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The cognitive profiles for different samples of mathematical learning difficulties and their similarity to typical development: Evidence from a longitudinal study

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ABSTRACT

Several cognitive deficits have been suggested to induce mathematical learning difficulties (MLD), but it is unclear whether the cognitive profile for all children with MLD is the same and to what extent it differs from typically developing (TD) children. This study investigated whether such a profile could be distinguished when cognitive skills and math performance are compared between TD children and children with MLD. This was accomplished by employing two-way repeated-measures analyses of covariance in 276 10-year-old participants (60 with MLD) from fourth and fifth grades. In addition, we investigated whether more restrictive selection criteria for MLD result in different mathematical and cognitive profiles by means of independent-samples *t* tests. Results revealed that cognitive mechanisms for math development are mostly similar for children with MLD and TD children and that variability in sample selection criteria did not produce different mathematical or cognitive profiles. To conclude, the cognitive mechanisms for math development are broadly similar for children with MLD and their TD counterparts even when different MLD samples were selected. This strengthens our idea that MLD can be defined as the worst performance on a continuous scale rather than as a discrete disorder.

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Introduction

Math development does not come naturally for every child. Whereas many children meet the national requirements for math proficiency throughout primary school, some encounter severe and persistent difficulties with learning mathematics, namely mathematical learning disabilities (MLD) (Desoete, 2007; Van Luit, 2019). The deficiencies displayed by children with MLD—sometimes also referred to as dyscalculia—can be particularly broad. They can be manifested in different domains of mathematics as well as in different underlying cognitive skills, which reflects the heterogeneity of MLD (Geary, 2004). The precise cognitive mechanisms that give some children the opportunity to develop their math performance more than others remain unclear. Variability in sample characteristics among studies (e.g., Murphy, Mazzocco, Hanich, & Early, 2007) further complicates the identification of the underlying cognitive mechanisms in MLD and, as a consequence, hinders the search in identifying the correct building blocks to establish adequate interventions for children with delays in mathematics. Therefore, the aim of the current study was to examine which cognitive mechanisms predict the math development of children with MLD and their typically developing (TD) peers and to what extent sample selection criteria lead to different profiles of mathematical and cognitive skills.

Mathematics in the upper grades of primary school demands both the understanding and application of common principles and strategies to math problems. The emphasis of the current study was on fourth- and fifth-grade mathematics because children in those grades typically expand their math knowledge from simple arithmetic skills to solving more complex math problems (Mullis, Martin, & Loveless, 2016). Mathematics in this study was operationalized into three domains that are present in the Dutch curriculum as well as in many other industrialized Western cultures: numbers, geometry, and proportions. First, the numbers domain represents children's proficiency in calculations with (large) whole numbers. Second, the geometry domain entails the ability to calculate the perimeter or area of a figure as well as to convert units of measure (e.g., centimeters to kilometers). Third, the proportions domain involves calculations with rational numbers such as fractions and decimals (Noteboom, Aartsen, & Lit, 2017). These are separate domains of mathematics, as shown by the notion that number systems are different from geometry systems (Spelke, Lee, & Izard, 2010) and the fact that children experience major difficulties when moving from whole numbers to rational ones (Wearne & Kouba, 2000).

Computational proficiency in any domain of mathematics may rely on a specific set of cognitive skills. According to previous research, a theoretical model on the development of mathematics in TD children can be derived, with the most relevant factors including number sense (Schneider et al., 2017), working memory (Friso-van den Bos, Van der Ven, Kroesbergen, & Van Luit, 2013), phonological awareness (Simmons & Singleton, 2008), rapid naming (Willburger, Fussenegger, Moll, Wood, & Landerl, 2008), and nonverbal reasoning (Seethaler, Fuchs, Star, & Bryant, 2011).

To elaborate on those cognitive skills, number sense—or the ability to recognize and understand quantities, number words, and Arabic digits (Dehaene, Piazza, Pinel, & Cohen, 2003)—is involved in math achievement in TD children by supporting their insight into relations between numbers and hence the performance of mental calculations. To elaborate, number sense, for instance, is involved in math tasks where number words, symbols, or numerosities are actively used to represent quantities (e.g., counting and ordering) as well as in tasks that require insight into the magnitude of numbers (e.g., the number 96 consists of the elements "ninety" and "six"). Second, working memory—defined as the ability to maintain verbal and visuospatial information in Engle's (2018) revisited working memory model and as the executive function of updating in an extension of Baddeley's working memory model (Miyake et al., 2000; Miyake & Friedman, 2012)—enables children to employ stepwise solutions during math problem solving. Both models state that working memory accounts for the storage of intermediate results and revising this information based on new input. Thus, when children solve a math problem such as 28 + 13 = __, they first need to store the answer to 28 + 3 = 31 and then revise that information by adding 10, resulting in the final answer of 41. Furthermore, rapid naming and phonological awareness facilitate quick access to and the effective manipulation of number facts retrieved from long-term memory, respectively (Vellutino, Fletcher, Snowling, & Scanlon, 2004;

Willburger et al., 2008). For example, children can have quick access to the answer to a math problem such as 4 * 7 = __ by memorization or can easily manipulate a problem such as 14 * 7 = __ into (10 * 7) + (4 * 7) by using their rapid naming and phonological awareness skills. Finally, nonverbal reasoning is used to draw inferences and consider alternative solutions in TD children (Stock, Desoete, & Roeyers, 2009). Stronger cognitive performance is generally associated with higher proficiency in mathematics because these cognitive skills are involved in understanding and processing formulas and procedures during math problem solving. This enables children to solve a complex task such as 3/8 ÷ 1/2 = __ because they can execute the correct procedure (i.e., flip the second fraction upside down, change the division sign into a multiplication sign, and multiply the tops and bottoms). Likewise, children with a weakness in any of those cognitive skills are at risk for developing MLD (Slot, van Viersen, de Bree, & Kroesbergen, 2016).

