METHODS

Increasing robustness of pairwise methods for effective connectivity in magnetic resonance imaging by using fractional moment series of BOLD signal distributions

Natalia Z. Bielczyk1,2, Alberto Llera1,3, Jan K. Buitelaar1,2, Jeffrey C. Glennon1,2, and Christian F. Beckmann1,2,3,4

1Donders Institute for Brain, Cognition and Behavior, Nijmegen, the Netherlands
2Department of Cognitive Neuroscience, Radboud University Nijmegen Medical Centre, Nijmegen, the Netherlands
3Radboud University Nijmegen, Nijmegen, the Netherlands
4Oxford Centre for Functional MRI of the Brain, University of Oxford, Oxford, UK

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ABSTRACT

Estimating causal interactions in the brain from functional magnetic resonance imaging (fMRI) data remains a challenging task. Multiple studies have demonstrated that all current approaches to determine direction of connectivity perform poorly when applied to synthetic fMRI datasets. Recent advances in this field include methods for pairwise inference, which involve creating a sparse connectome in the first step, and then using a classifier in order to determine the directionality of connection between every pair of nodes in the second step. In this work, we introduce an advance to the second step of this procedure, by building a classifier based on fractional moments of the BOLD distribution combined into cumulants. The classifier is trained on datasets generated under the dynamic causal modeling (DCM) generative model. The directionality is inferred based on statistical dependencies between the two-node time series, for example, by assigning a causal link from time series of low variance to time series of high variance. Our approach outperforms or performs as well as other methods for effective connectivity when applied to the benchmark datasets. Crucially, it is also more resilient to confounding effects such as differential noise level across different areas of the connectome.

AUTHOR SUMMARY

Estimating causal interactions from functional magnetic resonance imaging (fMRI) data is a formidable task. Recent advances in this field include methods for pairwise inference. In the first step of this procedure, connections are revealed by means of functional connectivity. In the second step, every detected connection is analyzed separately to reveal the direction of the causal links. We introduce an advance to the second step of this procedure by building a classifier based on the novel concept of fractional moments of the BOLD distribution combined into cumulants. The classifier is trained on datasets generated under the dynamic causal modeling (DCM) generative model. Using fractional cumulants gives a measure resilient to confounding effects such as differential noise levels across different areas of the connectome.
Effective connectivity: Causal relations between nodes of the investigated network. In fMRI, effective connectivity (as opposed to directed functional connectivity) is typically derived from a model that additionally considers the underlying neuronal processes (see: dynamic causal modeling).

Dynamic Causal Modeling: The most popular generative model in the fMRI connectivity research. It assumes that the observable BOLD fMRI signal is a product of an underlying neuronal communication smoothed with a slow hemodynamic response.

Functional connectivity: Statistical associations between activity in nodes of the investigated network. In fMRI, functional connectivity is most often evaluated by means of Pearson/partial correlation or mutual information.

Pairwise inference: A two-step causal inference procedure that reduces causal inference in a large graph to studying two-node interactions.

INTRODUCTION

In the context of functional magnetic resonance research, effective connectivity refers to the process of estimating causal interactions between distinct regions within the brain. Several characteristics of fMRI data impose severe restrictions on the possibility of estimating such effective connectivity (Valdes-Sosa, Roebroeck, Daunizeau, & Friston, 2011; Friston, 2011; Bielczyk et al., 2019). First, the temporal resolution of the image acquisition is low (sampling rate typically <1 Hz). Furthermore, blood oxygen level-dependent (BOLD) activity is delayed with respect to neuronal firing, with a delay of 3–6 s in the adult human brain (Arichi et al., 2012). The delayed hemodynamic response can also induce spurious cross-correlations between two BOLD time series (Ramsey et al., 2010; Bielczyk, Llera, Buitelaar, Glennon, & Beckmann, 2017). Both subject-to-subject and region-to-region variability in the shape of hemodynamic response (Devonshire et al., 2012) provide a general limitation to the methods for effective connectivity research in fMRI: when the hemodynamic response in one region is faster than in another, the temporal precedence of the peak of the hemodynamic response can easily be mistaken for causation. Secondly, fMRI data is characterized by a relatively low signal-to-noise ratio. Within gray matter and at field strengths of 3 T, the task-induced signal changes range within 2–3% of the mean signal depending on the task (Kruger, 2018). Furthermore, the stochastic noise in the brain has been shown to have a scale-free spectral characteristic (He, 2014; Bédard, Kröger, & Destexhe, 2006; Dehghani, Bédard, Cash, Halgren, & Destexhe, 2010), which additionally hinders identifiability of causal structures derived from fMRI data (Bielczyk, Llera, Buitelaar, Glennon, & Beckmann, 2017). Moreover, typical fMRI protocols involve a relatively short time series (a few hundred samples), in which estimation of conditional probabilities between variables, and higher order statistic in the time series becomes difficult.

Multiple methods were proposed to estimate effective connectivities from fMRI data (Friston, 2011). In the computational study by Smith et al. (2011), a range of methods for effective connectivity were tested on synthetic datasets derived from a dynamic causal modeling forward model (DCM; Friston, Harrison, & Penny, 2003). In this study, most methods for estimating causal interactions remained at chance level. One method highlighted as relatively successful at identifying causal links is based on Patel’s tau measure (PT; Patel, Bowman, & Rilling, 2006; Smith et al., 2011). PT entails a two-step approach in which the first step involves identifying the (undirected) connections by means of functional connectivity. This is achieved on the basis of correlations between the time series in different regions, which is also referred to as Patel’s kappa (Patel et al., 2006; Smith et al., 2011).

One note to make is that, PT and other pairwise inference procedures assume that causation implies correlation. This assumption is necessary in order to perform the first step of the inference procedure, that is, to select reliable connections for further classification into upstream and downstream regions. However, although this assumption is often true, this is not always the case as under certain circumstances (e.g., in control systems) causation might not be associated with correlation (Kennaway, 2015). In the recent study by Di & Biswal (2019), the authors investigated task-modulated whole-brain functional connectivity in six block-design and one event-related cognitive task. By using psychophysiological interactions between pairs of regions of interest (ROIs) from the whole brain, the authors identified statistically significant task modulations in functional connectivity, and reported, “task modulated connectivity was found not only between regions that were activated during the task but also regions that were not activated or deactivated.” This suggests that only considering pairs of regions in which activity is correlated might neglect some of the underlying effective connections.

There are multiple strategies for implementing and thresholding functional connectivity estimates (Varoquaux & Craddock, 2013). Since Pearson correlation typically returns dense...
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connectomes that contain spurious links (spurious links X-Z appearing as a consequence of links X-Y and Y-Z; Aldrich, 1995), most often, partial correlation (Marrelec et al., 2006) is employed as a method of choice to build functional connectomes on the basis of fMRI datasets. In partial correlation, for each pair of nodes X and Y, the time linear input from all the remaining nodes in the network is partialled out from the time series before taking Pearson correlation. For sake of computational efficiency, partial correlation is often computed as normalized inverse covariance.

Although partial correlation can be too conservative with respect to the underlying functional links or even induce spurious negative correlations in some cases (an effect known as Berkson’s paradox; Nie, Yang, Matthews, Xu, & Guo, 2015; Berkson, 1946), to date, it remains the state of the art for estimating functional connectivity. Proper thresholding of partial correlation matrices is also an open research problem. A popular strategy for thresholding involves permutation testing (Smith et al., 2011; Hyvärinen & Smith, 2013), in which subject labels are shuffled between samples in multiple random iterations, and a separate null distribution is created for each connection in the network on the basis of these permutations (leading to a single-thresholded connectome matrix for the entire subject population). Recently, a thresholding strategy employing mixture modeling has also been proposed (Bielczyk et al., 2018). These procedures allow for deriving individualized sparse connectomes per subject. Other alternative thresholding schemes involve proportional thresholding (van den Heuvel et al., 2017) and shrinkage estimates, such as Ledoit and Wolf (2004) and graphical Lasso (Friedman, Hastie, & Tibshirani, 2008).

