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Possible Lightest $\Xi$ Hypernucleus with Modern $\Xi N$ Interactions

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Experimental evidence exists that the $\Xi$-nucleus interaction is attractive. We search for $NN\Xi$ and $NNN\Xi$ bound systems on the basis of the AV8 $NN$ potential combined with either a phenomenological Nijmegen $\Xi N$ potential or a first principles HAL QCD $\Xi N$ potential. The binding energies of the three-body and four-body systems (below the $d + \Xi$ and $^3\text{He} + \Xi$ thresholds, respectively) are calculated by a high precision variational approach, the Gaussian expansion method. Although the two $\Xi N$ potentials have significantly different isospin ($T$) and spin ($S$) dependence, the $NNN\Xi$ system with quantum numbers ($T = 0, J^\pi = 1^+$) appears to be bound (one deep for Nijmegen and one shallow for HAL QCD) below the $^3\text{He} + \Xi$ threshold. Experimental implications for such a state are discussed.

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One of the major goals of hypernuclear physics is to understand the properties of hyperon-nucleon ($YN$) and hyperon-hyperon ($YY$) interactions; they are related not only to possible dibaryon states such as $H$ [1] but also to the role of hyperonic matter in neutron stars. Unlike the case of the $NN$ interactions, hyperon interactions are not well determined experimentally due to insufficient number of scattering data. Nevertheless, high-resolution $\gamma$-ray experiments [2–5] analyzed by the shell model [6] as well as the accurate few-body method [7] have provided valuable constraints on the $YN$ interaction in the strangeness $= -1$ sector such as the $\Lambda N$ force. Also, the $\Lambda\Lambda$ interaction in the strangeness $= -2$ sector receives some constraints from the binding energies of hypernuclei such as $^6\Lambda\Lambda\text{He}$ [8], $^{10}_{\Lambda\Lambda}\text{Be}$ [9], and $^{13}_{\Lambda\Lambda}\text{B}$ [10]. In addition, the femtoscopic analyses of the two-particle correlations in high-energy $pp$, $pA$, and $AA$ collisions at RHIC [11] and LHC [12,13] have started to give information on the low-energy $\Lambda\Lambda$ scattering parameters.

Recently, the KEK-E373 experiment showed the first evidence of a bound $\Xi^-$ hypernucleus, $^{15}_\Xi\text{C}$ ($^{14}\text{N} + \Xi$), the “KISO” event [14], which provides useful information on the attractive $\Xi N$ interaction in the strangeness $= -2$ sector. It was suggested experimentally two possible $\Xi$ binding energies $B_\Xi = E(^{15}_\Xi\text{C}) - E(^{14}\text{N})$: 4.38 ± 0.25 MeV and 1.11 ± 0.25 MeV. The latest femtoscopic data from $pA$ collisions at LHC [15] also indicate that the spin-isospin averaged $\Xi N$ interaction is attractive at low energies.
Motivated by the above observations on the \( \Xi N \) interaction, we address the following questions in this Letter: (i) What would be the lightest bound \( \Xi \) hypernucleus? and (ii) Which \( \Xi N \) spin-isospin channel is responsible for such a bound system? In particular, we consider three-body \( NN\Xi \) and four-body \( NNN\Xi \) systems simultaneously using a high-precision Gaussian expansion method (GEM) [16,17], which is one of the most powerful first principle methods to solve three- and four-body problems. We employ two modern \( \Xi N \) interactions, a phenomenological potential based on the meson exchanges, the Nijmegen \( \Xi N \) potential (ESC08c) [18], and a potential based on first principle lattice QCD simulations, the HAL QCD \( \Xi N \) potential (HAL QCD) [19].

We calculate the \( \Xi N \) effective central interactions at the imaginary-time distances \( t/a = 11, 12, 13 \), in which coupled-channel effect from higher channels as \( \Lambda \Sigma, \Sigma \Sigma \) are effectively included, whereas the effect from the lower channel (\( \Lambda \Lambda \) in the \( 1^{13}\Sigma_0 \) channel) is explicitly handled by the coupled-channel formalism [28,29].