These cognitive skills have also been identified as predictors of mathematics development over time, but a comprehensive model including the aforementioned domains of mathematics and cognitive skills is lacking. As a case in point, working memory and alphanumeric rapid naming predicted the numbers domain in second to fifth grades (Fung & Swanson, 2017). Second, it has been shown that verbal and visuospatial working memory, in addition to nonverbal reasoning, are predictors of geometry in fourth and fifth grades (Giofrè, Mammarella, & Cornoldi, 2014). Third, arithmetic, symbolic number sense, and verbal working memory predicted the proportions domain in third and fifth grades (Jordan et al., 2013). Finally, in a study that assessed both geometry and proportions in fifth grade, verbal working memory (mediated by arithmetic) and nonverbal reasoning were again identified as predictors (Kleemans, Segers, & Verhoeven, 2018). Next to the specific relations between the cognitive skills and the domains of mathematics, they have also been identified as predictors of mathematics in general (Slot et al., 2016).

To elaborate, the emphasis of the mathematics curriculum is on becoming proficient in retrieval practices such as simple addition and multiplication in the lower grades of primary school, which is mainly supported by cognitive retrieval skills such as number sense, phonological awareness, and rapid naming (Cowan et al., 2011). In contrast, the emphasis of the curriculum shifts toward procedural steps and logical thinking as mathematics becomes more complex in the upper grades of primary education. These mathematical problem-solving tasks are in turn facilitated by procedural cognitive skills such as working memory and nonverbal reasoning (Baroody, 2003). Thus, for children to be able to develop their math achievement throughout primary school, it seems evident that a proficiency in supporting retrieval and procedural cognitive skills is necessary. Correspondingly, math performance is described as a proficiency in a combination of cognitive skills in the current study.

The development of mathematics varies as a function of interacting factors at the neurocognitive level (i.e., connectivity between the frontal and parietal cortexes) combined with environmental opportunities (e.g., education) that induce learning (Nelson, 1999). From substantive theories such as the multiple deficit model by Pennington (2006), it follows that neurodevelopmental disabilities are predicted by multiple risk factors. In other words, MLD arises through a combination of (cognitive) deficits. Indeed, it has become evident that cognitive deficits play a detrimental part in mathematics performance. Andersson, 2010 evidenced in a longitudinal study that children with MLD in third and fourth grades developed their mathematical skills at similar rates as their TD peers until fifth and sixth grades. However, the children with MLD lagged behind in working memory growth, which prevented them from catching up in math performance with their TD peers. Similar findings were obtained in another longitudinal study from kindergarten to fifth grade (Geary, Hoard, & Bailey, 2012a). Thus, due to cognitive deficits, children with MLD start behind when they enter primary education and generally stay behind during the following years of formal schooling.

MLD is a heterogeneous disability, which can be accounted for by the many possible combinations of cognitive risk factors that could induce math difficulties (see Kroesbergen, Huijsmans, & Friso-van den Bos, 2021, for a meta-analysis). A pivotal factor related to this heterogeneity is the amount of variability in selection criteria among research studies. Indeed, the question has been raised concerning how these inconsistent choices affect the comparability of research outcomes: How different are the mathematical and cognitive profiles of more restricted samples of MLD from less stringent MLD samples (Kroesbergen et al., 2021; Murphy et al., 2007; Peters & Ansari, 2019). With this study, we aimed to lay the groundwork for more research on this important question by assessing the mathematical and cognitive profiles of different samples based on the most commonly used selection criteria for MLD: (a) *severity,* or level of performance on a standardized math task (Desoete, 2007; Geary, 2004; Van Luit, 2019), (b) *persistency,* or duration of math problems (Desoete, 2007; Geary, 2004; Van Luit, 2019), (c) *specificity,* or no comorbid learning or behavioral disabilities (Landerl, Göbel, & Moll, 2013), (d) *core deficit in number sense* (Butterworth, Varma, & Laurillard, 2011; Dehaene, Bossini, & Giraux, 1993; Shalev, 2004), and (e) *IQ discrepancy,* where math performance is below what could be expected based on intelligence (Van Luit, 2019).

An MLD sample with narrow criteria for severity and persistency (e.g., lowest 10th percentile and multiple assessments) will by definition consist of children with more serious math difficulties than more broadly selected samples (e.g., lowest 25th percentile and single assessment), but empirical studies have not revealed whether these children rely on different cognitive mechanisms for math development. Likewise, a sample selected on number sense or intellectual impairments will be weaker on these skills compared with a sample that uses broader definitions, but it has not been demonstrated whether this corresponds to different mathematical deficits in both groups and whether related cognitive skills are also affected in these children.

Furthermore, comparative research showed that different cutoff criteria for MLD result in similar cognitive profiles; consistent differences across profiles were not found, but rather difficulties appeared on many of the same skills (Murphy et al., 2007). To elaborate, IQ, visuospatial ability, and rapid naming significantly predicted current math achievement, and rapid naming also significantly predicted growth in mathematics performance for children with less severe (<25th percentile) and more severe (<10th percentile) MLD. However, when comparing performance on those skills across groups, it turned out that the most severe MLD group obtained the weakest performance across all cognitive measures. In other words, the patterns of how and which cognitive skills are related to mathematics do not differ across both groups. This notion seems to converge with the idea, as proposed in a recent opinion article by Peters and Ansari (2019), that children with a specific learning disability cannot be discriminated from TD children in terms of specific cognitive markers.

The current study

From research conducted so far, it remains unclear whether children with MLD rely on a different set of cognitive skills than TD children (which would confirm that MLD should be acknowledged as a distinct disability) or that the cognitive predictors of math development for all children are too similar to be recognized as separate groups. Therefore, the first research question was "How are cognitive skills over time related to math performance in the domains of numbers, geometry, and proportions, and what are the differences between children with MLD and TD children?" Children with MLD as a group are expected to mostly rely on the same set of cognitive skills as TD children for all math domains even though their cognitive skills will generally be weaker (Peters & Ansari, 2019).

Another point of concern is that previous research has often merely based the identification of MLD on one selection criterion, but given the fact that much heterogeneity exists within these children, it is unclear how different these samples are in terms of their mathematical ability and associated cognitive skills. Another reason why previous studies have not found profiles of MLD could be that they did not select their samples using specific criteria, resulting in samples that were too broad and heterogeneous to identify profiles. Consequently, the second research question was "To what extent can different criteria for MLD explain mathematical or cognitive profiles in children?" In line with extant research, we expected that a more stringent selection of children with MLD (i.e., more severe, persistent, and specific and with a core deficit and IQ discrepancy) will result in weaker math performance. However, based on previous literature, it is difficult to predict what cognitive profile will emerge for each criterion, but it can be expected that the type of criterion used will have consequences for the performance on the cognitive skills included in the current study. This study is the first to combine longitudinal data on mathematics learning with different selection criteria for the difficulties that can emerge when learning mathematics. Learning a skill such as mathematics is something that takes place over a prolonged period of time, which makes insights obtained from longitudinal data crucial. In addition, using multiple measurements increases the reliability of the data.