Thresholding a functional connectome obtained with use of partial correlation results in a binary graph of connections, where all other possible edges identified as absent are disregarded from further considerations. The second step of PT procedure determines the directionality in each one of the previously detected connections. In this step, effective connectivity boils down to a two-node Bayesian network. The concept is based on a simple observation: if there is a causal link X → Y, Y should get a transient boost of activity every time X increases activity. Vice versa: if there is a link Y → X, X should react to the activation in Y. Therefore, one can threshold the signals X(t), Y(t) in order to obtain binary series of events X_1(t), Y_1(t), and compute the difference between conditional probabilities P(Y_1 | X_1) and P(X_1 | Y_1). Three scenarios are possible:

1. P (Y_1 | X_1) equals P (X_1 | Y_1): it is a bidirectional connection X ↔ Y (since empty connections were sorted out in the previous step).
2. The difference between P (Y_1 | X_1) and P (X_1 | Y_1) is positive: the connection X → Y is likely.
3. The difference between P (Y_1 | X_1) and P (X_1 | Y_1) is negative: the connection Y → X is likely.

More recently, the pairwise likelihood ratios (PW-LR; Hyvärinen & Smith, 2013) approach was proposed. PW-LR builds on the concept of PT. The authors improved on the second step of the inference by analytically deriving a classifier to distinguish between two models X → Y and Y → X, which corresponds to the LiNGAM model (Shimizu, Hoyer, Hyvärinen, & Kerminen, 2006) for two variables. The authors compare the likelihood of these two competitive models derived under LiNGAM’s assumptions (Hyvärinen, Zhang, Shimizu, & Hoyer, 2010) and provide a cumulant based approximation to the likelihood ratio. Within the PW-LR family, there are three methods based on a third cumulant built for two signals X(t) and Y(t):

1. “PW-LR skew,” which is a classic third cumulant
2. “PW-LR tanh” building an alternative approximation to third cumulant using a tanh-based nonlinear correlation
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3. “PW-LR skew” which contains an additional term discounting the outliers
4. “PW-LR kurt” based on the fourth cumulant

The PW-LR approach clearly outperforms all the previously tested methods on the synthetic benchmark datasets (Hyvärinen & Smith, 2013). However, each one of the PW-LR methods is based on a single higher order cumulant based on the BOLD distributions in two signals $X(t)$ and $Y(t)$ (either third or fourth order). It is not robust, as lower moments can also account for possible differences in local signal-to-noise ratio (SNR). Furthermore, the SNR magnitude can differ with respect to the various features of the underlying neuronal time series, and if this is the case, these methods can erroneously flip the direction of the connection.

There are also other statistical methods for pairwise causal inference on the basis of a combination of marginal and conditional probabilities, such as the information-geometric approach by Janzing et al. (2012) or unsupervised inverse regression by Sgouritsa et al. (2015). However, we will not compare our method directly to these methods, as they have not been applied to fMRI datasets to date.

Therefore, we further expand on the concepts of PT and PW-LR by proposing a classifier based on complex cumulants derived from multiple, possibly fractional, moments of the distribution of BOLD recordings. We compare the performance of our approach on synthetic benchmark datasets (Smith et al., 2011) relative to other methods for effective connectivity used in fMRI research. Furthermore, we compare performance of the methods using simple two-node simulations generated from the DCM model with varying signal magnitudes and noise variance in the projecting (upstream) and the receiving (downstream) node.

One needs to make a note about the bidirectionality of connections here. As introduced above, the PT approach (Patel, Bowman, & Rilling, 2006) involves a two-step procedure. In the first step of this analysis, functional connectivity by means of partial correlation is used to find the position of connections in the connectome (regardless of their directionality). The idea is, once a connection is found by means of functional connectivity, we can assume it can either be univ- or bidirectional. In the second step of the analysis, the directionality of a connection is determined through classification of two nodes into an “upstream” and a “downstream” node. Then, under the assumption that the tested method can correctly estimate the directionality of a connection, if the method does not give a univocal answer, we can assume that the connection is bidirectional (as in the first step of the analysis, connection was already detected). However, in case there are reciprocal connections between the two nodes, but one of them is stronger than the other, PT will only detect the “net” effect, namely, it will indicate the stronger between the two connections. In that sense, reciprocal connection will be reduced to a univocal connection. The same effect holds for other pairwise inference methods, namely PW-LR (Hyvärinen & Smith, 2013) and for our approach. In fact, reciprocal connections are ubiquitous in neuronal networks (Kötter & Stephan, 2003). Therefore, proper representation of these bidirectional connections remains an important challenge in network neuroscience.

We demonstrate that our classifier shows higher robustness against these confounds than other methods. In the section Fractional Moment Series of BOLD Signal Distributions, we introduce the concept of fractional moments of the distribution, and in Complex Cumulants of the Distribution, we report on the procedure of combining fractional moments into fractional cumulants. In Combining Fractional Cumulants into a Classifier, we give details on the classifier built on the basis of a set of fractional cumulants, and in Supervised Learning Using Synthetic Benchmark Datasets, we describe the supervised learning procedure. In Selection of Other Approaches for Effective Connectivity Research in fMRI, we list popular methods.
for effective connectivity research in fMRI used for comparison. Finally, in Testing Robustness of the Methods Against Confounds, we describe the generation of additional synthetic data with confounds often encountered in fMRI datasets, which we further use to benchmark the methods.

In Supervised Learning Using Synthetic Benchmark Datasets, we describe the results of the validation of our method using synthetic benchmark datasets (Smith et al., 2011). In the Supporting Information 1, section 5, we present the detailed results of this validation. Furthermore, in Robustness of the Methods with Respect to Confounds, we describe the results of an additional validation performed with the use of the DCM generative model, but in the presence of confounds such as a background noise and variability in hemodynamic responses between upstream and downstream region. Finally, in Discussion, we critically discuss the results.

MATERIALS AND METHODS

Fractional Moment Series of BOLD Signal Distributions

In this work, we propose to estimate causal links from BOLD recordings by analyzing the dependence between an expanded set of (fractional) moments of the BOLD distribution. We keep the same two-stage scheme for the causal discovery as proposed by Patel et al. (2006) and implemented by Hyvärinen and Smith (2013). However, we improve on the second step of the causal inference—a two-node classification problem—by utilizing all moments of the BOLD fMRI distributions and combining them into cumulants.

Figure 1 presents a graphical representation of a dynamical system with just two nodes. In this problem, one region (“upstream”) is sending information to another region (“downstream”) through a connection of weight $A_{12}$. Both regions receive region-specific signal $u_{i}(t)$. This signal can both relate to experimental input, as well as input from other regions. Both nodes also are influenced by a background neuronal noise $\sigma_{i}(t)$ that influence local signal-to-noise ratios.

The BOLD dynamics of such a simple two-node networks can be simulated with the DCM generative forward model (Friston et al., 2003). Given the inputs to the network, the connection strength and sets of parameters characterizing local hemodynamic response within the nodes, the DCM generative model makes prediction on the BOLD dynamics in both nodes. The details of the model are provided in Supporting Information 1. The resulting BOLD time series can then be normalized and characterized in terms of its central moments:

$$M_{k} = \frac{1}{N} \sum_{i=1}^{N} \hat{x}^{k}_{i},$$  \hspace{1cm} (1)

where $k$ in $Q$, $\hat{x}^{k}_{i}$ are the time series with the mean and variance normalized to 0 and 1, respectively, and N is the length of BOLD time series. The novelty in this approach is that a discrete set of moment orders $k$ typically used to characterize a distribution (in terms of mean, variance, skew, etc.), is converted into a (pseudo-) continuous dimension by sampling moment order $k$ from sets of rational numbers. These fractional moments of the distribution are not isomorphic with the moment generating function as we do not convert the distribution of BOLD values into a probability density function at any stage.

Since the original BOLD time series is normalized to mean of 0 (and variance of 1), it contains negative values therefore, the fractional moments will become complex. Since Equation

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**Neuronal noise:**
Background signal in the neuronal network that cannot be explained by the experimental manipulation. This signal can relate to stochastic variability in the neuronal activity, or to the background cognitive processes in the brain that co-occur with the experiment.

