Possible Lightest Ξ Hypernucleus with Modern ΞN Interactions

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Experimental evidence exists that the Ξ-nucleus interaction is attractive. We search for $NN\Xi$ and $NNN\Xi$ bound systems on the basis of the AV8 NN potential combined with either a phenomenological Nijmegen ΞN potential or a first principles HAL QCD ΞN potential. The binding energies of the three-body and four-body systems (below the $d+\Xi$ and ${}^3\mathrm{H}/{}^3\mathrm{He}+\Xi$ thresholds, respectively) are calculated by a high precision variational approach, the Gaussian expansion method. Although the two ΞN potentials have significantly different isospin (T) and spin (S) dependence, the $NNN\Xi$ system with quantum numbers ($T=0$, $J^\pi=1^+$) appears to be bound (one deep for Nijmegen and one shallow for HAL QCD) below the ${}^3\mathrm{H}/{}^3\mathrm{He}+\Xi$ threshold. Experimental implications for such a state are discussed.

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One of the major goals of hypernuclear physics is to understand the properties of hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions; they are related not only to possible dibaryon states such as H [1] but also to the role of hyperonic matter in neutron stars. Unlike the case of the NN interactions, hyperon interactions are not well determined experimentally due to insufficient number of scattering data. Nevertheless, high-resolution γ-ray experiments [2–5] analyzed by the shell model [6] as well as the accurate few-body method [7] have provided valuable constraints on the YN interaction in the strangeness $=-1$ sector such as the ΛN force. Also, the $\Lambda\Lambda$ interaction in the strangeness $=-2$ sector receives some constraints from the binding energies of hypernuclei such as ${}^{6}_{\Lambda\Lambda}\mathrm{He}$ [8], ${}^{10}_{\Lambda\Lambda}\mathrm{Be}$ [9], and ${}^{13}_{\Lambda\Lambda}\mathrm{B}$ [10]. In addition, the femtoscopic analyses of the two-particle correlations in high-energy pp, pA, and AA collisions at RHIC [11] and LHC [12,13] have started to give information on the low-energy $\Lambda\Lambda$ scattering parameters.

Recently, the KEK-E373 experiment showed the first evidence of a bound Ξ^- hypernucleus, ${}^{15}_{\Xi}\mathrm{C}$ (${}^{14}\mathrm{N}+\Xi$), the “KISO” event [14], which provides useful information on the attractive ΞN interaction in the strangeness $=-2$ sector. It was suggested experimentally two possible Ξ binding energies $B_\Xi \equiv E({}^{15}_{\Xi}\mathrm{C}) - E({}^{14}\mathrm{N})$: 4.38 ± 0.25 MeV and 1.11 ± 0.25 MeV. The latest femtoscopic data from pA collisions at LHC [15] also indicate that the spin-isospin averaged ΞN interaction is attractive at low energies.

Motivated by the above observations on the ΞN interaction, we address the following questions in this Letter: (i) What would be the lightest bound Ξ hypernucleus? and (ii) Which ΞN spin-isospin channel is responsible for such a bound system? In particular, we consider three-body $NN\Xi$ and four-body $NNN\Xi$ systems simultaneously using a high-precision Gaussian expansion method (GEM) [16,17], which is one of the most powerful first principle methods to solve three- and four-body problems. We employ two modern ΞN interactions, a phenomenological potential based on the meson exchanges, the Nijmegen ΞN potential (ESC08c) [18], and a potential based on first principle lattice QCD simulations, the HAL QCD ΞN potential (HAL QCD) [19]. As explained below, these two potentials have significantly different spin-isospin dependence. For the NN potential, we use the AV8 potential [20] throughout this Letter.

In the following, we employ the spectroscopic notation $^{2T+1,2S+1}S_J$ to classify the S-wave ΞN interaction where T, S, and J stand for total isospin, total spin, and total angular momentum. Thus we have four channels to be considered, $^{11}S_0$, $^{13}S_1$, $^{31}S_0$, and $^{33}S_1$. As shown below, the largest attraction is in $^{33}S_1$ and $^{11}S_0$ for ESC08c and HAL QCD, respectively.