Method

Participants

The longitudinal study was conducted in the following periods: January to March 2018 (Time 1 [T1]; intermediate fourth grade), September to October 2018 (Time 2 [T2]; start fifth grade), and May to June 2019 (Time 3 [T3]; end fifth grade). Two articles with concurrent data from the first measurement point have previously been published (Huijsmans, Kleemans, & Kroesbergen, 2021; Huijsmans, Kleemans, Van der Ven, & Kroesbergen, 2020). The current study elaborates specifically on growth-related questions and includes the full range of tasks completed at each time point. The final sample included 276 children from 10 Dutch primary schools. Classrooms did not differ significantly on mathematics at T1 (F = 1.65, p = .09). A total of 60 children were identified as having MLD because they scored at or below the 25th percentile on Dutch national standardized tests for mathematics (*Cito Rekenen-Wiskunde;* Janssen, Verhelst, Engelen, & Scheltens, 2010) at T3. Sample characteristics are displayed in Table 1. All children were fluent in Dutch. Both TD children and children with MLD with any intellectual, physical, or behavioral disabilities were excluded from the data.

Procedure

The test battery was administered three times at the children's schools. Ethical approval was granted by the institutional review board within the boundaries of which this research was conducted. Schools were recruited using phone and e-mail, and active parental informed consent was obtained prior to data collection. Classroom paper-and-pencil measures (mathematics, arithmetic, phonological awareness, and nonverbal reasoning), self-reliant computerized measures (number sense and working memory), and individual read-aloud measures (rapid naming and word decoding) were administered over approximately 3 h per child (spread across several school days) by trained experimenters using a standardized protocol. Tasks were performed in the same order in each session, but the order of the sessions was counterbalanced across classrooms.

Instruments

Mathematics

A combined test with open-ended computational problems was used to assess mathematics; items from the *Schoolvaardigheidstoets Rekenen-Wiskunde* [School Achievement Test for Mathematics] (De Vos & Milikowski, 2012), an older version of the Dutch national test for mathematics (Janssen, Scheltens, & Kraemer, 2005), and the Fraction Competency Test (Brown & Quinn, 2007) were selected to obtain a comprehensive measurement of mathematics in the domains of numbers (19 items), geometry (24 items), and proportions (18 items). Each correct answer yielded 1 point. Internal consistency in the current study was good (α = .89).

Arithmetic

The *Tempo Test Automatiseren* [Rapid Arithmetic Test] (De Vos, 2010) was used to assess arithmetic by means of four subtests. Children started with the addition subtest, followed by subtraction, multi-

Table 1
Sample characteristics for TD children and children with MLD.

	TD	MLD
n	216	60
Mean age (years)	9.67	9.83
Gender (% boys)	51%	43%
SES[a] (% [applied] university)	36%	20%

Note. TD, typically developing; MLD, mathematical learning difficulties; SES, socioeconomic status.
[a] Parental educational level.

plication, and division subtests. Each subtest included 50 arithmetic operations of increasing complexity where as many items as possible needed to be solved within a 2-min time limit. Each correct answer yielded 1 point. Internal consistency for all subtests was at least sufficient (α > .78; Evers et al., 2009–2012).

Symbolic number sense

Number sense was assessed with the computerized Dutch Assessment Battery for Number Sense (Friso-Van den Bos, Schoevers, Slot, & Kroesbergen, 2015). Two subtests were used: symbolic comparison and number line estimation. Stimuli ranging from 1 to 100 were presented at random using E-Prime software (Version 2.0). The symbolic comparison task required participants to rapidly indicate by keypress which of two numbers was larger. Average reaction time in milliseconds for the correct trials was used for further analysis. In the number line estimation task, participants needed to position a target digit onto the number line using a lever. Mean absolute errors (MAEs) were computed by averaging the absolute difference between target numbers and their estimated positions. After training blocks, testing blocks with 33 and 30 items were administered. The scores on both tasks were combined for further analyses; recoded reaction times on the symbolic comparison task and MAEs on the number line estimation task were standardized and averaged into one symbolic number sense score. Internal consistency of both tasks was good (α > .79; Kline, 1999).

Working memory

Visuospatial and verbal working memory were assessed using the online computerized tasks Lion game and Monkey game, respectively (Van de Weijer-Bergsma, Kroesbergen, Jolani, & Van Luit, 2016; Van de Weijer-Bergsma, Kroesbergen, Prast, & Van Luit, 2015). In the Lion game, pictures of colored lions (red, blue, yellow, green, or purple) were presented subsequently within a 4 $\times$ 4 matrix, and children were asked to indicate the final locations of lions of a specific color. In the monkey game, children listened to familiar spoken words and needed to recall those words backward in the correct order by mouse clicks on written words in a 3 $\times$ 3 matrix. The average proportion of correctly recalled items was computed from the 20 items per task. Internal consistency for both tasks was good (α > .87; Van de Weijer-Bergsma et al., 2015, 2016).

Phonological awareness

Deletion and segmentation were assessed using two subtests from a phonological awareness task (Knoop-van Campen, Segers, & Verhoeven, 2018). In the 18-item deletion task, a letter (e.g., "s") needed to be deleted from a spoken word (e.g., "small") by crossing the corresponding picture (e.g., "mall" with distracters "ball" and "wall") within 4 s. In the 12-item spoonerism task, the first letter of two verbally presented words needed to be switched to form new words (e.g., "mouse" and "heat" became "house" and "meat") by crossing two of five corresponding pictures within 5 s. Each correct answer yielded 1 point. Internal consistency was sufficient (α = .70; Knoop-van Campen et al., 2018).

