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Linguistic precursors of advanced math growth in first-language and second-language learners

Tijs Kleemans*, Eliane Segers

Behavioural Science Institute, Radboud University, Montessorilaan 3, P.O. Box 9104, 6500 HE Nijmegen, the Netherlands

A R T I C L E I N F O

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A B S T R A C T

Background: Language plays an important role in the development of mathematics. Previous research has shown that both basic and advanced linguistic skills relate to fifth-grade advanced mathematics (i.e., geometry and fractions), but these effects have not yet been investigated longitudinally or in a linguistically diverse population.
Aims: The present study first examined the differences between first-language and second-language learners in advanced mathematics. Second, we investigated the extent to which the basic and advanced linguistic skills of first-language and second-language learners directly and indirectly (through arithmetic) predict their growth in advanced mathematics from fifth to sixth grade.
Methods and procedures: Participants were 153 first-language and 80 second-language learners from 10 to 12 years of age. Classroom as well as individual measures were administered.
Outcomes and results: First, the results showed lower scores for second-language learners on advanced mathematics. Second, for both groups of language learners, basic linguistic skills were found to indirectly predict the growth in advanced mathematics via arithmetic skills, whereas advanced linguistic skills directly predicted the growth in geometry and fractions.
Conclusions and implications: These results highlight the general need for opportunities to learn the basic and advanced linguistic skills associated with mathematics over individual native language background.

What this paper adds

Previous research has shown that both basic and advanced linguistic skills relate to fifth-grade advanced mathematics (i.e., geometry and fractions), but these effects have not yet been investigated longitudinally or in a linguistically diverse population. The results of the present study contribute to ongoing theory in several ways. First, we extended previous research in showing that the effect of basic linguistic skills on the growth in advanced mathematics is mediated by arithmetic skills. Furthermore, the present study was the first to show how each of the advanced linguistic skills have a unique direct effect on advanced mathematics, after controlling for the autoregressive effect of geometric skills and fraction skills at an earlier point in time. This is an important contribution, as studies so far were mainly based on composite language measures, which makes it difficult to understand how each of the basic and advanced linguistic skills uniquely predict growth in advanced mathematics. Overall, the results suggest that ongoing theoretical frameworks on learning advanced mathematics in the upper grades of primary education should incorporate academic vocabulary and verbal reasoning as key variables in predicting growth in advanced mathematics. Finally, as an extension of previous

* Corresponding author.
E-mail address: m.kleemans@bsi.ru.nl (T. Kleemans).

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research, the results indicate that the patterns of relations in predicting growth in advanced mathematics are similar between L1- and L2-learners. However, as the L2-group had lower overall scores on linguistic and mathematics measures, a general need for opportunities to learn the basic and advanced linguistic skills associated with mathematics over individual native language background is implied.

1. Introduction

Theoretical frameworks on the development of mathematics in the lower grades (e.g., LeFevre et al., 2010; Sowinski et al., 2015) have consistently pointed out that language plays an important role in the development of mathematics. This has been especially shown for arithmetic skills in the lower grades with basic linguistic skills such as phonological skills (De Smedt, Taylor, Archibald, & Ansari, 2010) and grammatical ability (Kleemans, Segers, & Verhoeven, 2018a) as key factors. However, compared to the lower grades, the framework on the role of language in the development of advanced mathematics (i.e., geometric skills and fraction skills) in the upper grades of primary education is less developed, while further insight into its underlying mechanisms is important with an eye on designing adequate interventions for children with problems in learning advanced mathematics (Chow & Jacobs, 2016). Moreover, the increasing complexity of tasks that comes with advanced mathematics builds on advanced linguistic skills as well. To be more precise, it has been found that advanced linguistic skills such as academic vocabulary and verbal reasoning relate to fifth-grade advanced mathematics (Kleemans, Segers, & Verhoeven, 2018b) but these effects have not yet been investigated longitudinally or in a linguistically diverse population. Therefore, the present study will focus on the role of linguistic precursors in advanced math *growth* in the upper grades of primary education in first-language (L1) and second-language (L2) learners.

In primary education, arithmetic, geometry, and fractions lie at the heart of the mathematics curriculum (Mullis, Martin, & Loveless, 2016), and these skills have been found to be hierarchically organized (Träff, Olsson, Skagerlund, & Östergren, 2018). In the lower grades of primary education, the emphasis is on the development of arithmetic skills (i.e., the basic operations of addition, subtraction, multiplication, and division), whereas in the upper grades these skills need to be applied in solving geometric (Mullis et al., 2016) as well as fraction skills (Vukovic et al., 2014). Building on previous studies that have been conducted in children in the upper grades, both phonological skills (De Smedt & Boets, 2010) and grammatical ability (Kleemans et al., 2018b) as basic linguistic representations were found to account for the variation in arithmetic skills. Phonological skills have been found to predict arithmetic as the solution of arithmetic problems relies on verbal codes that have a phonological nature (Simmons & Singleton, 2008). Grammatical ability plays a role in arithmetic skills as both rely on a common syntax (Baldo & Dronkers, 2007). To be more specific, the sequence of elements in a sentence determines the outcome (e.g., the cat chased the dog versus the dog chased the cat), which also goes for an arithmetic problem ($32 - 12 =$ versus $12–32 =$). In a cross-sectional study in fifth-grade children, Kleemans et al. (2018b) found phonological skills and grammatical ability to be directly related to arithmetic skills. In addition, indirect effects of phonological skills and grammatical ability through arithmetic skills were found in predicting fifth-grade geometric and fraction skills. In other words, arithmetic skills mediated the relation between basic linguistic skills on the one hand and advanced mathematics on the other, suggesting the significance of these skills (albeit indirectly via arithmetic skills) in learning advanced mathematics.