Before entering the detailed discussions on the three- and four-body systems, let us first summarize key features of our ΞN potentials. The ESC baryon-baryon potential is designed to describe NN, YN, and YY interactions in a unified way [21]. In its recent version of ESC08c [18], a $\pi\omega$-pair exchange potential $V_{\pi\omega}$ is introduced so as to provide extra attraction in the $T = 1$ ΞN channel and to be consistent with the attractive nature of Ξ-nucleus potential indicated by the (K^-, K^+) experiments [22] and the KISO event [23]. Because of the strong (ΞN-$\Lambda\Sigma$-$\Sigma\Sigma$) central + tensor couplings in the $^{33}S_1$ channel, a ΞN (deuteronlike) bound state, D^*, is generated in ESC08c. (The ΞN-ΞN sector composed of central and tensor terms is also attractive but is not sufficient to form a two-body bound state.) The $^{13}S_1$ channel is weakly attractive, and the $^{11}S_0$ and $^{31}S_0$ channels are, on the other hand, repulsive in ESC08c. In this Letter, we represent the ESC08c by a ΞN-ΞN single-channel potential with central and tensor components: In the $^{33}S_1$ channel, the ΞN-$\Lambda\Sigma$-$\Sigma\Sigma$ coupling effects are renormalized into a ΞN-ΞN central potential by adding a single-range Gaussian form $V_2 \cdot \exp(-(r/\beta)^2)$ with $V_2 = -233$ MeV and $\beta = 1.0$ fm.

The HAL QCD potential is obtained from first principles $(2+1)$-flavor lattice QCD simulations in a large spacetime volume, $L^4 = (8.1\ \text{fm})^4$, with nearly physical quark masses, $(m_\pi, m_K) = (146, 525)$ MeV, at a lattice spacing, $a = 0.0846$ fm. Such simulations together with the HAL QCD method [24,25] enable one to extract the YN and YY interactions with multiple strangeness, e.g., $\Lambda\Lambda$, ΞN [19], ΩN [26], and $\Omega\Omega$ [27].

We calculate the ΞN effective central interactions at the imaginary-time distances $t/a = 11$, 12, 13, in which coupled-channel effect from higher channels as $\Lambda\Sigma$, $\Sigma\Sigma$ are effectively included, whereas the effect from the lower channel ($\Lambda\Lambda$ in the $^{11}S_0$ channel) is explicitly handled by the coupled-channel formalism [28,29].

To make the few-body calculation feasible, we fit the lattice QCD result of the potentials with multiple Gaussian forms at short distances and the Yukawa form with form factors at medium to long distances [19]. As for the pion and Kaon masses which dictate the long range part of the potential, we use $(m_\pi, m_K) = (146, 525)$ MeV to fit the lattice data, and take $(m_\pi, m_K) = (138, 496)$ MeV for calculating the Ξ-nucleus systems. In the $^{11}S_0$ channel, the analysis of the $\Lambda\Lambda$ and $N\Xi$ scattering phase shifts shows that a ΞN interaction is moderately attractive. Also, deeply bound H-dibaryon is not found below the $\Lambda\Lambda$ threshold. Moreover, the channel coupling between $\Lambda\Lambda$ and ΞN is found to be weak [19]. On the basis of this evidence, we introduce an effective single-channel ΞN potential in which the coupling to $\Lambda\Lambda$ in $^{11}S_0$ is renormalized into a single-range Gaussian form $U_2 \cdot \exp(-(r/\gamma)^2)$ with $\gamma = 1.0$ fm with $U_2 (< 0)$ chosen to reproduce the ΞN phase shifts obtained with channel coupling. On the other hand, the ΞN interactions in other channels are found to be much weaker: The $^{13}S_1$ and $^{33}S_1$ channels are weakly attractive and the $^{31}S_0$ channel is weakly repulsive.

In Fig. 1, we show the ΞN phase shifts calculated with (a) the ESC08c potential and (b) the HAL QCD potential at $t/a = 12$ for comparison. The statistical and systematic errors are not shown in Fig. 1(b), but are taken into account in the few body calculations below. From the figure, one immediately finds a qualitative difference between (a) and (b): The $^{33}S_1$ channel is attractive in ESC08c to form a bound state with the binding energy of 1.59 MeV, while it has only weak attraction in HAL QCD. On the other hand, the $^{11}S_0$ channel is repulsive in ESC08c, while it is moderately attractive in HAL QCD. It is therefore interesting to see how such differences are reflected in the energy levels of the few-body Ξ hypernuclei.