**Generative model:**
A model representing prior knowledge of how underlying causal structures are manifested in the experimental datasets.
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Figure 1. A two-node network with one directed connection. (A) The upstream node, $x(t)$, is sending information to the downstream node, $y(t)$, through a single connection of weight $A_{12}$. Both regions received a binary signal $u_i(t)$ and neuronal noise $\sigma_i(t)$. The proportion between the amplitude of $s_i(t)$ and the variance of the noise $\sigma_i(t)$ defines the SNR in the network. (B) All the fractional moments for $k$ in $[0; 5]$, for the BOLD fMRI time series from a simulated two-node network, in the noiseless case. Blue: upstream region. Red: downstream region. The curve starts from $(1, 0)$ for $k=0$, traverses the upper half-plane and arrives at $(0, 0)$ for $k=1$. It travels back through the lower half-plane toward $(1, 0)$ for $k=2$ as the BOLD variance was fixed to 1 through the normalization. Every time $k$ becomes an integer, the curve crosses the real axis.

In Figure 1B, we present a phase diagram for all moments in range $k$ in $[0.0, 5.0]$ for a long simulated BOLD fMRI representing a simple two-node network in the noiseless case. The moments are computed separately for the upstream region (blue) and the downstream region (red). The curve starts at $(1, 0)$ for $k=0$. Then, it traverses the upper half-plane and for demeaned data arrives at $(0, 0)$ for $k=1$. Subsequently, it goes back through the lower half-plane and comes back to $(1, 0)$ for $k=2$ since the variance is equal to 1. Every time $k$ becomes an integer, the curve crosses the real axis. Note that the imaginary axis characterizes the left half of the BOLD distribution, since fractional moments give nonzero imaginary part for negative values of distribution of the BOLD values.
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Complex Cumulants of the Distribution

For two time series \( x(t) \) and \( y(t) \), not only the sole fractional moments but also the asymmetry between the moments can indicate the directionality of a connection. This asymmetry can be quantified by “fractional cumulants”:

\[
C_{kl} = \frac{1}{N} \sum_{i=1}^{N} \left( x_i^k y_i^l - \bar{x}_i^k \bar{y}_i^l \right)
\]

where \( k, l \) in \( Q \).

In this particular problem, \( x(t), y(t) \) denote the BOLD time series in the two-node system. In order to make a prediction (upstream vs. downstream) we learn the dependencies between moment time series by using simulations on the basis of the DCM generative model. We have run 1,000 two-node DCM simulations with \( F_s = 200 \) Hz for a duration of 10 min. In order to marginalize out the influence of the hemodynamic parameters from our results, we sampled the parameters independently for the two nodes and from the empirical distributions (Friston et al., 2003). In order to marginalize out the effect of different input strengths and frequencies, we also sampled the input magnitudes and frequencies (probabilities of switch from on- to off-state and vice versa) from a Gamma distribution with mean and variance of 1.

The input signals driving the upstream and the downstream region were also sampled independently from each other, as trains on- and off-states governed by Poissonian processes. The background neuronal noise was set to 0 in these simulations. In order to obtain a precise estimation of fractional cumulants, we did not subsample our synthetic BOLD every 2–3 s as is typically done for the synthetic BOLD fMRI.

We performed this simulation twofold. First, we fed in an empty connection in order to create a null distribution of cumulants. Second, we added a connection \( A_{12} \) with a weight of 0.9 to the two-node system. In Figure 2, we demonstrate the mean values for all cumulants of indexes \( k, l \) in \([0.0, 5.0]\) obtained from the simulations of a connection, in Cartesian coordinates (we also performed similar computation in polar coordinates; however, it did not give a substantial improvement to the classification performance—the results are presented in the Supporting Information 3). Figure 2A shows the mean values for all cumulants in range \( k, l \) in \([0.0, 5.0]\), over 1,000 simulations of a two node network with one connection (Figure 1). Figure 2B shows the same maps are zoomed into the range \( k, l \) in \([0.0, 3.0]\). Finally, for every cumulant, Figure 2C presents the sign of this cumulant for the majority of 1,000 instantiations of the simulation (we further refer to these binary maps as \( S_r \) and \( S_i \)).

These binary maps do not represent confidence intervals, or discriminability, for a particular cumulant, as “majority” could mean 51% as well as 100% simulations. In order to choose cumulants which can best discriminate between a “connection” and “no connection” case, we created distributions of cumulant values across the 1,000 simulations in the null case and compared against the distributions derived from simulations with a nonzero connection. We smoothed these distributions with kernel smoothing function and, for each cumulant, we computed the percentile of samples falling beyond 95th percentile of the null distribution (in case the mean for the given cumulant is negative as in Figure 2C, we took samples falling lower than the bottom 5th percentile of the null distribution, and higher than the 95th percentile otherwise).
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Figure 2. Mean values for all cumulants in the range k, l in [0.0, 5.0] and their respective signum maps. (A) Mean values for all cumulants, over 1,000 simulations of a two-node network with one connection (Figure 1), for k, l in [0.0, 5.0]. Since cumulants are antisymmetric with respect to indexing k, l, the heat maps for the real and the imaginary component are also antisymmetric. (B) A zoom of A into a smaller range of [0.0, 3.0]. The imaginary component is equal to zero for cumulants of integer orders: k, l in N, but exchanges the sign in the intervals between in a systematic way. (C) Sign of the cumulants in the majority of 1,000 instantiations of the generative model. Red: positive. Blue: negative. Green: zero. We further denote the “signum maps” for real and imaginary components as Sr and Si.

The results of the discriminability analysis in Cartesian coordinates are shown in Figure 3 (the results for polar coordinates are presented in Supporting Information 3). We can observe that whenever one of the indexes k, l equal to zero, that is, the cumulant reduces to a simple moment, it has lower discriminative value than the full cumulants. Therefore, we will disregard moments from further analysis and fully concentrate on full cumulants, that is, asymmetry between moments (k, l > 0).

Interestingly, the range of high discriminability is different for real and imaginary components. Maximal discriminability in both real and imaginary cumulants exists in the range of k + l > 5.0. Additionally, in real cumulants there is a region of high discriminability in the range k + l < 2.0, and in imaginary cumulants there is such region in the range 2.0 < k + l < 5.0 (Figure 3, both boundary lines marked with a white line). This different characteristic along the imaginary axis illustrates that moving from integer to fractional moments of the distribution provides additional predictive power. Note also that as we are deriving the discriminability maps from the DCM generative model. These maps are fingerprints of the particular problem of effective connectivity in fMRI; when derived from another generative model simulating another dataset, the maps would be different.
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Figure 3. Discriminative power for all cumulants in range \( k, l \) in \([0.0, 5.0]\), in the ideal case of a very long BOLD time series and no background neuronal noise. Maximal discriminability in both real and imaginary cumulants exists in the range of \( k + l > 5.0 \). Additionally, in real cumulants there is a region of high discriminability in the range \( k + l < 2.0 \), and in imaginary cumulants there is such region in the range \( 2.0 < k + l < 5.0 \) (both boundary lines marked with a white line).

In order to investigate how the performance of classification based on single cumulants changes when a single connection is embed in a bigger network, we evaluated their success rate in estimating connectivity for benchmark synthetic datasets (Smith et al., 2011). Figure 4 presents the grand mean success rate achieved by every cumulant separately, across all 28 benchmark synthetic datasets (Smith et al., 2011).

Figure 4. Success rate for all the individual cumulants, averaged over 28 simulations from the synthetic benchmark datasets (Smith et al., 2011). White-edged square denotes a single cumulant used by Hyvärinen and Smith (2013). The performance of this cumulant is shown in the color bar, white band. Black-edged squares denote cumulants that give the highest performance on this dataset. Their performance is presented in the color bar, black band. For the cumulants of high indexes \( k + l > 4.0 \) (white line), the success rate is not as high as the discriminability presented in Figure 3 would suggest. The high success rate is not preserved for high-indexed cumulants that achieved high discriminability on two-node simulations. The maximal grand mean performance equals 0.847 for the real components and 0.814 for imaginary components.
The success rate for each of the 28 separate synthetic datasets (Smith et al., 2011) is presented in Supporting Information 4. The maps of simulation-dependent success rate relate to the maps of discriminative power (Figure 3), but they are not identical and differ between simulations. One difference is that for the cumulants of high indexes \( k + l > 4.0 \), the success rate is not as high as the discriminability presented in Figure 3 would suggest. This is because Figure 3 represents the limit of a system of two isolated nodes with infinite SNR, and a very long BOLD time series, whereas benchmark synthetic datasets refer to a more realistic case when for each pair of nodes, the time series is short, there are confounding signals from other nodes in the network, and there is a certain degree of noise in the communication (see Supporting Information 2). Altogether, these factors make the high moments hard to estimate in practice.

