Reply to “Comment on ‘Inverse Square Lévy Walks are not Optimal Search Strategies for $d \geq 2$’” The central result of our Letter [1] is that (i) the capture rate $\eta$ of Lévy walks with Poisson distributed targets goes linearly with the target density $\rho$ for all values of the Lévy exponent $\alpha$ in space dimension $d \geq 2$. This contradicts results in [2] and has important consequences; (ii) the optimal gain $\eta_{\text{max}}/\eta$ achieved by varying $\alpha$ is bounded in the limit $\rho \to 0$ so that tuning $\alpha$ yields a marginal gain; (iii) the optimum is realized for a range of $\alpha$ and is controlled by the model-dependent parameters $\alpha$ (detection radius), $l_c$ (restarting distance), and $s$ (scale parameter) (Fig. 1).

First, and most importantly, [3] states that our main result (i) is correct, thereby acknowledging that the determination of $\eta$ in [2] is wrong.

Second, [3] proposes that claim (iii) is not new because earlier publications reported that optimal Lévy strategies can be realized for $\alpha \neq 1$. We did acknowledge such observations in [1], where we in fact show that they result from the linear scaling of $\eta$ with $\rho$ for $d \geq 2$; this is novel.

Last, [3] disputes claim (ii). Technically, claim (ii) is correct and by no means compromised by [3]. It states that for fixed values of $s, l_c$, the optimal gain $\eta_{\text{max}}/\eta$ is bounded when $\rho \to 0$. This comes from the linear scaling of $\eta$ with $\rho$ (Eq. (5) in [1], whose validity is acknowledged by [3]) and is independent of any determination of $K_0(\alpha, s, l_c)$. In [1], Eq. (3) is used only to derive the scaling of $\eta$ with $\rho$; we make no prediction regarding $K_0(\alpha, s, l_c)$. Attempting to deduce $K_0(\alpha, s, l_c)$ from Eq. (3) is the initiative of [3], not ours. In fact, we agree that Eq. (3) is unsuitable to study $l_c \to a$, which falls out of the validity regime given in [4]. This is certainly not a problem in [1], as argued by [3], simply because we nowhere aimed at determining $K_0(\alpha, s, l_c)$.

Finally, the only aspect in (ii) that [3] disputes is rhetorical: our qualification of the optimum as marginal. The comment is based only on the analysis of the singular limit $s \to 0$ and $l_c \to a$, which can indeed lead to arbitrarily large values of $\eta_{\text{max}}/\eta$ for $\alpha \to 1$. This is actually a mere 1$d$ limit (Fig. 1), as noted in [1]; it is thus expected, and consistent with our findings, to recover the 1$d$ optimum. This by no means contradicts claim (ii) of boundedness when $\rho \to 0$ for fixed $s, l_c$. Last, we summarize the conditions of optimality (CO) of inverse square Lévy walks for $d \geq 2$:

—Upon each capture event, a spherical target reappears infinitely fast at the same position.
—The searcher starts the new search infinitely close to the target boundary ($l_c - a \ll a$).

—The typical scale of its displacements is infinitely smaller than the target ($s \ll a$). If any of these conditions is not met, $\alpha = 1$ is not optimal. Given that $s$ and $l_c$ are system-dependent parameters with arbitrary values, the CO are generically not met, and our conclusion that inverse square Lévy walks are not optimal is justified. Additionally, if $l_c, s$ are allowed to vary, as done in [3], the obvious optimal strategy is $l_c = a$, leading to immediate recapture of the same target. The limit $l_c \to a^+$ in the CO is thus artificial.

To our knowledge, the CO have never been stated explicitly nor verified in any experimental system. Given that the CO are a mere 1$d$ limit of the problem, the claim that [3] restores the optimality of $\alpha = 1$ for $d \geq 2$ is unfounded, and given that [3] acknowledges that the scaling of $\eta$ with $\rho$ is wrong in [2], stating that [3] restores the validity of [2] is also unfounded.

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