In this Letter, we consider the $NN\Xi$ and $NNN\Xi$ systems simultaneously by using the GEM [16,17]. (We note that the $NN\Xi$ system was recently studied by the Faddeev method [30] with an effective ΞN potential inspired by ESC08c. Two bound states are found $B_\Xi = 13.5$ MeV with $(T, J^\pi) = (1/2, 3/2^+)$ and $B_\Xi = 0.012$ MeV with $(T, J^\pi) = (1/2, 1/2^+)$ with respect to the $d + \Xi$ threshold. In addition, one bound state is found to be 1.33 MeV with $(T, J^\pi) = (3/2, 1/2^+)$ with respect to the $D^* + N$ threshold.) The GEM is a variational method with Gaussian bases, which achieves similar accuracy for bound state problems to other methods such as the Faddeev method and the Green function Monte Carlo method [31]. The GEM has been applied successfully up to five-body problems. For ordinary nuclei without strangeness, we will not

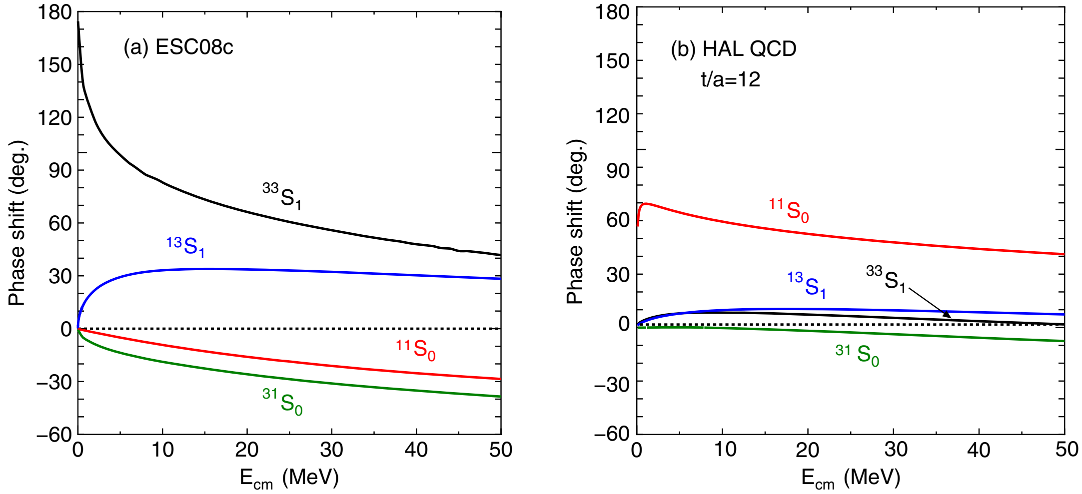

FIG. 1. ΞN phase shifts in the ${}^{33}S_1$, ${}^{13}S_1$, ${}^{11}S_0$, and ${}^{31}S_0$ channels using (a) the ESC08c potential and (b) the HAL QCD potential $(t/a = 12)$.

consider the isospin breaking from strong interaction nor the Coulomb interaction, so that T is a good quantum number. For the $N\Xi$ interaction, however, we take into account both strong interaction and the Coulomb interaction, since the latter effect may not be negligible for weakly bound Ξ nuclei. Accordingly, possible isospin breaking such as the mixing between $T = 0$ and $T = 1$ for $NNN\Xi$ may occur.

In the GEM, three and four Jacobi coordinates are introduced to describe $NN\Xi$ and $NNN\Xi$, respectively. Shown in Fig. 2 are the four rearrangement channels in $NNN\Xi$. The four-body wave function is given as a sum of $c = 1$–4 in Fig. 2 with the LS coupling scheme:

$$\begin{aligned}\Psi_{JM} = &\sum_{c=1}^{4}\sum_{\alpha I}\sum_{ss'Stt'T} C^{(c)}_{\alpha I s s' S t t' T}\\ &\times \mathcal{A}\{[\phi^{(c)}_{\alpha I}(r_c, R_c, \rho_c)\{[\chi_s(12)\chi_{\frac{1}{2}}(3)]_{s'}\chi_{\frac{1}{2}}(\Xi)\}_S]_{JM}\\ &\times \cdot[\{\eta_t(12)\eta_{\frac{1}{2}}(3)\}_{t'}\eta_{\frac{1}{2}}(\Xi)]_{T,T_z}\}. \qquad (1)\end{aligned}$$

