Factorial correlators from $\pi^+p$ and $K^+p$ collisions at 250 GeV/c

EHS/NA22 Collaboration

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Factorial correlators are studied in 250 GeV/c $\pi^+p$ and $K^+p$ collisions as a function of the distance in rapidity. The correlators are found to increase with decreasing correlation distance, independently of the rapidity resolution. The increase approximately follows a power law, but the power is considerably larger than expected from a log-normal approximation in the simple model of intermittency. Also, the FRITIOF results are independent of the resolution, but slopes and $p_T$ behavior cannot be reproduced by this model.

Intermittency [1] has now been studied in $e^+e^−$ [2–5], $\mu$ [6], $\nu\bar{\nu}$ [7], hadron–hadron [8,9], hadron–nucleus [10,11] and nucleus–nucleus [10,12–15] collisions. Recent reviews are given in refs. [16,17]. The study has been performed in terms of a power-law dependence of the scaled factorial moments

$$\langle F_n \rangle = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_m-1)\ldots(n_m-i+1) \rangle}{\langle n_m \rangle^i}, \quad (1)$$

where $M$ is the number of phase-space bins of size $\delta x = \delta y = \delta E$. The power $\alpha$ is regarded as a characteristic of the event. The averages under the sum are over the events in the sample. While in the early stage of the analysis of interest mainly arose from the difficulties of currently used models to reproduce the effect, a number of alternative interpretations is available now. These range from conventional short-range correlations and Bose–Einstein interference, via pencil jets and extended parton cascades, to a possible signal for quark–gluon plasma. Different data support different interpretations, so that more discriminative information is needed experimentally.

While the results defined in (1) measure local density fluctuations in phase space, additional information is contained in the correlation between these
fluctuations within a given event. This correlation can be extracted by means of the factorial correlators [1]

\[
\langle F_{m, m'}^{m m'} \rangle = \langle n_m (n_m - 1) \ldots (n_m - i + 1) \rangle \\
\times \langle n_{m'} (n_{m'} - 1) \ldots (n_{m'} - j + 1) \rangle \\
\times \left[ \langle n_{m - i} (n_{m - i + 1}) \ldots (n_{m - j + 1}) \rangle \right]^{-1},
\]

(2)

where \( n_m \) is the multiplicity in bin \( m \) and \( n_{m'} \) that in bin \( m' \) (see fig. 1). The correlators are calculated at given \( S \) for each combination \( m m' \) and then averaged over all combinations with given \( D \), as shown in fig. 1.

According to the simple intermittency model (\( \alpha \)-model) described in ref. [1], the \( \langle F_{m} \rangle \) should depend only on \( D \) and not on \( \delta \), the dependence should be according to the power law

\[
\langle F_{m} \rangle \propto (D/D)^{\alpha}.
\]

(3)

For the power \( f_0 \) (slope in a log-log plot) the following relation has been derived [1]:

\[
f_0 = f_{i+1} - f_i = i f_{i+1},
\]

(4)

where the first equality sign is due to the \( \alpha \) model, the second to the log-normal approximation. Since according to (4) \( f_{i} = f_{i+1} \), this can also be written in the form

\[
f_0 = f_{i},
\]

(5)

Preliminary results for pseudo-rapidity resolution \( \delta y \geq 0.2 \) have been reported by the HELIOS Collaboration [18], where, however, the multiplicities had to be estimated from the transverse energy.

The data presented here come from the NA22 experiment performed at CERN. In this experiment the European Hybrid Spectrometer (EHS) is equipped with the Rapid Cycling Bubble Chamber (RCBC) as a vertex detector and exposed to a 230 GeV/c tagged positive, meson enriched beam. In data taking, a minimum bias interaction trigger is used. The details of the spectrometer and the trigger can be found in previous publications [19,20].

Charged particle tracks are reconstructed from hits in the wire- and drift-chambers of the two lever-arm magnetic spectrometer and from measurements in the bubble chamber. The average momentum resolution \( \langle \Delta p/p \rangle \) varies from a maximum of 2.5% at 30 GeV/c to around 1.5% above 100 GeV/c. In the rapidity region \( \Delta y \) under consideration \((-2.0 < y < 20.0, \alpha \approx 0.2 \text{ to } 0.5)\), the experimental resolution varies between 0.01 and 0.05 units.

