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White dwarf binaries and neutron star mergers in the stellar halo of the Milky Way in a cosmological context

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Introduction

With so much irregular structure in the outer Galaxy, it looks as though the Milky Way is still growing, by cannibalizing smaller galaxies that fall into it.

- Mario Juric -

What is a star?

The Sun, which rises every morning and sets every evening, is just one example of the many stars the Universe hosts. It seems so much brighter than all the other stars, because on Earth we are so close to it. A fraction of the light it emits now, hits the Earth’s surface only eight minutes later. This is in sharp contrast to all other stars, which are at least lightyears and easily thousands or millions of lightyears away. A star distinguishes itself from a planet, like the Earth, by the fact that it radiates energy from an internal source. During 90% of a star’s life, it generates energy by converting the hydrogen in its core into helium. Throughout the star, a pressure-gradient force points from the interior of the star into the direction of outer space. Simultaneously, since a star is bound by its own gravity, there is also a force pointing in the opposite direction, towards the centre of the star. The balance of these two forces is called hydrostatic equilibrium. The luminosity of a star at a given radius equals the total energy being generated interior to that layer due to the star’s continuous production of energy. During most of its life, a star is therefore also in thermal equilibrium.

1 A lightyear is the distance light travels in one year. The distance to a nearby star can be measured using its apparent displacement, with respect to the distant background, because of a change in the observer’s point of view. For example, when Earth is on opposite sides of the Sun in its orbit. The semi-angle of inclination between the sight-lines to an observed star determined half a year apart is called the parallax. The parallax can be used to determine distances, since nearby objects show larger parallaxes than objects that are farther away. The parsec (pc) is a unit of length used to express large distances in astronomy. One pc, defined as the parallax of an arcsecond, is about 3.26 lightyears. A megaparsec (Mpc) is $10^6$ pc.
**Introduction**

The mass of a star is arguably its most important property [e.g. Massey & Meyer, 2001]. Stars can have masses in the approximate range \(0.1 - 100 \, M_\odot\), where \(M_\odot\) is the mass of the Sun. Only the very first stars are thought to have been more massive [e.g. Bromm et al., 1999]. Later generations of stars are pulsationally unstable if they are more massive than \(\sim 100 \, M_\odot\) [Ledoux, 1941; Schwarzschild & Härm, 1959]. However, this value is somewhat debated, other authors e.g. Figer [2005] observationally determined 150 \(M_\odot\) to be the upper limit to the mass of stars. On the other hand, astronomical objects with a mass smaller than 0.1 \(M_\odot\) do not have core temperatures high enough to fuse hydrogen into helium, the primary energy source of stars. In the first \(\sim 20\) minutes after the Big Bang, the helium mass fraction of the Universe grew to the 25% level, with the remaining 75% being hydrogen and a negligible amount of heavier elements [Wagoner et al., 1967]. At the present day, stars start their lives off with 1 – 2% heavier elements. This fraction of the total stellar mass, which originates from earlier generations of stars, is called the metallicity of the star.

Dependent on their (initial) mass \(m\), stars can be categorized as follows:

1. Very low mass stars \([m \lesssim 0.3 \, M_\odot]\), e.g. Prialnik, 2010], which are fully convective apart from their outermost shell (the photosphere). The helium produced by nuclear fusion in the stellar core is distributed uniformly throughout the star.

2. Low mass stars \((0.3 \, M_\odot \lesssim m \lesssim 2 \, M_\odot)\), which have increasingly smaller outer convective zones towards the higher mass end of this mass range [e.g. Kippenhahn & Weigert, 1990]. These stars build up an electron-degenerate helium core. Once the core will become sufficiently dense and hot (\(\sim 10^8\) K), it will ignite and burn 2 – 3% of its mass within a few minutes. This phenomenon is known as the helium flash: it is the beginning of the end of such a star’s life. The corresponding fusion process is triple-\(\alpha\): three helium nuclei are fused into a carbon nuclei in a two-stage reaction. Some carbon nuclei fuse with additional helium to produce oxygen.

3. Intermediate-mass stars \((2 \, M_\odot \lesssim m \lesssim 10 \, M_\odot)\) have higher core temperatures than low mass stars and can therefore ignite helium fusion more gradually in the Carbon-Nitrogen-Oxygen (CNO) cycle, instead of with a helium flash. For the rest their evolution is similar to that of low mass stars. After the central helium burning phase they form a carbon-oxygen core that becomes degenerate.

4. Massive stars \((m \gtrsim 10 \, M_\odot)\) undergo all the major burning stages: after hydrogen and helium, also carbon, oxygen, neon and eventually silicon are burned. An onion-skin structure with an iron core is developed, which collapses and leads to a supernova (Type II) shortly afterwards.

**Stellar lifetimes**

The main sequence (MS) is a continuous and distinctive band of stars that appears on plots of stellar luminosity versus effective temperature. These diagrams are known as Hertzsprung-Russell diagrams after their co-developers, Ejnar Hertzsprung and Henry Norris Russell, who
**Introduction**

![Figure 1: The lifetime of stars, as a function of their TAMS mass. The blue solid line is a fit to theoretical models from Buzzoni [2002], the cyan dot-dashed line indicates the simple theoretical estimate from equation (2). With horizontal lines, the time since the extinction of the dinosaurs (dashed line) and the age of the Solar system (dotted line) are indicated. Note that it is debated whether $10^2 \, M_{\odot}$ TAMS mass stars actually exist.](image)

Independently initiated these plots in 1911 and 1913 respectively. Stars on this band are known as main-sequence stars. Main-sequence stars are in the hydrogen-burning phase, and a power-law correlation exists between their mass $M$ and their luminosity $L$:

\[
\left( \frac{L}{L_{\odot}} \right) = \left( \frac{M}{M_{\odot}} \right)^{\nu}
\]

with $\nu$ approximately ranging between 3 and 5. The $\odot$-sign indicates solar quantities. Since on the main-sequence, the consumption of nuclear fuel equals the rate of energy release $L$, and all stars fuse approximately the same fraction of their mass, the lifetime of a star on the main sequence is inversely proportional to its mass cubed

\[
\tau_{\text{MS}} \propto \frac{M}{L} = M^{-3}
\]

where a $\nu$ of 4 is assumed.

With the lifetime of a star, astronomers often mean its lifetime on the main sequence. At the moment a star enters the main sequence, it is referred to as a Zero Age Main Sequence (ZAMS) star. When it leaves the main sequence, it is called a Terminal Age Main Sequence (TAMS) star. Buzzoni [2002] fitted a line through several sets of theoretical models for the MS lifetime of stars with solar metallicity over the stellar mass range between 0.5 and 100 $M_{\odot}$, as a function of their TAMS mass $M_{\text{TAMS}}$, and found

\[
\log \tau_{\text{MS}} = 0.825 \log^2 (M_{\text{TAMS}}/120) + 6.43
\]
INTRODUCTION

Although the TAMS mass is significantly lower than the ZAMS mass for high-mass stars, which lose a large fraction of their mass by stellar winds, for the majority of stars it can be assumed that the mass of the star stays roughly constant while it is on the main sequence.

The main sequence lifetime of the Sun can be estimated using Einstein’s famous equation $E = mc^2$, after estimating that the inner core that is hot enough for nuclear burning contains 10% of the Sun’s mass. The total amount of mass that is transferred into energy is proportional to the mass difference between four hydrogen atoms and a helium atom, which is 0.72%. Substituting the values $M_\odot = 1.99 \times 10^{30}$ kg, $L_\odot = 3.828 \times 10^{30}$ W and using that the speed of light $c = 2.998 \times 10^8$ m/s, we find

$$\tau_\odot \approx 0.0072 \times \frac{1}{10} \times \frac{M_\odot c^2}{L_\odot} = 3.36 \times 10^{17} \text{ s} = 1.07 \times 10^{10} \text{ yr}$$

Combining this result with equation (2) makes it possible to compare our earlier estimate of the lifetime of a star as a function of its stellar mass with the fit of Buzzoni [2002], equation (3). This is done in Figure 1, where we furthermore plot two horizontal lines indicating the time since the extinction of the dinosaurs and the formation of the solar system. The age of the solar system, which can be derived from the study of meteorites, is near $5 \times 10^9$ years. Dinosaur fossils have revealed that these animals went extinct approximately $65 \times 10^6$ years ago. The top of the Figure indicates the lifetime of the Universe, $\sim 14 \times 10^9$ years. This figure indicates that the $\tau_{\text{MS}} \propto M^{-3}$ relation is quite accurate for stars with a (TAMS) mass between 1 and 5 times the mass of the Sun. We also see that stars with a mass of 0.8 $M_\odot$ have a MS lifetime which is as long as the present age of the Universe.

Stellar remnants

After the main sequence lifetime, a star can follow several evolutionary paths, depending on its initial mass.

Low and intermediate mass stars become red giants. In the red giant phase, the star’s radius becomes tens to hundreds times larger than it was on the main sequence (Figure 2). Its luminosity increases also, but the temperature of the outer envelope decreases, giving the star its reddish color. The most common red giants are stars on the red-giant branch (RGB) of the Hertzsprung-Russell diagram, which are still fusing hydrogen into helium in a shell surrounding an inert helium core. Other red giants populate the so called asymptotic-giant-branch (AGB). These stars have a helium burning shell outside a degenerate carbon-oxygen core, and a hydrogen burning shell beyond that.

Red giants lose a large fraction of their mass by stellar winds. Their mass is insufficiently high to generate a core temperature required for carbon fusion, and an electron-degenerate carbon-oxygen (CO) core is developed. In the final stages of their lives, red giants shed their outer layers, which become a so called planetary nebula. The internal energy source that is still providing energy to the star is a thin nuclear burning shell left on top of the CO-core. After all the nuclear fuel is consumed, the left-over core is the remnant white dwarf.

As can be concluded from the end of the last section, we have not yet seen the evolutionary endstage of very low mass stars, which live longer than the Universe is old. In theory however,
these stars with a mass $m < 0.3 \, M_\odot$ do not become red giants, but they would evolve directly into white dwarfs with a helium (He) core.

Most spectacular is the evolution of massive stars, which end their lives in a core-collapse supernova. Stars with initial masses up to 20 solar masses leave behind neutron star remnants after the supernova. For stars with higher initial masses the chance of black hole formation increases. If the supernova explosion is only weak, some of the ejected material may fall back onto the proto-neutron star. A stellar black hole is formed from the accretion of this material when the neutron star mass limit of $\sim 2 - 3$ solar masses is reached.

**White dwarf cooling**

Once a low- or intermediate-mass star has evolved into a white dwarf, it no longer produces energy. However, it still radiates energy and slowly cools down, becoming dimmer and dimmer as time passes. Eventually, when the white dwarf is so cool that it does not longer emit a significant amount of light, one could speak of a “black dwarf”. However, the Universe is not old enough to form black dwarfs, and strictly speaking, never will be.

Mestel [1952] showed that, in the most simple model, the cooling time of white dwarfs is a power law function of their luminosity:

$$\tau_{\text{cool}} \propto L^{-5/7}$$

Mestel assumed that white dwarfs behave like an electron-degenerate gas with an isothermal core and a non-degenerate radiative envelope. This results in a reasonable description of the cooling behavior of real white dwarfs at intermediate luminosities [Althaus et al., 2010], e.g. $-1 \geq \log(L/L_\odot) \geq -3$ (Figure 3).
In reality, Coulomb interactions between the ions in the white dwarf become increasingly more important with time and eventually dominate over the thermal ion energy. The white dwarf core undergoes a transition from ion gas to a Coulomb liquid in the luminosity range $-1.5 \geq \log(L/L_\odot) \geq -3$ [Isern & García–Berro, 2004], and crystallizes at lower luminosities. For the faintest white dwarfs, a process called Debye cooling becomes important, as quantum effects come into play after the crystallization: the crystal lattice causes coherent vibrations, which promotes further energy loss. It depends on the white dwarf mass at what luminosities the onset of Debye cooling occurs. For a 0.6 $M_\odot$ white dwarf, this is only at $\log(L/L_\odot) \lesssim -7$, whereas for a 1.2 $M_\odot$ white dwarf, Debye cooling starts at $\log(L/L_\odot) \lesssim -4.1$ [Althaus et al., 2010].

**Binary stars**

A binary star consists of two stars orbiting a common centre of mass. The brighter star is called the primary, while the dimmer of the two stars is called the secondary. Stars are also known to exist in triple systems or with even higher multiplicity, but these are less common than binaries. The overall observed fractions of single, double, triple, and higher-order systems of Sun-like stars is determined to be 56% ± 2%, 33% ± 2%, 8% ± 1%, and 3% ± 1%, respectively [Raghavan et al., 2010]. Throughout this thesis, I generally assume a binary fraction of 0.5, meaning that 50% of the stars are born in a binary system, thus that two out of every three stars reside in a binary. Although this is a good approximation for solar-like stars (G stars), the multiplicity is lower for
stars with a lower mass and higher for stars with a higher mass [e.g. Sana et al., 2012]. For the recent status of this field of research, see e.g. Duchêne & Kraus [2013] and Moe & Di Stefano [2017].

While for a single star its mass is the most important parameter for describing its evolution, for binary stars the orbital separation between the two stars is at least as important as the individual masses of the stars. Wide binary systems with orbital separations \( \gtrsim 10 \) AU and corresponding orbital periods \( \gtrsim 30 \) years evolve independently from each other, while stars in a close binary interact and mass transfer between the two stars can occur\(^2\). Due to mass loss and angular momentum loss a binary star can experience orbital shrinking, and a merger may eventually occur.

The region around a star in a binary system within which orbiting material is gravitationally bound to that star is called the Roche lobe. Kippenhahn et al. [1967] first suggested that low-mass red giants filling their Roche lobes in binary systems undergoing substantial mass loss could result in helium core white dwarfs. Close binary stars thus provide a natural explanation for the existence of helium core white dwarfs in our 14 billion year old Universe, which as discussed in the previous subsection is too young to host fully evolved very low mass single stars.

**Type Ia Supernovae**

In case a close binary system contains at least one CO white dwarf, the other star being another white dwarf or a non-degenerate star, binary evolution may result in a Type Ia supernova. Due to mass accretion from the binary companion, the temperature and pressure in the CO white dwarf increases, which leads to carbon ignition once the mass of the white dwarf core exceeds \( \sim 1.37M_\odot \) [Arnett, 1969]\(^3\). If only one of the supernova Type Ia progenitors is a white dwarf, the supernova is said to have a single-degenerate formation, whereas the formation channel is double degenerate if two white dwarfs are involved.

Type I supernovae differ from Type II’s by the lack of hydrogen lines in their spectra. In the spectrum of a Type Ia supernova, silicon shows up very dominantly. Furthermore, a Type Ia supernova yields \( \sim 0.7M_\odot \) of iron to the interstellar medium, with a similar complementary amount of mass in the combined yields of mainly carbon, oxygen, neon, magnesium, silicon, sulfur, argon, and calcium [see Maoz et al., 2014].

**The Milky Way**

The Earth orbits around the Sun in exactly one year, and similarly the Sun (and about one hundred billion stars with it) revolves around the Galaxy, which contains a supermassive (\( \sim 4 \times 10^6M_\odot \)) black hole in its center [e.g. Genzel et al., 2010], with an orbital period of approximately 230 million years [Oort, 1927]. This collection of about one hundred billion stars, combined with

\(^2\)To determine the orbital period, a combined mass of 1.1 \( M_\odot \) is assumed and Kepler’s third law is applied. One AU, or astronomical unit is \( 1.495978707 \times 10^{11} \) metres (exactly), roughly the distance from Earth to the Sun.

\(^3\)This is the classical Chandrasekhar mass scenario, but there are also other possible explosion mechanisms behind Type Ia supernovae [e.g. Hillebrandt et al., 2013].
Introduction

Figure 4: Photograph of the Milky Way as it can be seen in the Netherlands, photo taken by Martijn van Geloof in the Loonse en Drunense Duinen.

Figure 5: Schematic view of the Milky Way, figure from Sparke & Gallagher [2000].
the gas and dust around and in between them, and the supermassive black hole in the centre, is called a galaxy\textsuperscript{4}.

Galaxies exist in various shapes and sizes. Besides the so called irregular galaxies, a galaxy can be classified as an elliptical or a disk galaxy. Elliptical galaxies have an ellipsoidal shape, because of which they always appear as an ellipse on the sky, whereas disk galaxies have a disk build up of gas and (mostly young) stars. Elliptical galaxies mostly contain old populations of stars. The galaxy that contains our Solar system is a disk galaxy [Kapteyn, 1922] with spiral arms, as we can infer from radio observations [Oort et al., 1958].

Almost all stars that can be seen with the naked eye from Earth are part of the same galaxy as the Sun, however a large hazy white band on the sky marks the area where most stars are positioned (Figure 4). This is how our Galaxy, the Via Lactea (Latin), better known as the Milky Way, got its name. This so called Galactic plane is apparent because the galaxy is seen from within the disk. If we observe a distant spiral galaxy edge on, also a galactic bulge in the centre of the galaxy (the middle of the plane) is visible. On the southern hemisphere of the Earth, the galactic bulge of the Milky Way is also visible.

Schematically, the Milky Way galaxy can be drawn as in Figure 5. The Galactic plane consists of a thin disk (shown in grey) and a thick disk (shown with dashed lines). The Sun lies approximately two thirds between the edge of the bulge and the edge of the disk, $\sim 8$ kpc ($\sim 26$ thousand lightyears) from the Galactic centre. The outermost dashed line in the shape of an ellipse surrounding the Galaxy indicates the Galactic halo. It contains $\sim 1\%$ of the stars in our Galaxy, among which many of the oldest. Many halo stars are contained in so called (metal-poor) globular clusters. Towards the Galactic bulge, the globular clusters get more metal-rich.

A globular cluster is a nearly spherical collection of $10^5 - 10^6$ stars orbiting the Galactic centre. Unlike satellite galaxies which also orbit around in the Galactic halo, they do not or hardly contain any dark matter. This form of matter, which is also present everywhere in the halo, can not directly be seen through visible light, and is only known to exist because of its gravitational effect.

The metallicity of a star reflects the environment in which it is born, and in a given environment metallicity grows with time. In an old-fashioned description, stars are categorized in three generations. Population III stars are the very first stars in the Universe, which were probably massive, therefore had short lifetimes and exploded as supernovae, enriching the Universe with its first metals. The metal-poor (Population II) stars that can be found in the Galactic halo have a wide metallicity distribution and formed out of gas that was enriched by several (a varying number of) previous generations. The more metal-rich stars such as the Sun (Population I) are enriched by many generations of stars and are typically found in the Galactic plane. However, this description is old-fashioned since it is unclear in which category some stars should be placed, e.g. metal-rich bulge stars.

\textsuperscript{4}If Galaxy is written with a capital astronomers mean our own galaxy.
Introduction

Gaia

The Gaia spacecraft was launched on the 19th of December 2013 with the aim to make the most accurate catalogue of positions, parallaxes and proper motions of \( \sim \) one billion astronomical objects, mostly stars in the Milky Way, but also planets, comets, asteroids and quasars amongst others. This mission of the European Space Agency determines positions at the microarcsecond (\( \mu \)as) level. This accuracy is equivalent to measuring the thickness of hair at a thousand kilometers distance.

Distances are needed to determine physical parameters such as the intrinsic luminosity and number densities of stars. By using the inverse square law, the intrinsic luminosity of an object can be calculated from its observed brightness. Once an object from a class known to have a characteristic luminosity is observed, such as the class of Type Ia supernovae, the distance to this object can be determined even if it is outside our Galaxy. In this way, distances to galaxies and other distant extragalactic objects, and the rate of expansion of the Universe, can be inferred from distances to galactic and nearby extragalactic objects, which is why it is very important for astronomy to determine them accurately.

Gaia will help us to understand the dynamics and assembly history of our galaxy, by measuring the motions of so many astronomical objects. This may shed light on the nature of dark matter. But it will also help us to learn more about stellar evolution, by improving estimations of stellar luminosity functions, i.e. the number of stars per luminosity interval. Comparing these observed luminosity functions with predictions from theory, will help to improve the theoretical models.