Here $\mathcal{A}$ denotes the antisymmetrization operator for the nucleons. Spin and isospin functions are denoted by χ's and η's, respectively. Total isospin T can in principle take the values 0,1,2. However, $T = 2$ corresponds to the $3N$ state of $t' = 3/2$ in the continuum, so that its contribution is negligible. The spatial wave functions have the form $\phi_{\alpha IM'}(\boldsymbol{r}, \boldsymbol{R}, \boldsymbol{\rho}) = \{[\phi_{n\ell}(\boldsymbol{r})\psi_{NL}(\boldsymbol{R})]_K \xi_{\nu\lambda}(\boldsymbol{\rho})\}_{IM'}$ with a set of quantum numbers, $\alpha = (n, \ell; N, L; K; \nu, \lambda)$, and the radial components of $\phi_{n\ell m}(\boldsymbol{r})$ are taken as $r^{\ell} e^{-(r/r_n)^2}$, where the range parameters r_n are chosen to satisfy a geometrical progression. Similar choices for $\psi_{NL}(\boldsymbol{R})$ and $\xi_{\nu\lambda}(\boldsymbol{\rho})$ are taken. These four-body basis functions are known to be sufficient for describing both the short-range correlations and the long-range tail behavior of the few-body systems. The $3N$ binding energy with the present AV8 NN potential becomes 7.78 MeV which is less than the observed binding energy 8.48 MeV of ^{3}H. This discrepancy is attributed to the three-body force, so that a phenomenological attractive three-body potential defined by $W_3 \cdot \exp[-\sum_{i>j}(r_{ij}/\delta)^2]$ is introduced, where r_{ij} are the relative distances between the three nucleons N_i, with $W_3 = -45.4$ MeV and $\delta = 1.5$ fm.

In Table I, we summarize the binding energies of $NN\Xi$ and $NNN\Xi$ systems, where we omit atomic states which are (almost) purely bound by the Coulomb interaction. We note that the isospin mixing by the Coulomb interaction is found to be small, so that the states can be labeled by T in good approximation.

Let us now discuss the results with the ESC08c ΞN potential. The binding energy of the $NN\Xi$ system with $(T, J^{\pi}) = (1/2, 3/2^{+})$ with respect to the $d + \Xi$ threshold is 7.20 MeV, while the $NN\Xi$ with $(T, J^{\pi}) = (1/2, 1/2^{+})$ is unbound. Such channel dependence can be easily understood in the following manner: For $NN\Xi(1/2, 3/2^{+})$, nucleon and Ξ spins are all aligned. Since the nuclear

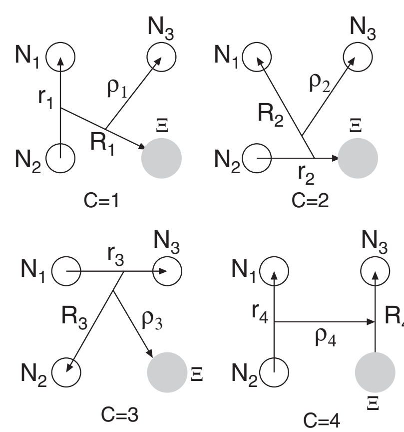

FIG. 2. Jacobi coordinates for the rearrangement channels of the $NNN\Xi$ system.

TABLE I. The calculated binding energies (in units of MeV) of $NN\Xi$ and $NNN\Xi$ with ESC08c potential and with HAL QCD potential with respect to the $d+\Xi$ and ^{3}H/^{3}He $+\ \Xi$ threshold, respectively.