To make the few-body calculation feasible, we fit the lattice QCD result of the potentials with multiple Gaussian forms at short distances and the Yukawa form with form factors at medium to long distances [19]. As for the pion and Kaon masses which dictate the long range part of the potential, we use \( (m_\pi, m_K) = (146, 525) \) MeV to fit the lattice data, and take \( (m_\pi, m_K) = (138, 496) \) MeV for calculating the \( \Xi \)-nucleus systems. In the \( 1^{11}\Sigma_0 \) channel, the analysis of the \( \Lambda \Lambda \) and \( N\Xi \) scattering phase shifts shows that a \( \Xi N \) interaction is moderately attractive. Also, deeply bound \( H \)-dibaryon is not found below the \( \Lambda \Lambda \) threshold. Moreover, the channel coupling between \( \Lambda \Lambda \) and \( \Xi N \) is found to be weak [19]. On the basis of this evidence, we introduce an effective single-channel \( \Xi N \) potential in which the coupling to \( \Lambda \Lambda \) in \( 1^{11}\Sigma_0 \) is renormalized into a single-range Gaussian form \( U_2 \cdot \exp(-r^2/\gamma^2) \) with \( \gamma = 1.0 \) fm with \( U_2(<0) \) chosen to reproduce the \( \Xi N \) phase shifts obtained with channel coupling. On the other hand, the \( \Xi N \) interactions in other channels are found to be much weaker: The \( 1^{13}\Sigma_1 \) and \( 33\Sigma_1 \) channels are weakly attractive and the \( 33\Sigma_0 \) channel is weakly repulsive.

In Fig. 1, we show the \( \Xi N \) phase shifts calculated with (a) the ESC08c potential and (b) the HAL QCD potential at \( t/a = 12 \) for comparison. The statistical and systematic errors are not shown in Fig. 1(b), but are taken into account in the few body calculations below. From the figure, one immediately finds a qualitative difference between (a) and (b): The \( 33\Sigma_1 \) channel is attractive in ESC08c to form a bound state with the binding energy of 1.59 MeV, while it has only weak attraction in HAL QCD. On the other hand, the \( 1^{11}\Sigma_0 \) channel is repulsive in ESC08c, while it is moderately attractive in HAL QCD. It is therefore interesting to see how such differences are reflected in the energy levels of the few-body \( \Xi \) hypernuclei.

In this Letter, we consider the \( NNN \) and \( NNN\Xi \) systems simultaneously by using the GEM [16,17]. (We note that the \( NNN \) system was recently studied by using the Faddeev method [30] with an effective \( \Xi N \) potential inspired by ESC08c. Two bound states are found \( B_2 = 13.5 \) MeV with \( (T, J^P) = (1/2, 3/2^-) \) and \( B_2 = 0.012 \) MeV with \( (T, J^P) = (1/2, 1/2^+) \) with respect to the \( d + \Xi \) threshold. In addition, one bound state is found to be 1.33 MeV with \( (T, J^P) = (3/2, 1/2^+) \) with respect to the \( d' + N \) threshold.) The GEM is a variational method with Gaussian bases, which achieves similar accuracy for bound state problems to other methods such as the Faddeev method and the Green function Monte Carlo method [31]. The GEM has been applied successfully up to five-body problems. For ordinary nuclei without strangeness, we will not
consider the isospin breaking from strong interaction nor the Coulomb interaction, so that $T$ is a good quantum number. For the $NN\Xi$ interaction, however, we take into account both strong interaction and the Coulomb interaction, since the latter effect may not be negligible for weakly bound $\Xi$ nuclei. Accordingly, possible isospin breaking such as the mixing between $T = 0$ and $T = 1$ for $NNN\Xi$ may occur.

In the GEM, three and four Jacobi coordinates are introduced to describe $NN\Xi$ and $NNN\Xi$, respectively. Shown in Fig. 2 are the four rearrangement channels in $NNN\Xi$. The four-body wave function is given as a sum of $c = 1$–4 in Fig. 2 with the LS coupling scheme:

$$\Psi_{JM} = \sum_{c=1}^{4} \sum_{\alpha l_1 \lambda_1 \lambda_2} \sum_{\beta l_2} C^{(c)}_{\alpha l_1 \lambda_1 \lambda_2 \beta l_2} \chi_{l_1, l_2} \chi_{\lambda_1, \lambda_2} \chi_{\lambda_1, \lambda_2}$$

where $\chi$'s and $\eta$'s, respectively. Total isospin $T$ can in principle take the values 0,1,2. However, $T = 2$ corresponds to the $3N$ state of $t' = 3/2$ in the continuum, so that its contribution is negligible. The spatial wave functions have the form $\phi_{\alpha l_1 \lambda_1 \lambda_2}(R, \rho) = \{\phi_{\alpha l_1 \lambda_1 \lambda_2}(R)\}^{(c)}_{\alpha l_1 \lambda_1 \lambda_2 \beta l_2}$, with a set of quantum numbers, $\alpha = (n, \ell; N, L; K; \nu, \lambda)$, and the radial components of $\phi_{\alpha l_1 \lambda_1 \lambda_2}(R)$ are taken as $r^{'j}e^{-\delta r}$, where the range parameters $r_\alpha$ are chosen to satisfy a geometrical progression. Similar choices for $\psi_{NL}(R)$ and $\tilde{\xi}_{l_1}(\rho)$ are taken. These four-body basis functions are known to be sufficient for describing both the short-range correlations and the long-range tail behavior of the few-body systems.