**Combining Fractional Cumulants into a Classifier**

We propose to combine information contained in multiple cumulants by building the classifier based on a “voting” scheme between the cumulants. This classifier determines whether the map of cumulants obtained for a pair of time series \( X(t), Y(t) \) is closer to the benchmark maps presented in Figure 2A (which is an evidence for a connection \( X \rightarrow Y \)), or their inverse (which is an evidence for a flipped connection \( Y \rightarrow X \)). Each of the cumulants \( C_{kl} \) votes due to sign \( S_{rk}, S_{ik} \) (Figure 2C). If the sign of the cumulant is the same as in Figure 2C, it adds to the evidence for a connection \( X \rightarrow Y \), and against this connection otherwise.

Since in realistic conditions (short datasets, large TRs), high index cumulants, \( k + l > 3.0 \), yield the aforementioned estimation problem, we discount their contribution in the voting by using a nonlinearity of a form (further referred to as weighting throughout the manuscript):

\[
f(x) = \log\left(\cosh(\max(x,0))\right)
\]

A similar function was proposed to discount the outliers present in the BOLD time series in the work by Hyvärinen and Smith (2013). The final classifier yields:

\[
\begin{cases} 
X \rightarrow Y & \text{if } \sum_{k,l} [S_{rk} f(C_{rk}) + S_{ik} f(C_{ik})] \geq 0 \\
Y \rightarrow X & \text{otherwise}
\end{cases}
\]

in the weighted case, and

\[
\begin{cases} 
X \rightarrow Y & \text{if } \sum_{k,l} [S_{rk} C_{rk} + S_{ik} C_{ik}] \geq 0 \\
Y \rightarrow X & \text{otherwise}
\end{cases}
\]

in the unweighted case.

**Supervised Learning Using Synthetic Benchmark Datasets**

In this work, we derive the classifier by using sign maps \( S_{rk}, S_{ik} \) (Figure 2C) developed using multiple realizations of a two-node simulation under the DCM generative model, which is a form of supervised learning. In a different application and under a different generative model, these maps would look differently.
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We know that cumulants differ with respect to discriminability (Figure 3), and that the success rate of cumulants differs depending on the range \( k + l \leq \text{Ind}_{\max} \) (Figure 4). Therefore, we optimize the performance of the classifier with respect to these two dimensions by finding a combination that gives the best grand mean performance across the 28 simulations from the synthetic benchmark datasets, as they represent the variety of experimental conditions encountered in real-life fMRI setups.

First, we fix \( \text{Ind}_{\max} = \max(k, l) = 3.1 \), and consider cumulants on a triangle \( k; l \geq 0.1, k + l < \text{Ind}_{\max} \). We then choose only cumulants of discriminability exceeding a particular value to be fed into the classifier. For instance, a cutoff value of 0.1 means that we include the vote from all cumulants for which the discriminative value is not less than 0.1. We can then evaluate the grand mean success rate (as the mean success rate over all 28 benchmark synthetic datasets) in the function of the thresholding discriminability value. Figure 5A, demonstrates that including

![Figure 5. Dependence of the grand mean performance on synthetic datasets on the choice of cumulants. (A) Grand mean performance for unweighted cumulants in range \( k + l \leq 3.1 \), in the function of the cutoff discriminative value. The higher cutoff, the less cumulants we take into account while voting for the directionality of the link. The results clearly show that in order to maximize the success rate in estimating effective connectivity, all the cumulants should be taken into account, except for the diagonal of \( k = l \). (B) The grand mean performance based on cumulants of indexes \( k, l \) between 0.1 and \( k + l \leq \text{Ind}_{\max} \), in the function of that maximal index. The optimal performance in unweighted case equals 0.835 for \( [\text{Ind}_{R_{\max}}; \text{Ind}_{I_{\max}}] = (2.4, 1.7) \), and 0.886 for \( [\text{Ind}_{R_{\max}}; \text{Ind}_{I_{\max}}] = (2.1, 3.7) \) in weighted case.](#)
all the cumulants with a positive discriminative value (all cumulants except for \( k = l \), for which discriminability is always zero) gives the best classification performance.

Second, we optimize the window \( \text{Ind}_{\text{max}} \) for indexes \( k, l \) and compare between classifier with and without weighting with the discount function introduced in Equation 3. Since discriminability is generally higher for low indexes \( k, l \) (Figure 3), we will evaluate the grand mean performance based on cumulants of indexes between 0 and a maximum \( \text{Ind}_{\text{max}} \) in the function of that maximum. We consider the maximal indexes along real and imaginary dimension separately. The results are presented in Figure 5B. The optimal performance in an unweighted case equals 0.835 for \( [\text{Ind}_{R_{\text{max}}}, \text{Ind}_{I_{\text{max}}}] = (2.4, 1.7) \), and 0.886 for \( [\text{Ind}_{R_{\text{max}}}, \text{Ind}_{I_{\text{max}}}] = (2.1, 3.7) \) in a weighted case, which exceeds both the grand mean performance of the “PW-LR skew r” method by Hyvärinen (0.845) and the maximal grand mean performance of any single cumulant in our study (Figure 4, the maximum of 0.847).

**Selection of Other Approaches for Effective Connectivity Research in fMRI**

In order to benchmark our classifier, we compare the performance against other methods. As mentioned in the Introduction, the field of effective connectivity in fMRI is very wide (Smith et al., 2011); therefore, we restricted ourselves to the most popular approaches (other than DCM itself; Friston et al., 2003):

1. **State-space implementation of Granger causality (GC; Granger, 1969; Seth, Barrett, & Barnett, 2015)** is a multivariate method inferring effective connectivity between a pair of time series under the assumption that both of them can be expressed as autoregressive processes. We used a simple version of GC featuring ordinary least square regression with lag of 1, implemented in Multivariate Granger Causality Toolbox (Barnett & Seth, 2014), obtained from http://www.sussex.ac.uk/sackler/mvgc. For GC based on VAR process, as in our study, the state-space implementation is more robust than spectral GC (Geweke, 1982, 1984), because the frequency-domain version has a bias-variance trade-off (a function of the VAR model order that can induce spurious conditional GG causality estimates, such as erroneous peaks in the frequency domain, as indicated in the recent work by Stokes and Purdon, 2017). Furthermore, the state-space formulation of GC is the most robust, mitigating effects of bias and variance due to the fact that the reduced model is VAR rather than VARMA (Barnett & Seth, 2015). In order to compare performance with the methods for pairwise inference, we used GC in a bivariate rather than multivariate fashion: by applying GC to each of the previously found connections separately.

2. **Partial Directed Coherence (PDC; Baccalá & Sameshima, 2001)** is known as a method conceptually close to Directed Transfer Function (Kamiński & Blinowska, 1991; Baccalá & Sameshima, 2001). Both these methods are derivatives from Geweke spectral measures of GC (Geweke, 1982, 1984), and all three methods have similar limitations (Chicharro, 2011). However, PDC is used substantially more often in fMRI studies than the other two methods, especially when compared with spectral GC. For this reason, we chose PDC as a method representing this class of approaches. We used PDC implementation from the Extended Multivariate Autoregressive Modelling Toolbox (Faes, Erla, Porta, & Nollo, 2013; http://www.lucafaes.net/emvar.html). As in case of GC, we applied PDC in a bivariate fashion.

3. **Patel’s tau (PT; Patel et al., 2006)**, as described in the Introduction, is implemented similarly as in Smith et al. (2011) by recalculating each time series into the range \([0, 1]\), setting samples under the 10th percentile to 0, over the 90th percentile to 1, and linearly
mapping the remaining samples to the range \([0, 1]\). Then, we infer the directionality of connection from the difference between \(P(X|Y)\) and \(P(Y|X)\). In addition to the previous implementation, however, we also integrate the results over all the possible thresholds in order to eliminate the thresholding problem while calculating the conditional probabilities \(P(X|Y)\), \(P(Y|X)\).