Rapid naming

Alphanumeric and non-alphanumeric rapid naming were assessed using the continuously naming part of the *Continu Benoemen en Woorden Lezen* [Continuously Naming and Word Reading] test (van den Bos & Lutje Spelberg, 2007). The task comprises four cards with five highly frequent items: colors (black, yellow, red, green, and blue), digits (2, 4, 5, 8, and 9), pictures (tree, chair, duck, scissors, and bike), and letters (d, o, a, s, and p). Each item was visually presented 10 times in random order, resulting in 50 items per card. Children were instructed to accurately name these items as quickly as possible. Averaged overall naming time in seconds was used as their raw score. Split-half reliability and test–retest reliability were sufficient (α > .75; Evers et al., 2009–2012).

Nonverbal reasoning

Raven's Standard Progressive Matrices were used to assess nonverbal reasoning (Raven, 1976). This paper-and-pencil task consists of five sets with 12 visual patterns in each set with increasing difficulty. Children had sufficient time to complete the task. In the first set, one part of each item was missing. Children needed to select the missing part to logically complete the design out of six alternatives. In the other sets, four to nine patterns were presented, from which the final figure was missing. Children selected the missing figure from between six and eight alternatives. Each correct answer yielded 1 point. Internal consistency was good (α > .90; Raven, Styles, & Raven, 1998).

Word decoding

Word decoding was measured with the *Eén Minuut Test* [One Minute Test; Brus & Voeten, 1999). Within 1 min, children needed to accurately read as many unrelated words as they could. Difficulty level was altered by increasing word length from one to four syllables. The number of correctly read words (max = 116) was taken as the raw score. Internal consistency was good (α = .90; Evers et al., 2009–2012).

Data analysis

Preliminary analysis. Measures included in the current study were the cognitive predictors at T1, arithmetic as a mediator at T1, and a latent growth factor (i.e., intercept and slope) for mathematics from T1 to T3. Outliers (>|3.29|) were winsorized; no strong violations of the normality assumption were identified (standardized |skewness| and |kurtosis| < 3.0). The observed variables of all latent constructs were significantly correlated ($p < .05$). This was the case for arithmetic ($r > .59$ for addition, subtraction, multiplication, and division), number sense ($r = .26$ for symbolic comparison and number line estimation), working memory ($r = .16$ for verbal and visuospatial working memory), phonological awareness ($r = .35$ for deletion and spoonerism), and rapid naming ($r = .64$ for alphanumeric and non-alphanumeric rapid naming). The cognitive predictors were not related to each other; thus, covariances were set to zero in further analyses.

Statistical analysis. To examine whether similar cognitive mechanisms are involved in children with MLD and TD children during math development, a repeated-measures two-way analysis of covariance (ANCOVA) was conducted with measurement point (T1, T2, or T3) as a factor, mathematics performance on numbers, geometry, and proportions as within-participants variables, group (MLD or TD) as a between-participants factor, and covariates arithmetic, number sense, working memory, phonological awareness, and rapid naming at T1. In addition, independent-samples *t* tests were used to compare different sample selection criteria (Table 2) for all mathematical and cognitive variables at T1 to T3.

Results

Cognitive mechanisms and mathematical development

Numbers

The interaction effect between time and group was not statistically significant for numbers. However, there was a trend for a main effect for time, $F(2, 274) = 2.63$, $p = .07$, $\eta_p^2 = .02$, and a significant main effect for group, $F(2, 284) = 39.59$, $p < .001$, $\eta_p^2 = .21$. Performance at T1 was significantly weaker than at T2 and T3, and performance at T2 was significantly weaker than at T3 (*ps* < .001). In addition, the TD group significantly outperformed the MLD group ($p < .001$) (see Fig. 1). Children with MLD

Table 2
Overview of selection criteria for MLD used in this study.

Criterion	Group 1	Group 2
Severity	≤ 10% on math	≤ 25% on math
Persistency	≤ 25% at T1–T3	≤ 25% only at T3
Specificity	≤ −1 SD on word decoding at T1	≥ mean on word decoding at T1
Core deficit	≤ −1 SD on number sense at T1	≥ mean on number sense at T1
IQ discrepancy	≥ 1 SD difference at T1	≤ 0.5 SD difference at T1

Note. MLD, mathematical learning difficulties; T1, Time 1; T3, Time 3.

scored significantly weaker than TD children on arithmetic and nonverbal reasoning (*ps* < .001, η_p^2 > .11). The other covariates of number sense, working memory, phonological awareness, and rapid naming did not differ across groups (*ps* > .27, η_p^2 < .01).

Geometry

A statistically significant interaction effect was observed between time and group for geometry, $F(2, 274) = 10.78$, $p < .001$, $\eta_p^2 = .07$. TD children improved more over time on math performance compared with children with MLD (see Fig. 2). Children with MLD scored significantly weaker than TD children on arithmetic and nonverbal reasoning (*ps* < .001, η_p^2 > .08). The other covariates of number sense, working memory, phonological awareness, and rapid naming did not differ across groups (*ps* > .16, η_p^2 < .01).

Proportions

There was a statistically significant interaction effect between time and group for geometry, $F(2, 274) = 15.10$, $p < .001$, $\eta_p^2 = .09$. TD children increased more over time on math performance compared with children with MLD (see Fig. 3). The performance of children with MLD was significantly weaker than that of TD children on arithmetic and nonverbal reasoning (*ps* < .02, η_p^2 > .03), and there was a trend for number sense ($p = .07$, $\eta_p^2 = .02$). The other covariates of working memory, phonological awareness, and rapid naming did not differ across groups (*ps* > .13, η_p^2 < .01).

