Genuine higher-order correlations in $\pi^+p$ and $K^+p$ collisions at 250 GeV/c

EHS/NA22 Collaboration

N.M. Agababyan$^h$, I.V. Ajinenko$^e$, M.R. Atayan$^h$, F. Botterweck$^{d,1}$, M. Charler$^{d,2}$, P.V. Chliapnikov$^e$, E.A. De Wolf$^{a,3}$, K. Dziunikowska$^{b,4}$, A.M.F. Endler$^f$, Z.Sh. Garutchava$^g$, G.R. Gulkanyan$^h$, R.Sh. Hakobyan$^h$, D. Kiselewska$^{b,4}$, W. Kittel$^d$, S.S. Mehrabyan$^h$, Z.V. Metreveli$^g$, L.C.S. Olivier$^0,3$, K. Olkiewicz$^{b,4}$, F.K. Rizatdinova$^e$, E.K. Shabalina$^c$, L.N. Smirnova$^e$, M.D. Tabidze$^8$, L.A. Tikhonova$^e$, A.V. Tkablazde$^8$, A.G. Tomaradze$^8,1$, F. Verbeure$^h$, S.A. Zotkin$^c$

$^a$ Department of Physics, Universitaire Instelling Antwerpen, B-2610 Wilrijk and Inter-University Institute for High Energies, B-1050 Brussels, Belgium

$^b$ Institute of Physics and Nuclear Techniques of Academy of Mining and Metallurgy and Institute of Nuclear Physics, PL-30055 Krakow, Poland

$^c$ Nuclear Physics Institute, Moscow State University, RU-119899 Moscow, Russia

$^d$ University of Nijmegen/NIKHEF, NL-6525 ED Nijmegen, The Netherlands

$^e$ Institute of Nuclear Physics, RU-119899 Moscow, Russia

$^f$ University of Nijmegen/NIKHEF, NL-6525 ED Nijmegen, The Netherlands

$^g$ Institute for High Energy Physics, RU-142284 Protvino, Russia

$^h$ Centro Brasileiro de Pesquisas Físicas, BR-22290 Rio de Janeiro, Brazil

$^i$ Institute for High Energy Physics of Tbilisi State University, GE-380086 Tbilisi, Georgia

Received 6 May 1994

Editor: L. Montanet

Abstract

Genuine higher-order correlations could be established up to fifth order in multiparticle production in $\pi^+p$ and $K^+p$ collisions at 250 GeV/c. At decreasing interparticle distance in invariant phase-space, these correlations follow approximate power-law scaling. The scaling behavior is different for non-exotic and like-charge particle combinations. It cannot be reproduced by FRITIOF without Bose-Einstein correlations. Including Bose-Einstein correlations according to JETSET7 reproduces the slope of like-charge two-particle correlations, but overestimates those of the higher orders.

1. Introduction

Multiparticle production in high energy collisions is one of the rare fields of physics where higher-order correlations are directly accessible in their full multi-
dimensional characteristics under well controlled experimental conditions.

Although two-particle correlations were studied in great detail in the past [1] and have attracted renewed interest in recent years (for a recent review see [2]), the study of (genuine) higher-order correlations has been hampered by the lack of a suitable method that makes optimal use of the statistics available in an experiment. Three-particle correlations also have been observed in the form of short-range rapidity correlations [3–7] and higher-order Bose-Einstein (BE) correlations [4,6,8–10], but evidence for genuine higher-order correlations (i.e. after subtraction of all lower-order contributions) has not been established so far.

Here, we report on an application of a recently developed method unambiguously isolating genuine multiparticle correlations and their bin-size dependence for various charge combinations, up to order 5.

2. The data

In this CERN experiment, the European Hybrid Spectrometer (EHS) was equipped with the Rapid Cycling Bubble Chamber (RCBC) as an active vertex detector and exposed to a 250 GeV/c tagged positive, meson enriched beam. In data taking, a minimum bias interaction trigger is used. The details of the spectrometer and the trigger can be found in previous publications [11,12].

Charged particle tracks are reconstructed from hits in the wire- and drift-chambers of the two lever-arm magnetic spectrometer and from measurements in the bubble chamber. The average momentum resolution $\langle \Delta p/p \rangle$ varies from a maximum of 2.5% at 30 GeV/c to around 1.5% above 100 GeV/c.

Events are accepted for the analysis when measured and reconstructed charge multiplicity is the same, charge balance is satisfied, no electron is detected among the secondary tracks and the number of badly reconstructed (and therefore rejected) tracks is 0. The loss of events during measurement and reconstruction is corrected for by means of the topological cross section data [11]. Elastic events are excluded. Furthermore, an event is called single-diffractive and excluded from the sample if the total charge multiplicity is smaller than 8 and at least one of the positive tracks has $|x_F| > 0.88$. After these cuts, the inelastic non-single-diffractive sample consists of 59 200 $\pi^+p$ and $K^+p$ events.

