large \( n_m \) tail of the multiplicity distribution and are, therefore, particularly sensitive to density fluctuations at the various scales \( \delta y \) used in the analysis.

The authors show that the \( \langle F_i \rangle \) are independent of \( \delta y \) if the \( y \) distribution is smooth (probability density continuous), but follow the power law

\[ \langle F_i \rangle \propto \delta y^{-f_i}, \quad f_i > 0 \]  

if the distribution is fractal (probability density is “intermittent”). The powers \( f_i \) (slopes in a log-log plot) are related to the anomalous dimensions \( \delta_i = f_i/(i-1) \) giving the deviation \( d_f = d - \delta_i \) of the fractal dimension \( (d_f) \) from the integer one \( (d) \), due to fluctuations.

The first step has, therefore, been the search for a more or less linear increase of \( \ln \langle F_i \rangle \) as a function of \( -\ln \delta y \). Within a surprisingly short time, this search has been performed in \( \epem \) [6-9], \( \mu p \) [10], \( \nu A \) [11], \( hh \) [12,13], \( hA \) [14,15] and \( AA \) [14,16,17,18,19] collisions. Intermittency is indeed seen in all types of collision. Recent reviews are given in refs 20 and 21.

Anomalous dimensions \( \delta_i \) fitted over \( 0.1 < y(\eta) < 1.0 \) are compared to each other in Fig. 2 [20]. They typically lie between 0.01 and 0.1, so that the fractal dimensions are close to one. They are larger and grow faster with increasing order \( i \) in \( \mu p \) and \( \epem \) (Fig. 2a) than in \( hh \) (Fig. 2b) collisions and are small and almost independent of \( i \) in heavy ion collisions (Fig. 2c). Within \( hh \) collisions, the growth is considerably faster for NA22 (at \( \sqrt{s}=22 \) GeV) than for UA1 (at 630 GeV).

Furthermore, the \( \delta_i \) are larger and intermittency is cleaner when studied in two dimensions.
Fig. 2 Anomalous dimension $d_i$ as a function of the order $i$, for a) $\mu p$ and $e^+e^-$ collisions, b) NA22 and UA1, c) KLM [20].

What do presently used models say about intermittency?

As shown in ref. 21, presently used models for particle production in hadron-hadron collisions do not reproduce the intermittency observed in the data.

In Fig. 3a the EMC data [10] ($4 < W < 20 \text{ GeV}$) are compared to what is expected from an extrapolation of conventional short range and long range correlations [22]. At low $\delta y$, the data are consistently above these expectations. In Fig. 3b, the slopes $f_i$ of the same data are seen to exceed considerably the expectations from the Webber and Lund models. Similarly, Fig. 3c shows too low $\ln(F_3)$ from Lund for $\nu Ne$ and $\nu D_2$ data [11] ($\langle W \rangle = 6.5 \text{ GeV}$).

So, also presently used lepton-hadron models cannot reproduce the intermittency observed in these types of collision.

In $e^+e^-$ collisions, the situation is far less unanimous. While a first (indirect) analysis [6] of HRS data and an analysis of the TASSO data [7] has shown a similar deviation from model expectations as observed in $lh$ and $hh$ data, recently CELLO [8] and in particular DELPHI [9] show reasonable agreement of their data with the parton shower version of the Lund Monte Carlo.

If we forget about the discrepancy between CELLO and TASSO for the moment, it could be thinkable that the parton shower (which is a cascade process and therefore expected to give intermittency) is too short in the models at PEP/PETRA energies and only fully developed at LEP. A comparison of the log-log plots on parton and hadron level in Figs. 4a,b [23], however, shows that in the standard JETSET 6.3 version the increase of $\ln(F_3)$ at large $-\ln \delta y$ is not due to the parton shower, but to hadronization!

Only if the parton shower is allowed to continue down to very low $Q^2$ values (Fig. 7c,d for $Q^2=0.4 \text{ GeV}^2$, implying local hadron parton duality), intermittency is becoming visible also at the parton level.

On the other hand, intermittency seems to be fully developed on the parton level already at 91 GeV in the Webber model, and even smeared out by hadronization there [24].
The sensitivity to the cut-off for the perturbative QCD decade and the rôle of both hard and soft phases has also been discussed in terms of the Ariadne dipole radiation model [25]. Intermittency can be increased in the soft phase by an increase of the $\pi/\rho$ ratio also required from direct measurements by NA22 [26] and EMC [27]. The direct pions resolve the underlying parton structure better than the more massive resonances. From a tunneling production mechanism, these pions are expected to have smaller $p_T$ than other particles, a property which has been neglected in the MC programs until now.

In any case, it will be necessary to approach the problem on the exact origin of intermittency in shower MC's from two sides:

1. With the limitations mentioned above, parton showering has become a good candidate to explain intermittency in the hard phase and should be tried for other types of collision. Before that, however, its exact origin will have to become clear in the Monte Carlo versions and the model will have to be checked on more sensitive distributions to be discussed below. Further studies are, therefore, needed at high energies where parton showering is fully developed and expected to dominate over the soft phase.

2. On the other hand, intermittency turns out to be particularly sensitive to the exact treatment of the soft phase. This phase will have to be studied in detail with high statistics at intermediate energies, where it is not dominated by parton showering in the hard phase.

Fig. 3 a) EMC results [10] compared to expectations from ref. 22, b) slopes $f_i$ for EMC data as well as Webber and Lund models, c) $\nu A$ data [11] in comparison to Lund model expectations.