The Local Group

An aggregation of about 50 galaxies that is gravitationally bound within a region of typically a few Mpc extend is called a galaxy group. The galaxy group that contains the Milky Way is called the Local Group. The three largest galaxies in this group are M31 (better known as the Andromeda galaxy), the Milky Way and M33 (also known as the Triangulum galaxy). As can be seen in Figure 6, many other galaxies in the Local Group are satellite galaxies of the Milky Way, such as the Large Magellanic Cloud (LMC), the Small Magellanic Cloud (SMC), and the dwarf galaxies Draco, Ursa Minor, Carina, Sextans, Sculptor, Fornax, Leo I and Leo II or of the Andromeda galaxy, e.g. M32, NGC 147, NGC 185, Andromeda I and Andromeda II.

The term Local Group was coined by the astronomer Edwin Hubble in 1936 and initially listed 12 known nearby galaxies. In 2003, about 36 Local Group galaxies were known [van den Bergh, 2003]. Today, the list comprises more than 50 galaxies, many of which are faint dwarf galaxies [e.g. McConnachie, 2012]. The Local Group itself is part of the larger Local Supercluster (also called Virgo Supercluster), which contains at least one hundred galaxy groups and has an extent of about 30 Mpc [Tully, 1982]. It has its centre near the Virgo cluster of galaxies in the constellation Virgo.
Figure 6: The 31 local group galaxies that were known in the beginning of the 21st century. For the 22 which are numbered in red in the legend, a photo is shown. ©2005 Cetin BAL.
Introduction

This thesis

In this thesis, the formation of the Galactic stellar halo is studied. The first formation scenario of the Galactic halo was that proposed by Eggen et al. [1962], who suggested that “the oldest stars with the lowest abundance of the heavy elements must have been formed in the collapsing protogalaxy when its size was at least ten times its present diameter and, furthermore, that the collapse since first star formation was very rapid, taking place in only a few times $10^8$ years”. Later, Searle & Zinn [1978] proposed an alternative scenario, in which the Galactic halo formed from “transient protogalactic fragments that continued to fall into dynamical equilibrium with the Galaxy for some time after the collapse of its central regions had been completed”. Nowadays, the standard model of hierarchical structure formation has elements of both scenarios. The theory predicts that gravitationally bound structures containing the first stars and small galaxies formed first, subsequently merging with gas and dark matter to form larger galaxies, followed by galaxy groups, clusters and superclusters of galaxies.

In the first chapter of this thesis, we do not use a detailed galaxy formation model. Instead, we explore plausible star formation histories for the stars in the halo of the Milky Way and for the shape of the so called initial mass function, i.e. the distribution of the initial (ZAMS) stellar masses. Single and binary white dwarfs are the astronomical objects that are being investigated in this so called population synthesis study. The binary population synthesis model SeBa [Portegies Zwart & Verbunt, 1996; Toonen et al., 2012] is used for this. Predictions are made about the number of stellar halo white dwarfs that can be observed with the Gaia satellite, as well as about the shape and normalization of the white dwarf luminosity function (WDLF). Especially including the binaries in such a study is new.

The hierarchical formation of the stellar halo is the topic of the second chapter, where the “building block” galaxies from which the stellar halo is formed are studied. The evolution of dark matter haloes as simulated in the Aquarius project [Springel et al., 2008], in the framework of a cold dark matter cosmology with a cosmological constant, is one of the two pillars that form the foundation of this study. The Munich-Groningen semi-analytic model of galaxy formation [Starkenburg et al., 2013a, and references therein] is the other pillar. The properties of these building block galaxies in terms of mass, age and metallicity are compared with those of (simulated) surviving satellite galaxies of the Milky Way.

It is not until the third chapter of this thesis, that the stellar population synthesis model SeBa and the Munich-Groningen semi-analytic model of galaxy formation are combined. Again using the Aquarius simulations as a backbone, binary and single white dwarfs in the building blocks of a Milky Way-like stellar halo are investigated. The difference between a halo white dwarf population simulated with simple assumptions on the stellar metallicities and star formation history (chapter 1) and one that is fully consistent with a cosmological framework is studied in this chapter. Furthermore, we explore whether the individual building block properties are still reflected in the halo white dwarf population if observed with an astronomical instrument like Gaia.

In the fourth chapter of this thesis, another binary population of astronomical objects is being studied: that of merging neutron stars in the Milky Way. This is a particularly interesting population, because it might be the source of the rapid neutron capture process elements. The elements in the periodic table around iron (Fe) have the highest nuclear binding energy per
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nucleon. These so called iron peak elements are the most massive elements that can still be
formed through nuclear fusion. All heavier elements are formed through either a slow (s-) or a
rapid (r-) neutron capture process, see Figure 7. The physical origin behind these two processes
is very different. Slow neutron capture, which typically takes place on timescales of hundreds or
thousands of years, originate from thermally pulsing low- to intermediate mass AGB stars [e.g.
Travaglio et al., 2004, and references therein]. Very convincing evidence for merging neutron stars
as production site for rapid neutron capture elements came with the detection of gravitational
waves from a neutron star merger and the associated signal emitted by r-process material [Abbott
et al., 2017b,a].

In this chapter, a population of merging neutron stars is studied in three of the six Aquarius
stellar haloes. The production of the element Europium (Eu) is followed, which is almost exclu-
sively formed through the r-process channel. At formation, the binary neutron star may receive
a kick velocity, which may be similar to or even higher than the escape velocity of the building
block galaxy the binary resides in. By the time the neutron star merges and enriches its environ-
ment in r-process material, it may thus already have left the galaxy. The next generation of stars
in that galaxy is then not enriched in r-process elements due to that binary neutron star which
was formed there a stellar generation earlier. By updating the Munich-Groningen semi-analytic
model of galaxy formation, and comparing simulated [Eu/Mg] versus [Mg/H] abundance ratios\(^5\)
to observations, constraints are put on the kick velocities of the neutron stars, when assuming
that Eu is solely produced by merging neutron stars.

The final chapter of this thesis contains a summary of the main results and a short outlook.

\(^5\)The comparison of the number densities of two elements A and B \((N_A \text{ and } N_B)\) in a star (indicated by the
symbol *) to those in the Sun, by calculating \(\log_{10}(N_A/N_B)_* - \log_{10}(N_A/N_B)_{\odot}\), is abbreviated as \([A/B]\).
**Introduction**

**Figure 7:** Periodic table showing the origin of the elements. The cells are filled with colors according to the ratio by which the corresponding sources contribute to their origin. The elements Technetium (Tc) and Promethium (Pm), as well as all elements heavier than Uranium (U) do not have long-lived or stable isotopes. This figure is slightly oversimplified, with “dying low mass stars” referring to the source of s-process elements (thermally pulsing low- to intermediate mass AGB stars) and “merging neutron stars” being the source of r-process elements. However, the latter is still being debated in the literature (see also the introduction of Chapter 4). Type II supernovae or “exploding massive stars” as they are called in this figure, could also contribute to the r-process enrichment of the Galaxy.
Aims: We study single and binary white dwarfs in the inner halo of the Milky Way in order to learn more about the conditions under which the population of halo stars was born, such as the initial mass function (IMF), the star formation history, or the binary fraction.

Methods: We simulate the evolution of low-metallicity halo stars at distances up to $\sim 3$ kpc using the binary population synthesis code SeBa. We use two different white dwarf cooling models to predict the present-day luminosities of halo white dwarfs. We determine the white dwarf luminosity functions (WDLFs) for eight different halo models and compare these with the observed halo WDLF of white dwarfs in the SuperCOSMOS Sky Survey. Furthermore, we predict the properties of binary white dwarfs in the halo and determine the number of halo white dwarfs that is expected to be observed with the Gaia satellite.

Results: By comparing the WDLFs, we find that a standard IMF matches the observations more accurately than a top-heavy one, but the difference with a bottom-heavy IMF is small. A burst of star formation 13 Gyr ago fits slightly better than a star formation burst 10 Gyr ago and also slightly better than continuous star formation $10 - 13$ Gyrs ago. Gaia will be the first instrument to constrain the bright end of the field halo WDLF, where contributions from binary WDs are considerable. Many of these will have He cores, of which a handful have atypical surface gravities ($\log g < 6$) and reach luminosities $\log(L/L_\odot) > 0$ in our standard model for WD cooling. These
so called pre-WDs, if observed, can help us to constrain white dwarf cooling models and might teach us something about the fraction of halo stars that reside in binaries.

1.1 Introduction

The Galactic halo is the oldest component of our Galaxy, containing metal-poor stars with high velocity dispersion. It contains a small percent of the total stellar mass of the Galaxy. Many questions about the formation of the halo and the Milky Way’s oldest stars are still to be answered, such as What is its star formation history (SFH)?, What is the initial mass function (IMF)?, and What fraction of halo stars resides in binaries?. In this paper, we will investigate all three of these questions by studying the population of halo white dwarfs with a population synthesis approach.

White dwarfs (WDs) are an increasingly important tool used to study Galactic populations. Because they are the end product of low and intermediate mass stars, WDs are interesting objects of study for age determinations [eg. Hansen et al., 2007; Bedin et al., 2009]. Since we have entered the era of large sky surveys, a huge amount of high-quality observational data of these stars is now or will soon become available. Physically, WDs are rather well understood, and they have been used as cosmic chronometers to study our Galaxy, as well as open and globular clusters, for more than two decades (Winget et al. 1987; see reviews by Fontaine et al. 2001 and Althaus et al. 2010, for example). Since in general, halo WDs are cool and faint, we confine ourselves to studying the ones in the solar neighbourhood.

The halo WD luminosity function (WDLF) was first derived by Liebert et al. [1989], based on six observed WDs with tangential velocities $v_t$ exceeding 250 km s$^{-1}$. The most recent estimate is based on observations of 93 WDs with $v_t > 200$ km s$^{-1}$ in the SuperCOSMOS Sky Survey [Rowell & Hambly, 2011, hereafter RH11]. Theoretical halo WDLFs have been determined by, amongst others, Adams & Laughlin [1996]; Isern et al. [1998]; Camacho et al. [2007]. Predictions for Gaia's performances on WDs have been made by Torres et al. [2005]. For a recent paper on this topic, see Carrasco et al. [2014]. However, the effect of binary stars has never been studied in great detail. Furthermore, different initial parameters, stellar evolution codes, and WD cooling models were used in most of these papers.

For different assumptions about the IMF, SFH, and binary fraction, as well as for two different WD cooling models, we determine the WDLF and compare it with the observed halo WDLF in RH11. We derive both its shape and its normalization from an independent mass density of low-mass halo stars. We include not only single stars, but also focus on the contribution from WDs in binaries and WDs that are the result of a binary merger. Furthermore, we predict the properties of the population of binary WDs in the halo for a standard model and derive the number of halo WDs that can be detected by the Gaia satellite.

The setup of this paper is as follows: in section 1.2 we explain our methods, in section 1.3 we discuss our results, and our conclusions can be found in section 1.4. In the concluding section we try to answer the question: What can Gaia observations of halo WDs teach us about the IMF, SFH, and binary fraction in the halo?
1.2 Model ingredients

We aim to derive the WDLF from first principles, i.e. not normalizing it to the observed WDLF, but using an independent estimate of the local stellar halo mass density to deduce a WDLF. A very important ingredient of our model is therefore the relation between this local density $\rho_0$, the stellar halo mass in the solar neighbourhood (the region that we simulate) and the IMF. In the next subsection, the expected number of halo stars is derived for three different IMFs. More details on this calculation can be found in the two appendices of this paper.

1.2.1 Initial Mass Functions

As a standard assumption, the IMF $\phi(m)$ can be written as a power law

$$\phi(m) \equiv \frac{dN}{dm} \propto m^{-(\gamma+1)}$$

(1.1)

with $N$ being the number of stars formed in the mass range $m, m + dm$ and $\gamma$ the IMF slope. We assume $\phi(m)$ to be independent of Galactic age or metallicity. Unless specified otherwise, $N$ here represents the number of stars in the case that all stars are single (a binary fraction of 0). In section 1.2.2 we explain how these numbers change with a nonzero binary fraction.

In a classical paper, Salpeter [1955] estimated $\gamma = 1.35$, and the corresponding IMF is nowadays referred to as a Salpeter IMF. Although not our standard model, one of the IMFs that we investigate in this paper is a Salpeter IMF for the whole mass range of stars ($0.1 - 100 \, M_\odot$). It is nowadays generally believed that the IMF flattens below $1.0 \, M_\odot$, so this can be considered a bottom-heavy IMF.

In our standard model we split the IMF up into three power laws, following Kroupa et al. [1993]:

$$\phi(m) \propto \begin{cases} 
\frac{35}{19} m^{-1.3} & \text{if } 0.1 \leq m < 0.5, \\
m^{-2.2} & \text{if } 0.5 \leq m < 1.0, \\
m^{-2.7} & \text{if } 1.0 \leq m < 100.
\end{cases}$$

(1.2)

The third IMF that we investigate in this paper is the top-heavy IMF suggested by Suda et al. [2013]. These authors argued that the IMF for stars with $[\text{Fe/H}] < -2$ is lognormal

$$\phi(m) \propto \frac{1}{m} \exp \left[ -\frac{\log_{10}(m/\mu)^2}{2\sigma^2} \right]$$

(1.3)

with median mass $\mu = 10$ and dispersion $\sigma = 0.4$. Originally, this IMF was proposed by Komiya et al. [2007] for stars with $[\text{Fe/H}] < -2.5$ to explain the observed features of carbon enhanced metal poor stars, therefore we refer to it as the Komiya IMF. The higher metallicity stars would be formed according to a Salpeter IMF. Following the metallicity distribution function (MDF) of a two-component halo model [An et al., 2013], 24% of the zero-age main-sequence (ZAMS) stars with masses between 0.65 and 0.75 $M_\odot$ are formed according to a Komiya IMF. Therefore, when normalizing the WDLF properly, we expect more signatures from high-mass WDs (which cool fast and are thus faint) when choosing this IMF.

In order to determine the actual number of stars in a population $N$, one has to integrate $\phi(m)$, thereby setting the integration boundaries and the normalization constant. For example,
integrating equation (1.1) with normalization constant $A$ yields

$$N = \int_{m_{\text{low}}}^{m_{\text{high}}} A m^{-(\gamma+1)} \, dm. \quad (1.4)$$

Hereafter, $m_{\text{low}} = 0.1$ and $m_{\text{high}} = 100$ for single stars and binary primaries. The value of $A$ can be determined from an observed mass or number density of stars. We use the estimated local stellar halo mass density $\rho_0 = 1.5 \cdot 10^{-4} \, M_\odot \, \text{pc}^{-3}$, based on observations of 16 halo stars in the mass range $0.1 \leq m < 0.8$ by Fuchs & Jahreiß [1998]. These authors derived $\rho_0 = 1 \cdot 10^{-4} \, M_\odot \, \text{pc}^{-3}$ as a firm lower limit. For a discussion on the correctness of this value compared to for example the lower estimate $\rho_0 = 6.4 \cdot 10^{-5} \, M_\odot \, \text{pc}^{-3}$ [Gould et al., 1998], see Digby et al. [2003] and Helmi [2008]. Since 0.8 $M_\odot$ is roughly the mass below which all stars can be considered unevolved, and 0.1 $M_\odot$ is our assumed lower mass boundary of all stars that are formed, this mass density is directly related to the total mass in unevolved stars $M_{\text{unev}}$ in our simulation box. Our top-heavy IMF has two normalization constants, one for the very metal-poor stars and one for the higher metallicity stars. The normalization constants are derived in Appendix 1.B.

Our simulation box represents the stellar halo in the solar neighbourhood, which we parameterize in a principal axis cartesian coordinate system as [Helmi, 2008]

$$\rho(x, y, z) = \frac{\rho_0}{r_0^n} \left( x^2 + y^2 + \frac{z^2}{q^2} \right)^{-n/2}, \quad (1.5)$$

with $r_0$ the distance from the Sun to the Galactic centre, $q$ the minor-to-major axis ratio and $n$ the power law exponent of the density profile. Throughout this paper, an oblate stellar halo ($q < 1$) is assumed. A sphere with radius $\xi < r_0$ around the Sun defines the minimum width and height of our simulation box. We show in section 1.3.3 that $\xi = 2.95$ kpc is sufficient for our study of halo WDs. Furthermore, we choose $r_0 = 8.0$ kpc [Moni Bidin et al., 2012, an average of 16 literature measurements], $n = -2.8$ and $q = 0.64$ [Jurić et al., 2008]. The simulated area with these values of $r_0$, $\xi$, $n$ and $q$ is shown in Figure 1.1. We note that although $n = -2.8$ and $q = 0.64$ are the formal best-fit parameters of Jurić et al. [2008], one should keep in mind the ranges $-3 \leq n \leq -2.5$ and $0.5 \leq q \leq 0.8$ as their fit results. Substituting the above-mentioned value of $\rho_0$ into equation (2.2) and integrating over the volume of our simulation box, we find $M_{\text{unev}} = 3.6 \cdot 10^7 \, M_\odot$ (see Appendix 1.A).

A crucial part of the normalization is the mass function of low-mass stars. The above-mentioned three IMFs predict drastically different numbers of stars in the range of masses $0.1 \leq m < 0.8$, which may or may not be in agreement with the observed sample from which the local halo mass density is determined [Fuchs & Jahreiß, 1998]. Since the mass function cannot be determined indisputably from this sample of 16 stars, we assume for each individual modelled IMF that it holds also in the low-mass regime. However, the resulting number of expected halo WDs depends strongly on the normalization of these low-mass stars (as we will see in section 4.6), so we also investigate the effect of different slopes of the IMF at $0.1 \leq m < 0.8$.

Since the mass in low-mass stars is fixed by our normalization, the number of evolved stars and thus of WDs depends on the ratio of evolved stars to unevolved stars. The flatter the slope of the IMF for unevolved stars, the fewer unevolved stars there are, i.e. the higher this ratio$^1$. Most

$^1$This statement also holds for the combined Salpeter+Komiya IMF, but not for the Komiya IMF by itself.
1.2 Model ingredients

Figure 1.1: Simulation box containing a sphere with radius $\xi = 2.95$ kpc centred at the position of the Sun $(x, y, z)_\odot = (8.0, 0, 0)$ kpc, $n = -2.8$ and $q = 0.64$ (left panel) and a density map of the simulation box (right panel).

studies of low-mass stars suggest that the slope of this part of the IMF flattens [eg. Bonnell et al., 2007, $\gamma_{\text{unev}} \approx 0$]. This is why we take the Kroupa mass function as our standard. Furthermore, we calculate the number of evolved stars, which have initial masses $0.8 \leq m < 100$, for a flat ($\gamma_{\text{unev}} = -1$) IMF, yielding a robust upper limit on the number of evolved stars, $N_{\text{ev,upper}}$ (see Appendix 1.B). In this way we derive a range of possible values for the number of evolved stars in our simulation box, between $N_{\text{ev}}$ (where $\gamma_{\text{unev}}$ is set consistently by the IMF, also in the low-mass regime) and $N_{\text{ev,upper}}$ (derived by setting $\gamma_{\text{unev}} = -1$). These numbers for the three different assumptions about the IMF are given in the first three columns of Table 1.1, assuming all stars are single. In section 1.2.2 we give a description of the last three columns of Table 1.1.

1.2.2 Population synthesis

The evolution of the halo stars is calculated with the binary population synthesis code SeBa [Portegies Zwart & Verbunt, 1996; Toonen et al., 2012; Toonen & Nelemans, 2013]. In SeBa, stars are generated with a Monte-Carlo method, using the following distributions:

- Binary primaries are drawn from the same IMF as single stars (see section 1.2.1).
- Flat mass ratio distribution between 0 and 1, thus for secondaries $m_{\text{low}} = 0$ and $m_{\text{high}} = m_{\text{primary}}$.
- Initial separation: flat in $\log a$ (Öpik’s law) between $1 R_\odot$ and $10^6 R_\odot$ [Abt, 1983], provided that the stars do not fill their Roche lobe.
- Initial eccentricity: chosen from the thermal distribution $\Xi(e) = 2e$ between 0 and 1 as proposed by Heggie [1975] and Duquennoy & Mayor [1991].