Although little is known about which linguistic skills directly predict the development of advanced mathematics, previous studies that have been conducted in the upper grades point out that language as a composite measure is still directly related to advanced mathematics. For example, in a comprehensive longitudinal study from first to fourth grade, Vukovic and Lesaux (2013) found significant effects of a composite language measure on the growth in fourth-grade advanced mathematics. Comparable results were found by Cirino, Tolar, Fuchs, and Huston-Warren (2016) in predicting sixth-grade mathematics achievement. However, more insight is needed into the individual components of linguistic skills to determine their unique contribution to advanced mathematics in higher grades (Chow & Jacobs, 2016). Building on these studies, it was recently suggested that more advanced forms of language, such as academic vocabulary and verbal reasoning should be considered as relevant factors (Kleemans et al., 2018b). Academic vocabulary comprises domain-specific words (e.g., denominator) to comprehend and solve mathematical problems as well as a domain-general vocabulary (e.g., argumentation) to understand scientific phenomena (Snow, 2010). Next to academic vocabulary, verbal reasoning has been shown to play a critical role in learning mathematics in secondary education (Otten, Gilbertson, Males, & Clark, 2014; Stylianides & Stylianides, 2008). Verbal reasoning encompasses the ability to correctly apply a solution procedure, after having considered and executed a given number of rules and other possible alternatives (Khemlani & Johnson-Laird, 2012). Kleemans et al. (2018b) found direct relationships between academic vocabulary and geometric skills as well as verbal reasoning and fraction skills, after controlling for arithmetic skills and cognitive skills (i.e., working memory and nonverbal intelligence), suggesting that the increasing complexity of tasks that comes with advanced mathematics is associated with advanced forms of linguistic skills as well. However, the extent to which advanced linguistic skills predict the *growth* in advanced math has not been investigated yet.

To sum up, cross-sectional evidence suggests that basic linguistic skills (i.e., phonological skills and grammatical ability) indirectly through arithmetic skills relate to advanced mathematics, whereas advanced linguistic skills (i.e., academic vocabulary and verbal reasoning) directly predict advanced mathematics. An important limitation of previous research, however, is that the relative contribution of each of these linguistic skills over time is still unexplored, which leaves the question unanswered to what extent language directly predicts the development of advanced mathematics in the upper grades of primary education, after controlling for arithmetic skills and advanced mathematics at an earlier point in time. Therefore, we investigated in the present study the extent to which basic linguistic skills and advanced linguistic skills directly and indirectly (through arithmetic skills) predict the growth in advanced mathematics (i.e., geometry and fractions), while taking into account relevant cognitive factors, such as working memory and nonverbal intelligence (Cirino et al., 2016; Vukovic et al., 2014), and controlling for SES and gender. More specifically, we studied the growth in advanced mathematics from fifth to sixth grade as in these grades advanced mathematical problems require active calculation with measurement or fraction units, which has a direct link with geometry and algebra in the beginning of secondary education (DeWolf, Bassok, & Holyoak, 2015). Given the fact that cross-sectional evidence in fifth-grade children (Kleemans et al., 2018b) suggested that advanced linguistic skills differentially relate to advanced mathematics (i.e., academic vocabulary was found to be directly related to geometric skills whereas verbal reasoning directly predicted fraction skills) both geometry and fractions were used as separate outcome measures in the present study.