What is needed there is a fully three dimensional analysis, with in particular the study of the transverse momentum dependence of the effect, and an analysis in terms of identical and non-identical particles to isolate the rôle of Bose-Einstein correlation.

At the level of the physical interpretation of the intermittency phenomenon, it is convenient to introduce the space-time description of a scattering process, such as represented in Figs. 5. In Fig. 5a, one considers the "conventional" picture of quark
hadronization; once created with the speed of light, a quark (or an antiquark) comes out of the interaction region by emitting hadrons. This region is typically of order 1 fermi in the proper-time of the system. As a consequence, this scale is expected to show up in the fluctuation pattern. By contrast, the intermittency phenomenon in principle requires the absence of typical scales of fluctuations, at least in some range from 1 fermi to 10 fermis or more.

As a qualitative illustration of such a physical process Fig.5b shows a cascading mechanism which has been proposed [4] to conciliate the scale-invariance of intermittent fluctuation patterns with the correlation length of 1 fermi which is reasonably supposed to remain a feature of the fundamental interaction. Fluctuations are expected when the interacting system, submitted to the relativistic expansion, becomes larger than the correlation length. However, instead of emitting hadrons directly, the two (or more) sub-systems may in turn expand and break later into pieces, generating a new, superimposed, fluctuation and so on, so forth. Indeed, such an hypothesis has been shown [28] to be realized in the semi-classical approximation of \( q\bar{q} \) pair creation by a Schwinger tunnelling mechanism. In this case, the tunnelling rate (within one-fermi distance) is not enough to transform the whole energy of the color field into \( q\bar{q} \) pairs, leaving room to subsequent cascading steps of pair formation in a fractal type process.

Note, however, that such a longer process has not to be present in all the events, or in the whole of one event. It is a fluctuating mechanism, which can (and in fact should) keep the average unchanged, and well represented by Fig.5a.

If this physical interpretation is correct, it can have far-reaching consequences on the study of hadronization mechanisms at long space-time distances. Indeed, the existence of intermittency patterns suggests that the density fluctuations of hadrons

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**Fig.4** \( \ln(F_i) \) as functions of \(-\ln \delta y\) for JETSET 6.3 parton shower at \( \sqrt{s}=91 \) GeV [23a] at the a) parton, b) hadron level, both with cut-off \( Q_0^2=1 \) GeV\(^2\), c) d) with cut-off \( Q_0^2=0.4 \) GeV\(^2\).
Fig. 5. Space-time interpretation of intermittent fluctuations.

In Fig. 5a-b, one shows the space-time frame in which a collision takes place, at least projected on the \((t, z)\) plane, namely (time, longitudinal distance). The causal conus \((t \pm z \geq 0, t \geq 0)\) is displayed, together with causal hyperbolae. \(\tau \equiv (t^2 - z^2)^{1/2} = \text{cste}\), where \(\tau\) is the proper time.

a) In-out picture of hadron production

The Figure shows the conventional picture of hadron production after a high-energy collision. At the origin (O) an interacting “string” of partonic or hadronic matter (hatched region) is found. After a proper time \(\tau\) (of order 1 fermi) of relativistic expansion, the “string” breaks into pieces measured (in average) by the conventional correlation length \(\xi\), giving rise to hadrons.

b) Random cascading in space-time

In the \((1+1)\) space-time frame, Fig. 5b shows the (random) generation of intermittent fluctuations following the logical structure of the \(\alpha\)-models [4], namely a random cascading model of hadronization. At each step of the cascade, one iterates the process shown in Fig. 5a. A system (or string) is stretched by the relativistic expansion to a length larger than the correlation length \(\xi\), and breaks into \(\lambda\) pieces with fluctuating density \(\rho\). The fluctuations appear at successive proper-time values \(\tau_s\), leading to structures at different values of rapidity size \(\delta y = \xi/\tau_s\). In the Figure, one has chosen \(l = -2, \tau_s = 2^l\), and random factors \(W_+\) and \(W_-\).

are sensitive to a succession of different scales. Thus, the intermittency parameters, such as the anomalous dimensions \(d_i\), are expected to teach something on the deep nature of the hadronization mechanism. It has already been proposed [29] that the intermittent fluctuation and correlation patterns may distinguish the presence of a phase transition — such as the quark-gluon plasma formation in heavy-ion collisions — from a genuine cascading mechanism. In the same spirit, we want to suggest that intermittency features give an interesting and complementary information on the space-
time development of hadronization in a nuclear environment. The example of deep-inelastic scattering of electrons on nuclei at an energy of 20 GeV or more deserves a particular study; it is quite possible that the study of hadronic density fluctuations in the final state represents a valuable tool of physical investigation for quarks in nuclei.

As a final comment, the intermediate range of energies — compared with the one usually considered for intermittency studies — which is involved in the present case calls for some remarks. It is true that high multiplicities are welcome for a better determination of the anomalous dimensions of the fluctuations. However, the factorial moment and correlator method has been proven to give interesting information at rather low values of the average multiplicity [30]. However, it seems required to develop better tools for handling statistical deviations, such as the "empty-bin" effect which appears when the maximal number of particles per bin is of the order of the factorial moment under study. Work is under way along this line, either by simulation [31] or by evaluation of extrapolation procedures for high moments [32]. It is our opinion that further progress will be made in the intermittency analysis at intermediate energies, giving its price to the fluctuation study of hadronization.

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