4. Pairwise likelihood ratios methods (PW-LR; Hyvärinen & Smith, 2013) are represented by “PW-LR r skew,” as it gives the highest performance among all the PW-LR methods when applied to synthetic fMRI data. We obtained the codes for the PW-LR methods from https://www.cs.helsinki.fi/u/ahyvarin/code/pwcausal/ (Hyvärinen used the cumulant \(k; l = (2; 1)\) weighted with covariance for synthetic benchmark datasets). Therefore, for this comparison, we use the classifier based on fractional cumulants weighted with covariance. For our study, we chose a PW-LR r skew version of the method, which involves an inference based on a third cumulant with a discount for outliers (Hyvärinen & Smith, 2013).

As in Hyvärinen & Smith (2013), we performed the first step of the inference as inverse covariance thresholded with permutation testing. All the methods, including multivariate methods such as GC and PDC, were then applied in a pairwise fashion (i.e., separately for each two-node network representing a single connection found in the previous step).

Furthermore, we did not include the DCM procedure in this comparison, for the same reasons as Smith et al. (2011): DCM is not an exploratory method and using it in this context, namely for exploratory causal research on the set of benchmark synthetic datasets (where the smallest network consists of five nodes) is not computationally feasible. Furthermore, the characteristics of this synthetic benchmark dataset is that input signals (Figure 1A) represent random events and can therefore emulate all types of fMRI experiments: classic task-fMRI studies, event-related responses or resting-state BOLD time series. In DCM, however, the inputs must be strictly specified in the block design, otherwise DCM inference cannot be initiated. Therefore, assumptions of DCM do not fit a research problem formulated in this particular way.

Testing Robustness of the Methods Against Confounds

In addition to evaluating our approach against the existing simulations from Smith et al. (2011), we further evaluate the performance under additional yet typical modes of variation in the data. Specifically we are interested in characterizing the discriminative performance relative to (i) more complex forms of stochastic noise in the data and (ii) unequal levels of SNR per node.

The benchmark synthetic datasets involve temporally uncorrelated, white background noise of a low magnitude on the neuronal level (Smith et al., 2011). This type of noise is not physiologically plausible, as it is known from physiological studies that in the neuronal networks, the background noise has a scale-free power spectrum (He, 2014; Bédard et al., 2006; Dehghani et al., 2010; Bielczyk et al., 2017). Therefore, we simulated a two-node system and introduced scale-free (pink) noise to the system. Then, we varied the variance of the noise in the range of \([0.2, 5.0]\) while keeping the amplitude of the inputs \(s_i(t)\) fixed to 1.0. We performed 500 realizations of 10 min simulation at high temporal resolution of \(F_s = 200\) Hz, for each configuration of the noise variances.

Furthermore, in the original version of the DCM procedure (Friston et al., 2003), as well as in most computational studies (Smith et al., 2011), equal stimulus strengths to both nodes \(s_i(t)\) are assumed. This assumption might not hold true in the real fMRI datasets. Therefore, we performed another, noiseless simulation, in which we varied signal strengths between the
upstream and downstream region. We performed 500 realizations of 10-min simulation at high
temporal resolution of $Fs = 200$ Hz for each configuration of input strengths in the range of
$[0.2, 5.0]$.

RESULTS

Supervised Learning Using Synthetic Benchmark Datasets

The best version of the classifier was obtained for voting between cumulants in the range
$[\text{IndR}_{\text{max}}; \text{IndI}_{\text{max}}] = (2.1; 3.7)$, and with the discount for high moment indexes (Equation 3). The comparison of this classifier against four other methods, GC, PDC, PT, and PW-LR r skew, on the benchmark simulation no. 2 is presented in Figure 6. The violin plots denote the distribution of the z-scores for connections as compared to the null distribution. Blue dots denote the percentage of correct assignments for the true connections, as in Smith et al. (2011). In most of the other 27 benchmark datasets, our classifier outperforms all the other methods (Supporting Information 5. As in the original study by Smith et al. (2011), lagged methods, GC, and PDC perform worse than the structural methods. PW-LR r skew and fractional cumulants both outperform PT, most probably because PT is based on the thresholded signal and therefore contains a free parameter.

In general, in the benchmark synthetic datasets, fractional cumulants outperform all other

techniques in almost all cases, although the difference between performance of the fractional
cumulants and PW-LR methods is small.

Figure 6. Comparison between the classifier based on the fractional cumulants and several other methods on benchmark simulation no. 2. The violin plots denote the distribution of the z-scores for connections as compared with the null distribution. Blue dots denote the percentage of correct assignments for the true connections (Smith et al., 2011). The difference in performance between the classifier based on fractional cumulants and PW-LR r skew (Hyvärinen & Smith, 2013) is small.
Robustness of the Methods with Respect to Confounds

Figure 7 presents the comparison between the fractional cumulant classifier and various other methods on a two-node simulation under the addition of varying levels of physiologically plausible, scale-free neuronal noise across various levels of SNR for the upstream and downstream regions. The results suggest that all previously tested methods show low levels of robustness toward such additional sources of variability in the data. GC as well as PDC are at the lowest performance, and give results on a chance level across the whole parameter space. In PT, the success rate in proper classification between upstream and downstream node rapidly drops toward 50% along with a decrease in SNR in the upstream node. However, in case SNR in the upstream node is higher than 1 (left half of the Patel’s tau panel, Figure 7), PT is resilient to the variance of SNR in the downstream node. The performance of PW-LR r skew drops down to the chance level with respect to both the noise level in the upstream and downstream region, whereas GC and PDC are performing almost equally poorly under any combination of noise variances (probably because the variance of the noise from the fitted autoregressive model is used to establish the directionality of the causal influence in GC).

Figure 7. Robustness of the methods against the background scale-free neuronal noise. The variance of the background noise s(t) differs between upstream and downstream region in the range of [0.2; 5.0]. As signal magnitude is constant and equal to 1 in these simulations, the signal-to-noise ratio (SNR) was calculated as \(1/\text{var}(\sigma)\).
Figure 8. Robustness of different methods to the change in signal strengths. The variance of the signal differs between upstream and downstream region, both in the range of [0.2, 5.0]. GC and PDC give performance around the chance level across the whole parameter space, whereas PW-LR r skew and PT exhibit certain resilience toward this variability in the inputs. However, the classifier based on fractional cumulants is the only method whose performance does not fall toward the chance level within the parameter space.

DISCUSSION

Summary

This work provides an advance to the effective connectivity research in fMRI, by utilizing the additional information contained in the BOLD time series with use of fractional moments of the BOLD distribution combined into cumulants. Usage of this additional information (embedded within a classifier) significantly increases the robustness toward plausible sources of variability.
in fMRI, namely presence of physiologically realistic (scale-free) background noise as well as disproportion in the inputs strength, either due to differences in the amount of neuronal activity locally induced and/or due to effective differences induced by, for example, regional variations in the coil sensitivity profiles. This is where the value of added information coming from fractional cumulants becomes apparent: among the methods tested in this work, only the classifier based on the fractional cumulants gives a performance better than chance across the whole parameter space in these experimentally realistic conditions.

Effective connectivity is a research problem directly related to the notion of causality. Causality is, in general, difficult both to define (Pearl, 2000) and to measure. In the most basic formulation, in X causes Y, it means that without X, Y would not occur. The picture is far less clear in complex dynamic systems such as brain networks: for any event, a high number of potential causes can be defined, and these causes most often interfere with each other. This research problem was recently discussed by Albantakis, Marshall, Hoel, and Tononi (2017) who decomposed causality into independent dimensions: realization, composition, information, integration, and exclusion. Also, interpretation of a causal component in a given process depends on the context. For example, respiratory movement is typically considered a confound in fMRI experiments, unless we are interested in the influence of respiration speed on the activity of neuronal populations. Furthermore, in brain networks, temporal ordering of the cause of cause and effect is hard to maintain, as information is circulating in recurrent rather than feed forward networks (Schurger & Uithol, 2015).

Furthermore, if we have an interventional study at disposal, establishing causality becomes straightforward, but this is rarely the case in human brain research. In human fMRI, all the studies are observational rather than interventional. In such cases, causation can never be observed directly, just correlation (Hume, 1772). When a correlation is highly stable, we are inclined to infer a causal link. Additional information is then needed to assess the direction of the assumed causal link, as correlation indicates for association and not for causation (Altman & Krzywinski, 2015).