The $3N$ binding energy with the present AV8 $NN$ potential becomes 7.78 MeV which is less than the observed binding energy 8.48 MeV of $^3H$. This discrepancy is attributed to the three-body force, so that a phenomenological attractive three-body potential defined by $W_3 \exp[-\sum_{i>j}(r_{ij}/\delta)^2]$ is introduced, where $r_{ij}$ are the relative distances between the three nucleons $N_i$, with $W_3 = -45.4$ MeV and $\delta = 1.5$ fm.

In Table I, we summarize the binding energies of $N\Xi N$ and $NNN\Xi$ systems, where we omit atomic states which are (almost) purely bound by the Coulomb interaction. We note that the isospin mixing by the Coulomb interaction is found to be small, so that the states can be labeled by $T$ in good approximation.

Let us now discuss the results with the ESC08c $\Xi N$ potential. The binding energy of the $NN\Xi$ system with $(T, J^p) = (1/2, 3/2^+)$ with respect to the $d + \Xi$ threshold is 7.20 MeV, while the $N\Xi N$ with $(T, J^p) = (1/2, 1/2^+)$ is unbound. Such channel dependence can be easily understood in the following manner: For $NN\Xi(1/2, 3/2^+)$, nucleon and $\Xi$ spins are all aligned. Since the nuclear
force is most attractive in the spin-1 pair, and the $\Xi N$ force in ESC08c is also attractive for spin-1 pairs as shown in Fig. 1(a), this channel is most attractive to bring the bound state. On the other hand, in $NN\Xi(1/2, 1/2^+)$, one of the nucleon spins or $\Xi$ spin is antiparallel to the others, so that one or two spin-0 $\Xi N$ pairs appear in the wave function. Since such a pair is repulsive in ESC08c as shown in Fig. 1(a), this channel becomes unbound. Note here that our results of $NN\Xi$ are qualitatively similar to but numerically different from those in Ref. [30] due to a different $NN$ potential and different treatment of ESC08c. In the $T = 3/2$ $NN\Xi$ channel we do not find a bound state with respect to the $D^* + N$ threshold, while one bound state is found with $(3/2, 1/2^+)$ in Ref. [30].

For the $NN\Xi$ system in ESC08c, the state in $(T, J^\pi) = (0, 0^+)$ is unbound with respect to the $^3$H/$^3$He + $\Xi$ threshold, while the states in $(T, J^\pi) = (0, 1^+), (1, 0^+)$ and $(1, 1^+)$ are bound by 10.20, 3.55, and 10.11 MeV, respectively, as shown in Table I. The effect of the $\Xi N$ Coulomb interaction to these binding energies is only 10%–20% of those numbers. The physical reason behind such channel dependence is more involved than the case of $NN\Xi$ due to various combinations of the pairs. Nevertheless, we find that the dominant $\Xi N$ pair in the $(T, J^\pi) = (0, 0^+)$ system is the repulsive $^{11}S_0$ channel in ESC08c, which leads to the unbinding of this system. On the other hand, the dominant $\Xi N$ pairs in $(T, J^\pi) = (1, 1^+)$ and $(0, 1^+)$ systems are $^{33}S_1$ and $^{13}S_1$ channels so that the binding energies of these $NN\Xi$ systems are large.

Let us now turn to the $NN\Xi$ and $NN\Xi$ systems with the HAL QCD $\Xi N$ potential. We found that none of the potentials ($t/a = 11, 12,$ and 13) support bound states for $N\Xi$ and $NN\Xi$ systems. Only for the four-body $NN\Xi$ system with $(T, J^\pi) = (0, 1^+)$, we have a possibility of a shallow bound state with the binding energies of 0.63 ($t/a = 11$), 0.36 ($t/a = 12$), 0.18 ($t/a = 13$) MeV with respect to the $^3$H/$^3$He + $\Xi$ threshold. In Table I, we quote the number 0.36 (16)(26) MeV where the first parenthesis shows the error originating from the statistical error of the $\Xi N$ potential at $t/a = 12$ and the second parenthesis shows the systematic error. The former is estimated by the jackknife sampling of the lattice QCD configurations and the latter is estimated from the data at $t/a = 11$ and 13.