Events are accepted for the present analysis when measured and reconstructed charge multiplicity are consistent, charge balance is satisfied, no electron is detected among the secondary tracks and the number of badly reconstructed (and therefore rejected) tracks is 0. Elastic events are excluded. Furthermore, an event is called single-diffractive and excluded from the sample if the total charge multiplicity is smaller than \( 8 \) and at least one of the positive tracks has \( |x| > 0.88 \). After these cuts, our “cleaned” inelastic non-single-diffractive sample consists of 59 232 \( \pi^+ p \) and \( K^+ p \) events. The averages in (1) and (2) are normalized of this sample, including events with no track in \(-2.0 < y < 2.0\).

For momenta \( p_{\text{lab}} < 0.7 \text{ GeV/c} \), the range in the bubble chamber and/or the change of track curvature is used for proton identification. In addition, a visual ionization scan has been used for \( p_{\text{lab}} < 1.2 \text{ GeV/c} \) on the full \( K^+ p \) and 62% of the \( \pi^+ p \) sample. Particles with momenta \( p_{\text{lab}} > 1.2 \text{ GeV/c} \) are not identified in the present analysis and are treated as pions.

As mentioned above, no losses due to bad reconstruction are allowed in our sample. It has been checked by a comparison to an event sample with track losses that the slopes \( f_0 \) are hardly affected by track losses. For further analysis of possible biases see ref. [21].

The \( \ln \langle F_{m} \rangle \) are shown as a function of \( -\ln D \) in
Fig. 2. In $\langle F_{ij} \rangle$ as a function of $-\ln D$ for four values of $\delta y$, as indicated. Data in numerical form can be obtained by bitnet from U632007 at HNYKUN11.

Figs. 2a–2d, for four values of $\delta y$ (corresponding to $M = 10, 20, 30$ and 40), respectively. Statistical errors (estimated from the dispersion of the $F_{ij}$ distribution) are in general smaller than the size of the symbols. While an estimate of $\langle F_{ij} \rangle$ is possible up to third order in $i$ and $j$ for $\delta y = 0.4$ binning (fig. 2a), we restrict ourselves to first and second order at $\delta y = 0.1$ (fig. 2d). Since the smallest possible value for $D$ is the bin size $\delta y$, fig. 2a extends to $D = 0.4$ and fig. 2d to $D = 0.1$. In all cases an increase of $\ln \langle F_{ij} \rangle$ is observed with increasing $-\ln D$.

In fig. 3a we compare the $\ln \langle F_{ij} \rangle$ at fixed $D = 0.4$ for the four different values of $\delta y$. The dashed lines correspond to a horizontal line fit through the points. As expected from the $\alpha$ model, the $\langle F_{ij} \rangle$ indeed do not depend on $\delta y$. However, this property is not unique to the $\alpha$-model. Fig. 3b shows that the $\delta y$ independence is also reproduced by the FRITIOF model [22] (using a sample of 60 000 FRITIOF Monte Carlo events). The dashed lines correspond to horizontal line fits through the points.

For $\langle F_{11} \rangle$, the $\delta y$ independence can be extracted from the integral over the two-particle density, with two integration domains of size $\delta y$ separated by $D$. Using exponential short-range order [23], this gives

$$\langle F_{11} \rangle - 1 \propto \frac{1}{a^2} \exp(-D/L) \left( e^{a} - 1 \right) \left( 1 - e^{-a} \right),$$