For laboratory-momenta $p_{LAB} < 0.7$ GeV/c, the range in the bubble chamber and/or the change of track curvature is used for proton identification. In addition, a visual ionization scan has been used for $p_{LAB} < 1.2$ GeV/c on the full $K^+p$ and 62% of the $\pi^+p$ sample. Positive particles with $p_{LAB} > 150$ GeV/c are given the identity of the beam particle. Other particles with momenta $p_{LAB} > 1.2$ GeV/c are not identified in the present analysis and are treated as pions.

In spite of the electron rejection mentioned above, residual Dalitz decay and $\gamma$ conversion near to the vertex still contribute to the two-particle correlations. Their influence on our results has been investigated in detail in [13].

3. The method

The method used is that of normalized cumulant moments

$$K_q'(\epsilon) \equiv k_q(\epsilon)/q^{\text{norm}}(\epsilon)$$

evaluated by the star integration [14]

$$k_q(\epsilon) = \int C_q(x_1, \ldots, x_q) \times \Theta_{12}\Theta_{13} \ldots \Theta_{1q} dx_1 \ldots dx_q$$

of the (factorial) cumulants, or “connected” correlation functions

$$C_2(x_1, x_2) = p_2(x_1, x_2) - p_1(x_1)p_1(x_2),$$

$$C_3(x_1, x_2, x_3) = p_3(x_1, x_2, x_3) - p_1(x_1)p_2(x_2, x_3) - p_1(x_2)p_2(x_3, x_1)$$

etc.

In (3) and (4), the $p_q$ are the $q$-particle densities at the phase space point $\{x_1 \ldots x_q\}$. In (2) the $\Theta_{ij}$ are defined by the Heaviside unit step-function

$$\Theta_{ij} \equiv \Theta(\epsilon - |x_i - x_j|)$$

and restrict all $q - 1$ coordinates $x_j$ to lie within a distance $\epsilon$ of $x_i$. 
The cumulant moments $f_q(\epsilon)$ are normalized by the integrals

$$
\xi_q^{\text{norm}}(\epsilon) = \int \rho_1(x_1) \cdots \rho_1(x_q)
\times \Theta_{12}\Theta_{13} \cdots \Theta_{1q} dx_1 \cdots dx_q
$$

(6)
evaluated with the help of (5) with particles $j$ taken randomly from different events ("event mixing").

Unbiased estimators for cumulant moments and normalization are given in [15]. The non-trivial modifications needed for events with non-uniform weights are derived in [16].

Inclusive $q$-particle densities $\rho_q(x_1, \ldots, x_q)$ in general contain uninteresting contributions from lower-order densities. It is, therefore, advantageous to consider cumulant functions which have the property to vanish whenever one of their arguments becomes statistically independent of the others.

The star-integral method presents the advantage of optimal use of available statistics and minimal use of

---

*Fig. 1.* $\ln K^*_q(Q^2)$ as a function of $-\ln Q^2$ for all charged particles, compared to the expectations from the FRITIOF model.
Table 1
Slopes $\phi_q$ of the power-like scaling law for the various charge combinations, fitted in the range $0 \leq -\ln Q^2 \leq 5$

<table>
<thead>
<tr>
<th>charge comb.</th>
<th>$q = 2$</th>
<th>$q = 3$</th>
<th>$q = 4$</th>
<th>$q = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>all</td>
<td>$0.205 \pm 0.005$</td>
<td>$0.72 \pm 0.03$</td>
<td>$1.2 \pm 0.2$</td>
<td>$2.0 \pm 1.0$</td>
</tr>
<tr>
<td>like-charged</td>
<td>$0.387 \pm 0.009$</td>
<td>$1.03 \pm 0.08$</td>
<td>$1.8 \pm 0.3$</td>
<td></td>
</tr>
<tr>
<td>unlike-charged</td>
<td>$0.096 \pm 0.004$</td>
<td>$0.61 \pm 0.03$</td>
<td>$2.0 \pm 0.5$</td>
<td></td>
</tr>
</tbody>
</table>

computer time.
Since higher accuracy is obtainable, dynamical structures in the correlations can be studied in greater detail than with conventional methods.

A detailed and systematic study of normalized factorial moments $F_q$ and normalized cumulant moments $K_q$, as obtained from various methods, is given in [17].

4. The results

The results for the normalized cumulant moments $K_q^*$ are given in Fig. 1 for the sample of all charged particles, in the form of a double-logarithmic plot. The particle 4-momenta $p_i$ are used as coordinates $x_i$ and $Q_{ij}^2 = -(p_i - p_j)^2$ as a distance measure, so that (5) is replaced by

$$\Theta_{ij} \equiv \Theta(Q^2 - Q_{ij}^2).$$

Because of (3), (4), the $K_q^*$ can become negative and/or the errors can become very large. Such data points are not shown.

For the first time, positive genuine multi-particle correlations can be unambiguously established up to order five. The star-integral method used here indeed gives a clear improvement over an earlier analysis based on the same data [13,18]. For all orders $q$, the correlations become stronger with decreasing “distance” $Q^2$.