Furthermore, to fully address the impact of language on advanced mathematics, representative samples of both L1- and L2-learners were included in the present study. The comparison between L1-and L2-learners is important, as the relations between linguistic skills and advanced mathematics may have repercussions for linguistically disadvantaged groups (e.g., Vukovic & Lesaux, 2013). The present study was conducted in The Netherlands, in which bilinguals originating from Turkey and Morocco form the largest minority group (Central Bureau of Statistics, 2016). Turkish-Dutch and Moroccan-Dutch children tend to live in native-language speaking homes, and therefore, the Dutch language only gradually enters their lives through playmates and Dutch school (Scheele, Leseman, & Mayo, 2010). Children from a linguistic minority background are thus confronted with the challenging task of acquiring advanced mathematical skills in a language in which they are lagging behind. Given that L2-learners tend to score below their monolingual speaking peers on grammatical ability (Vukovic & Lesaux, 2013) as well as academic vocabulary (Haag, Roppelt, & Heppt, 2015) and verbal reasoning (Cocking & Mestre, 2013), it can therefore be expected that they also score below L1-learners on advanced mathematics. However, as compared to what is known on the arithmetic skills of bilingual children in the lower grades of primary education (e.g., Kleemans, Segers, & Verhoeven, 2014) as well as in secondary school (e.g., Van Rinsveld, Brunner, Landerl, Schiltz, & Ugen, 2015; Van Rinsveld, Schiltz, Landerl, Brunner, & Ugen, 2016; Van Rinsveld, Schiltz, Brunner, Landerl, & Ugen, 2016), few studies so far have been conducted on the advanced mathematical skills of L2-learners in the upper grades of primary education. Furthermore, results regarding the advanced mathematical skills of L2-learners are still inconclusive. Some studies found L2-learners to score below their monolingual peers on advanced mathematics (Martin, Liem, Mok, & Xu, 2012; Mullis et al., 2016), whereas others did not find any differences between the two groups of language learners (Vukovic & Lesaux, 2013). Furthermore, although it could be argued that because of their delays in academic vocabulary and verbal reasoning, L2-learners may differ on the extent they rely on their basic and advanced linguistic skills when predicting growth in advanced mathematics, results so far have not provided support for this line of reasoning. As a case in point, in a cross-sectional study in younger children, Kleemans et al. (2014) found no differences between Dutch L1- and L2-learners when predicting basic arithmetic skills. Comparable results were found in a longitudinal study by Vukovic and Lesaux (2013) who did not find differences between L1- and L2-learners in predicting fourth-grade advanced mathematics. And finally, Haag et al. (2015) indicated that the impact of academic language demands (i.e., advanced linguistic skills) on mathematics performance seems to depend on grade level rather than on language minority student status. To shed further light on whether language plays a similar role in the development of advanced mathematics in L1- and L2-learners, differences between the two groups of language learners in predicting advanced mathematics were considered in the present study as well.

The following research questions were addressed:

1) How do L1- and L2-learners differ in their advanced mathematics?
2) To what extent do the basic and advanced linguistic skills of L1- and L2-learners directly and indirectly (through arithmetic skills) predict their growth in advanced mathematics, after taking nonverbal intelligence and working memory into account, and controlling for SES and gender?

With respect to the first research question, we expected lower scores of the group of L2-learners on advanced mathematics (i.e., geometry and fractions). With respect to the second research question, we expected for both groups that basic linguistic skills would be related to advanced mathematics via arithmetic skills and that advanced linguistic skills would be directly related to the growth in geometry and fractions.

2. Method

2.1. Participants

The group of participants originated from twelve schools for mainstream primary education throughout the Netherlands. Informed consent was given by 95 % of the parents, which resulted at Time 1 in a group of 157 L1-learners (M_{age} = 131.40, age range 120–143 months) and 83 L2-learners (M_{age} = 131.60, age range 120–143 months), from 14 mixed classrooms in which the proportion of L2-learners varied from 35 to 65 %. At Time 2, one year later, seven children dropped out of the study due to having moved (n = 5) or staying an extra year in fifth grade (n = 2); thus, in sixth grade the sample contained 153 L1-learners (M_{age} 143.32 months, age range 132–155 months) and 80 L2-learners (M_{age} 143.51 months, age range 132–155 months). No differences were found between the group of children who dropped out after fifth grade and the group of children who participated in sixth grade in terms of the predictor and criterion measures (all p's > .106).

A questionnaire was used to determine children's language use at home. All L1-learners (72 boys, 81 girls) were born in the Netherlands and were monolingual speakers of Dutch. The group of L2-learners (42 boys, 38 girls) consisted of Turkish-Dutch and Moroccan-Dutch children. To be more precise, within the group of L2-learners, children indicated to speak Turkish (51.25 %) or Moroccan (48.75 %) at home with at least one parent, while they heard their parents speaking mainly Turkish or Moroccan to each other. Therefore, no L2-children were excluded from the present study. Furthermore, the majority (75 %) of L2- learners was born in the Netherlands, which is in line with the general population of Turkish and Moroccan L2-learners in the Netherlands (Central Bureau of Statistics, 2016).

Socioeconomic status (SES) was measured by the educational background of both parents, using a 4-point scale (1 = no post-secondary education was completed, 2 = the parent completed intermediate postsecondary education, 3 = the parent completed higher education, 4 = the parent completed university). Within the group of L1-learners, the average SES for fathers was 2.16 (SD = .765), and for mothers 2.09 (SD = .906). For the group of L2-learners, the average SES for fathers was 1.98 (SD = .981) and for mothers 1.89 (SD = .981). Preliminary analyses showed no differences between the group of L1- and L2-learners on fathers' SES (p = .137), mothers' SES (p = .114), age (p = .818), and cognitive abilities (i.e., nonverbal intelligence; p = .923, and working memory; p = .475). Finally, no differences were found between boys and girls in terms of the predictor and criterion measures (all p's > .096).

2.2. Materials

2.2.1. Predictor measures (Time 1: Fifth grade)

2.2.1.1. Phonological skills. In the upper primary grades, phonological skills have reached ceiling levels for most children (Vloedgraven & Verhoeven, 2009). We therefore used the Doorstreepleestoets [Cross Out Reading Test] (Van Bon, 2007) as a proxy for phonological skills as this task measures decoding skills by means of lexical decision. Indeed, given the fact that numerical and lexical symbols rely on the same underlying processes of encoding and manipulation (e.g., LeFevre et al., 2010), the use of lexical decision tasks can be seen as a valid alternative to measure phonological skills. In this test, a paper with 120 words (i.e., 60 words interspersed with 60 pseudo words) spread over four columns was presented to the participant. The task of the participant was to silently read as many words as possible within one minute while crossing out the pseudo words they encountered. The number of words read minus the number of errors resulted in the overall score on this test (Cronbach's alpha > .80, Van Bon, 2007).