All simulated halo stars have metallicity $Z = 10^{-3}$ ($0.05 Z_\odot$), unless a top-heavy IMF is assumed. In that case, all stars that are born following a Komiya IMF are generated with metallicity $Z = 2 \cdot 10^{-4}$ ($0.01 Z_\odot$). The common-envelope (CE) prescription of the standard model in SeBa [$\gamma_\alpha$, Toonen et al., 2012] is used. With SeBa, we calculate the stellar evolution up to the
Table 1.1: Number of stars in our simulation box as a function of the IMF.

<table>
<thead>
<tr>
<th>IMF</th>
<th>N_{unev}</th>
<th>N_{ev}</th>
<th>N_{ev,upper}</th>
<th>N_{wd(0)}</th>
<th>N_{wd(0.5)}</th>
<th>N_{wd(1)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kroupa</td>
<td>12</td>
<td>1.9</td>
<td>5.9</td>
<td>1.7</td>
<td>1.1</td>
<td>0.70</td>
</tr>
<tr>
<td>Salpeter</td>
<td>17</td>
<td>1.1</td>
<td>6.7</td>
<td>0.97</td>
<td>0.63</td>
<td>0.41</td>
</tr>
<tr>
<td>Top-heavy</td>
<td>16</td>
<td>80</td>
<td>330</td>
<td>25.4</td>
<td>21.2</td>
<td>18.3</td>
</tr>
<tr>
<td>– Komiya</td>
<td>0.24</td>
<td>79</td>
<td>326</td>
<td>24.5</td>
<td>20.6</td>
<td>17.9</td>
</tr>
<tr>
<td>– Salpeter</td>
<td>16</td>
<td>1.0</td>
<td>4.3</td>
<td>0.93</td>
<td>0.61</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Notes: For three different IMFs are indicated: in the first three columns, the number of stars in our simulation box ($\times 10^7$) for a binary fraction of 0 (all stars are single). The border mass below which all stars are considered to be unevolved is $0.8 \, M_\odot$. The numbers $N_{ev,upper}$ come from taking $\gamma_{unev} = -1$. The resulting number of WDs with three different assumptions on the binary fraction (0, 0.5, or 1) are listed in the last three columns, with our standard assumptions on the SFH and WD cooling. For the top-heavy IMF, the first of the three lines lists the sum of the contributions from the Komiya and the Salpeter IMF, which are given in the second and third line.

point where the stars become WDs, neutron stars, or black holes, as well as the evolution of the binary systems until the end time of the simulation. For the WD cooling a separate code is used (see section 4.3).

Having determined the total stellar mass in the simulated area, we still need to make an assumption on the binary fraction in order to arrive at the total number of stars in our simulation box. Because we assume a flat mass ratio distribution, the mass of the secondary is on average half the mass of the primary. The total number of binary systems in our simulation box if all stars are in binaries is therefore 1.5 times less than the total number of ZAMS stars if the binary fraction is zero. As a standard assumption we adopt a binary fraction of 0.5. This means that there are as many binary systems as there are single stars, thus that two out of every three stars are in a binary system. The total number of single stars (which is the same as the total number of binary systems) in this case can be found by dividing the numbers in the first three columns of Table 1.1 by 2.5.

The resulting number of WDs in our simulation box is listed in the last three columns of Table 1.1 for three different values for the binary fraction (0, 0.5, or 1) and standard assumptions about the SFH and WD cooling (see the next subsections). These numbers are quite sensitive to the assumed binary fraction, especially for a Kroupa or Salpeter IMF, because most of the binary primaries are unevolved stars in this case. This means that even more secondaries are unevolved stars, which do not become WDs within the age of the Galaxy, if the binary fraction is larger. Thus the total number of WDs is smaller if the binary fraction is larger. For a top-heavy IMF this effect is obviously less significant.

In our simulation we distinguish between carbon-oxygen core (CO) WDs, helium core (He) WDs and oxygen-neon core (ONe) WDs. He WDs must have undergone episodes of mass loss in close binary systems in order to be formed within a Hubble time. They are thus only found in binary systems, or as a result of two components of a binary system that merged. In Table 1.2,
1.2 Model ingredients

Figure 1.2: Comparison of cooling tracks of 0.2 M$_\odot$ (dashed lines), 0.5 M$_\odot$ (solid lines) and 1.2 M$_\odot$ (dotted lines) WDs, between the models from the Althaus group (dark blue lines) and the Bergeron group (light blue lines). The 3 lines that represent cooling tracks from the Althaus models correspond to 3 WDs with different core compositions: He for the 0.2 M$_\odot$ WD, CO for the 0.5 M$_\odot$ WD and ONe for the 1.2 M$_\odot$ WD, whereas the 3 lines from the Bergeron models all correspond to WDs with a CO core. The age of the universe is indicated with a vertical thin black line.
1.2.3 Star Formation Histories

We make three different assumptions about the star formation history of the halo, based on observational indications that the vast majority of halo stars are old [Unavane et al., 1996; Kalirai, 2012]:

(a) one burst of star formation 13 Gyr ago (our standard);

(b) continuous star formation from 13 until 10 Gyr ago, no star formation afterwards;

(c) one burst of star formation 10 Gyr ago;

where a burst is assumed to last 250 Myr. After SeBa is run, all simulated halo stars have the same age, i.e. all stars are evolved for 10 Gyr or for 13 Gyr. To account for the SFH of the halo, we therefore shorten their lifetime by an amount of time randomly chosen between 0 and 250 Myr in case of assumptions (a) and (c) or with a number between 0 and 3 Gyr in case of assumption (b). If, by doing so, the lifetime of the star is reduced below the time it takes that star to become a WD, it is removed from our sample of WDs.

1.2.4 White dwarf cooling models

To determine the temperature, surface gravity and luminosity of a WD with a certain mass and cooling time, we use the cooling tracks published by Althaus et al. [2013] for He WDs and those from Renedo et al. [2010] for CO WDs. For ONe WDs, we use the cooling tracks from Althaus et al. [2007], both to determine their luminosities and temperatures, and their colours and magnitudes. We will refer to this set of cooling models as the Althaus models (our standard for WD cooling). The metallicities of the He and CO WDs in the Althaus models are assumed to be $Z = 0.01$, that of the ONe WDs $Z = 0.02$. Colours and magnitudes for CO WDs come from
1.2 Model ingredients

Althaus [2012], whereas colour tables of He WDs with high-metallicity progenitors (Z = 0.03) [Althaus et al., 2009] were used to determine the colours and magnitudes of He WDs. In all these cooling tracks and colour tables, the WDs have a higher metallicity than the ones in our simulation box (Z = 0.001). However, from the cooling tracks that were available for different metallicities [Althaus et al., 2009; Panei et al., 2007] we conclude that the effect of metallicity on WD cooling is smaller than other effects, such as the core composition (He or CO) of the WD, at least for large cooling times.

Alternatively, we also use the WD cooling tracks that are published on the website www.astro.umontreal.ca/~bergeron/CoolingModels/ [Bergeron et al., 2011; Holberg & Bergeron, 2006; Kowalski & Saumon, 2006; Tremblay et al., 2011], hereafter called the Bergeron models. The main difference between these two sets of cooling models is that in the Althaus models the evolution prior to WD formation is taken into account to arrive at a WD with a certain core composition, whereas in the Bergeron models the ad hoc assumption is made that all WDs have a CO core.

In order to compare the two cooling models, we take the low-mass end of the CO WDs in the Bergeron models as the “He core” WDs in their models, and the high-mass end as their “ONe core” WDs, since non-CO core WD cooling models from Bergeron et al. were unfortunately not available in the literature. A comparison between these cooling models is given in Figure 1.2. For details about the CNO flashes, which are very prominent on the cooling branch of the He WD in the left panel of Figure 1.2, we refer to Althaus et al. [2013]. Because the global specific heat of the He WDs is larger in the Althaus models, at a given cooling time the luminosity of such a low-mass WD is higher than that of a WD with the same mass in the Bergeron models (compare the two dashed lines in Figure 1.2). Similarly, the global specific heat of ONe WDs is smaller in the Althaus models, resulting in lower luminosity WDs at a given cooling time compared to the high-mass WDs in the Bergeron models (the dotted lines in Figure 1.2).

The explanation of the difference between the two solid lines in Figure 1.2 is a bit more complicated, since in this case the WD core composition is the same in both models. Whether the prior evolution of the WD is or is not taken into account, will affect the onset of crystallization and the magnitude of the energy released by CO phase separation, a process that affects the cooling times below log(L/L⊙) ≈ −4. This could account for part of the difference in the cooling times between both models for the CO WDs. In addition, at low luminosities, the WD evolution is sensitive to the treatment of the outer boundary conditions and the equation of state at low densities [Althaus, 2012].

Having chosen a set of cooling tracks, we want to determine the present-day luminosites for all the WDs in our model. Per WD type (He, CO, or ONe), typically only ten cooling tracks are available; these are interpolated and extrapolated over mass and cooling time to cover the whole parameter space that is sampled by our population synthesis code. The interpolation is linear, both in mass and in cooling time. For WDs that are more massive than the most massive WD of which a cooling track is available in the literature, we assume the cooling to be the same as for the most massive that is available. Similarly, the cooling track of the least massive WD is taken for WDs with a lower mass. In the Althaus models, this “extrapolation in mass” is done for He
Chapter 1: Binary white dwarfs in the halo of the Milky Way

WDs with \( m < 0.155 \) or \( m > 0.435 \), for CO WDs with \( m < 0.505^2 \) or \( m > 0.934 \), and for ONe WDs with \( m > 1.28 \). At the faint end of the cooling track, for CO WDs with \( m > 0.878 \) and \( t_{\text{cool}} \gtrsim 10^{10} \) years and ONe WDs with \( t_{\text{cool}} \gtrsim 6 \cdot 10^9 \) years (\( \pm 2 \cdot 10^9 \) years, depending on the WD mass), the luminosity is extrapolated using Mestel [1952] cooling. If the cooling tracks that we use in this study do not give a value for the luminosity of the WD at birth, we keep the luminosity constant at the value corresponding to the first given cooling time (for ONe WDs, this is \( \sim 10^5 \) years). This yields lower limits to the luminosities of very young ONe WDs in the Althaus models and for all types of very young WDs in the Bergeron models.

In this way we construct three grids of WD cooling (one for each WD type; He, CO, or ONe), which are shown in the three different panels of Figure 1.3. In this catalogue, the luminosity of every star in our simulation box can be found. The mass, type and cooling time of every WD in our simulation box was matched to the nearest catalogue grid point using the K3Match software [Schellart, 2013]. The dashed lines in Figure 1.3 indicate the boundaries beyond which extrapolation was done as described above. Dotted lines similarly indicate the region beyond which the \( M_V \) and \( M_I \) magnitudes were determined using extrapolation. The luminosity and colour regions that are covered by the available cooling tracks in the literature overlap for ONe WDs, and so does the \( m = 0.505 \) boundary for CO WDs. This is indicated by the light grey dashes in between the dark grey vertical lines at the overlapping points. The scattered points in Figure 1.3 visualize the position of the halo WDs that are observable with Gaia according to our standard stellar halo model. They will be discussed in section 1.3.3.

1.2.5 Preparation of the WDLF

In this paper, we distinguish between spatially resolved and unresolved binaries. For each binary system, from the assigned distance and orbital separation a separation on the sky can be determined. Assuming thus two stars in a binary should be separated by at least 0.1 - 0.2 arcsec in order to be spatially resolved by Gaia [Arenou et al., 2005], we assign all binaries with a separation larger than or equal to 0.3 arcseconds to the group of resolved binaries, those with a smaller separation to the group of unresolved binaries. Unresolved binaries are included as a single WD in the WDLF with a luminosity equal to the sum of the luminosities of the individual WDs in the binary.

To obtain a WDLF that can be compared with the observed one by RH11, we transform the luminosities of the WDs in our simulation box to bolometric magnitudes (using \( M_{\text{bol,}} = 4.75 \)). We divide the total magnitude range into 2 bins per magnitude, ending up with bins such as \( M_{\text{bol,}} = [3.0, 3.5] \) and \( [3.5, 4.0] \), etc. The total number of stars per bin is then divided by the effective volume of our simulation box (\( V_{\text{eff}} = M_{\text{unev}}/\rho_0 \)) to arrive at \( N PC^{-3} M_{\text{bol}}^{-1} \).

There are also observational selection effects that need to be taken into account. Because RH11 only included halo WDs with tangential velocities \( v_t > 200 \) km s\(^{-1}\), we reduce the number of WDs in each luminosity bin by a factor \( P(v_t > 200) \), which represents the probability that the tangential velocity of a halo star exceeds 200 km s\(^{-1}\). The tangential velocities of halo stars [Chiba & Beers, 2000] along the line of sight to one of the SuperCOSMOS survey fields is shown.

\({}^2\)The cooling track corresponding to the 0.5 M\(_{\odot}\) CO WD in the Althaus models that is plotted in Figure 1.2 is thus assumed to be equal to that of an 0.505 M\(_{\odot}\) WD in these models.
1.2 Model ingredients

Figure 1.3: From left to right: constructed grids of He, CO and ONe WD cooling in the Althaus models. The width of the panels corresponds to the maximum possible mass range per panel in our simulation, see Table 1.2. Vertical (horizontal) dashed lines indicate masses (cooling times) beyond which luminosities are obtained from extrapolation of the used cooling tracks. Dotted lines indicate the boundaries beyond which \( M_v \) and \( M_i \) magnitudes are obtained by extrapolation. For ONe WDs these regions completely overlap, for CO WDs the lower mass boundary partly overlaps, which is indicated by the light-grey-dark-grey dashed line. Scattered points indicate halo WDs that are observable with Gaia in our standard model. An explanation of the symbols is given section 1.3.3.
in Figure 16 in RH11. From this figure, we estimate that $P(v_t > 200)$ lies between 0.4 and 0.5, therefore we take 0.45. In our results section we will show the effect of choosing $P(v_t > 200) = 0.4$ or 0.5 instead.

### 1.2.6 Gaia magnitudes and extinction

The light from distant stars gets absorbed and reddened by interstellar dust. Following Toonen & Nelemans [2013], we assume that the dust follows the distribution

$$P(z) \propto \text{sech}^2 \left( \frac{z}{z_h} \right),$$

(1.6)

where $z_h$ is the scale height of the Galactic dust (assumed to be 120 pc) and $z$ the cartesian coordinate in the $z$-direction. The interstellar extinction $A_V$ between the Milky Way and a distant Galaxy in the $V$-band is assumed to be the extinction between us and a star at a distance $d = \infty$ [Sandage, 1972], for Galactic latitude $b = \arcsin(z/d)$:

$$A_V = \begin{cases} 0.165(\tan 50^\circ - \tan b) \csc b & \text{if } |b| < 50^\circ \\ 0 & \text{if } |b| \geq 50^\circ \end{cases} \equiv A_V(\infty).$$

(1.7)

The fraction of the extinction between us and a star at a distance $d$ with Galactic latitude $b \neq 0$ and this extinction is then

$$\frac{A_V(d)}{A_V(\infty)} = \frac{\int_0^{d \sin b} P(z)dz}{\int_0^\infty P(z)dz} = \tanh \left( \frac{d \sin b}{z_h} \right).$$

(1.8)

Equation (1.7) is used to calculate the apparent magnitude $V$ of a star at distance $d$ and Galactic latitude $b \neq 0$:

$$V = M_V + 5 \left( \log_{10}(d) - 1 \right) + A_V(\infty) \tanh \left( \frac{d \sin b}{z_h} \right).$$

(1.9)

Because $A_I(d) = 0.6 \ A_V(d)$ [Schlegel et al., 1998], we similarly calculate $I$ from $M_I$ and equation (1.7), after which Gaia magnitudes are calculated using [Jordi et al., 2010]

$$G - V = a_1 + a_2(V - I) + a_3(V - I)^2 + a_4(V - I)^3$$

(1.10)

with $a_1 = -0.0257$, $a_2 = -0.0924$, $a_3 = -0.1623$ and $a_4 = 0.0090$. We expect Gaia to detect all WDs with $G < 20$ [Brown, 2013].

### 1.3 Results

We start our results section with an analysis of the theoretically determined WDLF in our standard halo model and compare it with the observationally determined one by RH11. In the second part of this section, we will compare the WDLFs predicted by models that were introduced in section 1.2 and discuss our findings. In section 1.3.3, we examine the halo WD population in more detail, again for our standard model. We derive the number of halo WDs that will be detectable by the Gaia satellite, and also predict properties of the whole population of (binary) WDs in the halo.
1.3 Results

1.3.1 Standard model WDLF: theory vs. data

In section 4.3 we have seen that the cooling tracks for He core WDs are quite different from those with CO cores, which in turn differ from ONe core WD cooling tracks. The effect of this can be seen partly in Figure 1.4, where we present how the WDLF for our standard halo model is built up from contributions of the various WD types. We first note the monotonic increase in the WDLF, which occurs because the cooling of WDs is a simple gravothermal process [Isern et al., 2013]. The drop in the number of stars at $M_{\text{bol}} \approx 16$ is a consequence of the finite age of the universe. As was suggested by e.g. Winget et al. [1987], the observation of this drop can be used to constrain the age of our Galaxy. A second peak in the WDLF (the dotted curve around $M_{\text{bol}} \approx 19$ in Figure 1.4) is expected to consist of ONe WDs, due to their fast cooling times, as was first pointed out by Isern et al. [1998].

The contribution from the He WDs to the WDLF is shown with a dot-dashed line in Figure 1.4. These He WDs have an unseen neutron star (NS) or black hole (BH) companion or they are the

---

Figure 1.4: Build-up of the WDLF in our standard halo model (50% binaries). All WDs are included in the black solid line (the total WDLF). The lower solid line shows the contribution from unresolved binary WDs. The dashed, dot-dashed and dotted lines show the contributions from CO, He and ONe WDs respectively. The light blue line with error bars is the halo WDLF determined from the SuperCOSMOS Sky Survey [Rowell & Hambly, 2011], shown for comparison.
resulting merger product of two stars in a binary. However, there are many more He WDs that contribute to the WDLF: those in unresolved binary WD pairs (the lower solid line in Figure 1.4). Since they have slow cooling times, the contribution of He WDs to the WDLF is largest at the bright end. The unresolved binaries that end up in the second peak of the WDLF (around $M_{\text{bol}} \approx 18.5$) are systems in which at least one of the two WDs has an ONe core. The main contributors to the WDLF are CO WDs, visualized with a dashed line in Figure 1.4, which is just below the black solid line. These can be single CO WDs, CO WDs in wide binary WD pairs, but also CO WDs with a NS or BH companion or merger products. WDs with a main-sequence star as companion are not included in the WDLF, because the light from the main-sequence star will dominate the spectrum in that case.

Figure 1.4 shows that our standard model WDLF lies below the observed WDLF (RH11; the light blue line with error bars in Figure 1.4), however we shall see in the next subsection that this discrepancy disappears when we vary the normalization. Our integrated standard model luminosity function (the black solid line in Figure 1.4) yields $n_{\text{Halo WDs}} = 2.08 \cdot 10^{-5} \text{pc}^{-3}$. This value is lower than the integrated value of the RH11 WDLF, $(1.4 \pm 5.6) \cdot 10^{-4} \text{pc}^{-3}$, mainly because of their higher estimate of the number of WDs in the luminosity bins around $M_{\text{bol}} \approx 17$. Our models predict that there are no WDs in these bins. Although the present-day estimate of the number of WDs with $M_{\text{bol}} \approx 17$ should be regarded as an upper limit because of the large error bars, future observations on the shape of this faint end of the WDLF will help to constrain WD cooling models and the SFH of the halo, whereas the normalization of the WDLF, especially at the faint end, will help to constrain the IMF and binary fraction (see section 1.3.2).