Originally, it was advocated [19] that the normalized factorial moments $F_q$ might follow a power-like scaling law. Later [20], it was suggested on the basis of two-particle correlation data, that normalized cumulant moments, rather than factorial moments, might show power-behaviour, so that

$$K_q^*(Q^2) \propto (Q^2)^{-\phi_q}. \quad (8)$$

Fig. 1 shows that the data for $q = 2$ increase faster than a simple power in $Q^2$ and can only be represented by a function of that type in a restricted range of independent variable. Higher-order cumulants show a tendency to level off after an initial power-like increase at larger $Q^2$.

As a qualitative measure of the rise of the cumulants, we have fitted the powers $\phi_q$ in (8) over the range $0 \leq -\ln Q^2 \leq 5$. The results are given in Table 1 and are seen to increase with $q$.

The experimental data are compared to the expectations from the FRITIOF model [21]. Since version 2.0 is better tuned to our data [18] than the most recent version of this model (version 7.0) we show the comparison for the former. To include BE-correlations, we use the (ad-hoc) algorithm developed for JETSET 7.3 [22] with exponential parametrization $^5$ in $Q$ and measured parameters $r$ and $\lambda$ [13]. Furthermore, we allow for Dalitz decay in the model and add undetected $\gamma$-conversion (0.25%) [13]. The generated events are subjected to the same selection criteria as the real data.

For $q = 2$, FRITIOF only reproduces the data after inclusion of BE correlations and $\gamma$ conversion. For larger $q$, the influence of $\gamma$-conversions is negligible, but the model predictions curve upwards, contrary to what is seen in the data. Consequently, we conclude that Bose-Einstein parametrization as implemented in

$^5$ FRITIOF 2.0 with BE describes the shape of the $Q^2$ dependence of the second order factorial moment, but overestimates the increase with increasing $-\ln Q^2$ of the third order factorial moment [18].
JETSET strongly overestimates the correlations, especially at small $Q^2$.

The charge dependence of the correlation is studied in a comparison of like-charged (Fig. 2) to unlike-charged (Fig. 3) particle combinations. Both charge combinations show non-zero genuine higher-order correlations and an increase of the correlation function with decreasing "distance" $Q^2$. The correlations among unlike-charged combinations (i.e. combinations to which resonances contribute) are relatively strong near $\ln Q^2 \sim 0$, but the increase for larger $-\ln Q^2$ is relatively slow. Correlations among like-charged particles are small at $\ln Q^2 \sim 0$, but increase rapidly to reach, or even cross, those of the unlike-charged combinations. This difference diminishes with increasing order $q$.

For the like-charged particle combinations (Fig. 2), $K_f^2$ is well described by FRITIOF if B.E. correlations are included. An exception may be the $-\ln Q^2 = 0$ region for positives, where an influence of the leading particles is present. However, the overestimate of the higher-order correlations by FRITIOF + BE, al-
5. Conclusions

Genuine multiparticle correlations up to fifth order have been established. The correlations increase in strength with decreasing inter-particle distance $Q^2 = -(q_1 - q_2)^2$ in invariant phase-space. The dependence on $Q^2$ of the data is more complicated than the simple power law advocated in previous intermittency studies.

The behavior of the cumulants is different for non-exotic and like-charge particle combinations. It can-

---

Fig. 3. $\ln K_s^\gamma(Q^2)$ as a function of $-\ln Q^2$ for unlike-charged particle combinations, compared to the expectations from the FRITIOF Model.

ready observed in Fig. 1 for the sample of all charged particles, is also evident here.

For the unlike-charged particle combinations, $K_s^\gamma$ is reproduced only after including $\gamma$ conversion. For higher orders, the effect of $\gamma$ conversion is small and the data are reproduced reasonably well.
not be reproduced by FRITIOF without Bose-Einstein correlations. Including Bose-Einstein correlations according to the JETSET7 prescription, FRITIOF reproduces the $Q^2$-dependence of like-charged two-particle correlations. However, cumulants of higher orders are strongly overestimated, especially at the smallest values of $Q^2$.

Acknowledgments

It is a pleasure to thank the EHS coordinator L. Montanet and the operating crews and staffs of EHS, SPS and H2 beam, as well as the scanning and processing teams of our laboratories for their invaluable help with this experiment. We are grateful to the III. Physikalisches Institut B, RWTH Aachen, Germany, the DESY-Institut für Hochenergiephysik, Berlin-Zeuthen, Germany, the Department of High Energy Physics, Helsinki University, Finland, and the University of Warsaw and Institute of Nuclear Problems, Poland for early contributions to this experiment. We, furthermore, would like to thank B. Buschbeck and P. Lipa for clarifying discussions.

This work is part of the research programme of the "Stichting voor Fundamenteel Onderzoek der Materie (FOM)", which is financially supported by the "Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO)". We further thank NWO for support of this project within the program for subsistence to the former Soviet Union (07-13-038).

References