To summarize, the performance of children with MLD was weaker on each domain of mathematics compared with their TD peers, and children in mid-fourth grade demonstrated a weaker performance on those domains than children in late fifth grade. TD children also improved more in the domains geometry and proportions (especially in fifth grade, e.g., from T2 to T3) compared with children with MLD. Regarding the predictors of math development, it became evident that differences in growth on mathematics could be explained only by differences on arithmetic and nonverbal reasoning at T1

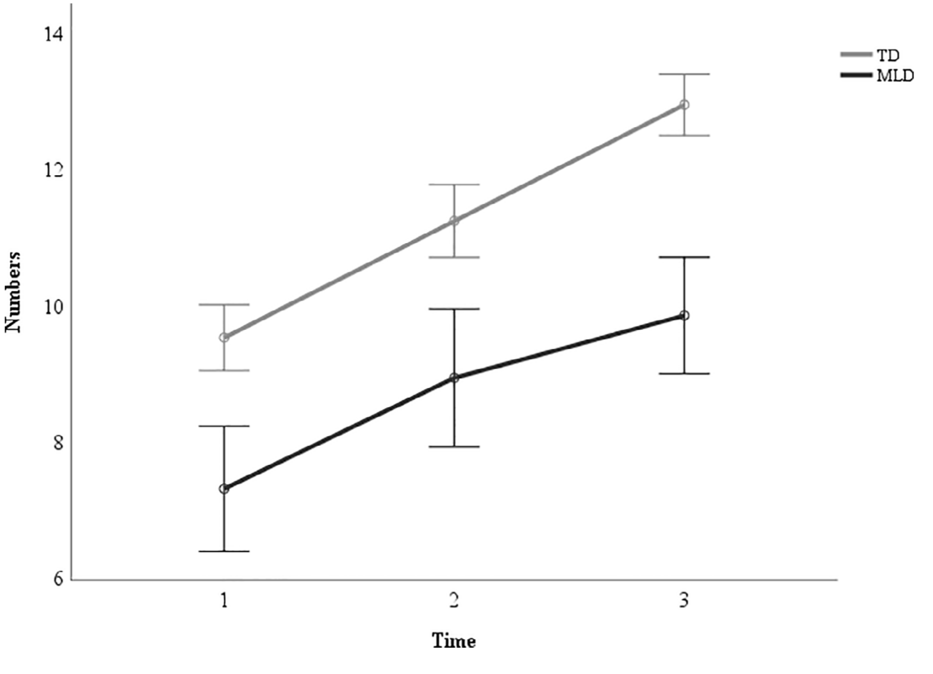

Fig. 1. Mean scores for children with mathematical learning difficulties (MLD; depicted in black) and typically developing (TD) children (depicted in gray) for the mathematics domain numbers at Time 1 to Time 3. Error bars represent ± 2 standard errors.

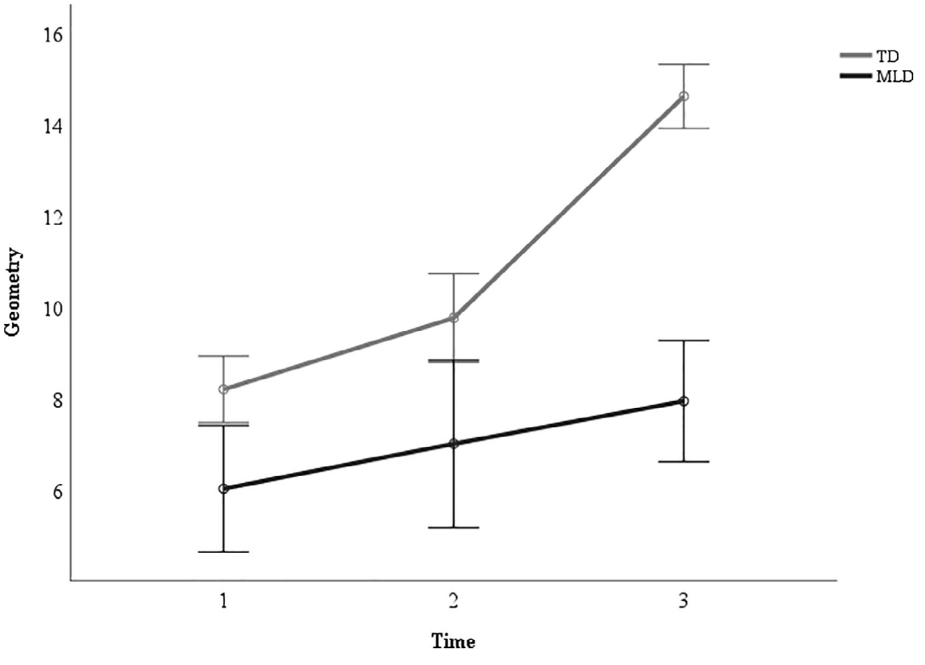

Fig. 2. Mean scores for children with mathematical learning difficulties (MLD; depicted in black) and typically developing (TD) children (depicted in gray) for the mathematics domain of geometry at Time 1 to Time 3. Error bars represent ± 2 standard errors.

between children with MLD and TD children. Other cognitive mechanisms did not surface to explain variations in growth.

Selection criteria for MLD

To answer the second research question, "To what extent can different criteria for MLD explain mathematical or cognitive profiles in children?", several different selection criteria were applied to the MLD sample and mathematical and cognitive profiles were plotted for each criterion. Performance on each of the measures was compared using independent-samples *t* tests for severity (lowest 10% vs. lowest 25%), persistency (one measurement vs. three measurements), specificity (comorbidity with reading learning difficulty [LD] vs. specific LD), core number sense deficit (yes vs. no), and IQ discrepancy (yes vs. no).

More stringent criteria (i.e., a more severe LD or a persistent LD) resulted in the weakest performance in mathematics, especially for the numbers and geometry domains (Table 3). In contrast, a comorbid reading LD resulted in weaker arithmetic skills but did not affect mathematics performance. Whether children with MLD had a core deficit in number sense failed to affect mathematics performance. Finally, children with an IQ discrepancy performed more poorly on geometry than children whose nonverbal reasoning was in line with their math achievement.