2.2.1.2. Grammatical ability. The Zinsleestaak [Sentence Reading Task] from the Taaltoets Allochtone Kinderen Bovenbouw [Language Test for Immigrant Children, Upper Grades] (Verhoeven & Vermeer, 1993) was used to measure grammatical ability. In this paper-and-pencil test, the participant was presented with 50 items, each item containing three sentences. The participant was asked to determine whether one of the presented sentences contained a grammatical error. All correct answers were counted (Cronbach's alpha = .86; Verhoeven & Vermeer, 1996).

2.2.1.3. Academic vocabulary. An academic vocabulary test, derived from the Dutch Word List on Academic Vocabulary (Giezenaar & Schouten, 2002), was used to assess children's academic vocabulary. Each of the multiple-choice items (54 in total) assessed children's vocabulary skills within the domains of science and technology (i.e., physics, engineering, biology, math, earth and space) and science in general. On each item, children were asked to select the correct meaning of an underlined word, out of four alternate answers. All correct answers received one point (Cronbach's alpha = .86).

2.2.1.4. Verbal reasoning. A syllogistic reasoning test (Segers & Verhoeven, 2016) was used to measure verbal reasoning. In this test, short stories, consisting of a maximum of three sentences, were presented to the child along with a few questions that need to be answered. For each question, the child had to determine which answer out of three alternatives (i.e., "yes", "no", or "maybe") was correct (e.g., "All children who do their homework receive good grades". (A) "Jan always does his homework. Will he receive a good grade?" (answer: yes). (B) "Sanne received a good grade. Did she do her homework?" (answer: maybe). All correct answers received one point (Cronbach's alpha > .72; Segers & Verhoeven, 2016).

2.2.1.5. Working memory. Working memory was assessed using the subtest number recall from the Kaufman Assessment Battery for Children, second edition (Kaufman & Kaufman, 2004). In this task, a sequence of digits was verbally presented to the child and he or she was asked to repeat this sequence in reverse order. The series of digits increased as testing progresses. After three consecutive mistakes, testing was terminated. All correctly recalled sequences received one point (Cronbach's alpha = .85; Kaufman, Lichtenberger, Fletcher-Janzen, & Kaufman, 2005).

2.2.1.6. Nonverbal intelligence. The Raven's Standard Progressive Matrices (Raven, 1976) was used to measure children's nonverbal intelligence. A total of 60 items, divided over five sets (A-E) of 12 items were presented to the child. In set A, a patterned figure was shown with one part missing. The task of the child was to select the missing part by choosing one of six alternatives. In set B-E, each item contained a pattern of four (B) to nine (C-E) figures, in which the last figure was missing. The child was asked to select the missing figure by choosing from six (A-B) to eight (C-E) alternatives. All correct answers were counted (Cronbach's alpha > .90).

2.2.2. Criterion measures: mathematics

2.2.2.1. Arithmetic skills (Time 1: Fifth grade). Children's arithmetic skills were assessed using the Tempotest Automatiseren [Speed Test for Arithmetic Fluency; De Vos, 2010]. This test consisted of four cards (50 items each) with on each a card a different arithmetic operation. First, the children started with addition, followed by subtraction, multiplication, and division. On each card, the task of the child was to solve as many arithmetical problems as possible within a time limit of two minutes. On each card, all correct answers were counted (Cronbach's alpha > .78).

2.2.2.2. Advanced mathematics (Time 1: Fifth grade and Time 2: Sixth grade). Two tasks derived from the Schoolvaardigheidstoets Rekenen-Wiskunde [School Achievement Test for Mathematics] (De Vos & Milikowski, 2012) were used to measure advanced mathematics in the domains of geometry and fractions. In this test, children were asked to solve 30 items that assessed geometric skills (e.g., 2 m – 2 cm = ...cm), followed by 30 items that required calculations with fractions (e.g., 1/2 × 1/4 =). In both tasks, difficulty increased as testing progresses. All correct answers received one point (Cronbach's alpha geometry = .84; Cronbach's alpha fractions = .85).

2.3. Procedure

Schools were asked for written consent to participate in the present study. Before data collection, four graduate students in educational science received a two-hour training by an educational psychologist (i.e., the first author). Each of the students received course credit for their assistance in data collection. Testing was done within the classroom at each school with the children's teacher present. The predictor and criterion measures were divided over three blocks (A-B-C; 45 min each) that were randomly distributed among the various classrooms. The tests within the blocks, however, were administered in the same order. Furthermore, children's working memory was administered individually in a quiet room at the school.

3. Results

Before answering the research questions, descriptive statistics for all predictor and criterion measures were computed to check for normality. The results showed that all variables were normally distributed, with acceptable values of skewness and kurtosis (i.e., values between −1.5 and 1.5; Tabachnick & Fidell, 2013). Furthermore, after standardization ($M = 0$; $SD = 1$) of each of the predictor and criterion measures, the results showed no extreme scores ($z > 3.29$). Furthermore, the percentages of potential outliers ($z > 1.96$) and possible outliers ($z > 2.58$) were within acceptable ranges (i.e., < 5 % and < 1 %, respectively; see also Field, 2013).