When comparing our theoretically determined WDLF with the observed one by RH11 apart from the missing faint end (which is not reached by SuperCOSMOS because it is a magnitude-limited survey), also the missing bright end catches the eye. Here, another selection effect plays an important role:

RH11 only included WDs with a tangential velocity larger than $v_{t,\text{min}} = 200 \text{ km s}^{-1}$, to filter out thin and thick disk WDs. Due to the mean lower proper motion completeness limit $\mu_{\text{min}} = 40 \text{ mas yr}^{-1}$ across the SuperCOSMOS Sky Survey, the sample of RH11 is becoming less complete at a distance of approximately

$$d_{\text{max}} \frac{\text{pc}}{\text{pc}} = \left( \frac{p_{\text{min}}}{\text{arcsec}} \right)^{-1} = \frac{v_{t,\text{min}}}{4.74 \text{ km s}^{-1}} \left( \frac{\mu_{\text{min}}}{\text{arcsec yr}^{-1}} \right)^{-1} \approx 1 \cdot 10^3. \quad (1.11)$$

Here, $p_{\text{min}}$ is the minimum parallax that is reached and we have used that a proper motion of 1 arcsec yr$^{-1}$ corresponds to a tangential velocity of 1 AU yr$^{-1} = 4.74$ km s$^{-1}$ at 1 pc. Because at distances larger than $\sim 1 \text{ kpc}$, young and bright halo WDs contribute more to the WDLF than fainter WDs, the bright end of the WDLF is not reached by SuperCOSMOS.

We expect this latter bias to be resolved by Gaia, which can do microarcsecond astrometry and, as we will show in section 1.3.3, is expected to detect intrinsically bright WDs to distances of $\sim 2.5 \text{ kpc}$. Although the bright end of the WDLF has already be determined from an empirical measure of the WD cooling rate in a globular cluster (Goldsbury et al. 2012), we will soon have a new window on the Galactic halo, when we start to explore bright field halo WDs with Gaia.
Figure 1.5: Comparison of halo WD luminosity functions corresponding to different assumptions about the IMF (top left), WD cooling (top right), binary fraction (bottom left) and SFH of the halo (bottom right). The WDLF corresponding to our standard model, indicated by the blue solid line in all panels, is constructed using the $1.2 \times 10^7$ WDs in our simulation box (see Table 1.1).
1.3.2 Comparing the WDLFs of different halo models

Eight different model WDLFs are visualized in Figure 1.5, comparing the effect of a different IMF, cooling track, binary fraction and SFH. As explained at the beginning of section 1.2, we calculate not only the shape of the WDLF, but also derive its normalization from an independent mass density estimate of halo stars. For each model in Figure 1.5 a band is given rather than a single line, which comes from the two different normalizations explained in section 1.2.1. To arrive at the upper lines, the lower line is simply multiplied with a normalization factor corresponding to the used IMF \((5.9/1.9\) for the Kroupa IMF, \(6.7/1.1\) for the Salpeter IMF and \(330/80\) for the top-heavy IMF, see Table 1.1). The blue band in each panel represents our standard model, labelled “Kroupa”, “Althaus”, “50% binaries, 50% singles” and “13 Gyr burst” respectively.

The left panels of Figure 1.5 show that a different IMF or binary fraction affects the normalization of the WDLF, as expected from sections 1.2.1 and 1.2.2. Regarding the shape of these five different WDLFs in the left panels of Figure 1.5, the differences are the largest at the extremely faint and the extremely bright end. We see that the WDLF corresponding to a model with a larger binary fraction resembles more closely the shape of the lower solid line in Figure 1.4, where the contribution to the WDLF from unresolved binary WDs is shown.

From the top right panel in Figure 1.5 it is clear that there is a significant difference in the shape of the faint end of the WDLF when a different assumption is made about WD cooling. The drop between the two peaks of the WDLF is less prominent when the Bergeron models are used compared to the Althaus models, because CO WDs with a luminosity \(\log(L/L_\odot) < -4\) cool faster in the Bergeron models than in the Althaus models (see the right panel of Figure 1.2).

The logarithmic scale on the vertical axis implies that it will be observationally challenging to distinguish the three different models of SFH (shown in the bottom right panel). These differ slightly from each other at the the faint end of the WDLF. As expected, the WDLF of a 10 Gyr halo drops off at lower magnitudes than a 13 Gyr old halo. Furthermore, the gap between the two peaks of the WDLF is more prominent in the models with a SF burst compared to models with a continuous SFH.

There is a quite good overall agreement between our theoretically predicted WDLFs and the observed one by RH11, except for the model with a top-heavy IMF, which overpredicts the number of WDs per luminosity bin at the faint end of the WDLF. This also follows from the reduced \(\chi^2\)-test that we conducted to compare the agreement between the different model WDLFs in Figure 1.5 with the observationally determined WDLF quantitatively (see Table 1.3).

From the first column of Table 1.3 we see that our standard model and the model with 100% singles have the lowest reduced \(\chi^2\) values \(\chi^2 = 2.29\) and \(2.28\) respectively, closely followed by the models with alternative SFHs \(\chi^2 = 2.31\) for the model with uniform SF between 10 and 13 Gyr, and \(\chi^2 = 2.35\) for the model with a SF burst 10 Gyr ago). The fact that all these values are so close together can also be determined from Figure 1.5, where these four curves almost completely overlap. The models with 100% binaries or a Salpeter IMF do slightly worse, due to their lower normalization. In all cases the line corresponding to the upper limit of the number of stars has worse agreement with the observed WDLF \(\chi^2_{\text{upper}}\) second column) than the lower line corresponding to that same model. This seems to indicate that the low-mass part of the IMF does not turn over at \(\sim 1.0\ M_\odot\) to become completely flat, but rather has a negative slope.
1.3 Results

Table 1.3: Reduced $\chi^2$ values for eight halo models.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\chi^2$</th>
<th>$\chi^2_{\text{upper}}$</th>
<th>$\chi^2_{\text{min}}$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>2.29</td>
<td>7.07</td>
<td>2.26</td>
<td>1.28</td>
</tr>
<tr>
<td>Salpeter</td>
<td>2.99</td>
<td>7.76</td>
<td>2.27</td>
<td>2.40</td>
</tr>
<tr>
<td>Top-heavy</td>
<td>5.74</td>
<td>140</td>
<td>2.67</td>
<td>0.44</td>
</tr>
<tr>
<td>Bergeron</td>
<td>2.74</td>
<td>5.68</td>
<td>2.68</td>
<td>1.38</td>
</tr>
<tr>
<td>100% singles</td>
<td>2.28</td>
<td>16.8</td>
<td>2.33</td>
<td>0.88</td>
</tr>
<tr>
<td>100% binaries</td>
<td>2.61</td>
<td>3.57</td>
<td>2.25</td>
<td>1.76</td>
</tr>
<tr>
<td>10 Gyr burst</td>
<td>2.35</td>
<td>13.5</td>
<td>2.43</td>
<td>0.95</td>
</tr>
<tr>
<td>Uniform 10–13 Gyr</td>
<td>2.31</td>
<td>9.50</td>
<td>2.37</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Notes: Reduced $\chi^2$ values for all model WDLFs (first column), reduced $\chi^2$ values corresponding to the upper limits (second column), minimum value that the reduced $\chi^2$ can become ($\chi^2_{\text{min}}$) by multiplying the model WDLF with a factor $f$ (third and fourth column). In the first two columns there are 28 degrees of freedom, in the last column there is one degree of freedom less.

We varied the normalization of the WDLFs by multiplying them with a free parameter $f$, to see how well we can fit the shape of the WDLF. We kept $f$ as a free parameter, because there are many ways in which we could adapt the normalization, for example choosing a different $\gamma_{\text{unev}}$ (as we did for calculating the upper limits), a different binary fraction or a different mass density in unevolved stars $\rho_0$. The results of this analysis (summarized in the parameter $\chi^2_{\text{min}}$) are given in the third column of Table 1.3 for each model with the corresponding $f$ value in the fourth column. Without normalizing the model WDLFs, the model with 100% binaries comes out best, with a reduced $\chi^2_{\text{min}}$ value of 2.25. Although these minimum $\chi^2_{\text{min}}$ values lie close together for most of the models, the Top-heavy and Bergeron models still have the worst agreement with the WDLF observed by RH11. For the two models with alternative SFHs and the model with 100% singles the $\chi^2_{\text{min}}$ values are larger than the $\chi^2$ value corresponding to our preferred normalization, because there is one degree of freedom less if we fix the normalization of the WDLF.

The $\chi^2$ values are also affected by our assumption of $P(v_t > 200)$. If we had chosen the value $P(v_t > 200) = 0.4$ or 0.5, our standard model $\chi^2$ value would change to 2.39 or 2.22 respectively, and how this other choice affects the other curves can be determined from the parameter $f$. If $f$ is larger than 1, the larger value $P(v_t > 200) = 0.5$ would reduce the $\chi^2$ value, if $f$ is smaller than one $P(v_t > 200) = 0.4$ would yield a better match.

1.3.3 Halo white dwarfs detectable by Gaia

In this subsection of our results, we take a closer look at the population of halo WDs in our standard model and what fraction of this population can be seen by the Gaia satellite.

An important point to keep in mind when studying halo WDs, is that one is biased towards young and bright WDs in a magnitude-limited survey. Since the bright part of the WDLF is to
a large extent built up by unresolved binary WD pairs (see Figure 1.4), we first look at their properties. The right-hand side panels of Figure 1.6 show the properties of all unresolved binary WD pairs in our simulation box, whereas the left-hand side panels of this Figure focuses on the \( \sim 300 \) unresolved binary WD pairs with \( G < 20 \). We note that this also includes binaries with large orbital separations which have never undergone interaction, because at large distances these can still be unresolved. Of the two WDs in each binary, the properties of the brightest are plotted. A distinction is made between CO+He WDs and He+CO WDs, the second of the two WD types in each group is the brightest WD in the system. For most of the systems, this is also the youngest WD. However, in some CO+He systems the He WD was formed first and is still brighter than the later formed CO WD, which is possible since He WDs can be intrinsically brighter than CO WDs at birth and they in general have longer cooling times (see section 4.3). In the legend of each panel on the left-hand side of Figure 1.6, the number of systems of that particular kind is given in brackets. Due to the low number of halo WDs we expect to find in the Milky Way, there is some statistical noise in this Figure. What we want to show here, are the global positions of the WDs in this diagram, without focussing on their individual positions.

A particular aspect of the cooling models that we use as our standard, is that the luminosity of He WDs stays constant for a long time \((10^5 - 10^9 \text{ yr})\), before cooling starts. This can be seen from the dark dashed line in the left panel of Figure 1.2. As a consequence of this feature, He WDs that are on this part the cooling track will be seen more often than CO WDs with the same cooling time. We will refer to them as pre-WDs, to indicate that these objects do not look like standard WDs, because they are brighter and have smaller surface gravities.

In the top panels of Figure 1.6 we show the \( \log g - \log T_{\text{eff}} \) diagrams. The pre-WDs (plotted with star symbols) are clearly distinguishable from the other WDs because of their low surface gravities. We define pre-WDs as those double WDs in which the brightest of the two has \( 4.3 < \log g < 6 \). There are more He WDs with even lower surface gravities, indicated by the label “other” in the legends. However, these will be hard to distinguish from main-sequence stars or giants, which lie on or above the dashed line at \( \log g \approx 4 \) in Figure 1.6 [Allen, 1973]. Because pre-WDs are only apparent when we use the Althaus cooling tracks for He WDs (see Figure 1.2), Figure 1.6 would not have any points with \( \log g < 6 \) if we use the models with Bergeron cooling instead of our standard model. In the top right-hand side panel of Figure 1.6 we see that more than 95% of the halo WDs have \( \log g > 7.0 \) and \( \log T_{\text{eff}} < 4.2 \), which is also the part of the diagram where most of the single WDs and resolved WD binaries are expected to be situated. Furthermore, we see from the top left-hand side panel of Figure 1.6 that there is a narrow gap between the \( \log g \) and \( \log T_{\text{eff}} \) values of systems in which the brightest WD has a CO core and those in which the brightest of the two has a He core. In this way, these systems can in principle be distinguished from their positions in the \( \log g - \log T_{\text{eff}} \) diagram.

The middle panels of Figure 1.6 show the IFMR for halo WDs, i.e. the mass of the brightest WD in the binary system \( (M_{\text{bright}}) \) is plotted as a function of its corresponding initial mass \( (M_{\text{ZAMS}}) \). In the middle left-hand side panels we see that in most of these unresolved binary WDs the brightest of the two stars has a main-sequence progenitor star with a mass \( M_{\text{ZAMS}} \approx 0.84 \, M_{\odot} \). Because in our standard model halo stars are born about 13 Gyr ago, these stars have just become WDs, thus will be very abundant in the Gaia catalogue of halo WDs. There are very few high-mass
FIGURE 1.6: Caption on next page.
Figure 1.6: Left panels: Properties of unresolved binary WD pairs with $G < 20$ for our standard model. Shown are double He WDs (circles), double CO WDs (squares), CO+He systems in which the He WD is the brightest (upward pointing triangles), He+CO systems in which the CO WD is the brightest (downward pointing triangles), pre-WDs (filled stars), and “other” stars, which are pre-WDs that are indistinguishable from MS stars or giants. After the label discriptions in the legend, the total number of WD binaries of that particular kind is given. Due to the low number of halo WDs with $G < 20$ we expect to find in the Milky Way, there is some statistical noise in these panels. Right panels: Properties of all $1.3 \cdot 10^6$ unresolved binary WD pairs in our simulation box (maximum magnitude $G = 35$), for our standard model. Contour lines mark the regions in which 33%, 67% and 95% of the binaries are located. Higher density regions are given darker colours. The 1% binaries in the tail of the distribution are indicated by the scattered points. Compared to the left-hand side panels, these panels hardly have statistical noise because of the large number of halo WDs we expect to find in the Milky Way halo if we would go up to magnitude $G = 35$.

WDs in our sample, mainly because their progenitor stars have shorter main-sequence lifetimes and they have thus cooled down much more. In the middle right-hand side panel, the global IFMR for halo WDs is shown. There is no focus on only the brightest WDs, with the result that the highest density region is shifted towards a line that resembles the IFMR of single stars, populated by the unresolved binary WDs that have not undergone interaction.

In the bottom panels of Figure 1.6 the $M_{\text{bright}} -$ orbital period relation for halo WDs is shown. We see that the unresolved binary WD pairs with $G < 20$ lie on three distinct lines, where each line is mostly populated by one of the different binary types. The majority of double CO systems have not interacted and thus evolve to systems with wide periods. Because most stars are low-mass they form WDs with similar masses, while the periods are determined by the initial period distribution. The short-period branch shows systems that are formed via CE evolution. In our model, this CE between a giant and a WD is always described by a the energy balance [$\alpha$, see Toonen et al., 2012]. The correlation they show between WD mass and orbital period can then be understood from the relation between the core mass and the radius of giants. Systems that start the CE phase in a more compact orbit will have giants with smaller radii and thus lower-mass cores. This means both a spiral in to shorter final periods and a final WD mass that is lower. The branch with longer periods shows systems that are formed via a second phase of mass transfer that was stable. During the mass transfer the orbit widens, which stops when the whole envelope of the giant has been transferred to the first formed WD. Due to the same relation mentioned above, giants with larger core masses (that form more massive WDs) are bigger and thus end their evolution in binaries with longer orbital periods. The same relation is seen in the WD companions to millisecond radio pulsars [Savonije, 1987]. From the right-hand side panels of Figure 1.6 it is clear that the left-hand side panels only resembles a small part of the complete parameter space, but it constitutes a representative selection of the low-mass part of this diagram.

White dwarfs in unresolved binaries are of course not the only halo WDs we expect Gaia to observe. As we already mentioned in section 4.6, single WDs, resolved double WDs, WDs with a NS or BH companion, and WDs that are the result of a merger also contribute to the WDLF.
The number of WDs in each of these five groups is specified per WD type (pre-WD, He, CO, or ONe core) in Table 1.4. We see that all single WDs and all brightest WDs in a resolved binary system have a CO core with the limiting magnitude $G < 20$. If we look at fainter magnitudes, e.g. $G < 23$ or $G < 26$, Table 1.4 shows that ONe WDs will be detected, although there are still very few of them compared to CO WDs. The same is true for WDs with a NS or BH companion.

When selecting halo WDs from the Gaia catalogue, selection effects are expected, like the factor $P(v_t > 200)$ that we multiplied our theoretically determined WDLFs with in the previous subsections to compare our results with that of RH11. Here, we do not include this factor, since it is not yet clear how large it will be. For example, for some fraction of the stars ($V < 17$), radial velocities will also be available. Therefore it should be possible to obtain a larger number of halo WD stars than just with a cut in $v_t$. Furthermore, the determination of the initial number of stars in our simulation box has a greater effect on the number of halo WDs than these selection effects have.

In the top and bottom rows of Table 1.4, the total number of halo WDs in two spheres around the Sun with respective radii 400 pc and 2.95 kpc is given, as well as the total mass these halo WDs constitute. From these we calculate the number densities of halo WDs $n_{\text{Halo WDs}} = 5.89 \cdot 10^{-5} \text{ pc}^{-3}$ (within 400 pc) and the slightly higher value $n_{\text{Halo WDs}} = 6.00 \cdot 10^{-5} \text{ pc}^{-3}$ (within 2.95 kpc). These values are more than a factor of two larger than the number density we derived by integrating the WDLF in section 4.6. This difference is due to the factor $P(v_t > 200)$ which is not taken into account here. Furthermore, here all halo WDs are counted within spheres of a certain radius around the Sun, whereas in section 4.6 we estimated the number density including the edges of our simulation box (which is not a sphere, see Figure 1.1).

Torres et al. [2005] also estimated the number of (single) halo WDs with $G < 20$ within 400 pc, and found 542 (their Table 3). We found slightly more halo WDs within 400 pc: 621, including both singles and binaries, see the top row of Table 1.4. However, it is not strange that these numbers differ from each other, given the large number of uncertainties in our estimate of the number density of halo WDs from the observed mass density in unevolved stars (section 1.2.1) and the selection effects. The latter are implicitly taken into account by Torres et al. [1998], because they normalized the number density of halo WDs within 400 pc to the local observed value [Torres et al., 1998].

To check that our simulation box is large enough and make the claim that Gaia can detect approximately $1.5 \cdot 10^3$ halo WDs with $G < 20$ (Table 1.4), we plotted the distances to all these WDs as a function of their bolometric magnitude in Figure 1.7. With the same markers as in Figure 1.6 the unresolved binaries are visualized, additional markers are used for single WDs, resolved binary WDs and WDs that are merger products. We see that apart from a few outlying “other” WDs, all WDs fit well within the sphere with radius $\xi = 2.95 \text{ kpc}$ around the Sun, which validates the size of our simulation box. As explained above, the “other” WDs were excluded from our luminosity function anyway because of their low surface gravities.

Projected onto the vertical axis of Figure 1.7 is the number of halo WDs that can be found in every luminosity bin of the WDLF from the Gaia catalogue. From this right panel of Figure 1.7 we get an idea of the statistical errors that are to be expected per luminosity bin of the WDLF. We see that the faint end of the WDLF will stay underdetermined since we expect to detect
Table 1.4: Number of halo WDs in our simulation box.