In the simulations, we used multiple realizations of the dynamics for every network pattern. We needed to run multiple instantiations of a noisy system in order to evaluate the mean success rate of the methods under noisy circumstances. We also simulated long runs of the dynamics aiming to find an upper bound on the methods’ performance in a function of the SNR disproportion and background noise levels. We did not investigate the effects of the sample length on the results. This is because in our study, we focused on the confounding factors which—unlike the duration of the study—cannot be influenced by the researcher.

Although fractional moments of a distribution, as a mathematical concept, were studied before (Dremlin, 1994; Matsui & Pawlas, 2013), this concept was not applied to biomedical sciences to date. One reason for this lack of applications might be that the fractional moments become complex numbers for the normalized time series, and that subsequently, the features characterized by these moments cannot be conceptualized as easily as the features characterized by the integer moments (e.g., skewness can be interpreted as a measure of “asymmetry” of the distribution, and kurtosis can be interpreted as its “flatness”). However, although the fractional moments of a distribution are a mathematical concept with limited practical interpretation, nevertheless they could still contain valuable, in our case causal, information. In this work, we demonstrate that fractional moments provide important additional information about the distribution of BOLD values. We first perform supervised learning of the classifier on the set of benchmark synthetic datasets, and then validate the classifier on two-node
Improving pairwise inference in fMRI using fractional moments of BOLD simulations with biologically realistic confounds. We believe that confounding factors such as a physiological background noise of a magnitude varying between the nodes is important to overcome for any method for causal inference in fMRI. This is because every network in the brain is embedded in larger networks, and therefore the background activity of other interconnected networks can be interpreted as “noise” (Deco, Jirsa, & McIntosh, 2011). We demonstrate that our approach can increase the robustness of the methods for pairwise inference in fMRI to the main sources of variability in BOLD fMRI.

Unlike the previous methods for pairwise inference in fMRI (Hyvärinen & Smith, 2013), the classifier defined in this study is informed by the dynamic causal modeling generative model, therefore it incorporates the priors derived from the neurophysiological studies (Buxton, Wong, & Frank, 1998). Deriving benchmark signum maps for the classifier from the multiple instantiations of the DCM generative model allows for marginalizing out all the parameters unimportant for the effective connectivity research: the classification procedure focuses only on classifying a pair of regions into upstream and downstream instead of fitting all the hyperparameters as is done in the classic DCM inference procedure. Therefore, this approach is a reduction of the problem of effective connectivity in a large network to a two-node classification problem on one hand, and an extension of the feature space from integer to fractional moments on the other hand.

The signum maps derived in the training process are dependent on the generative model. In this study, we chose the canonical, original formulation of the DCM (Friston et al., 2003). There are also newer formulations of the DCM, for example, the canonical microcircuit approach (Pinotsis et al., 2017; Friston et al., 2017), in which layer-specific neuronal populations in the cortex are modeled with use of a neural mass model. In this case, we assume that the inference is performed on mesoscale level, in which ROIs represent brain regions (cortex regions or subcortical nuclei) rather than cortex components. Furthermore, although versions of DCM containing higher order, nonlinear effects (Stephan et al., 2008) are also developed, we believe that a (bi)-linear model is a good simplification to describe the underlying neuronal dynamics, as it refers to the linear part of the sigmoidal transfer functions between neuronal populations in the brain (Silver, 2010; Bielczyk, Buitelaar, Glennon, & Tiesinga, 2015). Modeling communication between nodes in the network with use of linear transfer functions is a common practice in modeling effective connectivity in fMRI (see, e.g., structural equation models (McIntosh & Gonzalez-Lima, 1994) or LiNGAM-ICA (Shimizu et al., 2006; Smith et al., 2011). The bilinear version of DCM, often referred to as the general linear model approach, is still a very popular tool for finding effective connectivity patterns from fMRI in clinical practice (see, e.g., recent work by Zhang et al., 2018; Nackaerts et al., 2018; Arioli et al., 2018; Pool et al., 2018).

We reproduced the DCM generative model after Smith et al. (2011). This implementation is acknowledged in the field as a benchmark setting for testing new methods for functional and effective connectivity in fMRI (see, e.g., Hyvärinen & Smith, 2013; Hinne et al., 2015; Bielczyk et al., 2019). In this implementation, bilinear effects (namely, modulation of connections by experimental inputs) are not modeled. In pairwise inference this omission is justified, as modulation of connection only affects the strength of connection weight $A_{12}$, which will influence cumulant values quantitatively and not qualitatively, which does not influence the outcome signum maps.

Evaluating methods with the use of synthetic datasets as the ground truth is typically the first step in the validation of any new data analytic framework. Validating new methods with use of
Improving pairwise inference in fMRI using fractional moments of BOLD

synthetic datasets is a state of the art technique across the whole field of neuroimaging, from single cell imaging to whole-brain imaging with fMRI or EEG/MEG. This tradition has a long history, starting from the Nobel-winning Hodgkin and Huxley model for initiation and propagation of action potential (Hodgkin & Huxley, 1952). Today, methods for effective connectivity between neuronal assemblies measured with multielectrode arrays are still validated on synthetic datasets generated from this classical model, including recent approaches: nonlinear data assimilation (Hamilton, Berry, Peixoto, & Sauer, 2013) and differential covariance (Lin, Das, Krishnan, Bazhenov, & Sejnowski, 2017). In cognitive neuroimaging, testing methods on synthetic datasets from generative models is also a standard. In EEG/MEG research, there are multiple classes of generative models generating different type of dynamics, depending on the purpose of the modeling study, for example, the nonlinear lumped-parameter model for generating alpha rhythms and its neural mass extension by (David & Friston, 2003), the Wong-Wang model for winner-take-all dynamics (Wong & Wang, 2006), the Hindmarsh-Rose model for epileptor dynamics (Hindmarsh & Rose, 1984), and DCM for EEG/MEG (Kiebel, Garrido, Moran, Chen, & Friston, 2009; Steen, Almgren, Razi, Friston, & Marinazzo, 2018; Moran, Pinotsis, & Friston, 2013). Furthermore, the Human Neocortical Neurosolver simulator developed at Brown University (HNN, https://hnn.brown.edu) is a complex tool simulating local field potentials measured with EEG/MEG by bottom-up modeling of clusters of neurons. All these tools can serve to validate new methods for functional and effective connectivity in EEG/MEG (Valdes-Sosa et al., 2011; Wang et al., 2014).

In fMRI, the selection of generative models is narrower than in EEG/MEG: DCM (Friston, Moran, & Seth, 2013; Smith et al., 2011) achieved a status of the standard generative model. With use of this synthetic data generated from this model, new methods for effective connectivity in fMRI are validated, for example, the third- and fourth-cumulant method by Hyvärinen and Smith (2013) and artificial immune algorithm combined with the Bayes net method (AIAEC; Ji, Liu, Liang, & Zhang, 2016).

To date, DCM is the most biologically relevant generative model proposed in the field of functional magnetic resonance imaging. The implementation of benchmark synthetic datasets based on DCM by Smith et al. (2011) has gained a lot of attention and following in the field, but it has also gained its critics. For instance, according to Smith’s results, PT (Patel, Bowman, & Rilling, 2006) is one of the methods giving best performance in retrieving directed connectivity patterns from synthetic benchmark datasets. In a recent work, Wang, David, Hu, and Deshpande (2017) performed a modeling study on datasets derived from an experiment by David et al. (2008) in which fMRI activity was measured in genetically modified rats suffering from epilepsy. Activity from the same set of regions was recorded in an associated intracerebral EEG study in order to provide ground truth information flow. The authors chose primary somatosensory cortex barrel field (S1BF), thalamus, and striatum (caudate-putamen; CPu) as ROIs and demonstrated that Patel’s tau is no better than chance in recovering directional connectivity patterns from this data in both raw and deconvolved fMRI datasets. On the contrary, DCM and Granger causality proved to correctly estimate the directionality of the information flow on the group level and on the deconvolved data.