3.1. Differences between first-language and second-language learners

To answer the first research question on differences between L1- and L2-learners, t-tests for independent samples using Holm-Bonferroni correction (Holm, 1979) were computed. The results showed the L2-learners to score below the L1-learners on grammatical ability, academic vocabulary, verbal reasoning and mathematics (i.e., arithmetic skills, geometric skills, and fraction skills), but not on nonverbal intelligence, working memory, and phonological skills. Effect sizes (Cohen's *d*) varied between 0.378 and 0.944 (see Table 1).

3.2. Predicting growth in advanced mathematics

Before turning to the second research question, a confirmatory factor analysis with varimax rotation was conducted on the four cards (i.e., addition, subtraction, multiplication, and division) that measured arithmetic skills. The results showed a clear one-factor

Table 1
Descriptive Statistics for First-Language (L1) Learners (n = 150) and Second-Language (L2) Learners (n = 83); Results of t-tests for Independent Samples; and Cohen's d.

Variable	L1		L2		*t*	*d*
	M	*SD*	*M*	*SD*		
Predictor measures						
Phonological skills	69.68	16.336	71.93	18.883	−.943	−0.124
Grammatical ability	23.57	5.664	20.36	5.731	4.086***	0.538
Academic vocabulary	37.67	5.303	32.83	7.307	5.248***	0.944
Verbal reasoning	12.36	3.490	10.53	4.063	3.598****	0.473
Working memory	7.90	2.022	7.70	2.089	.716	0.094
Nonverbal intelligence	41.97	5.723	42.06	7.097	−.097	−.017
Criterion measures						
Arithmetic skills (fifth grade)						
Addition	36.97	5.406	32.49	7.537	4.726***	0.854
Subtraction	34.21	6.513	28.86	9.102	4.666***	0.844
Multiplication	36.12	10.488	31.14	11.454	2.871**	0.439
Division	31.72	12.253	26.85	12.363	3.338**	0.378
Geometric skills (fifth grade)	19.05	4.533	15.68	5.423	5.072***	0.662
Geometric skills (sixth grade)	20.89	5.379	18.49	5.407	3.230**	0.425
Fraction skills (fifth grade)	12.22	5.615	9.71	4.648	3.637***	0.530
Fraction skills (sixth grade)	19.02	4.942	16.79	4.973	3.267**	0.430

* $p < .05$. ** $p < .01$. *** $p < .001$.

Table 2
Pearson Correlations among the Predictor Measures and Criterion Measures (n = 233).

Variable	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
1. Phonological skills	–										
2. Grammatical ability	.123	–									
3. Academic vocabulary	.205**	.451***	–								
4. Verbal reasoning	.050	.208**	.346***	–							
5. Working memory	.136**	.148*	.150*	.158*	–						
6. Nonverbal intelligence	−.052	.269***	.149*	.226***	.338***	–					
7. Arithmetic skills (fifth grade)	.295***	.307***	.230*	.170**	.276***	.154*	–				
8. Geometric skills (fifth grade)	.198**	.310***	.558***	.324***	.164*	.241***	.292***	–			
9. Geometric skills (sixth grade)	.158*	.347***	.461***	.355***	.256***	.373***	.487***	.558***	–		
10. Fraction skills (fifth grade)	.033	.224**	.343***	.543***	.198*	.360***	.300***	.439***	.419***	–	
11. Fraction skills (sixth grade)	.086	.251***	.376***	.499***	.260***	.370***	.414***	.434***	.590***	.670***	–

* $p < .05$. ** $p < .01$. *** $p < .001$.

solution accounting for 77.97 % of the variance in arithmetic skills. We therefore computed a composite factor score and used this score for further analyses.

We then computed Pearson correlations between the predictor and criterion measures. The results indicated that each of the predictor measures correlated significantly with the tasks that measured mathematics (see Table 2), with the exception of phonological skills and fifth-grade fraction skills ($r = .033$; $p = .621$) and phonological skills and sixth-grade fraction skills ($r = .086$; $p = .190$).

To answer the second research question, we performed two separate mediation analyses using the process add-on in SPSS, with a default bootstrapping at 5000 cycles (Hayes, 2013). In mediation models, the total effect (c) consists of the sum of the direct effect c' and the indirect effect ab. This indirect effect (ab) consist of the product of the effect of the independent variable on the mediator (a) and the effect of the mediator on the dependent variable (b) (Hayes, 2013).

In the first mediation analysis, sixth-grade geometric skills was the dependent variable, the six predictor measures (i.e., phonological skills, grammatical ability, academic vocabulary, verbal reasoning, working memory, and nonverbal intelligence) were the independent variable, and arithmetic skills was the mediator. Furthermore, the autoregressive effect of fifth-grade geometric skills was added as an additional mediator as well. To estimate the indirect and direct effects of each of the independent variables, the model was run six times with the other variables as covariates. To control for possible influences of SES and gender, both covariates were included in the mediation analysis as well. Given that fathers and mothers did not differ in SES ($t = 1.220$, $p = .224$), fathers' education was included as a covariate in the mediation analyses. However, it should be noted that when we used mothers' education as a covariate, we found similar results leading to identical conclusions.