where $L$ is a correlation length and $a = \delta y / L$. Accord-

---

1. A. L. C. 1.2
2. 0.8
3. 0.4
4. 0
Table 1
Slopes $f_i$ and $f_j$ from fits in the $D$ region indicated.

<table>
<thead>
<tr>
<th>$ij$</th>
<th>$\delta y=0.4$</th>
<th>$\delta y=0.2$</th>
<th>$\delta y=0.133$</th>
<th>$\delta y=0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.4 &lt; $D$ &lt; 1.2)</td>
<td>(0.2 &lt; $D$ &lt; 1.0)</td>
<td>(0.13 &lt; $D$ &lt; 0.93)</td>
<td>(0.1 &lt; $D$ &lt; 1.0)</td>
</tr>
<tr>
<td>11</td>
<td>$0.096 \pm 0.004$</td>
<td>$0.057 \pm 0.002$</td>
<td>$0.043 \pm 0.001$</td>
<td>$0.042 \pm 0.001$</td>
</tr>
<tr>
<td>21</td>
<td>$0.163 \pm 0.007$</td>
<td>$0.102 \pm 0.004$</td>
<td>$0.070 \pm 0.003$</td>
<td>$0.079 \pm 0.003$</td>
</tr>
<tr>
<td>31</td>
<td>$0.214 \pm 0.016$</td>
<td>$0.165 \pm 0.011$</td>
<td>$0.066 \pm 0.012$</td>
<td>$0.197 \pm 0.004$</td>
</tr>
<tr>
<td>22</td>
<td>$0.283 \pm 0.008$</td>
<td>$0.193 \pm 0.005$</td>
<td>$0.114 \pm 0.004$</td>
<td>$0.114 \pm 0.004$</td>
</tr>
<tr>
<td>32</td>
<td>$0.397 \pm 0.017$</td>
<td>$0.397 \pm 0.017$</td>
<td>$0.397 \pm 0.017$</td>
<td>$0.397 \pm 0.017$</td>
</tr>
<tr>
<td>33</td>
<td>$0.588 \pm 0.025$</td>
<td>$0.588 \pm 0.025$</td>
<td>$0.588 \pm 0.025$</td>
<td>$0.588 \pm 0.025$</td>
</tr>
<tr>
<td></td>
<td>($0.1 \leq \delta y \leq 1.0$)</td>
<td>($0.1 \leq \delta y \leq 1.0$)</td>
<td>($0.1 \leq \delta y \leq 1.0$)</td>
<td>($0.1 \leq \delta y \leq 1.0$)</td>
</tr>
</tbody>
</table>

According to (6), $<F_{ij}>$ becomes independent of $\delta y$ for $a < 1$. Since $\exp(-D/L) \rightarrow 1$ with $D \rightarrow 0$, this form also leads to deviations from (3) observed as a bending in fig. 2.

Because of the bending, fitted slopes $f_i$ only will be used as an indication for the increase over a certain range. They are given in table 1 for the fit ranges stated. For two values of $\delta y$ they are compared to FRITIOF predictions in figs. 4a and 4b, respectively. As observed earlier for the case of single moments [8], the FRITIOF slopes are too low also for the correlators $<F_{ij}>$. This shortcoming of FRITIOF is related [23] to the failure of this model to reproduce two-particle rapidity correlations in ref. [24] and other data [25]. Future improvements of the model should account for these results simultaneously.

According to (4), the ratio $f_i/f_j$ is expected to grow with increasing orders $i$ and $j$ like their product $ij$. In figs. 4c and 4d this is tested for $\delta y=0.4$ and $\delta y=0.2$, respectively. In both cases, the experimental results lie far above the dashed line corresponding to the expected $f_i/f_j = ij$. Since the dependence of $\ln <F_{ij}>$ on $-\ln D$ is not strictly linear, this comparison depends on the range of $\delta y$ and $D$ used to determine $f_i$ and $f_j$. In fig. 4d we, therefore, compare a number of fits. Slopes are reduced when reducing the upper limit in $D$, but do not reach the $\alpha$-model prediction (dashed line).

It can be verified from table 1 that at least for the higher orders the discrepancy with (4) is mainly due to the second equal sign, derived from a log-normal distribution.

![Fig. 4](image-url)

Fig. 4. (a), (b) The increase of the slopes $f_i$ with increasing order $ij$ compared to the expectation from FRITIOF, for two values of $\delta y$, respectively; (c), (d) the increase of $f_i/f_j$ with increasing order $ij$, compared to that expected from the $\alpha$ model (dashed line), for two values of $\delta y$, respectively.
Fig. 5. The dependence of $f_{ij}$ on the order $ij$ for the full sample and after $p_T$ cuts as indicated, (a) for the experimental data, (b) for the FRITIOF sample.

Recently, a transverse momentum dependence of the intermittency effect has been reported in our data [27], with essentially all of the effect being restricted to $p_T < 0.30$ GeV/c. In fig. 5 we therefore compare results for $p_T$ cuts with the full sample in terms of $f_{ij}$ for $\delta y=0.4$. In the data (fig. 5a), the slopes increase both when restricting to small and to large $p_T$. On the other hand, FRITIOF gives an increase of the slopes for small $p_T$, but a decrease for large $p_T$.

We conclude that the correlators $\langle F_{ij} \rangle$ increase with decreasing correlation length $D$, but only approximately follow a power law for $D \leq 1$. For fixed $D$ the values of $\langle F_{ij} \rangle$ do not depend on resolution $\delta y$, a feature expected from the $\alpha$ model, but also reproduced by FRITIOF and probably common to any model with short range order. The powers $f_{ij}$ increase linearly with the product $ij$ of the orders, but are considerably larger than expected from FRITIOF and from the simple $\alpha$ model. Even though FRITIOF has a strong effect at small $p_T$, the $p_T$ dependence of the effect is not fully reproduced by the FRITIOF model.

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