Regarding the cognitive skills, differential profiles were not detected for most of the selection criteria of MLD (see Table 4). A (visuospatial) working memory deficit was revealed for children with a more severe or persistent LD, but none of the other cognitive skills could be used to distinguish between those selection criteria. In contrast, children with a comorbid math and reading LD performed weaker on verbal working memory, phonological awareness, and rapid naming than children without a comorbid LD. In addition, a number sense core deficit did not lead to differences in any of the other cognitive skills. Finally, results for the IQ discrepancy criterion were somewhat inconsistent;

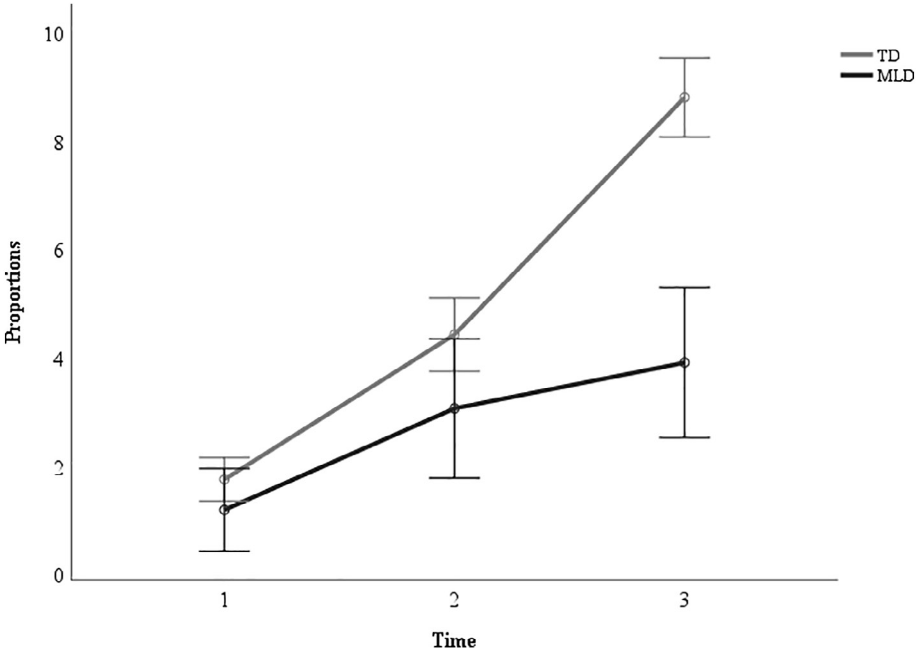

Fig. 3. Mean scores for children with mathematical learning difficulties (MLD; depicted in black) and typically developing (TD) children (depicted in gray) for the mathematics domain proportions at Time 1 to Time 3. Error bars represent ± 2 standard errors.

children with a discrepancy scored lower on alphanumeric rapid naming, whereas children with combined weak math and weak nonverbal reasoning had lower scores on working memory.

Discussion

The current study examined how cognitive functions are related to math performance in the domains of numbers, geometry, and proportions over time and how this differs between children with MLD and TD children. In addition, we investigated to what extent different criteria for MLD can explain mathematical or cognitive profiles in primary school children. A repeated-measures two-way ANCOVA was conducted to assess the effects of the cognitive skills (i.e., number sense, working memory, phonological awareness, rapid naming, and nonverbal reasoning) on the mathematics domains (i.e., numbers, geometry, and proportions) from mid-fourth grade to late fifth grade.

With respect to the first research question on the differences in cognitive mechanisms associated with growth in mathematics, an MLD group (n = 60) was compared with a TD group (n = 216). We ascertained that children with MLD achieved a weaker performance in mathematics in all grades than their TD peers, and although improvements over time on mathematics were observed for all children, those who started behind in mid-fourth grade tended to stay behind in late fifth grade. These findings are in line with previous research (Andersson, 2010; Geary, Hoard, & Bailey, 2012).

In addition, the results showed that children with MLD made less progress compared with their TD peers, especially in the geometry and proportions math domains. The shift from an emphasis on retrieval skills in the lower grades of primary school toward an emphasis on procedural and conceptual skills in the upper grades of primary school apparently poses challenges for children with MLD (Baroody, 2003; Cowan et al., 2011). Children with MLD mainly differed from TD children in cognitive predictor nonverbal reasoning (but note that the scores of all participants were within the normal range), and this could explain why children with MLD in the current study mainly showed a lack of growth in the procedural geometry and proportions domains. To elaborate, Engle (2018) proposed

Table 3
Standardized scores on the mathematics domains at T1 to T3 for each of the MLD selection criteria.

		Severity criterion		Persistency criterion		Specificity criterion		Core deficit		IQ discrepancy	
		≤10% math	≤25% math	MLD at T1–T3	MLD only at T3	Comorbid	Not comorbid	Deficit	No deficit	Discrepancy	No discrepancy
T1	Arithmetic	−1.26	−0.72	−0.91	−0.64	−1.28	−0.33***	−0.94	−0.54	−0.85	−0.79
	Numbers	−1.46	−0.92*	−1.43	−0.46***	−0.91	−0.79	−1.28	−0.92	−1.31	−0.85*
	Geometry	−1.10	−0.71*	−1.01	−0.46***	−0.81	−0.72	−0.98	−0.71	−1.02	−0.60*
	Proportions	−0.29	−0.38	−0.40	−0.33	−0.44	−0.31	−0.45	−0.31	−0.39	−0.37
T2	Arithmetic	−1.61	−0.70	−0.98	−0.52*	−1.15	−0.34***	−0.91	−0.59	−0.83	−0.69
	Numbers	−1.81	−0.88*	−1.34	−0.53***	−0.70	−1.01	−1.28	−1.11	−0.94	−1.00
	Geometry	−1.13	−0.72	−1.08	−0.39***	−0.70	−0.68	−0.87	−0.64	−0.92	−0.67
	Proportions	−0.91	−0.59	−0.66	−0.57	−0.54	−0.55	−0.60	−0.59	−0.54	−0.72
T3	Arithmetic	−1.24	−0.71	−0.88	−0.66	−1.36	−0.28***	−1.17	−0.49	−0.91	−0.64
	Numbers	−2.70	−0.85***	−1.22	−0.85	−1.20	−1.06	−1.40	−1.01	−0.83	−1.30
	Geometry	−1.94	−1.15***	−1.35	−1.12	−1.15	−1.38	−1.38	−1.14	−1.43	−1.10
	Proportions	−1.28	−1.04	−1.11	−1.03	−1.07	−0.99	−1.13	−1.06	−1.00	−1.01
n		6	50	30	30	14	24	12	22	16	16

Note. T1, Time 1; T2, Time 2; T3, Time 3; MLD, mathematical learning difficulties.
* $p < .05$.
*** $p < .001$.

Table 4
Standardized scores on the cognitive skills at T1 to T3 for each of the MLD selection criteria.