	$NN\Xi$		$NNN\Xi$			
(T, J^π)	$(\frac{1}{2}, \frac{1}{2}^+)$	$(\frac{1}{2}, \frac{3}{2}^+)$	(0, 0$^+$)	(0, 1$^+$)	(1, 0$^+$)	(1, 1$^+$)
ESC08c	⋯	7.20	⋯	10.20	3.55	10.11
HAL QCD	⋯	⋯	⋯	0.36(16)(26)	⋯	⋯

force is most attractive in the spin-1 pair, and the ΞN force in ESC08c is also attractive for spin-1 pairs as shown in Fig. 1(a), this channel is most attractive to bring the bound state. On the other hand, in $NN\Xi(1/2, 1/2^+)$, one of the nucleon spins or Ξ spin is antiparallel to the others, so that one or two spin-0 ΞN pairs appear in the wave function. Since such a pair is repulsive in ESC08c as shown in Fig. 1(a), this channel becomes unbound. Note here that our results of $NN\Xi$ are qualitatively similar to but numerically different from those in Ref. [30] due to a different NN potential and different treatment of ESC08c. In the $T = 3/2$ $NN\Xi$ channel we do not find a bound state with respect to the $D^* + N$ threshold, while one bound state is found with $(3/2, 1/2^+)$ in Ref. [30].

For the $NNN\Xi$ system in ESC08c, the state in $(T, J^\pi) = (0, 0^+)$ is unbound with respect to the ^{3}H/^{3}He $+\ \Xi$ threshold, while the states in $(T, J^\pi) = (0, 1^+), (1, 0^+)$ and $(1, 1^+)$ are bound by 10.20,3.55, and 10.11 MeV, respectively, as shown in Table I. The effect of the ΞN Coulomb interaction to these binding energies is only 10%–20% of those numbers. The physical reason behind such channel dependence is more involved than the case of $NN\Xi$ due to various combinations of the pairs. Nevertheless, we find that the dominant ΞN pair in the $(T, J^\pi) = (0, 0^+)$ system is the repulsive ${}^{11}S_0$ channel in ESC08c, which leads to the unbinding of this system. On the other hand, the dominant ΞN pairs in $(T, J^\pi) = (1, 1^+)$ and $(0, 1^+)$ systems are ${}^{33}S_1$ and ${}^{13}S_1$ channels so that the binding energies of these $NNN\Xi$ systems are large.

Let us now turn to the $NN\Xi$ and $NNN\Xi$ systems with the HAL QCD ΞN potential. We found that none of the potentials ($t/a = 11$, 12, and 13) support bound states for $N\Xi$ and $NN\Xi$ systems. Only for the four-body $NNN\Xi$ system with $(T, J^\pi) = (0, 1^+)$, we have a possibility of a shallow bound state with the binding energies of $0.63(t/a = 11), 0.36(t/a = 12), 0.18(t/a = 13)$ MeV with respect to the ^{3}H/^{3}He $+\ \Xi$ threshold. In Table I, we quote the number 0.36 (16)(26) MeV where the first parenthesis shows the error originating from the statistical error of the ΞN potential at $t/a = 12$ and the second parenthesis shows the systematic error. The former is estimated by the jackknife sampling of the lattice QCD configurations and the latter is estimated from the data at $t/a = 11$ and 13.

The reason why the bound state is so shallow is that, unlike the case of ESC08c, the HAL QCD potential is moderately attractive in ${}^{11}S_0$, while it is either weakly attractive or repulsive in other channels as shown in Fig. 1(b). If we switch off the Coulomb interaction, the bound state at $t/a = 12$ (and 13) disappears. Therefore, this is a Coulomb-assisted bound state. However, the contribution from the strong ΞN interaction is still substantially larger than that of Coulomb ΞN interaction as seen from their expectation values, $\langle V_{\Xi N}^{\rm strong} \rangle = -2.06$ MeV vs $\langle V_{\Xi N}^{\rm Coulomb} \rangle = -0.38$ MeV for $t/a = 12$. Also, the mixing of the $(T, J^\pi) = (1, 1^+)$ state to the $(T, J^\pi) = (0, 1^+)$ state due to Coulomb effect is less than 1% for $t/a = 12$.

Shown in Fig. 3 is a comparison of the $NNN\Xi$ binding energies calculated with ESC08c and HAL QCD. In both cases, $NNN\Xi$ in $(T, J^\pi) = (0, 1^+)$ [Fig. 3(a)] is a possible candidate of the lightest Ξ hypernucleus. The binding energy and the binding mechanism are, however, totally different between the two cases; the strong attraction in ${}^{33}S_1$ drives ∼10 MeV binding for the ESC08c potential, while