In addition, a moderated mediation analysis (Hayes, 2013) was conducted for each of the six independent variables to check whether the strength in direct and indirect effects differed for both groups (i.e., L1- and L2-learners). For the second mediation analysis, in which sixth-grade fraction skills was the dependent variable, the same procedure was followed, but with the autoregressive effect of fifth-grade fraction skills as additional mediator.

With respect to sixth-grade geometric skills, the results first showed significant direct effects of fifth-grade geometric skills, arithmetic skills, nonverbal intelligence, academic vocabulary, and verbal reasoning. Furthermore, both SES ($B = .351$, 95 % CI = [−.261 – .964], $p = .261$) and gender ($B = .598$, 95 % CI = [−.433 – 1.630], $p = .254$) did not show significant effects. In addition, the indirect effects of phonological skills (ab = .389, 95 %CI = [.175–.707]), grammatical ability (ab = .372, 95 % CI = [.144–.702]), and working memory (ab = .313, 95 %CI = [.118–.584]) via arithmetic skills were significant as the bootstrapped 95 % confident intervals did not cover 0. This also goes for the indirect effects of academic vocabulary (ab = .733, 95 % CI = [.319–1.455]), verbal reasoning (ab = .203, 95 % CI = [.020–.462]), and nonverbal intelligence (ab = .226, 95 % CI = [.025–.551]) via fifth-grade geometric skills. Finally, the results of the moderated-mediation analyses with each of the predictor variables as independent measures, did not show significant interactions in predicting arithmetic skills and fifth- and sixth-grade geometric skills (all p's > .077). Thus, the strength of the direct and indirect effects was the same for both groups of language learners. Note that none of the moderated-mediation analyses showed a significant Group effect, suggesting that the differences between L1- and L2-learners on sixth-grade geometric skills were explained by the predictor variables. The results are presented in Fig. 1A.

With respect to sixth-grade fraction skills, we found direct effects of fifth-grade fraction skills, arithmetic skills, nonverbal intelligence, academic vocabulary, and verbal reasoning. In addition, no effects were found for SES ($B = .154$, 95 % CI = [−.388 – .697], $p = .576$) and gender ($B = .188$, 95 % CI = [−.722 – 1.098], $p = .684$). Furthermore, the indirect effects of phonological skills (ab = .248, 95 % CI = [.092–.493]), grammatical ability (ab = .236, 95 % CI = [.081–.492]), and working memory (ab = .199, 95 % CI = [.064–.419]) via arithmetic skills were significant. Moreover, academic vocabulary (ab = .335, 95 % CI = [.102–.636]), verbal reasoning (ab = .967, 95 % CI = [.604–1.416]), and nonverbal intelligence (ab = .493, 95 % CI = [.231–.840]) all showed significant effects via fifth-grade fraction skills. Finally, we again did not find significant interactions in predicting fifth- and sixth-grade geometric skills (all p's > .082), suggesting identical direct and indirect effects for L1- and L2-learners. Moreover, none of the analyses showed a significant effect of Group. Fig. 1B depicts the results.