There are more caveats to be noted with respect to the benchmark DCM simulations by Smith et al. (2011). First, the synthetic datasets derived by the authors of the study involve a low noise condition, in which the background noise in the networks in as low as 5% of the signal magnitude. Given that the background activity in the brain networks includes not only noise but also echo of cognitive processes unrelated to the experiment, 5% of background activity seems to be on the lower end of the spectrum of possibilities. Second, networks
investigated by Smith et al. are sparse (a number of connections in a network of size N is of order of N) and almost acyclic, which also seems to be a very optimistic scenario. Third, Smith et al. used a TR of 3 s and time series of 200 data points. This TR is too long, and the time series length too short for generalizing the empirical datasets used today. Today, shorter TRs (1 s or less, e.g., 0.72 s as in Human Connectome Project datasets; Van Essen et al., 2013), and longer time series have become the norm (e.g., 4,800 samples in resting-state datasets from Human Connectome Project datasets; Van Essen et al., 2013). Last, Smith et al. used a fixed delays of 50 ms in the first layer of the DCM model, representing the underlying neuronal communication. This delay represents synaptic transmission delays and axonal transmission delays between nodes of the network. The constant value of delay is a crude estimation, especially given that pairs of brain regions positioned at different distances from each other should have different axonal transmission delays. Also, given polysynaptic connections, effective delays between neuronal populations might be much higher than the aforementioned 50 ms. For example, P300 potential appears after 300 ms (Polich, 2007), and some other cortical potentials have even slower latencies. This lack of attention toward modeling neuronal delays might favor nonlagged methods (i.e., PT or LiNGAM) and might be preferred in this analysis over the lagged methods. Altogether, there are reasons to believe that benchmark datasets derived by Smith et al. are, to some extent, not representative of the real fMRI datasets. For these reasons, results of validation on the benchmark datasets should be interpreted with care.

There are also other, competitive generative models in the field, for example, the model proposed by Seth et al. (2013). In this model, the authors used a simple VAR generative model in order to simulate neuronal dynamics in the testing network of five regions, based on work by Baccalá and Sameshima (2001). Subsequently, the VAR model output is convolved with five different HRF kernels generated with use of the difference-of-gamma approach as implemented in SPM8 (http://www.fil.ion.ucl.ac.uk/spm/software/spm8/). Another possibility, would be to use local field potentials (LFPs) instead of neuronal dynamics simulated as a system of differential equations with delay, and convolve LFPs with HRF (Deshpande, Sathian, & Hu, 2010). However, to date, Smith’s synthetic datasets derived from the DCM generative model remains the benchmark datasets.

Furthermore, in this work, we performed the inference on the full BOLD response, without deconvolving the BOLD time series into the neuronal time series. It has been shown in synthetic and empirical data that incorporating a physiologically based model of spatiotemporal hemodynamic response function into the preprocessing pipeline leads to an improvement in the estimated neuronal activation (Aquino, Robinson, Schira, & Breakspear, 2014). It was also shown that it is generally difficult to accurately recover true task-evoked changes in BOLD fMRI time series irrespectively of the method chosen for modeling HRF response (Lindquist et al., 2009). Hence, there is a long-lasting debate in the field of connectomics on whether or not a (blind) hemodynamic deconvolution is necessary to perform the (effective) connectivity research in fMRI (Wu et al., 2013). For instance, structural equation models (McIntosh & Gonzalez-Lima, 1994) are often applied without deconvolution to fMRI datasets (Schlösser et al., 2003; Zhuang, LaConte, Peltier, Zhang, & Hu, 2005). Our previous theoretical research in synthetic datasets generated from the DCM model suggests that deconvolution is not necessary in effective connectivity research in fMRI if the method used in the study is not lag dependent (Bielczyk et al., 2017). This is because under the assumption that the underlying signal on the neuronal level is in the low-frequency range, the hemodynamic response—as a low-pass filter—does not affect the signatures of different connectivity patterns present in the outcome BOLD response. Therefore, in this work, we did not perform the deconvolution step before assessing effective connectivity with any of the tested methods, including Granger causality. In fMRI
literature, GC is applied both with (David et al., 2008; Ryali, Supekar, Chen, & Menon, 2011; Ryali et al., 2016; Hutcheson et al., 2015; Wheelock et al., 2014; Sathian, Deshpande, & Stilla, 2013; Goodyear et al., 2016) and without (Zhao et al., 2016; Regner et al., 2016; Chen et al., 2017) use of deconvolution. Recent research suggests that all connectivity methods (including functional connectivity) will improve their estimation accuracy post-HRF deconvolution (Rangaprakash, Wu, Marinazzo, Hu, & Deshpande, 2018). However, in this work, we chose for implementation without deconvolution, to be consistent with (Smith et al., 2011).

Here, we would like to mention that in recent years a lot of progress has been made in the area of modeling local hemodynamics from fMRI datasets. For example, Havlicek et al. (2011) proposed a new approach to modeling hemodynamic response functions based on cubature Kalman filtering. Furthermore, Bush et al. (2015) proposed and validated a meta-algorithm for performing semiblind deconvolution of the BOLD fMRI by using bootstrapping. This method allows for estimating the timing of the underlying neural events stimulating BOLD responses, together with confidence levels. Sreenivasan et al. (2015) proposed a nonparametric blind BOLD deconvolution method based on homomorphic filtering.

Last, in our simulations we have set the connection strength to a fixed value of \( a = 0.9 \). On this stage, the output of the classifier (Equation 4 and Equation 5) is a binary response, that is, an indication for a connection, either \( X \rightarrow Y \) or \( Y \rightarrow X \). This indication is based on a linear combination the binary signum maps \( S_r, S_i \) (Figure 2C) with the values of the cumulants \( C_r, C_i \) computed for the given dataset \( X(t), Y(t) \), in either weighted or unweighted form. If the connection strength varies, the strength of the coupling between fractional moments in \( X(t) \) and \( Y(t) \) varies accordingly. Therefore, the absolute values of the associated fractional cumulants will also adjust. If cumulant values scale, then the RHS sum in Equations 4 and 5 will scale accordingly, but the signum of this sum should stay the same. Therefore, in a noiseless case, this classifier should give the same response regardless of the connection strength \( a \).

Limitations of the Method

It is also important to remember that there are always two independent aspects to a method for causal inference. First, the method should have assumptions grounded in a biologically plausible framework relevant for the given research problem. For instance, a method for causal inference in fMRI should respect (1) the confounding, region-, and subject-specific BOLD dynamics (Handwerker, Ollinger, & D’Esposito, 2004) and (2) co-occurrence of cause and effect (since the time resolution of the data is low compared with the underlying neural dynamics; the causes and their effects most likely happen within the same frame in the fMRI data). The new methods for pairwise inference such as classifying on the basis of fractional cumulants address this issue by (1) breaking the time order, and performing causal inference on the basis of statistical properties of the distribution of the BOLD samples, and not from the timing of events; and (2) using correlation in order to detect connections. A good counterexample here is GC, which has been proven useful in multiple disciplines. However, there is an ongoing discussion on whether or not GC is suited for causal interpretations of fMRI data. On the one hand, theoretical work by Seth, Chorley, and Barnett (2013) and Roebroeck, Formisano, & Goebel (2005) suggest that despite the slow hemodynamics, GC can still be informative about the directionality of causal links in the brain. Second, an estimation procedure needs to be computationally stable. Even if the generative model faithfully describes the data, it still depends on the estimation algorithm as to whether the method will return correct results. However, the face validity of the algorithms can only be tested in particular paradigms, in which the ground truth is known. If in the given paradigm the ground truth is unknown, which is most often the case in fMRI experiments, only reliability can be tested.
Our approach requires certain assumptions. For instance, we assume that on the neuronal level, effects of directed connectivity are linear. This is also an assumption underlying the original DCM model used in this study. However, it is known that this is not always the case in the neuronal dynamics. For instance, shunting inhibition (Alger & Nicoll, 1979) is a phenomenon in which excitatory potential is reduced by division rather than by subtraction. However, effects such as shunting inhibition typically happen in microscale and should not affect large-scale neuronal dynamics as measured by BOLD fMRI. Therefore, we do not consider effects such as shunting inhibition as plausible confounds to our approach.