		Severity criterion		Persistency criterion		Specificity criterion		Core deficit		IQ discrepancy	
		≤10% math	≤25% math	MLD at T1–T3	MLD only at T3	Comorbid	Not comorbid	Deficit	No deficit	Discrepancy	No discrepancy
T1	Number sense	−0.07	−0.30	−0.47	−0.04*	−0.71	0.09**	N/A	N/A	−0.28	−0.02
	Verbal WM	−0.49	−0.40	−0.45	−0.35	−0.50	−0.35	−0.69	−0.25	−0.76	−0.56
	Visual WM	−0.18	−0.07	−0.32	0.18	−0.02	−0.05	−0.13	0.08	0.06	−0.24
	Phonology	0.11	−0.30	−0.35	−0.15	−0.25	−0.18	−0.02	−0.24	−0.44	−0.42
	Alpha RAN	0.19	0.07	0.12	0.05	−0.43	0.47**	0.27	0.26	−0.34	0.40**
	Non-alpha RAN	0.61	−0.04	−0.01	0.04	−0.66	0.26*	0.02	0.12	−0.40	0.05
T2	Number sense	−0.11	−0.42	−0.52	−0.26	−0.61	−0.35	N/A	N/A	−0.12	−0.52
	Verbal WM	0.15	−0.15	−0.13	−0.11	−0.16	0.15	0.27	0.07	−0.08	−0.26
	Visual WM	−1.90	−0.42***	−0.81	−0.29*	−0.30	−0.51	−1.14	−0.45	−0.16	−0.95
	Phonology	−0.11	−0.32	−0.28	−0.34	−0.88	0.22**	0.17	−0.54*	−0.24	−0.45
	Alpha RAN	0.10	0.00	0.09	−0.08	−0.24	0.22	0.17	0.07	−0.26	0.32
	Non-alpha RAN	0.01	−0.12	−0.05	−0.19	−0.56	0.05	−0.02	−0.08	−0.41	−0.05
T3	Number sense	−0.71	−0.33	−0.48	−0.26	−0.35	−0.19	N/A	N/A	−0.29	−0.38
	Verbal WM	−0.57	−0.59	−0.93	−0.25**	−1.03	−0.26**	−1.10	−0.75	−0.53	−1.07**
	Visual WM	−2.20	−0.56***	−1.11	−0.32**	−0.57	−0.79	−1.17	−0.62	−0.27	−1.24**
	Phonology	−0.63	−0.56	−0.68	−0.46	−0.86	−0.44	−0.65	−0.67	−0.71	−0.54
	Alpha RAN	0.00	−0.12	−0.09	−0.13	−0.61	0.22*	0.08	−0.01	−0.19	0.20
	Non-alpha RAN	0.46	−0.17	−0.11	−0.13	−0.80	0.22**	−0.10	−0.05	−0.10	−0.07
n		6	50	30	30	14	24	12	22	16	16

Note. T1, Time 1; T2, Time 2; T3, Time 3; MLD, mathematical learning difficulties; WM, working memory; Phonology, phonological awareness; RAN, rapid automatized naming.
* $p < .05$.
** $p < .01$.
*** $p < .001$.

that a strong IQ is involved in mental processes to select task-relevant information. Nonverbal reasoning can be seen as a proxy for IQ. Thus, weaker nonverbal reasoning skills hamper children's ability to logically and systematically engage in mathematical problem solving. This is especially true in the upper grades of primary school, when proficiency in procedural and conceptual skills is required in order to perform well in mathematics.

In contrast, cognitive retrieval skills such as number sense, phonological awareness, and rapid naming did not predict differing mathematics developments for children with MLD or TD children. The theoretical body of literature has illustrated that these skills are the most important cognitive predictors of mathematics, and this appears to be the case for all children (i.e., either with or without MLD). Instead, it seems as if their mental representations of numbers, geometry, and proportions change over time. Such a representational perspective on learning mathematics in children with MLD may further our understanding thereof.

Regarding the (cognitive) mechanisms that could explain variations in growth in mathematics, it was found that only arithmetic and nonverbal reasoning skills were involved in differences in math development between children with MLD and TD children. The former are known to have weaknesses not only in mathematical problem solving (e.g., in the numbers, geometry, and/or proportions domains) but also in arithmetic fact retrieval (Huijsmans et al., 2020). In addition, it has been demonstrated that children with MLD have lower intelligence (where nonverbal reasoning is a proxy) than TD children, although their levels should be within the normal range (Fletcher et al., 1998).

In contrast, the number sense, working memory, phonological awareness, and rapid naming cognitive skills did not explain any variation in math development. These results suggest that the cognitive profile of math development is largely the same for children with MLD and TD children. In other words, the results seem to indicate that the cognitive predictors of math development for all children (i.e., either with or without MLD) are too similar to recognize them as separate groups. Thus, children with different levels of math performance (e.g., ranging from weak to average to strong) seem to rely on similar mechanisms, such as having acquired arithmetic skills (Cirino, Tolar, Fuchs, & Huston-Warren, 2016; Vukovic et al., 2014) and the ability to use reasoning in situations where problem solving is required (Giofrè et al., 2014), to increase their math performance in all domains throughout the upper grades of primary school. Therefore, we carefully propose that children with weak math performance cannot be categorized as having a learning disability based on their cognitive profile. Although children with math difficulties on average perform weaker on those cognitive skills compared with average- to high-achieving children, individual variations seem to be too large to arrange these differences into separate categories.

Study selection criteria

In short, when taking both mathematical and cognitive skills into account, we suggest that the selection criteria commonly used do not meaningfully distinguish between mathematical and cognitive profiles. Research often discriminates between MLD and low achievement, but the empirical data of our study did not support such a distinction. At first glance, this finding might seem contradictory to previous research in which the cognitive skills of children with more severe MLD differ from those of less severe low-achieving children (e.g., Geary, 2011; Geary, Hoard, Nugent, & Bailey, 2012b). What such studies do not show, however, is that although children with math problems perform weaker on associated cognitive skills compared with children with stronger math skills, the patterns of how and which cognitive skills are related to mathematics do not differ. Therefore, we tentatively conclude that although more stringent criteria select children with weaker math performance, their cognitive profiles are comparable to those of children with stronger math skills. Hence, although research has previously used these criteria to select more homogeneous samples of children with MLD (see, e.g., Peters & Ansari, 2019, for a review), evidence for employing such an approach was not provided by the current data. It seems as if these children have global deficits across all domains of mathematics and in associated cognitive skills (see also Murphy et al., 2007). Consequently, it may be redundant to use these criteria for scientific purposes.