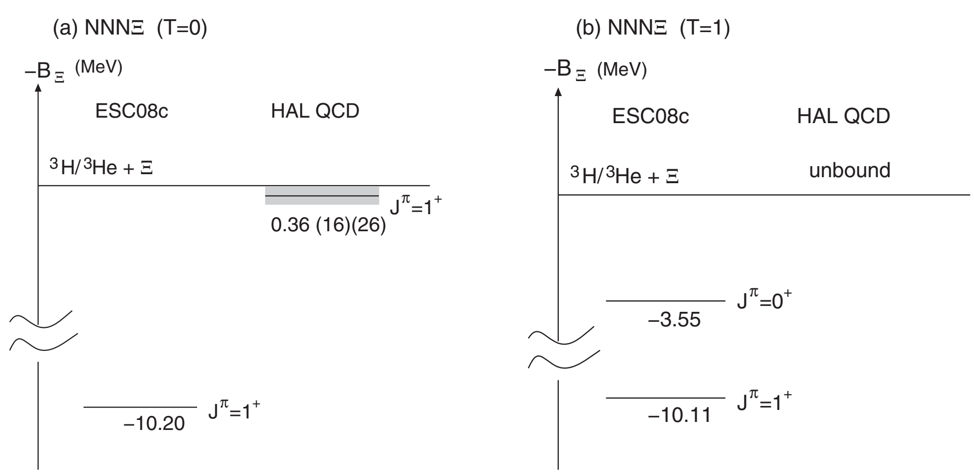

FIG. 3. Binding energies of the $NNN\Xi$ system using ESC08c and HAL QCD potentials for (a) $(T, J^\pi) = (0, 1^+)$ and (b) $(T, J^\pi) = (1, 0^+), (1, 1^+)$ states. The gray band for HAL QCD is obtained by the quadrature of the statistical and systematic errors.

the moderate attraction in ${}^{11}S_0$ leads to a binding less than 1 MeV for the HAL QCD potential.

Here, we note that all the $NNN\Xi$ states in Fig. 3 are the resonant states above the $N+N+\Lambda+\Lambda$ threshold. We estimate perturbatively the decay width Γ of $NNN\Xi$ by using the ΞN-$\Lambda\Lambda$ coupling potential and found that $\Gamma = 0.89,\ 0.43,\ 0.03$ MeV for $(0,1^+), (1,0^+), (1,1^+)$, respectively, with ESC08c. With HAL QCD, $\Gamma = 0.06$, 0.05, 0.03 MeV in $t/a = 11$, 12, 13, respectively, for $(0,1^+)$. In both cases, the decay widths are sufficiently small for those states to be observed.

To produce $NNN\Xi$ states experimentally, heavy ion reactions at GSI and CERN LHC would be useful. If there exists a bound $NNN\Xi(0,1^+)$, it decays into $d+\Lambda+\Lambda$ or a possible double Λ hypernucleus ${}^{4}_{\Lambda\Lambda}$H through the ΞN-$\Lambda\Lambda$ coupling. On the other hand, to produce $NNN\Xi(1,0^+)$ and $NNN\Xi(1,1^+)$ states as predicted by ESC08c, the (K^-,K^+) reaction with a ^{4}He target will be useful.

Finally, we remark that ${}^{4}_{\Lambda\Lambda}$H with $\Lambda\Lambda$-ΞN and ΛN-ΣN couplings has been studied before with phenomenological YN and YY interactions [32]. (Here we note that they used the observed data for the two-Λ separation energy of ${}^{6}_{\Lambda\Lambda}$He, $B_{\Lambda\Lambda} = 7.25 \pm 0.19^{+0.18}_{-0.11}$ MeV. Afterwards, the revised data $B_{\Lambda\Lambda} = 6.91 \pm 0.16$ MeV was reported, which implies that the $\Lambda\Lambda$ attraction is slightly weaker.) They reported possible existence of a weakly bound state below $d+\Lambda+\Lambda$ threshold, which has not yet been confirmed experimentally [33]. Also, Cottenssi *et al.* [34] have recently emphasized that the particle stability of $A = 5$ double Λ hypernuclei (${}^{5}_{\Lambda\Lambda}$He and ${}^{5}_{\Lambda\Lambda}$H) is robust. It is, therefore, tempting to revisit the ${}^{4}_{\Lambda\Lambda}$H system together with the $NNN\Xi(0,1^+)$ with modern coupled channel baryon-baryon interactions to answer the following question: What would be the lightest strangeness $= -2$ nucleus? The analyses and the results of the present work provide a first step towards the goal.

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