<table>
<thead>
<tr>
<th></th>
<th>single WD</th>
<th>merger→WD</th>
<th>WD+WD.unres</th>
<th>WD+WD.res</th>
<th>WD+NS/BH</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d &lt; 400$ pc</td>
<td>$N_{\text{tot}}$</td>
<td>395</td>
<td>83</td>
<td>85</td>
<td>58</td>
<td>621</td>
</tr>
<tr>
<td></td>
<td>$N_{\text{tot}}$</td>
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<td>2604</td>
<td>1340</td>
<td>1447</td>
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<tr>
<td></td>
<td>$M_{\text{tot}}$</td>
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<td>$1.83 \cdot 10^3$</td>
<td>$1.63 \cdot 10^3$</td>
<td>$2.09 \cdot 10^3$</td>
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</tr>
<tr>
<td>$G &lt; 20$</td>
<td>$N_{p\text{WD}}$</td>
<td>—</td>
<td>1</td>
<td>5</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$N_{b=\text{He}}$</td>
<td>—</td>
<td>99</td>
<td>164</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$N_{b=\text{CO}}$</td>
<td>872</td>
<td>97</td>
<td>128</td>
<td>157</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$N_{b=\text{On}}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
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</tr>
<tr>
<td></td>
<td>$N_{\text{tot}}$</td>
<td>872</td>
<td>197</td>
<td>297</td>
<td>157</td>
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</tr>
<tr>
<td>$G &lt; 23$</td>
<td>$N_{p\text{WD}}$</td>
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<td>29</td>
<td>365</td>
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<td>—</td>
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<tr>
<td></td>
<td>$N_{b=\text{He}}$</td>
<td>—</td>
<td>$4.63 \cdot 10^3$</td>
<td>$7.00 \cdot 10^2$</td>
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<td>12</td>
<td>13</td>
<td>9</td>
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<td>$8.97 \cdot 10^4$</td>
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<td>$d &lt; 2.95$ kpc</td>
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<td>405</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$N_{b=\text{He}}$</td>
<td>—</td>
<td>$9.21 \cdot 10^4$</td>
<td>$1.66 \cdot 10^5$</td>
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<td>—</td>
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<tr>
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<td>$N_{b=\text{CO}}$</td>
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<td>$9.60 \cdot 10^5$</td>
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<tr>
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<td>$1.61 \cdot 10^4$</td>
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<td>$2.05 \cdot 10^3$</td>
<td>$2.62 \cdot 10^3$</td>
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<td>$1.07 \cdot 10^6$</td>
<td>$7.66 \cdot 10^5$</td>
<td>$3.80 \cdot 10^5$</td>
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<tr>
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<td>$M_{\text{tot}}$</td>
<td>$2.89 \cdot 10^5$</td>
<td>$7.49 \cdot 10^5$</td>
<td>$9.76 \cdot 10^5$</td>
<td>$5.54 \cdot 10^5$</td>
<td>$9.47 \cdot 10^4$</td>
</tr>
</tbody>
</table>

Notes: The three middle rows indicate magnitude-limited selections of halo WDs in our simulation box. For the volume-limited selections ($d < 400$ pc and $d < 2.95$ kpc) the total mass in WDs is indicated by $M_{\text{tot}}$. These numbers are determined using our standard model (50% binaries). An asterisk (*) stands for $G < 20$, and a long dash (—) indicates that the particular combination does not occur. The following abbreviations are used in the table: $b$ for brightest, res for resolved, tot for total and pWD for pre-WD.
1.3 Results

Figure 1.7: Distances and bolometric magnitudes of the $1.5 \cdot 10^3$ halo WDs that can be observed with Gaia, for our standard model. Top panel: distribution of their distances. Right panel: distribution of their bolometric magnitudes, which gives an idea of the statistical errors that are to be expected per luminosity bin of the WDLF for Gaia. The yellow stars, labelled “other”, are pre-WDs that are indistinguishable from MS stars or giants. These are not included in the projected distribution histograms.
only a handful of WDs with $M_{\text{bol}} > 16$ with Gaia in our standard halo model. However, already with the few WDs in the lowest luminosity bins that can be reached, we can start comparing our halo models. It is not clear whether the drop at $M_{\text{bol}} \approx 16$ is a detection limit, since a cut-off of the luminosity function due to the age of the Galaxy is expected at approximately the same bolometric magnitude (see Figure 1.4).

As we explained at the end of section 4.6, most of the WDs at the bright end of the WDLF can be included with Gaia whereas they could not before, since Gaia has a lower mean lower proper motion completeness limit than previous surveys. From the long tail of the distance distribution (top panel of Figure 1.7), we see that there are many halo WDs with $G < 20$ beyond $\sim 1$ kpc, which all have absolute bolometric magnitudes $M_{\text{bol}} < 8$. It is because of the inclusion of these WDs that the bright end of the WDLF will probably be better constrained with Gaia than ever before.

The masses and cooling times of the $1.5 \cdot 10^3$ halo WDs with $G < 20$ are plotted on top of the interpolated cooling track panels of Figure 1.3. Again, the same markers are used as in Figures 1.6 and 1.7. For the halo WDs that are merger products, it is indicated what type of WD the merger product is (He WD or CO WD) by the particular panel the marker is drawn in. Plusses and diamonds represent single WDs and resolved binary WDs respectively, which all have a CO core, as can also be seen from Table 1.4. This is not surprising since He WDs can only be formed through binary interaction within the age of the universe and ONe WDs cool so fast that they pile up at the faint end of the WDLF, which will not be covered by Gaia.

It is interesting to see the narrow line at $m = 0.54 \, M_\odot$ in Figure 1.3, where the single and non-interacting binary WDs pile up. This can be explained by the evolution lifetime of single and non-interacting ZAMS stars with an initial mass of $0.84 \, M_\odot$, which is equal to the age of the halo in our standard model and we see in the middle right-hand side panel of Figure 1.6 that they become $0.54 \, M_\odot$ WDs. Kalirai [2012, hereafter K12] first pointed out that the mass determination of these bright single halo WDs can be used to determine the age of the inner halo. The determination of the masses of the brightest WDs in a globular cluster provides an anchor point on the IFMR for low metallicity stars, since their ages can be deduced independently from the cluster age. K12 then drew a straight line through this anchor point and the mass of the brightest halo WDs, yielding a linear IFMR for WD masses between $0.50$ and $0.58 \, M_\odot$, see Figure 1.8. The age of the field halo stars was subsequently determined using the Dartmouth Stellar Evolution Database [Dotter et al., 2008], in which $\sim 0.55 \, M_\odot$ WDs (with $0.83 \, M_\odot$ progenitors) are $\sim 11.4$ Gyr old.

Although K12 did a careful analysis on the binarity of field halo WDs and in the globular cluster, the binary fraction in globular clusters is generally lower than in the field [eg. Ji & Bregman, 2013, and references therein]. Especially for the latter, the observed single WDs could thus still have an unseen companion or be the result of a binary merger. Furthermore, the behaviour of the (metallicity-dependent) IFMR in this mass regime is not well determined yet theoretically. This can be seen from Figure 1.8, where we compare the low-metallicity IFMR of K12 with two IFMRs predicted by the detailed stellar evolution code MESA [Paxton et al., 2010] and with two IFMRs predicted by SeBa. The relation in age and mass in SeBa shows a strong upturn in age between WD masses with $0.54 \sim 0.53 \, M_\odot$, after which the slopes become shallower again. This difference in slopes is due to two different evolution paths for low-mass main sequence stars. The higher mass
stars follow a standard evolution path: they become WDs after the AGB phase. The lower mass stars on the other hand, lose their envelope on the RGB, whereafter they become WDs. In MESA this transition between these two evolution paths is implemented differently, yielding a more linear IFMR, which however is still steeper than the one inferred by K12. The observations seem to be consistent with all of these model lines. In fact, it will be challenging to observationally distinguish between the mass-age relations predicted by MESA and SeBa, since the difference between the two sets of lines is largest after a Hubble time. If on the other hand, \( \sim 0.51 \text{ M}_\odot \) single WDs will be found to follow the black solid line in Figure 9, as some data seems to imply [see eg. Table 1 of Renzini et al., 1996], there is a challenge for the theoretical modellers to explain how such low mass WDs can be formed by single stellar evolution within the age of the universe.

This comparison between MESA and SeBa was made using AMUSE [Portegies Zwart et al., 2009, 2013; Pelupessy et al., 2013]. See also Renedo et al. [2010] for a comparison between theoretically and observationally determined IFMRs with different metallicities.

### 1.4 Conclusions

The easiest way to constrain the IMF with halo WDs is to determine the halo WD number density, because the normalization of the WDLF is linked one-to-one to the IMF. From a comparison between our derived halo WDLFs with the WDLF observed by RH11, we conclude that a Kroupa IMF (\( \chi^2 = 2.29 \)) is slightly preferred over a Salpeter IMF (\( \chi^2 = 2.99 \)). A top-heavy IMF (\( \chi^2 = 5.74 \)) clearly overpredicts the number of faint halo WDs. Due to large uncertainties on the normalization, it is not yet possible to completely rule out a non-standard IMF. However, also the shape of the WDLF corresponding to a top-heavy IMF has worse agreement with the WDLF observed by RH11 than those of the WDLFs corresponding to models with a Kroupa or Salpeter IMF. Although most investigated halo models match the observed WDLF approximately equally well if we fix the normalization of our WDLF (2.3 \( \lesssim \chi^2_{\text{min}} \lesssim 2.7 \)), none of the models comes close to a reduced \( \chi^2 \) value of 1.

The exact number of halo WDs that Gaia can observe depends on how easily they can be distinguished from thin and thick disk WDs. In our standard model we find that Gaia will be able to detect approximately \( 1.5 \cdot 10^3 \) halo WDs, which is an order of magnitude more than the currently known number of halo WDs. Taking into account selection effects will probably not reduce this number by more than a factor of two. A wrong assumption on the mass function of unevolved stars has a stronger effect on the determined number density of halo WDs, but this will probably also be constrained by Gaia. If our assumptions are correct, the error bars on the observationally determined WDLF will become smaller with Gaia, at the part of the luminosity function that already has the smallest error bars (e.g. \( 5 \lesssim M_{\text{bol}} \lesssim 10 \)), but also for the fainter luminosity bins. This implies that we might soon be able to start ruling out IMFs on the basis of their predicted WD number densities, especially in the \( M_{\text{bol}} \gtrsim 15 \) luminosity bins.

Since the effect of the SFH of halo stars is the strongest at the faint end of the WDLF, where we expect only a handful of WDs to be detected by Gaia, it will be observationally challenging to put strong constraints on this parameter in the near future. Although the differences are small, from the current observational constraints, we find that a model in which there was a burst of
Figure 1.8: Age of the brightest single halo WDs (and thus of the Galactic halo) as a function of WD mass. Since for single stars every age corresponds one-to-one to an initial stellar mass and WD mass, this is an alternative representation of the IFMR. The IFMRs for single stars predicted by SeBa (light blue curves) and those predicted by MESA (dark blue curves) both differ from the black solid straight line through the data points with error bars of Kalirai [2012]. The dashed lines correspond to the metallicity value that we used as our standard for the Milky Way halo in this study (Z=0.001), while the dotted lines correspond to half that metallicity value (Z=0.0005).

SF 13 Gyr ago ($\chi^2 = 2.29, \chi^2_{\text{min}} = 2.26$) is slightly preferred over a burst of star formation 10 Gyr ago ($\chi^2 = 2.35, \chi^2_{\text{min}} = 2.43$) and over continuous star formation 10 – 13 Gyrs ago ($\chi^2 = 2.31, \chi^2_{\text{min}} = 2.37$).

A determination of the masses of the brightest halo WDs can in theory be used to determine the age of the halo as suggested by K12. However, at this point only preliminary conclusions can be drawn since the observational uncertainties are large and the effect of binarity and/or metallicity can be underestimated. It would be useful to have more anchor points on the low-metallicity IFMR than the current single one.

With Gaia it will be possible to constrain for the first time the bright part of the field halo WDLF, where contributions from (unresolved) binary WDs are considerable [that this can be done in star clusters, and for singly evolved WDs was shown by Goldsbury et al., 2012]. By
determining the periods of WDs with masses below \( \approx 0.5 \, M_\odot \) (which can safely be assumed to be in binaries or to be the result of a binary merger) with Gaia follow-up observations, we can start to explore how binary stars with low metallicities evolve. In this paper, we do not vary any binary evolution parameters, but deviations from the predictions we made in the bottom panel of Figure 1.6 are produced by different models of binary evolution.

It might be possible to put some constraints on the binary fraction by the number of pre-WDs that will be observed, although we expect this number to be small, since a large fraction of the pre-WD candidates will be indistinguishable from main sequence stars or giants. Furthermore, with the Bergeron models for WD cooling, pre-WDs are not expected to exist at all. However, pre-WDs can help us to constrain WD cooling models, because they are situated on an uncertain part of the WD cooling track, in the early phases of WD cooling.

If future observations on halo WDs go up to fainter magnitudes than Gaia can observe, we will be able to determine the validity of a top-heavy IMF or the Bergeron cooling models. In this respect, observations of the Large Synoptic Survey Telescope (LSST) will be very helpful [over \( 4 \cdot 10^5 \) halo WDs to \( r < 24.5 \); LSST Science Collaboration et al., 2009]. To improve this study, WD cooling tracks and corresponding colours and magnitudes over the whole parameter range of WD masses and cooling times would be useful, for WDs with low-metallicity progenitors. In the near future, we will couple a semi-analytic model for galaxy formation with a binary population synthesis code and study how this affects the halo WD population.

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1.A

Cartesian coordinates are related to spherical coordinates by

\[
\begin{align*}
    x &= r \sin \theta \cos \phi \\
    y &= r \sin \theta \sin \phi \\
    z &= r \cos \theta
\end{align*}
\]

with radius \( r \), polar angle \( \theta \) and azimuth angle \( \phi \). The Sun is assumed to be at position \( (x, y, z)_\odot = (r_0, 0, 0) \), or equivalently at \( (r, \theta, \phi)_\odot = (r_0, \pi/2, 0) \). We define the primed coordinates

\[
\begin{align*}
    x' &= r' \sin \theta' \cos \phi' \equiv x \\
    y' &= r' \sin \theta' \sin \phi' \equiv y \\
    z' &= r' \cos \theta' \equiv q \, z
\end{align*}
\]

such that the local halo density (equation (2.2)) can be expressed independent of a polar angle and azimuth angle:

\[
\rho(x', y', z') \equiv \rho(r') = \rho_0 \left( \frac{r'}{r_0} \right)^n.
\]
We note that in the Galactic plane, \( z = 0 \), thus primed radius \( r' = r \) and the primed polar angle \( \theta' = \theta = \pi/2 \). At the Galactic pole, \( \theta' = \theta = 0 \), and \( r' = z' = qz = qr \). In all other cases, the relation between the \( r' \), \( \theta' \) and their spherical equivalents is given by

\[
\theta' = \arctan\left(\frac{\tan(\theta)}{q}\right) \quad (1.19)
\]

\[
r' = r \frac{\sin \theta}{\sin \theta'}. \quad (1.20)
\]

Since we assume an oblate stellar halo (\( q < 1 \)), it follows from equations 1.19 and 1.20 that \( \theta' \geq \theta \) and \( r' \leq r \) for any given point in the spheroid. Because we want a sphere with radius \( \xi \) around the Sun to be contained in our simulated area, we set the boundary conditions,

\[
r_0 - \xi \leq r \leq r_0 + \xi \quad (1.21)
\]

\[
\delta \leq \theta \leq \pi/2 - \delta \quad (1.22)
\]

\[
-\epsilon \leq \phi \leq \epsilon, \quad (1.23)
\]

with \( \delta \leq \arctan(r_0q/\xi) \) and \( \epsilon \leq \arctan(\xi/r_0) \). These set the limits of integration in our determination of the stellar halo mass:

\[
M = \frac{\rho_0}{r_0^3} \int_{r_0-\xi}^{r_0+\xi} r^{n/2} dr \int_{\delta}^{\pi/2-\delta} \left( \frac{\cos^2 \theta}{q^2} + \sin^2 \theta \right)^{n/2} \sin \theta \, d\theta \int_{-\epsilon}^{\epsilon} d\phi. \quad (1.24)
\]

In order to solve the integral over \( \theta \), we now first make an estimation of \( \delta \). With the assumed values of \( \xi \), \( q \) and \( r_0 \) mentioned in the main text, we find \( \delta \leq 0.334 \pi \). Thus, we take \( \delta = \pi/3 \). The integral over \( \theta \) can now be expressed as the hypergeometric function \( _2F_1(1/2, -n/2; 3/2; 1/4 - 1/4q^2) \). Again with \( q = 0.64 \) and \( n = -2.8 \) for consistency with Jurić et al. [2008], we find \( _2F_1(0.5, 1.4; 1.5; -0.36) = 0.866 \). Because this value of \( n \neq -3 \), the integral over \( r \) can also be evaluated:

\[
\frac{1}{r_0^n} \int_{r_0-\xi}^{r_0+\xi} r^{n/2} dr = \frac{\left( r_0 + \xi \right)^{n+3} - \left( r_0 - \xi \right)^{n+3}}{(n+3) r_0^n} = 3.91 \cdot 10^{11} \text{ pc}^3. \quad (1.25)
\]

The integral over \( \phi \) yields \( 2\epsilon \), thus after choosing \( \epsilon = \arctan(\xi/r_0) \) this reads \( 2\arctan(\xi/r_0) = 0.707 \). The multiplication of an assumed value of \( \rho_0 = 1.5 \cdot 10^{-4} \text{ M}_\odot \text{ pc}^{-3} \) [Fuchs & Jahreiß, 1998] with these three integrals gives \( M_{\text{unev}} = 3.6 \cdot 10^7 \text{ M}_\odot \).

