Reply to "Comment on 'Inverse Square Lévy Walks are not Optimal Search Strategies for $d \geq 2$'" The central result of our Letter [1] is that (i) the capture rate η of Levy walks with Poisson distributed targets goes linearly with the target density ρ for all values of the Levy exponent α in space dimension $d \geq 2$. This contradicts results in [2] and has important consequences: (ii) the optimal gain $\eta_{\max}/\eta$ achieved by varying α is bounded in the limit $\rho \to 0$ so that tuning α yields a marginal gain; (iii) the optimum is realized for a range of α and is controlled by the model-dependent parameters a (detection radius), l_c (restarting distance), and s (scale parameter) (Fig. 1).

First, and most importantly, [3] states that our main result (i) is correct, thereby acknowledging that the determination of η in [2] is wrong.

Second, [3] proposes that claim (iii) is not new because earlier publications reported that optimal Levy strategies can be realized for $\alpha \neq 1$. We did acknowledge such *observations* in [1], where we in fact *show* that they result from the linear scaling of η with ρ for $d \geq 2$; this is novel.

Last, [3] disputes claim (ii). Technically, claim (ii) is correct and by no means compromised by [3]. It states that for *fixed* values of s, l_c, the optimal gain $\eta_{\max}/\eta$ is bounded when $\rho \to 0$. This comes from the linear scaling of η with ρ (Eq. (5) in [1], whose validity is acknowledged by [3]) and is *independent* of any determination of $K_d(\alpha, s, l_c)$. In [1], Eq. (3) is used only to derive the scaling of η with ρ; we make no prediction regarding $K_d(\alpha, s, l_c)$. Attempting to deduce $K_d(\alpha, s, l_c)$ from Eq. (3) is the initiative of [3], not ours. In fact, we agree that Eq. (3) is unsuitable to study $l_c \to a$, which falls out of the validity regime given in [4]. This is certainly not a problem in [1], as argued by [3], simply because we nowhere aimed at determining $K_d(\alpha, s, l_c)$.

Finally, the only aspect in (ii) that [3] disputes is rethorical: our qualification of the optimum as marginal. The comment is based only on the analysis of the singular limit $s \to 0$ and $l_c \to a$, which can indeed lead to arbitrarily large values of $\eta_{\max}/\eta$ for $\alpha \to 1$. This is actually a mere $1d$ limit (Fig. 1), as noted in [1]; it is thus expected, and consistent with our findings, to recover the $1d$ optimum. This by no means contradicts claim (ii) of boundedness when $\rho \to 0$ for fixed s, l_c. Last, we summarize the conditions of optimality (CO) of inverse square Levy walks for $d \geq 2$:

—Upon each capture event, a spherical target reappears infinitely fast at the same position.

—The searcher starts the new search infinitely close to the target boundary ($l_c - a \ll a$).

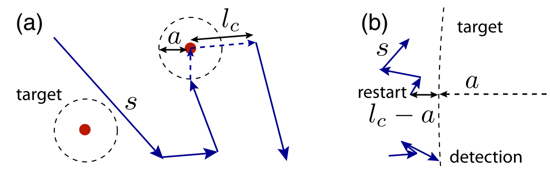

FIG. 1. (a) The Lévy walk search model in the generic $2d$ case. (b) Inverse square Levy walks are optimal only in the singular $1d$ limit $l_c - a \ll a$ and $s \ll a$.

—The typical scale of its displacements is infinitely smaller than the target ($s \ll a$). If *any* of these conditions is not met, $\alpha = 1$ is not optimal. Given that s and l_c are system-dependent parameters with arbitrary values, the CO are generically not met, and our conclusion that inverse square Levy walks are not optimal is justified. Additionally, if l_c, s are allowed to vary, as done in [3], the obvious optimal strategy is $l_c = a$, leading to immediate recapture of the same target. The limit $l_c \to a^+$ in the CO is thus artificial.

To our knowledge, the CO have never been stated explicitly nor verified in any experimental system. Given that the CO are a mere $1d$ limit of the problem, the claim that [3] restores the optimality of $\alpha = 1$ for $d \geq 2$ is unfounded, and given that [3] acknowledges that the scaling of η with ρ is wrong in [2], stating that [3] restores the validity of [2] is also unfounded.

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