One crucial limitation of our approach (as well as previous methods such as pairwise likelihood ratios) is that these techniques only retrieve the net connectivity. Namely, what these methods effectively pick up on is the difference between connectivity strengths, and in a scenario where the connectivity strengths X \rightarrow Y and Y \rightarrow X are equal, the outcome cumulant map for the system will have lower amplitudes than in case of a unidirectional connection X \rightarrow Y with the same connection strength. The significance of the cumulant values (whether or not the values are significantly different from zero) can be established with use of permutation testing. For more details, see Supporting Information 6, Figure 12B. However, since in the first step of the inference we select only strong functional links for further classification, we can interpret ambiguous output from the classifier as a bidirectional connection. Since the quality of the classifier depends on the ability to determine the directionality of a unidirectional connection, we used only unidirectional connections in the validation stage. One point to make here is that bidirectional connections are hard to disambiguate for many methods for effective connectivity, as they represent cyclic nets. This is an issue, for example, in applications of Bayesian nets (Pearl, 2000), in which joint probability for a certain graph is not tractable when propagation of information in the network is cyclic. Also, the third- and fourth-cumulant method by Hyvärinen and Smith (2013) falls into this category, as the asymmetry between third cumulants in the case of bidirectional connections will be equal to zero.

One interesting direction for the method development would be also classification between excitatory and inhibitory connectivity. This aspect is missing in our study, as we focus on the connections of a positive sign only deriving a classifier able to distinguish between excitatory and inhibitory connections would require deriving an additional set of benchmark S_r and S_i maps built on the basis of repetitive simulations of an inhibitory connection, and creating a new set of benchmark synthetic datasets, as the original datasets by Smith et al. (2011) involve excitatory links only. The reason why inhibition is not implemented yet, is because the cumulant patterns for inhibitory connection are different from patterns given in Figure 2 (we did not include the pictures in this manuscript though). The classifier in its current form gives a binary decision on whether the connection is going in the direction of X \rightarrow Y or Y \rightarrow X. The decision is based on whether the cumulant pattern obtained from the given dataset X(t), Y(t) is more similar to the signum maps derived from DCM for connection X \rightarrow Y (Figure 2C) or to the inverse of these signum maps. In order to add inhibition to the picture, one would need to extend the number of possible classes by adding signum maps derived from DCM for inhibitory connection X \rightarrow Y and introducing some metrics of distance to excitatory/inhibitory signum maps. This is the next step to take. One thing to note in addition is whether or not inhibitory effective connectivity is expected in large-scale networks investigated with fMRI; this is a matter for debate. On one hand, it is known that long-distance projections in the brain are mostly excitatory as inhibition is typically local, presented by groups of tightly connected interneurons within single brain regions (Markram et al., 2004), which is also modeled in the DCM generative model with use of the self-inhibition term (Friston et al., 2003). On the other hand, several anatomical and physiological studies indicate that there are also long-range inhibitory
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connections, for example, between hippocampus and entorhinal cortex in rats (Melzer et al., 2012).

Furthermore, we consider local variability in brain physiology by varying hemodynamic responses between realizations of the simulation and by studying the effects of the scale-free background noise on the resulting effective connectivity measures. The DCM generative model summarizes the current state of knowledge about the mechanisms underlying generation of the BOLD response (Friston et al., 2017; Havlicek et al., 2015; Havlicek, Ivanov, Roebroeck, & Uludag, 2017). Therefore, we do not have efficient ways of incorporating human brain physiology into our consideration in any more depth than this model allows for.

However, at the same time, we do not consider the influence of artifacts from the data acquisition, such as the effects of movement in the scanner. We believe that the influence of such confounders should be limited by a proper data preprocessing. For instance, motion artifacts can be reduced with use of new, data-driven protocols for motion artifacts removal such as ICA-AROMA (Pruim et al., 2015) or a censoring-based artifact removal strategy based on volume censoring (Power et al., 2014). Therefore, developing efficient strategies for controlling such confounders is beyond the scope of this paper.

Future Research

In the context of fMRI research, increasing the granularity of moments in order to better characterize the distribution is an especially useful application because the experimental fMRI datasets are short (a few thousand samples at most); therefore, the estimation error for the high-order integer moments of the distribution becomes high. However, the subsequent cumulants contain information redundant to a certain extent, as they are correlated for any given time series x(t). We chose the granularity that gives smooth patterns of discriminability (Figure 3), which is $\Delta_k = 0.1$. Choosing the optimal moment resolution is a subject to future research; although, we believe that increasing index resolution to substantially less than 0.1 would not be beneficial, yet it would substantially increase the computational cost for the method.

In this work, we validated our approach by using synthetic benchmark datasets derived from the DCM generative model. Using generative models is valuable in neuroimaging, in terms of validating new methods as mentioned in point 2, but also in applied research. Especially in the fields of applied computational psychiatry (Frässle et al., 2018) and network neuroscience in general (Betzel & Bassett, 2017), using generative models is valuable because these models enable inference on model parameters, which afford for some degree of mechanistic interpretability on the putative processes underlying the studied phenomena. Generative models, especially DCM, are acknowledged in multiple contexts in the field of cognitive neuroimaging, from method validation to application in clinical datasets. However, a valuable method should also give predictions testable in clinics (e.g., lead to more reliable estimation of directed causal influences during cognitive tasks, lead to better stratification of clinical populations in resting state, etc.), which we will also further investigate.

In this work, we faithfully reproduced the pipeline after Smith et al. (2011). However, since the original version of DCM (Friston, Harrison, & Penny, 2003) based on the original Balloon-Windkessel model (Buxton, Wong, & Frank, 1998) was published, substantial advancements to the hemodynamic model have been proposed. First, Obata et al. (2004) reported an error in the expression for the outcome BOLD response (Equation 8, Supporting Information 1: DCM forward model). Stephan, Weiskopf, Drysdale, Robinson, & Friston (2007) proposed a more
accurate expression for one of the terms in this formula. Second, Heinzle, Koopmans, den Ouden, Raman, & Stephan (2016) proposed an updated formula for field strengths higher than 1.5 T. In the following work, we will take these improvements into account.

Furthermore, the approach needs to further be tested and compared against other methods with use of experimental fMRI datasets, such as, for example, the Human Connectome Project data (Van Essen et al., 2013; Barch et al., 2013). Since little is known about causal connections in large-scale brain networks, especially in the resting state, such a validation might be quantitive (i.e., by means of reliability) rather than qualitative. Alternatively, one could perform such a validation in particular pathways in which one can make assumptions about the directionality of information flow, such as the dorsal and the ventral stream in the visual cortex. One possibility is testing using datasets coming from an interventional study in which neural activity was evoked and, therefore, the ground truth is known: a fused fMRI-optogenetic experiment (Ryali et al., 2016), in which intervention with use of optogenetics guarantees causal link between investigated neuronal populations.

CONCLUSIONS

The field of effective connectivity research in fMRI is still growing. For instance, in recent years, multiple algorithms for the graph network search have been developed and applied to fMRI datasets, including independent multiple-sample greedy equivalence search (IMaGES; Ramsey et al., 2010), group iterative multiple model estimation (GIMME; Gates & Molenaar, 2012), or fast greedy equivalence search (FGES; Ramsey et al., 2016). It is material for debate whether or not bivariate or multivariate inference serves a better purpose for effective connectivity research in fMRI. In our work, we focused on the pairwise inference, and we achieved a significant improvement over previous approaches for pairwise estimation of functional causal interactions. Most importantly, the robustness against known sources of variability (possible differences between up- and downstream noise magnitude and possible presence of non-Gaussian scale-free noise) significantly increases due to the simultaneous incorporation of multiple aspects of the associated BOLD distributions. We believe that, as this approach based on fractional moments of a distribution increases resilience of the methods for pairwise connectivity to potential confounds in the experimental data, it can become a generic method to increase the power of causal discovery studies, both in cognitive neuroimaging and beyond.

SUPPORTING INFORMATION

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AUTHOR CONTRIBUTIONS

Natalia Bielczyk: Conceptualization; Formal analysis; Methodology; Validation; Visualization; Writing – Original Draft; Writing – Review & Editing. Alberto Llera: Conceptualization; Formal analysis; Supervision; Writing – Review & Editing. Jeffrey Glennon: Funding acquisition. Jan Buitelaar: Supervision; Writing – Review & Editing. Christian Beckmann: Conceptualization; Formal analysis; Methodology; Supervision; Writing – Review & Editing; Funding acquisition.
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