### 1.B

In case \( \phi(m) \) is a single power law function between the upper and lower mass boundary of unevolved stars in our simulation box \( m_{\text{high,unev}} \) and \( m_{\text{low,unev}} \), the total mass in unevolved stars

\[
\frac{M_{\text{unev}}}{\text{M}_\odot} = \int_{m_{\text{low,unev}}}^{m_{\text{high,unev}}} A m^{-\gamma_{\text{unev}}} dm = \frac{A \left( m_{\text{high,unev}}^{1-\gamma_{\text{unev}}} - m_{\text{low,unev}}^{1-\gamma_{\text{unev}}} \right)}{1-\gamma_{\text{unev}}}. \quad (1.26)
\]

Given the mass in unevolved stars \( M_{\text{unev}} \) which was derived in Appendix 1.A, \( \gamma_{\text{unev}} = -1 \), \( m_{\text{high,unev}} = 0.8 \) and \( m_{\text{low,unev}} = 0.1 \), this results in a normalization constant belonging to the
lower limit on the number of unevolved (single) stars $N_{\text{unev}}$ in our simulation box $A_{\text{lower}} = 1.1 \cdot 10^8$. When substituted into equation (1.4), this yields

$$N_{\text{unev}} > A_{\text{lower}} (m_{\text{high,unev}} - m_{\text{low,unev}}) = 8.0 \cdot 10^7.$$  

(1.27)

We derive an upper limit on the number of evolved stars $N_{\text{ev}}$ in our simulation box, for the three different IMFs that we investigate in this paper by determining their normalization constants from the IMF at $m_{\text{high,unev}}$. For example, writing the normalization constant for the upper limit on the number of evolved stars in case of a Kroupa IMF as $B_{\text{upper}}$, the relation

$$\phi(m_{\text{high,unev}}) = A_{\text{lower}} = B_{\text{upper}} (m_{\text{high,unev}})^{-2.2}$$

leads to $B_{\text{upper}} = 7.0 \cdot 10^7$, from which follows

$$N_{\text{ev},\text{Kroupa}} < B_{\text{upper}} \cdot I_{\text{ev},\text{Kroupa}} = 5.9 \cdot 10^7,$$

(1.28)

where

$$I_{\text{ev},\text{Kroupa}} = \int_{0.8}^{1.0} m^{-2.2} dm + \int_{1.0}^{100} m^{-2.7} dm = 0.84.$$  

(1.29)

To obtain actual numbers instead of an upper limit, we assume that the low-mass part of the IMF is correctly given by equation (3.9), with normalization constant $B$,

$$\frac{M_{\text{unev}}}{M_\odot} = \int_{0.1}^{0.5} \frac{35}{19} B m^{-0.3} dm + \int_{0.8}^{0.5} B m^{-1.2} dm = 1.6 B,$$

(1.30)

again using the calculated total mass in unevolved stars $M_{\text{unev}} = 3.6 \cdot 10^7 M_\odot$, we find $B = 2.2 \cdot 10^7$. Now because

$$I_{\text{unev,\text{Kroupa}}} = \int_{0.1}^{0.5} \frac{35}{19} m^{-1.3} dm + \int_{0.8}^{0.5} m^{-2.2} dm = 5.5$$

(1.31)

we find

$$N_{\text{unev,\text{Kroupa}}} = B \cdot I_{\text{unev,\text{Kroupa}}} = 1.2 \cdot 10^8$$

(1.32)

$$N_{\text{ev,\text{Kroupa}}} = B \cdot I_{\text{ev,\text{Kroupa}}} = 1.9 \cdot 10^7.$$  

(1.33)

Assuming that the Salpeter IMF holds for masses $m > 0.8$ results in the same way into an upper limit on the number of evolved stars, whereas assuming that it is for the entire mass range $0.1 < m < 100$ gives the expected number of evolved stars. Since

$$I_{\text{ev,\text{Salpeter}}} = \int_{0.8}^{100} m^{-2.35} dm = 1.00,$$

(1.34)

the upper limit on the number of evolved stars in the case of a Salpeter IMF immediately follows from the normalization constant $C_{\text{upper}} = A_{\text{lower}}/m_{\text{high,unev}}^{-2.35} = 6.7 \cdot 10^7$,

$$N_{\text{ev,\text{Salpeter}}} < C_{\text{upper}} \cdot I_{\text{ev,\text{Salpeter}}} = 6.7 \cdot 10^7.$$  

(1.35)

The expected number of stars in our simulation box if the low-mass part of the mass function is also Salpeter

$$N_{\text{unev,\text{Salpeter}}} = C \cdot I_{\text{unev,\text{Salpeter}}} = 1.7 \cdot 10^8$$  

(1.36)

$$N_{\text{ev,\text{Salpeter}}} = C \cdot I_{\text{ev,\text{Salpeter}}} = 1.1 \cdot 10^7.$$  

(1.37)

43
with
\[ \frac{M_{\text{unev}}}{M_{\odot}} = \int_{0.1}^{0.8} C \ m^{-1.35} dm = 3.3 \ C, \]
thus \( C = 1.1 \cdot 10^7 \), and
\[ I_{\text{unev,Salpeter}} = \int_{0.1}^{0.8} m^{-2.35} dm = 15.6. \]

Finally, for the top-heavy IMF we derive the normalization constants for the Komiya IMF (indicated by the letter \( D \)) and the Salpeter IMF (indicated by the letter \( E \)) simultaneously, using the MDF of the halo described by An et al. [2013], who studied halo main-sequence stars with masses between 0.65 \( M_{\odot} \) and 0.75 \( M_{\odot} \) in the Sloan Digital Sky Survey. These authors found that the halo can be described by a two-component model, with 24% of the stars belonging to a low-metallicity population with a peak at [Fe/H] = \(-2.33\) (i.e. their calibration model). If this population of low-metallicity stars is born according to a Komiya IMF, we have
\[ \int_{0.65}^{0.75} D \exp \left[ -\frac{\log_{10}(m/\mu)}{2\sigma^2} \right] \frac{dm}{m} = 0.24 \int_{0.65}^{0.75} E m^{-2.35} dm, \]
which holds for and \( D \) and \( E \), as well as for \( D_{\text{upper}} \) and \( E_{\text{upper}} \). The normalization constants for the upper limit on the number of evolved stars in case of a top-heavy IMF follow again from
\[ \phi(m_{\text{high,unev}}) = \frac{D_{\text{upper}}}{m_{\text{high,unev}}} \exp \left[ -\frac{\log_{10}(m_{\text{high,unev}}/\mu)}{2\sigma^2} \right] + E_{\text{upper}} (m_{\text{high,unev}})^{-2.35} = A_{\text{lower}}. \]
From the standard integral
\[ \int \exp \left[ -\frac{\log_{10}(m/\mu)}{2\sigma^2} \right] \frac{dm}{m} = \frac{\sqrt{\pi/2}}{\sigma} \text{erf} \left[ \frac{\log_{10}(m/\mu)/\sqrt{2}\sigma}{\log_{10} e} \right] \]
it now follows that \( D_{\text{upper}} = 1.4 \cdot 10^9 \) and \( E_{\text{upper}} = 4.3 \cdot 10^7 \). Consequently, the number of evolved stars
\[ N_{\text{ev,Komiya}} < D_{\text{upper}} \cdot I_{\text{ev,Komiya}} = 3.3 \cdot 10^9 \]
\[ N_{\text{ev,Salpeter (top–heavy)}} < E_{\text{upper}} \cdot I_{\text{ev,Salpeter}} = 4.3 \cdot 10^7 \]
with
\[ I_{\text{ev,Komiya}} = \int_{0.8}^{100} \exp \left[ -\frac{\log_{10}(m/\mu)}{2\sigma^2} \right] \frac{dm}{m} = 2.29. \]
If the suggested top-heavy IMF holds in the low-mass regime,
\[ \frac{M_{\text{unev}}}{M_{\odot}} = \int_{0.1}^{0.8} D \exp \left[ -\frac{\log_{10}(m/\mu)}{2\sigma^2} \right] dm + \int_{0.1}^{0.8} E m^{-1.35} dm \]
\[ = 4.4 \cdot 10^{-3} D + 3.3 E, \]
where we used the standard integral:
\[ \int \exp \left[ -\frac{\log_{10}(m/\mu)}{2\sigma^2} \right] dm = \]
\[ -\frac{\sqrt{\pi/2}}{\mu \sigma} \exp \left( \frac{\sigma^2}{2 \log_{10} e} \right) \text{erf} \left[ \frac{\sigma^2 - \log_{10} e \log_{10}(m/\mu)}{\sqrt{2} \sigma \log_{10} e} \right]. \]
Combining equations 1.40 and 1.46, we find $D = 3.4 \cdot 10^8$ and $E = 1.0 \cdot 10^7$, as well as

$$N_{\text{unev}, \text{Komiya}} = D \cdot I_{\text{unev}, \text{Komiya}} = 2.4 \cdot 10^6 \quad (1.48)$$
$$N_{\text{unev}, \text{Salpeter (top−heavy)}} = E \cdot I_{\text{unev}, \text{Salpeter}} = 1.6 \cdot 10^8 \quad (1.49)$$
$$N_{\text{ev}, \text{Komiya}} = D \cdot I_{\text{ev}, \text{Komiya}} = 7.9 \cdot 10^8 \quad (1.50)$$
$$N_{\text{ev}, \text{Salpeter (top−heavy)}} = E \cdot I_{\text{ev}, \text{Salpeter}} = 1.0 \cdot 10^7, \quad (1.51)$$

where

$$I_{\text{unev}, \text{Komiya}} = \int_{0.8}^{0.1} \exp \left[ - \frac{\log_{10}(m/\mu)}{2\sigma^2} \right] \frac{dm}{m} = 7.0 \cdot 10^{-3}. \quad (1.52)$$
Chapter 2

Building Blocks of the Milky Way’s Accreted Spheroid

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Abstract

In the $\Lambda$CDM model of structure formation, a stellar spheroid grows by the assembly of smaller galaxies, the so-called building blocks. Combining the Munich-Groningen semi-analytical model of galaxy formation with the high resolution Aquarius simulations of dark matter haloes, we study the assembly history of the stellar spheroids of six Milky Way-mass galaxies, focussing on building block properties such as mass, age and metallicity. These properties are compared to those of the surviving satellites in the same models. We find that the building blocks have higher star formation rates on average, and this is especially the case for the more massive objects. At high redshift these dominate in star formation over the satellites, whose star formation timescales are longer on average. These differences ought to result in a larger $\alpha$-element enhancement from Type II supernovae in the building blocks (compared to the satellites) by the time Type Ia supernovae would start to enrich them in iron, explaining the observational trends. Interestingly, there are some variations in the star formation timescales of the building blocks amongst the simulated haloes, indicating that $[\alpha/$Fe] abundances in spheroids of other galaxies might differ from those in our own Milky Way.

2.1 Introduction

The formation and evolution of the Galactic spheroid, consisting of the central bulge and the stellar halo, has been studied for more than fifty years since the classical paper of Eggen et al.
[1962] on the origin of the Milky Way. Although it is still unclear to which extent accretion plays a role besides instabilities of the disc in the formation of the Galactic bulge [e.g., Combes, 2000; Gerhard, 2015; Di Matteo, 2016], there is growing consensus on the formation of the Galactic halo. Since the proposed scenario of Searle & Zinn [1978] in which the stellar halo formed via the merging of several protogalactic clouds, there have been many pieces of evidence suggesting indeed a hierarchical build-up of the Milky Way’s stellar halo [e.g., Ibata et al., 1994; Helmi et al., 1999; Belokurov et al., 2006; Bell et al., 2008; Starkenburg et al., 2009; Janesh et al., 2015]. Presently, we have a firm theoretical framework provided by the ΛCDM paradigm predicting a hierarchical formation scenario that can be simulated in much detail [e.g., Johnston, 1998; Bullock et al., 2001; Bullock & Johnston, 2005; Moore et al., 2006; Abadi et al., 2006]. On the other hand, the accretion history of our Galaxy in particular is not completely unravelled yet, although much progress is expected thanks to the Gaia mission [Perryman et al., 2001]. One particularly intriguing question is how the building blocks that formed our Milky Way’s accreted spheroid compare to the satellite galaxies that we see around us today.

In a pioneering paper, Unavane et al. [1996] attempted to constrain the accretion history of the stellar halo from comparisons of the age distribution and chemical abundances of halo stars with those of the stars in present-day dwarf spheroidal (dSph) galaxies. Numerous observational studies [Shetrone et al., 1998, 2001, 2003; Tolstoy et al., 2003; Venn et al., 2004; Koch et al., 2008; Tolstoy et al., 2009; Kirby et al., 2010] reported discrepancies between chemical abundances of satellite galaxies of the Milky Way and field halo stars. These studies show that the present-day satellites are, at least partly, unlike the building blocks of the Milky Way’s stellar spheroid. The dSphs that we see around the Milky Way in our Local Group are survivors and thus had naturally more time to form stars than the building block galaxies that already dissolved into the halo [e.g., Mateo, 1996]. Even when comparing equal age populations in both environments [as done by Fiorentino et al., 2015, using RR Lyrae stars] discrepancies are found between the typical dSphs that survived and those that contributed majorly to the build-up of the spheroid.

In this work we specifically focus on the properties (in terms of mass, age and metallicity) of the building blocks of our Milky Way’s accreted spheroid modelled within a fully cosmological framework. We investigate when they merged and how they relate to the surviving satellite population. In the past decade several efforts have already focussed on the build-up of Milky Way stellar haloes and/or their chemical evolution, either using hydrodynamical simulations or with semi-analytic techniques [e.g., Bullock & Johnston, 2005; Salvadori et al., 2007; Tumlinson, 2006, 2010; Zolotov et al., 2009; Cooper et al., 2010; Font et al., 2011; Tissera et al., 2013, 2014; Cooper et al., 2015; Pillepich et al., 2014; Lowing et al., 2015; Pillepich et al., 2015]. A specific focus on chemical evolution has been provided by Robertson et al. [2005]; Font et al. [2006a,b], using the hybrid semi-analytic plus N-body approach of Bullock & Johnston [2005]. Cooper et al. [2010, C10 hereafter] used the GALFORM semi-analytic galaxy formation model to study the disruption of satellite galaxies within the cosmological N-body simulations of the six galactic haloes of the Aquarius project [Springel et al., 2008], which have masses comparable to values typically inferred for the Milky Way halo.

We use here a different semi-analytic model to study the formation of our Galaxy and its spheroids’ building blocks than C10 [Starkenburg et al., 2013a, hereafter S13, and references
2.2 The Model

We use the semi-analytic model for galaxy formation that was originally established in Munich [Kauffmann et al., 1999; Springel et al., 2001; De Lucia et al., 2004; Croton et al., 2006; De Lucia & Blaizot, 2007; De Lucia & Helmi, 2008] and developed further in Groningen [Li et al., 2010, S13]. The merger history trees of the six Milky Way-like haloes of the Aquarius project, denoted A – F [Springel et al., 2008], and their substructures were constructed using the subfind algorithm [Springel et al., 2001], after which baryonic processes are modelled using simple but observationally and astrophysically motivated prescriptions [De Lucia et al., 2004; Li et al., 2010, S13, and references therein].

A galaxy merger tree was constructed to follow the galaxies that end up in the Milky Way’s stellar spheroid over time. Each building block of the spheroid undergoes three phases in this galaxy merger tree: a first phase where it is a main galaxy with its own dark matter halo; a second phase where it is a satellite galaxy (its dark matter halo becomes a subhalo of a more massive halo); and a so called orphan phase, where the dark halo of a satellite galaxy is tidally stripped down to below the subfind resolution limit of 20 particles. Up until this last point, where the galaxy has ‘lost’ its dark matter halo, the galaxy merger tree is identical to the dark matter merger tree.

As explained in detail by De Lucia & Helmi [2008], stellar spheroids grow via galaxy mergers and disc instabilities in our model. In situ star formation only takes place in discs of galaxies, not in spheroids. However, during a major merger, the disc of the galaxy gets completely destroyed and all its stars end up in the spheroid, including the stars that just have been formed in the starburst initiated by the collision. This could be regarded as in situ star formation in the spheroid. We classify a galaxy merger as major if the mass ratio (mass in stars and cold gas) of the merging galaxies is larger than 0.3. In a minor merger, for which this mass ratio of the merging galaxies is smaller than or equal to 0.3, the stars of the least massive galaxy are added therein], but using also the Aquarius simulations as a backbone. In Section 4.2.2 we briefly describe our model, followed by a detailed description of the resulting stellar spheroids in Section 2.3. We will focus on their accreted components, but in this section we will also show how they relate to the full spheroids in terms of stellar mass. In Section 2.4 we investigate the stellar mass – metallicity relation for the building blocks of the accreted spheroids and compare this to the observed stellar mass – metallicity relation for the surviving satellite galaxies of the Milky Way, and the simulated one by S13. In this section, we also show that the early star formation (i.e. over 12 Gyrs ago) in the accreted spheroid was dominated by its building blocks and was much lower in the satellite galaxies that survive until the present day. We apply our analysis to infer observable [α/Fe] trends in galaxies with various accretion histories in Section 2.5 and we conclude in Section 2.6.

Throughout this paper we name all accreted stellar material together the “accreted spheroid” of a galaxy. This definition is preferred over the term “halo” to clarify that this component is present at all radii. Only in Section 2.3, we furthermore use the term “accreted bulge” for the innermost 3 kpc of the accreted spheroid.
to the spheroid of the more massive one, thereby leaving the disc of the latter intact. Whenever a galaxy is disc-dominated (spheroid stellar mass / total stellar mass < 0.1) and meets the disc instability condition

\[ \frac{V_{\text{max}}}{(Gm_{\text{disc}}/r_{\text{disc}})^{1/2}} \leq 1.1 \]  

(Efstathiou et al., 1982; Mo et al., 1998), half the disc mass is transferred to the spheroid in our code to make the disc stable again. In this equation \( V_{\text{max}} \) is the maximum velocity of the main halo, \( m_{\text{disc}} \) and \( r_{\text{disc}} \) are the stellar mass and the radius of the disc respectively, and \( G \) is the gravitational constant.

Despite the semi-analytical nature of our model in which no stellar particles are explicitly modelled or tagged, our model includes some prescription of stellar stripping in merging satellites, but only when the dark matter halo of a satellite galaxy is so heavily stripped that its half-mass radius becomes smaller than the half-mass radius of the stars and cold gas. In this case, the stars and cold gas are removed up to the half-mass radius of the dark matter and added to the host spheroid (see S13, Appendix A1). Orphan galaxies - a class that is particularly difficult to handle well, because no information on them is present in the simulations anymore - either merge with the central galaxy on a dynamical friction timescale before redshift zero, or, if this timescale is longer, might survive. In the latter case, their survival depends on their average mass density compared to that of their host system (see S13, Appendix A2). We note that besides stripping of their dark matter mass and possibly some stellar content, surviving orphan galaxies are similar to any surviving dwarf galaxy including a chemical history of self-enrichment [which sets them apart from globular clusters for instance, Kruijssen et al., 2012; Leaman, 2012; Willman & Strader, 2012]. Some fainter orphan galaxies could potentially even still be dark matter dominated. Note that the tidal disruption of orphan galaxies is another way of growing the spheroid, as is stellar stripping.

To summarize, the five ways of spheroid growth in our model are: (1) major mergers, (2) minor mergers on a dynamical friction timescale, (3) disc instabilities, (4) stellar stripping and (5) tidal disruption of orphans. The four different types of galaxies that we model are: (1) Main galaxies that do survive until the present day. These galaxies have a dark matter halo and may have several satellite galaxies in subhaloes of this dark halo or orphan galaxies bound to them. The most massive of these main galaxies in our simulation is our model Milky Way. (2) Building block galaxies that once were main galaxies but went through the above-mentioned three phases of evolution before merging with our progenitor Milky Way galaxy. These do not survive until the present day. (3) Surviving satellite galaxies of a main galaxy, which do have a dark matter halo that is a subhalo of the dark halo containing the main galaxy. (4) Orphan galaxies that might eventually merge with a main galaxy but survive until the present day.

Our model assumes an instantaneous recycling approximation, i.e. we do not take into account finite stellar lifetimes. The abundance of an \( \alpha \)-element such as Mg for instance, mainly originating in short-lived supernovae (SNe) type II, can therefore better be compared with the metallicity predicted by our model than Fe, which is thought to originate mainly in the (delayed) population of type Ia SNe. No individual elements are explicitly traced in the model though, instead it returns a total mass in metals for any system in the gas and stars. Throughout this paper, we refer to the metallicity of any stellar system by \( \log[Z_{\text{stars}}/Z_{\odot}] \), i.e. the logarithm of the ratio of mass in
metals over the total mass in stars in that galaxy, divided by the solar metallicity, \( Z_\odot = 0.02 \). Two discrepancies however arise when comparing this modelled metallicity with observational data: (1) often in observational data \([\text{Fe/H}]\) is measured, rather than \([\text{Mg/H}]\), and (2) the average \([\text{Fe/H}]\) of a stellar system is calculated by taking the average of the logarithms of (a representative sample of) its individual stars’ iron abundance ratios compared to that of the Sun, whereas we take the logarithm of the average metallicity compared to that of the Sun. Thus the metallicity values that we find are higher. In our comparisons with data we attempt to compensate for both discrepancies. We refer the reader to S13 for details (i.e. see their Equation 2 and the paragraph below that equation), but in short we convert each measured \([\text{Fe/H}]\) into \([\text{Mg/H}]\) and apply an offset to correct for the difference in the averaging procedure.

2.3 The Stellar Spheroids

Our modelled stellar spheroids are part of galaxies that were already analysed in some detail in S13, who looked at the total stellar mass of the galaxies that developed in the six Aquarius haloes as well as their bulge/disc ratios. As pointed out by S13, if viewed as one component, the integrated or average values for the spheroids can best be compared with the Milky Way bulge since the stellar halo contains very few stars compared to the bulge. In terms of metallicity, spheroid B has the closest match to the observed bulge metallicity of the Milky Way of \([\text{Fe/H}] \sim -0.25\) [McWilliam & Rich, 1994; Zoccali et al., 2003; Bensby et al., 2013]. S13 found that the bulge/disc ratios of the simulated galaxies in all Aquarius haloes except E and F are close to the estimated value of 0.2 – 0.3 [Bissantz et al., 2004] for the Milky Way. We list the disc stellar mass and the spheroid stellar mass in Table 2.1. The simulated galaxies in Aquarius haloes B and E are the closest Milky Way analogs in terms of total stellar mass, i.e. the sum of these first two columns.