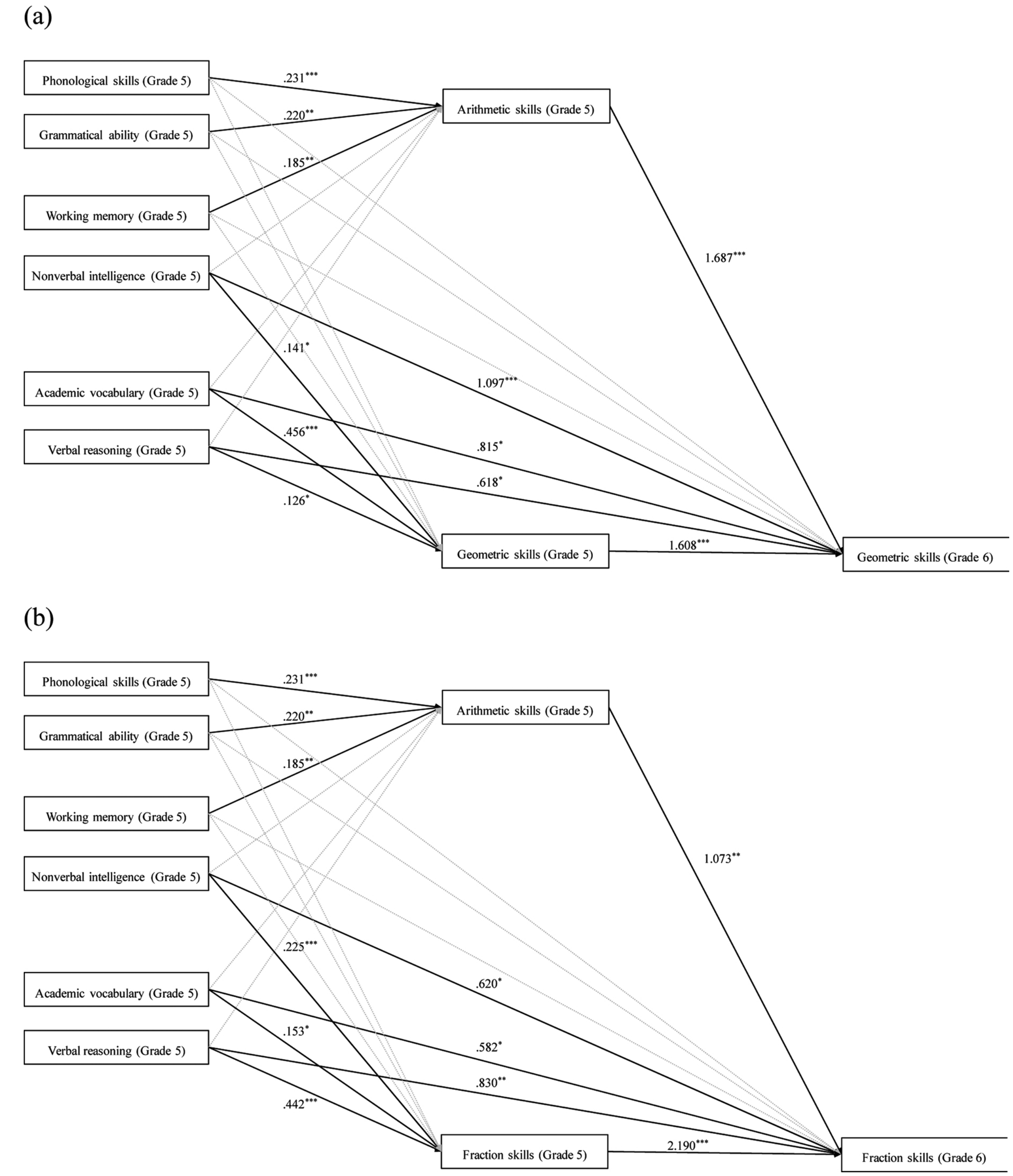

Fig. 1. Predicting growth in (a) geometric skills and (b) fraction skills, while controlling for SES and gender. Grey lines indicate non-significant relationships.
Note. Total R^2 geometric skills (grade 6) = .512 ($p < .001$); Total R^2 fraction skills (grade 6) = .550 ($p < .001$).
* $p < .05$. ** $p < .01$. *** $p < .001$.

4. Discussion

The present study focused on the role of basic linguistic and advanced linguistic skills in advanced math growth in the upper grades of primary education. Furthermore, representative samples of L1- and L2-learners were included to fully address the impact of language on advanced mathematics. Two research questions were addressed. The first question examined the differences between first and second language learners on advanced mathematics. The second question was on the extent the basic and advanced linguistic skills of L1- and L2-learners directly and indirectly (through arithmetic skills) predict their growth in advanced mathematics (i.e., geometry and fractions), after taking nonverbal intelligence and working memory into account, and controlling for SES and gender, and checking for possible group differences.

With respect to the first research question, we indeed found lower scores for L2-learners on both geometric and fraction skills. Previous research in younger children already indicated that Dutch L2-learners have lower arithmetic skills as compared to their monolingual peers (Kleemans et al., 2014). Because of the impact of arithmetic on advanced mathematics in the upper grades of primary education (Cirino et al., 2016; Vukovic et al., 2014), as well as the fact that L2-learners have lower scores on academic vocabulary (Haag et al., 2015) and verbal reasoning (Cocking & Mestre, 2013), the results of the present study indeed showed that the geometric and fraction skills of sixth-grade L2-learners lie behind as compared to their monolingual peers. This result suggests that L2-learners are faced with the challenge of simultaneously learning academic content and developing proficiency in the language of instruction which poses a threat to their mathematics proficiency.

With respect to the second research question, we first found the combinations of phonological skills, grammatical ability, and working memory to predict fifth-grade arithmetic, which is consistent with a previous cross-sectional study in fifth-grade monolingual children (Kleemans et al., 2018b). In addition, arithmetic skills were found to directly predict the growth in geometric and fraction skills in the upper grades, which supports the theoretical claim that arithmetic skills lie at the heart of learning advanced mathematics (Cirino et al., 2016; Vukovic et al., 2014). New, however, is the finding that the effect of basic linguistic skills on the growth in advanced mathematics is mediated by arithmetic skills. To be more specific, the solution of geometric and fraction problems relies on having access to arithmetic representations, but these representations, in turn, seem to have both a phonological (i.e., arithmetic fact retrieval; Simmons & Singleton, 2008) and grammatical (i.e., knowing the sequence of elements within an arithmetical problem to determine the outcome; Baldo & Dronkers, 2007) underpinning.

With respect to the role of advanced linguistic skills, we indeed found both academic vocabulary and verbal reasoning to directly predict growth in geometry and fractions, over and above arithmetic skills. As compared to earlier longitudinal research in the upper grades of primary education (e.g., Cirino et al., 2016; Vukovic et al., 2014), the present study was the first to show how each of the advanced linguistic skills have a unique direct effect on advanced mathematics, after taking into account the autoregressive effect of geometric skills and fraction skills at an earlier point in time. This is an important contribution, as studies so far were mainly based on composite language measures (Chow & Jacobs, 2016), which makes it difficult to understand how each of the basic and advanced linguistic skills uniquely predict growth in advanced mathematics. Overall, the results suggest that ongoing theoretical frameworks on learning advanced mathematics in the upper grades of primary education (e.g., Cirino et al., 2016; Vukovic et al., 2014) should incorporate academic vocabulary and verbal reasoning as key variables in predicting growth in advanced mathematics.