It should be noted, however, that different profiles can be identified for the specificity criterion (i.e., comorbidity with word decoding). Children with combined math and word reading difficulties performed weaker on arithmetic as well as on cognitive skills related to reading (i.e., verbal working memory, phonological awareness, and rapid naming; Slot et al., 2016) than children without a comorbid difficulty. Despite the similarities in performance within the mathematics domains between children with and without a word decoding deficit, the former might display an additional processing speed problem on top of their general math difficulties. Thus, delayed fact retrieval from long-term memory might explain their weak math performance (Geary, 2004; Koponen, Aunola, Ahonen, & Nurmi, 2007).

Limitations and suggestions for future research

Some limitations to the current study need to be acknowledged. First, we should recognize that effect sizes for growth in mathematics were only small to medium, suggesting that the cognitive skills included in this study are not the only factors involved in academic growth. Other aspects of learning mathematics have the potential to better explain why some children have more difficulties in learning mathematics than others. Therefore, future research could emphasize characteristics of the learning environment such as quality of instruction, access to (digital) resources, and time spent on mathematics per week. In addition, the current study included only the updating executive function, but inhibition and shifting have also been shown to be related to mathematics (Cragg, Keeble, Richardson, Roome, & Gilmore, 2017). Finally, other mechanisms related to learning, such as motivation and learning strategies, are likely to play a role as well (Murayama, Pekrun, Lichtenfeld, & Vom Hofe, 2013).

Another vital consideration is that we did not specifically sample children who have previously been diagnosed with developmental dyscalculia. Perhaps group differences would have become more evident if the MLD group comprised children with an official diagnosis. However, because our data showed that the more restrictive sample selection criteria—which are more in line with the clinical diagnosis of developmental dyscalculia—did not devolve into different mathematical or cognitive profiles compared with less restrictive criteria, we do not expect that this would have made a difference. In addition, the advantage of our approach is that it was easier to obtain larger samples, allowing for greater power to detect medium to large effects.

With respect to future research on mathematics and mathematical difficulties, the results of the current study call for an alternative perspective on how the process of learning mathematics—and potential difficulties along the way—should be perceived. It has become clear that children with MLD cannot be separated from their TD peers in terms of their cognitive profiles. Instead, proficiency in mathematics can be described as a continuum in which some children experience more difficulties than others due to their individual profiles of (cognitive) strengths and weaknesses. Such a perspective asks for a shift in methodologies or analysis techniques in future research; when investigating mathematics learning, one preferably employs a dynamic approach rather than making a comparison between groups. Therefore, we opt for an individual differences perspective on learning and learning difficulties (see, e.g., Dowker, 2005). In such an approach, math achievement is assessed on a continuum to overcome the problems associated with discrete categories of learning difficulties, such as variability in severity cutoffs and exclusion of comorbid disabilities (Peters & Ansari, 2019) but also the hazard of neglecting children who struggle with mathematics but just come short of the diagnostic criteria, and individual strengths and weaknesses on related cognitive skills can be taken into account. Using a continuous approach to assess development in that respect has a major advantage over stage theories, such as those of Piaget and Erikson, because this better accounts for the individual differences among children.

In addition, the current study has determined that most of the commonly used selection criteria for MLD do not result in distinct subgroups either. Subsequently, the research field should step away from the idea that all children with difficulties in learning mathematics can (or should) be separated into different categories (where at least one group would explicitly differ from TD children) because these children are simply too different from each other in order to be able to create meaningful groups or categories. Thus, a suggestion for future research could also be to specifically report on sample characteristics in a transparent manner.

Finally, future research should aim to replicate the findings of the current study to confirm our findings, preferably also in other age groups.

Conclusions

The cognitive mechanisms for math development in fourth and fifth grades were quite similar for children with math difficulties and TD children. Despite weaker performances in such cognitive skills, children with math problems do not seem to have a specific profile of cognitive deficits that distinguishes them from TD children. In addition, different ways of selecting children with MLD did not result in different cognitive profiles either. Taking these findings together strengthens the idea that MLD cannot be seen as a discrete disorder but rather that mathematics should be viewed on a continuous scale (see also, e.g., Hudziak, Achenbach, Althoff, & Pine, 2007; Moll, Kunze, Neuhoff, Bruder, & Schulte-Körne, 2014). Some children with weak math performance may advance less throughout primary school compared with other children with math difficulties based on individual variation in their underlying cognitive mechanisms, but these differences cannot be categorized into a separate disability. Thus, variations in mathematics performance cannot be differentiated by means of cognitive profiles for any of the selection criteria used in the current study.

As a result, we recommend applying an individual differences perspective in which a child's unique profile of cognitive strengths and weaknesses is taken into account when studying mathematics and the processes related to developing MLD. More specifically, per the finding that individual differences in nonverbal reasoning (as a proxy for IQ) mainly accounted for the variance in mathematical proficiency, we also recommend differentiating between children with lower IQ and those with higher IQ (as well as for their entire profile of skills) because they likely benefit from different instruction and tools (e.g., support in choosing and using alternative problem-solving strategies). This could be done by using adaptive educational tools (see, e.g., Molenaar & Knoop-van Campen, 2016) or by employing teaching assistants within the classroom (see, e.g., Sharples, Blatchford, & Webster, 2016).

In addition, an individual differences perspective can also be applied to intervention. Early identification and remediation that are adjusted to a child's unique profile of cognitive strengths and weaknesses can help to more quickly detect and adapt to children with (a risk of) MLD who slowly but surely fall behind their peers. Math difficulties are not general but could manifest themselves in either retrieval skills, procedural skills, or both depending on the child's strengths and weaknesses in related cognitive skills. Affectional aspects such as math motivation and anxiety could be accounted for as well given that individual differences likely exist in those skills as well. To summarize, when looking more closely at which mathematical skills are lacking—independent of labels or diagnoses—learning difficulties could be detected earlier, enabling timely intervention and hopefully remediation and prevention of more serious problems.

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