In Figure 2.1 we show the growth of the six Aquarius spheroids over time, including the stars that were moved to the spheroid through disc instability (solid lines), or excluding this and showing the contribution from accreted stellar mass only (dashed lines). The final spheroid masses can be read off from the vertical axis of each panel (and are listed in Table 2.1). They vary from \(1.1 \times 10^{10} M_\odot\) for spheroid B to \(5.2 \times 10^{10} M_\odot\) for spheroid C. Whereas spheroid B grows more gradual, the other Aquarius spheroids have undergone one or two major growths. Interestingly, whereas Aquarius haloes B and E contain the least massive galaxies in terms of total stellar mass (S13), they have the largest fraction of accreted spheroid stars compared to their total spheroid mass, since the contribution of disc instabilities to the build-up of the total spheroid is smallest in these haloes. The fraction is smallest in Aquarius haloes A and C, which also have the lowest accreted stellar mass in an absolute sense. Whereas the disc instability channel is mainly considered to lead to the formation of the galactic bar that is much more centrally concentrated, accreted material will contribute at all radii. The largest mass ratio (mass in stars and cold gas) that we find for galaxies merging with the central galaxy in our simulation is 0.23. Because we assign the label \textit{major} only to a merger in which this ratio is larger than 0.3, none of the Milky Way galaxies in our simulations have undergone a major merger during their lifetimes, and even their mergers with the most massive building blocks are classified as minor. Consequently, we do not find any stars that were formed in situ in our spheroids, since in situ star formation only
Figure 2.1: Build-up of the six Aquarius spheroids in stellar mass over time. Solid lines indicate the full spheroid growth, including stars that were moved from the disc to the spheroid by the disc instability channel (see Section 4.2.2). Dashed lines indicate the spheroid growth by accretion only.
2.3 The Stellar Spheroids

Table 2.1: The stellar mass of the disc, total spheroid stellar mass, the accreted spheroid stellar mass, the fraction of accreted spheroid stellar mass contributed by surviving satellite galaxies \( f_{\text{surv}} \), the percentage of surviving satellites that is orphan, and the number of significant progenitors \( N_{\text{prog}} \), per halo.

<table>
<thead>
<tr>
<th>Halo</th>
<th>( M_{*,\text{disc}} ) ( \times 10^{10} M_\odot )</th>
<th>( M_{*,\text{sph}} ) ( \times 10^{10} M_\odot )</th>
<th>( M_{*,\text{acc}} ) ( \times 10^{10} M_\odot )</th>
<th>( f_{\text{surv}} )</th>
<th>Orph %</th>
<th>( N_{\text{prog}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15.35</td>
<td>3.409</td>
<td>0.467</td>
<td>0.104</td>
<td>36</td>
<td>3.3</td>
</tr>
<tr>
<td>B</td>
<td>5.925</td>
<td>1.080</td>
<td>0.896</td>
<td>0.034</td>
<td>23</td>
<td>3.1</td>
</tr>
<tr>
<td>C</td>
<td>15.79</td>
<td>4.906</td>
<td>0.529</td>
<td>0.482</td>
<td>45</td>
<td>6.1</td>
</tr>
<tr>
<td>D</td>
<td>12.40</td>
<td>3.431</td>
<td>1.182</td>
<td>0.484</td>
<td>32</td>
<td>5.4</td>
</tr>
<tr>
<td>E</td>
<td>3.156</td>
<td>1.932</td>
<td>1.305</td>
<td>0.009</td>
<td>35</td>
<td>1.3</td>
</tr>
<tr>
<td>F</td>
<td>5.182</td>
<td>3.890</td>
<td>1.226</td>
<td>0.896</td>
<td>31</td>
<td>1.4</td>
</tr>
</tbody>
</table>

happens in major mergers in our model.

In Table 2.1 the stellar masses of these accreted spheroids are listed, as well as the fraction of the accreted spheroid stellar mass that is material stripped from surviving satellites \( f_{\text{surv}} \). This fraction is very large for Aquarius halo F, because the progenitor that almost completely accounts for its spheroids total stellar mass has a still surviving counterpart. For Aquarius haloes C and D we also find large values of \( f_{\text{surv}} \), which is in agreement with the result from C10. The difference between our results and those of C10 - in particular for halo F, but in lesser extent for the other haloes - is probably due to our differences in the treatment for the tidal disruption of orphan galaxies and stellar stripping (C10 make use of a particle tagging technique).

Table 2.1 also lists the percentage of surviving satellites that is orphan, when using the S13 prescriptions that we summarized in Section 4.2.2. We see that they constitute \( \sim \frac{1}{3} \) of the total population of surviving satellites in most haloes.

The stellar mass of the accreted spheroids is on average \( 9.3 \cdot 10^9 \, M_\odot \) in our models. The resulting accreted spheroid/disc mass ratios that we find are 0.03, 0.15, 0.03, 0.10, 0.41 and 0.24 for haloes A–F respectively. Using the same Aquarius haloes but a different semi-analytic model, C10 find on average accreted spheroids of \( 1.2 \cdot 10^9 \, M_\odot \), thus on average almost a factor 8 lower. To find differences between the two codes in these values is not so surprising; the stellar mass – halo mass relation in our model is quite different from that in GALFORM in this mass range due to their stronger feedback (see S13 for a discussion).

An observational estimate of the Galactic halo’s stellar mass can be made from the local halo mass density \( \rho_0 = 1.5 \cdot 10^{-4} \, M_\odot \, \text{pc}^{-3} \) [Fuchs & Jahreiß, 1998] combined with the density function

\[
\rho(x, y, z) = \frac{\rho_0}{r_0^n} \left( x^2 + y^2 + \frac{z^2}{q^2} \right)^{-n/2} \quad (2.2)
\]

[Helmi, 2008], where \( r_0 = 8.0 \, \text{kpc} \) [Reid, 1993] is the distance from the Sun to the Galactic centre, \( q = 0.64 \) the minor-to-major axis ratio and \( n = -2.8 \) the power law exponent of the density profile [Jurić et al., 2008]. Assuming this mass density profile holds for the stellar halo from a 3 kpc distance of the Galactic centre out to 20 kpc, an integration between these boundaries yields \( 1.2 \cdot 10^9 \, M_\odot \) as an analytical estimate of the stellar halo mass. Beyond 20 kpc the stellar mass
Total accreted stellar mass of that spheroid, followed by the stellar mass of the largest building blocks that were selected to contribute at least 10% of the spheroid's stellar mass. The first number in the legend of each panel is again the total accreted stellar mass of that spheroid. The main progenitors of the largest building blocks (dashed, dotted and dot-dashed lines) that contribute to them. The main progenitors of these main functions (black solid lines) in combination with the star formation rates of their main progenitor.
density slope $n$ steepens [Bell et al., 2008; Deason et al., 2014b] and the extra stellar mass that is obtained by integrating further is similar to the uncertainty caused by errors on the estimated values of the parameters used for the integration in the range $3 - 20$ kpc.

On average 38% of the accreted spheroid stars in the galform semi-analytic model of C10 is located in the innermost 3 kpc. They refer to this component as the accreted bulge. The spread between these fractions in the six Aquarius haloes is large within their model, i.e. 26, 59, 8.7, 12, 90 and 20% for haloes A–F respectively. Since we have no particle tagging scheme implemented, we can not make this distinction based on distances in our models, however if the accreted spheroid stars in our model were distributed among bulge (inner 3 kpc) and halo according to the percentages derived from C10, the accreted stellar haloes (excluding the accreted bulge) A–F would respectively be 3.4, 3.6, 4.8, 10, 1.2 and 9.7·10$^9$ $M_\odot$ in our model. Aquarius halo E, containing the lowest mass galaxy, also contains the lowest mass stellar halo. Note that this estimate of its stellar halo mass matches our analytical estimate above.

The rightmost column in Table 2.1 lists the number of significant progenitors $N_{\text{prog}}$ of the accreted spheroids. Following C10, this number is defined as $N_{\text{prog}} = M_{\text{tot,s}}^2 / \sum m_{\text{prog,i,*}}^2$, which is the total number of progenitors in the case where each contributes equal mass, or the number of significant progenitors in the case where the remainder provide a negligible contribution. $m_{\text{prog,i,*}}$ is the total stellar mass of a building block, or the total stellar mass stripped from one and the same surviving satellite. The sum of all progenitor masses thus equals the total accreted stellar mass of the spheroid ($\sum m_{\text{prog,i,*}} = M_{\text{tot,s}}$). While the full spheroids (including the disc instability mechanisms) are dominated by one or two major growths only, as shown in Figure 2.1, we see from Table 2.1 that the accreted spheroids are typically built out of several significant building blocks.

We find that the stellar spheroid is built almost completely by a few main progenitor galaxies, as C10 find for the stellar halo. However, the number that we find is for all Aquarius haloes, except halo A, higher than the result of C10. Most significantly for halo C (for which they find $N_{\text{prog}} = 2.8$), although only marginally for haloes E and F. For halo C, we find 9 building blocks with a stellar mass larger than 10$^8$M$_\odot$, which is more than in any of the other Aquarius haloes, and indeed points to a large number of significant progenitors, even though the total accreted mass of this spheroid is one of the lowest (see Table 2.1). Our accreted spheroids B, E and F have the lowest number of progenitors, in agreement with C10’s finding that the accreted haloes corresponding to these Aquarius haloes have the lowest $N_{\text{prog}}$. The larger number of significant progenitors that we find for haloes B, C and D could be due to our different selection of radii, i.e. C10 do not include any stars within a 3 kpc distance from the Galactic centre in their halo selection. The other differences between our model and that of C10 that could again play a role are the different stellar mass – halo mass relation, and the treatment of stellar stripping and the tidal disruption of orphan galaxies.

Figure 2.2 shows the spheroids’ star formation rates (SFRs, in $M_\odot$/yr) as a function of look-backtime. The six black solid lines in Figure 2.2 show the total SFRs of the accreted spheroids, i.e. the sum of the SFRs of all building blocks. With dashed, dotted, dot-dashed and coloured solid lines the SFRs of the main progenitor galaxies are shown. These main progenitors were selected to contribute at least 10% of the spheroid’s stellar mass. Note that this selection cri-
Chapter 2: Building Blocks of the Milky Way’s Accreted Spheroid

Figure 2.3: Age-Metallicity Maps of the six accreted spheroids. The colormap represents the stellar mass per bin, on a logarithmic scale. In the bottom left corner of each panel the total accreted stellar mass of that spheroid is indicated.
Comparison of Building Blocks and Surviving Satellites: Metallicity and SFR

The comparison of building blocks and surviving satellites is more simplistic than the non-integer statistical count of main progenitors as defined in C10 and presented in Table 2.1. The blue dashed lines represent the SFRs of the most massive progenitor galaxies, the green dotted lines the second most massive progenitors, red dot-dashed the third, and cyan solid the fourth (if applicable). Most of these main progenitors are completely disrupted, but for haloes C, D and F, respectively 69%, 94% and 54% of the original stellar mass of a still surviving satellite constitutes its most massive “building block”.

In Figure 2.3, the ages (in bins of 1 Gyr) and metallicities \( \log\frac{[Z_{\text{stars}}]}{Z_{\odot}} \), in bins of 0.5 dex of the accreted spheroids. The stars in the lowest luminosity bin \( \log\frac{[Z_{\text{stars}}]}{Z_{\odot}} < -6 \) in Figure 2.3 were formed without any metals; that these (still) exist in our model is due to the neglect of any kind of pre-enrichment from Population III stars in our model. The majority of our modelled accreted stellar spheroid population have near solar metallicity abundances. This is in agreement with the observed average \([Mg/H]\) value for bulge stars, eg. \([Fe/H] \sim -0.25\) [McWilliam & Rich, 1994; Zoccali et al., 2003; Bensby et al., 2013], which combined with \([Mg/Fe] \sim +0.25\) brings the observed average \([Mg/H]\) to solar. However, it is difficult to compare our model, which predicts the metallicity distribution function of spheroid stars over all radii, directly to an observed one [eg. An et al., 2013; Allende Prieto et al., 2014], because the observations are made in a certain direction and/or distance range. If we interpret the lower metallicity accreted spheroid stars as halo stars, we seem to underestimate the lowest metallicities \( (< \sim -2) \) compared to the observed distributions by An et al. [2013] and Allende Prieto et al. [2014].

From Figures 2.2 and 2.3, it is clear that the majority of spheroid stars is old. An exception is the stellar spheroid of F, but this galaxy is atypical as a Milky Way analog because its spheroid was built mainly by the recent stripping of stars from one still surviving (orphan) satellite, i.e. 3 Gyr ago (see the bottom right panel of Figure 2.1).

As expected, our total spheroid’s SFR values are very similar to those presented in Figure 8 in De Lucia et al. [2014] who use a slightly different version of the same semi-analytic code. Spheroid F looks very different between both models, but this is mainly due to the single massive progenitor galaxy mentioned above that either is counted as part of the Galactic spheroid (in our model) or not (in their model). In the corresponding erratum, De Lucia et al. [2015] moreover show that when they apply a new technique to account for finite stellar lifetimes in their model, rather than relying on an instantaneous recycling approximation, the total spheroid masses are typically lower by a factor of \( \sim 2 \). The implementation of finite stellar lifetimes does also have a significant effect on their overall spheroid metallicity; these are typically lowered by \( \sim 0.5 \) dex.

2.4 Comparison of Building Blocks and Surviving Satellites: Metallicity and SFR

Having presented the SFRs of the most massive building blocks in the previous section and Figure 2.2, we will now discuss in more detail the properties of all the building block galaxies and compare them with those of the surviving satellites. We would like to point out that although the number of building blocks is smaller than the number of surviving satellites (for spheroid E the number of building blocks is even less than half the number of surviving satellites), the total stellar mass in building blocks is much larger than the stellar mass in surviving satellites for most.
spheroids, for spheroid B even more than a factor 10. The exceptions are spheroid A (where the stellar mass is similar in the two populations) and spheroid F (which has more mass in surviving satellites).

In their Figure 5, S13 showed the luminosity-metallicity relation for the satellite galaxies of all Aquarius haloes and concluded from a comparison with observed average [Mg/H] values for the Milky Way satellites that the model resembles reality quite accurately. An exception are the model galaxies with metallicities \(< -3\). Many more of those are seen in the models than we observe around the Milky Way. However, as explained in S13, these typically have experienced star formation in less than 4 snapshots of our simulation, resulting in a clear signature of the neglect of pre-enrichment from the very first generation of stars. All stars formed in the first star formation event are modelled to have no metals at all and an identical initial mass function (IMF) to more metal-rich components. A different, more top-heavy, IMF for these first stars is however likely to enrich these galaxies easily to a metallicity floor of [Fe/H] \(\sim -3\) [e.g., Salvadori et al., 2008].

In Figure 2.4, we show the average metallicities (\(\log[Z_{\text{stars}}/Z_\odot]\)) of these satellites (blue circles), as a function of their stellar mass, and compare their values to those of the fully disrupted building blocks of the spheroids (yellow squares). The satellites / building blocks with less than or equal to 4 star formation snapshots (SFS) are plotted with open symbols in Figure 2.4. These indeed cover almost all galaxies below metallicities \(< -3\). Because of the neglect of pre-enrichment from the very first generation of stars, we do not trust the physical nature of the second line at metallicities \(\sim -4\) below the physical mass-metallicity relation which is starting at metallicities \(\sim -2\) (almost all symbols on this line are open). All satellites shown here are those within a 280 kpc radius from the centre of the central galaxy in our simulations, which is a proxy for the virial radius of the Milky Way [Koposov et al., 2008]. Only a few satellites that are still bound to the central galaxy in our simulations can be found outside of this radius. Observed Milky Way satellites [Mg/H] values, corrected to better compare with our model calculations (as described at the end of Section 4.2.2) within the same radius are plotted as red triangles with error bars, which for most galaxies fall within the symbol size. Values are taken from McConnachie [2012].

Figure 4 does not evidence much difference in the metallicities of the building blocks and those of the surviving satellites at a given stellar mass. In order to see if the two populations could be drawn from the same underlying distribution, we conducted a Kolmogorov-Smirnov (KS) test, thereby excluding the open symbols and the galaxies in the shaded areas, where we suspect the resolution of our simulation to limit the robustness of our results. Furthermore, we removed the mass dependence of the \(\log[Z_{\text{stars}}/Z_\odot]\) values by subtracting a quadratic polynomial fit from the data: \(\log[Z_{\text{stars}}/Z_\odot] = ax^2 + bx + c\) with \(x = \log(\text{Stellarmass}/M_\odot)\), \(a = -0.0208\), \(b = 0.682\) and \(c = -4.79\). The resulting \(D\) statistic of 0.09 and \(p\)-value of 0.38 indicate that the populations are consistent with each other. For this case, we conclude that the two populations follow the same underlying distribution.

Figure 2.5 shows the average SFR (in \(M_\odot/\text{yr}\)) versus stellar mass, with the same color coding as Figure 2.4. We have calculated the average SFR as the sum of the SFRs prior to infall into the halo of a larger galaxy (i.e. the galaxy was not yet a satellite) divided by the total number of timesteps during which there was star formation in this period.
2.4 Comparison of Building Blocks and Surviving Satellites: Metallicity and SFR

Figure 2.4: Stellar mass versus metallicity ($\log[Z_{\text{stars}}/Z_\odot]$) relation for the building blocks (yellow boxes) and for the surviving satellites (blue circles). Building blocks/surviving satellites with more than 4 snapshots during which there is star formation (star formation snapshots, or SFS) are shown with filled marker symbols, those with less than or equal to 4 SFS with open symbols. The 88 “building blocks” that are stripped material of a surviving satellite (17% of the total number of BBs) are not shown. With a grey zone we indicate what mass range of the building blocks/surviving satellites we do not trust due to the limiting resolution of our simulation. Observed satellite metallicity values [McConnachie, 2012], corrected to approximate a mass-weighted average of [Mg/H] for a better comparison to the models (see text for details) are plotted as red triangles with errorbars. The seven points in the gray area correspond to Leo IV, Bootes III (which nature is unclear), Ursa Major (I), Leo V, Pisces II, Canes Venatici II and Ursa Major II, from right to left respectively.
Figure 2.5: Stellar mass versus the average Star Formation Rate for the building blocks (yellow boxes) and for the surviving satellites (blue circles). Building blocks/surviving satellites with more than 4 SFS are shown with filled marker symbols, those with less than or equal to 4 SFS with open symbols. The 88 “building blocks” that are stripped material of a surviving satellite (17% of the total number of BBs) are not shown. With a grey zone we indicate what mass range of the building blocks/surviving satellites we do not trust due to the limiting resolution of our simulation.
2.4 Comparison of Building Blocks and Surviving Satellites: Metallicity and SFR

Figure 2.6: The star formations rates ($M_\odot$/yr) of the fully disrupted building blocks (black line with red band) and the surviving satellites (blue line with light blue band) in haloes A–E, weighted by the mass of that galaxy at that time of star formation. At each time, the sum of the weights of the fully disrupted building blocks as well as those of the surviving satellites add up to one for each of these five Aquarius haloes ($\sum w_i = 1$), after which the median is plotted (black and blue lines) as well as the 30–70 percentiles (red and light blue bands). The corresponding redshift is shown at the top axis. In the zoom-in panel the SFRs in the first Gyr are shown. Adding those satellites that are stripped by at least two thirds of their original stellar mass to the building blocks (to include also the main progenitors of spheroids C and D) did not significantly change this figure.
Figure 2.7: The same mass-weighting as in Figure 2.6 is used to show the 30–50–70 percentiles of haloes A–E virial masses of building blocks as a function of lookbacktime (black line with gray band) and those of surviving satellites (green line with light green band). Again, the zoom-in panel shows the first Gyr of galaxy formation.
2.4 Comparison of Building Blocks and Surviving Satellites: Metallicity and SFR

It is clear from Figure 2.5 that the average SFRs of the building blocks are typically higher at a given stellar mass than those of the surviving satellites. To quantify this difference, we again conducted a KS test on the two populations (open symbols, non-shaded area only) after subtracting a quadratic polynomial fit to remove the mass dependence of the SFR values: 

\[ y = ax^2 + bx + c \]

with 

\[ x = \log(\text{Stellar mass } / M_\odot), \quad y = \log(\text{average SFR } / (M_\odot/\text{yr})) \]

\[ a = 0.0479, \quad b = -0.0509, \quad c = -3.83. \]

We find a \( D \) statistic of 0.49 and a \( p \)-value of \( 2.7 \cdot 10^{-22} \). To check the sensitivity of this strong result on our choices of calculating the average SFRs described earlier, we additionally calculate the averages in various different ways: the sum of the SFRs in all timesteps during which there is star formation divided by the number of timesteps in which there is star formation (\( D: 0.51, p\)-value: \( 4.0 \cdot 10^{-24} \)); and the total stellar mass of the building block/satellite divided by the timespan of star formation (\( D: 0.64, p\)-value: \( 5.7 \cdot 10^{-37} \)). Furthermore, we compared the peak SFRs of the building blocks and the surviving satellites, which show the same trend again (\( D: 0.49, p\)-value: \( 8.0 \cdot 10^{-23} \)). Because the \( p \)-value is very low in all of these cases, we are convinced that our conclusion that the SFR in the building block population is higher than that in the surviving satellites is robust.