It is interesting to note that *both* academic vocabulary and verbal reasoning were found to predict growth in geometric as well as fraction skills, which differs from the cross-sectional study of Kleemans et al. (2018b) that was conducted in 167 monolingual fifth-graders. They found academic vocabulary to only directly predict geometric skills, whereas verbal reasoning was found to directly predict fraction skills, but not geometric skills. In fifth grade, children are expected to calculate with measurement units of the same order (e.g., "3km + 100 m = … m"), as well as to add and subtract a fraction from a whole number (e.g., "3 – 1/8 = …"), whereas in sixth grade children need to actively convert measurement units (e.g., "3 dm^3 = … cl"), and to also multiply and divide with fractions (e.g., "1/4 : 1/3 = …") (Van der Stap, 2014). This requires children in sixth grade to develop a deep and profound understanding of the transitive character of fractions in relation to their corresponding percentages or ratios. To be more specific, knowing that a fraction, percentage and ratio have the same meaning may be helpful to solve more complex fraction problems (e.g., 1/4 + 1/3 = 0.25 + 0.125 = 0.375 which equals 3/8). At the same time, verbal reasoning becomes increasingly important in calculating with measurements, as the conversion of measurement units requires children to correctly execute the action sequence based on a given set of rules (e.g., 10 cl = 1 dl; 1 dl = 1 dm^3). It might thus be the case that advanced linguistic skills become increasingly important when children move through the advanced mathematics curriculum. Considering the fact that fifth- and sixth-grade advanced mathematics have a direct link with learning geometry and algebra in the beginning of secondary education (DeWolf et al., 2015), it could be expected that such mathematical skills may rely on academic vocabulary and verbal reasoning as well. Based on the results of the present study, the claim to already pay attention to the development of advanced linguistic skills in primary education (Snow, 2010; Stylianides, 2007) is therefore further supported.

Finally, the results showed that the strength between linguistic skills on the one hand and advanced mathematics on the other hand did not differ between L1-learners and L2-learners. This result seems to be in line with the longitudinal study of Vukovic and Lesaux (2013) who found identical relationships between L1 and L2-learners in predicting fourth-grade advanced mathematics. As an extension of previous research, the results of the present study indicate that identical relationships between L1- and L2-learners also exist when it comes to learning fifth- and sixth-grade advanced mathematics. In general, these results support the theoretical claim that language ability is not only fundamental in the development of mathematics in the lower grades (e.g., LeFevre et al., 2010), but in the upper grades as well, regardless of linguistic background.

The aforementioned results have some limitations. First, the advanced mathematics measures consisted of Arabic digits only. Given the fact that in the upper grades of primary education, mathematical problems in general are found to be increasingly embedded within a realistic, authentic context (i.e., Realistic Math Education; Van den Heuvel-Panhuizen, 2003), other linguistic processes may play a role as well. For instance, reading comprehension skills were found to significantly impact the mathematical abilities of sixth-graders, especially when mathematical problems have a high level of semantic complexity (Boonen, De Koning, Jolles, & Van der Schoot, 2016). Future research could therefore focus on the extent the combinations of basic and advanced linguistic skills along with reading comprehension skills relate to growth in advanced mathematics when a realistic context is added to the mathematical problems. Furthermore, although the present study showed advanced linguistic skills to impact the growth in advanced mathematics from fifth to sixth grade, such a conclusion cannot be drawn from the effect of basic linguistic skills on arithmetic skills as both were measured in fifth grade. Although previous research has consistently shown a causal link between basic linguistic skills and arithmetic (e.g., De Smedt et al., 2010; Kleemans et al., 2018a), we cannot conclude from the results of the present study that basic linguistic skills impact arithmetic in the upper grades. To make such a claim, the basic and advanced linguistic skills of the present study should be already measured in fourth grade, which could be taken into consideration in future research. Finally, in the present study we only included domain-general predictors of advanced mathematics. As can be seen in Table 2, most correlations between the predictor and criterion measures varied between small to moderate, which leaves room to consider other more content-related relevant variables such as number sense as well (see e.g., Tosto et al., 2017). This could be a topic for future research.

To conclude, the results first point to the significance of both basic and advanced linguistic skills in predicting growth in advanced mathematics in the upper grades of primary education. To be more precise, phonological skills and grammatical ability had an indirect effect via arithmetic on the growth in geometry and fraction skills, whereas academic vocabulary and verbal reasoning directly predicted the growth in advanced math, over and above arithmetic skills. Second, the results indicate that language matters in the development of advanced mathematics regardless of linguistic background. These results have some practical implications. To begin with, when it comes to delays in advanced mathematics, educational professionals are recommended to test for problems in basic and advanced linguistic skills as well. In addition, follow-up interventions that are targeted at improving advanced mathematics, should consider both the basic and advanced linguistic skills children bring to the classroom. Finally, it would be recommended to incorporate academic vocabulary and verbal reasoning in the school curriculum.

CRediT authorship contribution statement

Tijs Kleemans: Conceptualization, Methodology, Validation, Resources, Formal analysis, Investigation, Writing - original draft, Project administration. **Eliane Segers:** Conceptualization, Methodology, Validation, Writing - review & editing.

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