The reason why the bound state is so shallow is that, unlike the case of ESC08c, the HAL QCD potential is moderately attractive in $^{11}S_0$, while it is either weakly attractive or repulsive in other channels as shown in Fig. 1(b). If we switch off the Coulomb interaction, the bound state at $t/a = 12$ (and 13) disappears. Therefore, this is a Coulomb-assisted bound state. However, the contribution from the strong $\Xi N$ interaction is still substantially larger than that of Coulomb $\Xi N$ interaction as seen from their expectation values, $V_{\Xi N}^{\text{strong}} = -2.06$ MeV vs $V_{\Xi N}^{\text{Coulomb}} = -0.38$ MeV for $t/a = 12$. Also, the mixing of the $(T, J^\pi) = (1, 1^+)$ state to the $(T, J^\pi) = (0, 1^+)$ state due to Coulomb effect is less than 1% for $t/a = 12$.

Shown in Fig. 3 is a comparison of the $NN\Xi$ binding energies calculated with ESC08c and HAL QCD. In both cases, $NN\Xi$ in $(T, J^\pi) = (0, 1^+)$ [Fig. 3(a)] is a possible candidate of the lightest $\Xi$ hypernucleus. The binding energy and the binding mechanism are, however, totally different between the two cases; the strong attraction in $^{33}S_1$ drives $\sim 10$ MeV binding for the ESC08c potential, while

![FIG. 3. Binding energies of the $NN\Xi$ system using ESC08c and HAL QCD potentials for (a) $(T, J^\pi) = (0, 1^+)$ and (b) $(T, J^\pi) = (1, 0^+), (1, 1^+)$ states. The gray band for HAL QCD is obtained by the quadrature of the statistical and systematic errors.](092501-4)
the moderate attraction in $^{11}\Sigma_0$ leads to a binding less than 1 MeV for the HAL QCD potential.

Here, we note that all the $NNN\Xi$ states in Fig. 3 are the resonant states above the $N + N + \Lambda + \Lambda$ threshold. We estimate perturbatively the decay width $\Gamma$ of $NNN\Xi$ by using the $\Xi N$-$\Lambda\Lambda$ coupling potential and found that $\Gamma = 0.89, 0.43, 0.03$ MeV for $(0, 1^{+}), (1, 0^{+}), (1, 1^{+})$, respectively, with ESC08c. With HAL QCD, $\Gamma = 0.06, 0.05, 0.03$ MeV in $\hbar\alpha = 11, 12, 13$, respectively, for $(0, 1^{+})$. In both cases, the decay widths are sufficiently small for those states to be observed.

To produce $NNN\Xi$ states experimentally, heavy ion reactions at GSI and CERN LHC would be useful. If there exists a bound $NNN\Xi(0, 1^{+})$, it decays into $d + \Lambda + \Lambda$ or a possible double $\Lambda$ hypernucleus $^{4}_{\Lambda\Lambda}H$ through the $\Xi N$-$\Lambda\Lambda$ coupling. On the other hand, to produce $NNN\Xi(1, 0^{+})$ and $NNN\Xi(1, 1^{+})$ states as predicted by ESC08c, the $(K^{-}, K^{+})$ reaction with a $^{4}He$ target will be useful.

Finally, we remark that $^{4}_{\Lambda\Lambda}H$ with $\Lambda\Lambda$-$\Sigma\Lambda$ and $\Lambda N$-$\Sigma N$ couplings has been studied before with phenomenological $YN$ and $YY$ interactions [32]. (Here we note that they used the observed data for the two-$\Lambda$ separation energy of $^{5}_{\Lambda\Lambda}He$, $B_{\Lambda\Lambda} = 7.25 \pm 0.19^{+0.18}_{-0.11}$ MeV. Afterwards, the revised data $B_{\Lambda\Lambda} = 6.91 \pm 0.16$ MeV was reported, which implies that the $\Lambda\Lambda$ attraction is slightly weaker.) They reported possible existence of a weakly bound state below $d + \Lambda + \Lambda$ threshold, which has not yet been confirmed experimentally [33]. Also, Cottenssi et al. [34] have recently emphasized that the particle stability of $A = 5$ double $\Lambda$ hypernuclei ($^{5}_{\Lambda\Lambda}He$ and $^{5}_{\Lambda\Lambda}H$) is robust. It is, therefore, tempting to revisit the $^{4}_{\Lambda\Lambda}H$ system together with the $NNN\Xi(0, 1^{+})$ with modern coupled channel baryon-baryon interactions to answer the following question: What would be the lightest strangeness $= -2$ nucleus? The analyses and the results of the present work provide a first step towards the goal.

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