Figure 2.6 shows the SFR of all fully disrupted building blocks weighted by their stellar mass at that time of star formation, versus the mass-weighted SFR of the surviving satellites, calculated separately for five of the Aquarius haloes, after which the median is plotted with a black line for the building blocks and with a blue line for the satellites. With a coloured band, the 30−70 percentiles are shown. The Aquarius simulation F is not included here, because it has a different time step size. The figure thus mainly represents the SFRs of the brightest objects in these simulations, which are most easily detected. We see that the SFRs are especially different at early times (i.e. in the first Gyr, which is shown in the zoom-in panel). This may have important implications, for example the contribution to the reionization of the local universe. The surviving satellites have not contributed significantly compared to the fully disrupted ones and we might no longer be able to see host galaxies of the sources that reionized the local universe [see also Weisz et al., 2014; Boylan-Kolchin et al., 2014, 2015]. Furthermore, our result implies that globular clusters may be forming more easily in building block galaxies than in galaxies that survive as satellites until the present day, since they are thought to be outcomes of large starbursts occurring in the early universe [Kruijssen et al., 2012].

To shed some light on the origin of the difference in SFR between building blocks and surviving satellites, we show in Figure 2.8 the virial mass versus SFR for all galaxies that still have their own dark matter halo at six different time steps in the early universe. Galaxies that end up as building blocks are marked as yellow squares, whereas galaxies that survive as a satellite galaxy to the present day are visualized as blue circles. From this figure, it is clear that until \( \sim 11.5 \) Gyr ago, the most massive galaxies are those that are building blocks at the present epoch. These building blocks are thought to be associated with high-density peaks that collapsed at higher redshifts, compared to the present-day surviving satellites that descended from more average density fluctuations in the early universe [Barkana & Loeb, 2001; Diemand et al., 2005]. At early epochs these satellites have lower virial masses than the building blocks, resulting in longer timescales for merging with the central galaxy and lower SFRs. This same trend is shown in Figure 2.7 where we apply the same mass weighting as we did to show the median SFR of the
Figure 2.8: Virial mass versus SFR for the galaxies that become building blocks (yellow boxes) and the ones that survive as satellites (blue circles) at six different time steps. The time and redshift labels are shown in the upper left corner of the panels. Also the total number of objects at that time step is indicated in the legend of each panel.
2.5 Comparison of Building Blocks and Surviving Satellites through the Star Formation Timespan

2.5.1 Timescales of growth

As already shown in the previous section, we find in our simulations that the average SFR in the surviving satellites is lower than in building blocks of the same mass (see Figure 2.5), so it generally takes them a longer time to form their stars. This is visualised in more detail in Figures 2.9 to 2.12.

The mass build-up of the accreted stellar spheroids (in percentage of the total accreted mass over time) since the beginning of star formation is shown for both building blocks and surviving satellites in Figures 2.9 and 2.10. For each galaxy, the moment at which the first stars in that galaxy begin to form is set as the zero Time $T$. If a surviving satellite or building block experienced a merger with another galaxy, the star formation histories of the two galaxies are added together. The galaxy after the merger is therefore always considered to have started forming stars as early as the earliest of the two galaxies started to form stars. We do not examine the complete merger histories of building blocks and satellites here, but Deason et al. [2014a] estimated that $\sim$10% of the dwarf galaxies with a stellar mass $>10^6M_\odot$ that are within the virial radius of the host experienced a major merger since $z = 1$.

Compared with the lookbacktime (used the previous sections in Figures 2.1, 2.2, 2.3 and 2.6) the onsets of star formation are shifted so that they all start at the same zero time, thus allowing us to address the enrichment histories of the building blocks and satellites on a similar internal time scale. This is particularly relevant since several enrichment processes have longer delay time than others; e.g., SNe type Ia originate from lower mass progenitors than SNe type II and thus the former can only contribute significantly to enrich the galaxy after a certain time since the onset of star formation. The relative contributions of these SN types will have their imprint on the chemical abundances of the next generations of stars; whereas SNe type II are producing lots of $\alpha$-elements such as Mg, Ca and Ti, SNe Ia are thought to be the main contributors for Fe for instance. The shifts that we applied, from the universal time to $T$, are no more than 0.1 Gyr for the most massive building blocks (of which we plotted the SFR in Figure 2.2) and no more than 0.5 Gyr for the three most massive satellites of each spheroid.

We see that in spheroids A and C (leftmost panel of Figure 2.9) $\sim$50% of the stars in building blocks are formed within one Gyr, whereas only $10-30\%$ of the stars in the surviving satellites are formed during this timespan. For spheroids D and E (middel panel), the difference between the two percentages is the largest after approximately two Gyr, when $\sim70-80\%$ of the stars in the building blocks were formed, compared to $\sim40\%$ in the satellites. For spheroids B and F
Figure 2.9: Percentage of stars in surviving satellites (solid lines) versus building blocks (dashed lines) that is formed since the onset of star formation. For spheroids A and C (left panel) the difference between the two populations is very large already after $\sim 1$ Gyr, for spheroids D and E (middle panel) the difference is largest after $\sim 2$ Gyr, whereas for spheroids B and F (right panel) the difference is negligible. All building blocks that form the stellar spheroid are included, also the stripped material from surviving satellites.
Figure 2.10: Percentage of stars formed since the onset of star formation in the three most massive surviving satellites that have lost less than 20% of their stars (top three panels) and the three most massive building blocks that have been fully disrupted (bottom three panels). As in Figure 2.9, spheroids A and C are visualized in the left panels, spheroids D and E in the middle panels, and spheroids B and F in the right panels. Each color indicates a separate spheroid, the same colorcoding is used as in Figure 2.9. The percentage of the total mass in satellites/building blocks that is contained in a particular satellite/building block is given in brackets in the legend of each panel. The most massive surviving satellites shown here (numbers 1) of spheroids B (indicated with the green solid line in the top right panel), D (the cyan solid line in the top middle panel) and E (magenta solid line in the top middle panel) lost respectively 6%, 2% and 2% of their initial mass through tidal stripping, the other satellites did not lose any mass.
(right panel) however, the difference between the build-up of the two populations of stars is much smaller.

In Figure 2.10 we show the same growth rates for the surviving satellites (top panels) and the building blocks (bottom panels) as in Figure 2.9, but now split out into contributions from three massive progenitors/surviving satellites in the population. As in Figure 2.9 spheroids A&C, D&E, and B&F are shown from left to right. The solid lines in the top panels and the dashed lines in the bottom panels correspond to the most massive satellites that have lost less than 20% of their stars and most massive building blocks that have been fully disrupted respectively, the second and third most massive galaxies are shown with other linestyles (see legend). The contribution of that particular object to the total stellar mass in satellites or building blocks is given in between brackets in the legend of each panel. This gives an estimate of the weighting of that line with respect to the total build-up of satellites or building blocks over time (Figure 2.9).

From Figure 2.10 we learn that the spread in stellar mass build-up over time for massive satellites is larger than for massive building blocks. Comparing Figures 2.9 and 2.10, we see that the most massive surviving satellites (shown with solid lines in the top three panels of Figure 2.10) resemble quite well the total build-up of the satellites in that spheroid over time (solid lines in Figure 2.9). The same is true for the most massive building blocks of spheroids B and E, i.e. the dashed lines in the bottom panels of Figure 2.10 match the dashed lines in Figure 2.9 for these spheroids quite well. For spheroid A on the other hand, the second most massive building block shown in Figure 2.10 forms many more stars in a short period of time than the most massive one, thereby increasing the total percentage of stars formed early on. For the other spheroids (in particular spheroid F), there is also a discrepancy between the dashed lines in the bottom panels of Figure 2.10 and those in Figure 2.9 because the most massive progenitor is not fully disrupted and therefore not included in Figure 2.10.

Focussing now on the time span during which the first fifty percent of the stars in a building block/surviving satellite are formed, we show an overview of this distribution in Figure 2.11. From this it is once again clear that the building blocks form the first 50% of their stars in a shorter time than the surviving satellites on average, as we expected from Figures 2.9 and 2.10. In Figure 2.11 the time since the onset of star formation (T, Gyr) is binned in 15 equal-size bins on a logarithmic scale from $10^{-2.4}$ to 10 Gyr. We left out the building blocks/surviving satellites that had star formation in less than or equal to 4 snapshots again, which would otherwise pile-up in the lowest bin. The contribution from stripped surviving satellites is divided among the two populations according to the mass fraction. For example, the most massive progenitor of spheroid D is counted as 0.94 building block and 0.06 surviving satellite. With a dark shading in the building block histogram, we show that this material follows a different distribution than the total building block population and is more similar to that of the surviving satellites.

Finally, in Figure 2.12 we split the total number of building blocks/surviving satellites up into three mass regimes, i.e. massive ($>10^7 M_\odot$, in blue), intermediate mass ($10^{5.5} < M/M_\odot \leq 10^7$, in green) and low-mass ($\leq 10^{5.5} M_\odot$, in red) shown from left to right respectively. For the building blocks, indicated again with filled histograms, the fraction of stars in the lowest time bin relative to the total in that mass regime becomes larger if they are less massive, thus the chance that they formed their stars in a short timespan becomes larger with decreasing mass. However, the
2.5 **Comparison of Building Blocks and Surviving Satellites through the Star Formation Timespan**

**Figure 2.11:** Fraction of building blocks (filled gold histogram) and surviving satellites (transparent histogram with thick blue edge) with more than 4 SFS that form 50% of their stars within the time since the onset of star formation ($T$, Gyr), for the six Aquarius haloes combined. “Building blocks” that are stars stripped from surviving satellites are divided among the two populations according to the mass fraction of the initial satellite mass. The dark shaded filled histogram shows how this material contributes to the building block distribution. The total number of building blocks and surviving satellites in these distributions are shown in brackets in the legend.
Figure 2.12: Number of building blocks (filled histograms) and surviving satellites (transparent histograms with thick edges) with more than 4 SFR per timespan it took to form all their stars (SF Timespan, binned in bins of 1.5 Gyr). The populations are split up into three different stellar mass regimes. High stellar mass building blocks/satellites ($M_* > 10^9 M_\odot$) are shown with blue colors in the left panel, intermediate stellar mass building blocks/satellites ($10^6.5 < M_* / M_\odot \leq 10^9 M_\odot$) with green colors in the middle panel, and low stellar mass building blocks/satellites ($M_* \leq 10^6.5 M_\odot$) with red colors on the right. With a dark shading, the material stripped from surviving satellites is indicated again.
2.5 Comparison of Building Blocks and Surviving Satellites through the Star Formation Timespan

The shape of the distribution is peaking at the shortest timespan in all three mass regimes for the building blocks. The opposite is true for the surviving satellites: the probability density function of the massive satellites peaks at large timespans, and the peak moves towards shorter timespans if the satellites become less massive, towards a distribution that is similar in shape to that of the building blocks for the lowest mass satellites. The “building blocks” that were stripped from surviving satellites are indicated again with a dark shading, and as in Figure 2.11, they are divided among the two populations according to the mass fraction of the initial satellite mass.

The three stellar mass bins chosen in Figure 2.12 represent the various classes of dwarf satellite galaxies surrounding the Milky Way (as illustrated in comparison with Figure 2.4). From the observed trends with stellar mass we can conclude that the population of the faintest (also called “ultra-faint”) dwarfs show slightly more overlap in their star formation properties with the building blocks of similar mass than the brighter “classical” dwarf galaxies.

2.5.2 Relation between Iron Abundance and Supernovae Type Ia Delay Time Distribution

One key observable of our Milky Way system is that although its halo population is $\alpha$-rich [Hawkins et al., 2014; Jackson-Jones et al., 2014], its (classical) satellites are predominantly $\alpha$-poor, with an exception for their most metal-poor components [e.g., Shetrone et al., 1998, 2001, 2003; Tolstoy et al., 2003; Venn et al., 2004; Koch et al., 2008; Kirby et al., 2010; Starkenburg et al., 2013b; Jablonka et al., 2015a; Frebel & Norris, 2015]. At metallicities of $[\text{Fe/H}] \approx -1$, stars in the local halo have $[\alpha/\text{Fe}]$ ratios ($[\alpha/\text{Fe}] \sim 0.2 - 0.4$) that are approximately $0.2 - 0.6$ dex higher than those in dwarf galaxies in the Local Group [see for a review Tolstoy et al., 2009].

Several modelling efforts have already pointed out that indeed such a discrepancy could arise in a stellar halo built out of few early-accreted, massive main progenitor galaxies. These would have had high SFRs and have been enriched primarily by type II SNe, whereas the surviving satellites had lower SFRs over a longer period of time, during which also type Ia SNe contributed to their metal content, resulting in a lower $[\alpha/\text{Fe}]$ abundance [Robertson et al., 2005; Font et al., 2006a,b; Geisler et al., 2007]. If type Ia SNe start to contribute significantly only after a certain timescale, this causes the $[\text{Fe/H}]$ ratio to increase with respect to $[\alpha/\text{H}]$, leading to a “knee” in the $[\text{Fe/H}]$ versus $[\alpha/\text{Fe}]$ diagram. Support for such a scenario comes from the position of the observed knee in various galaxies; the $\alpha$-element knee of the Sculptor dSph for example is estimated to be around $[\text{Fe/H}] \approx -1.8$ [Tolstoy et al., 2009], whereas for the more massive Sagittarius it takes place at $[\text{Fe/H}] \approx -1.3$ [De Boer et al., 2014]. This is consistent with our Figure 2.12, where we see that the higher mass satellites have a higher SF efficiency, especially at earlier times.

As discussed before, our semi-analytic model does not include finite stellar lifetimes, but is based on an instantaneous recycling approximation, and therefore the relative contribution from type Ia SNe and type II SNe are not modelled directly. However, from the distribution of star formation timespans in the various building blocks and satellites as shown in Figures 2.9 until 2.12 we can still infer some information on the $[\alpha/\text{Fe}]$ ratios expected. For instance, by making the (very rough, but common) assumption that all spheroid stars formed after a certain time $T$, say 1 Gyr, are significantly enriched in iron from type Ia SNe, and those before that time are $\alpha$-rich, we can use Figure 2.9 to predict that of the six Aquarius haloes, spheroids A and C have
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the most dominant high-α populations, spheroids B and F the least. De Lucia et al. [2014, 2015] have investigated the implementation of different delay time distributions for SN Ia explosions within a semi-analytical model that includes chemical evolution, but which is otherwise very similar to ours. They conclude that the Milky Way stellar disc metallicity distribution function is best represented for delay time distributions that are fairly broad, rather than strongly peaked at either short or intermediate delay times. In all these cases, the effective [O/Fe] yield (for a simple stellar population at fixed metallicity) is at a level ∼0.25 dex lower and most steeply dropping around a 1 Gyr timescale, consistent with our simple assumption.

We see from Figure 2.11 that 99% of the building blocks and 92% of the surviving satellites form the first 50% of their stars in 3.3 Gyrs. The observed discrepancy in [α/Fe] values between the stellar halo and the satellites of the Milky Way would therefore not be expected to show up in any of our modelled haloes, making 3.3 Gyr an upper limit on the delay time scale for SNe Ia to become significantly abundant in the chemical enrichment process. Should on the other hand the relevant contribution delay time be as small as \(6.5 \cdot 10^{-2}\) Gyr, then only 19% of the building blocks would be α-rich, versus 10% of the satellites. A difference that small would also be inconsistent with observations, making \(6.5 \cdot 10^{-2}\) Gyr a lower limit on this time scale.

We would like to note that although the exact distribution of type Ia’s as a function of time is highly uncertain [see e.g., Matteucci et al., 2009; Maoz et al., 2014], many authors find that the delay time distribution has a power-law form, \(\propto t^{-1}\), according to which ∼50% of the type Ia SNe occur within ∼1 Gyr. The time scale \(T\) discussed here represents a cumulative result of all SNe Ia explosions on the galaxy enrichment, capable of moving from a regime of forming predominantly high-α stars to low-α stars, not the time scale at which these stars start exploding. In the solar neighbourhood the change of the [O/Fe] slope around [Fe/H] = −1 in the [O/Fe] versus [Fe/H] diagram is consistent with galactic chemical evolution models in which the overall delay time scale for significant SNe Ia enrichment is ∼1 Gyr [eg. Matteucci & Recchi, 2001]. These timescales could be different in regions with a different SFR, such as the bulge or the outer disc [eg. Pipino & Matteucci, 2009].

The [α/Fe] distribution function of surviving satellites versus that of the inner halo was already modelled in detail by Font et al. [2006a], who also concluded that the bulk of the halo formed from massive satellites accreted early on. As noted by Font et al. [2006b] and Johnston et al. [2008] we see that in our models the net efficiency of star formation - and in particular the difference therein between the building blocks and satellites - shows some variations between the modelled Milky Way-like systems. This means that, largely independent of the assumptions made for the exact delay time distributions and/or Fe-enrichment mechanisms, we might expect to see different [α/Fe] distributions in various Milky Way-mass systems depending on their detailed formation histories. From an observational point of view, this is an exciting prospect offering us a different angle by which the history of a stellar halo can be unravelled. Currently, we do not have a clear picture on the [α/Fe] ratio in external Milky Way-like haloes (although Vargas et al. [2014] measured [α/Fe] abundances of four stars in the outer halo of the Andromeda Galaxy and found them α-enriched) but this will very likely change in the era of the E-ELTs [see e.g., Battaglia, 2011].
2.6 Conclusion

In this paper we have investigated the accreted stellar spheroids of Milky Way like galaxies with the Munich-Groningen semi-analytical model of galaxy formation, combined with the high-resolution Aquarius dark matter simulations. Typically, each of the accreted spheroids was built by only a few main progenitor galaxies and the majority of stars that end up in our Milky Way like stellar spheroids is $10^{-13}$ Gyr old. In three of our six galaxies (C, D and F) a large fraction of the spheroid stars is stripped from satellites that are surviving to the present day. For spheroids C&D these may be resembling the Sagittarius dwarf’s contribution to the Milky Way halo. Spheroid F is atypical as a Milky Way analog because it accreted $\sim 10^{10} M_\odot$ in stars over the last $\sim 3$ Gyr.

We compared the properties of the building blocks of the Milky Ways stellar spheroid to those of the surviving satellites and found that in terms of the stellar mass – metallicity relation, the difference between the two populations is small, but that the former have significantly higher star formation rates on average - they form comparable amounts of stars in a shorter time (see Figures 2.6 and 2.12). In particular, the more massive surviving satellites show a larger variety in stellar mass build-up over time than the massive building blocks (Figure 2.10). On the other hand, the faintest surviving satellites build up mass in a similar fashion to building blocks with similar mass (right panel of Figure 2.12).

From these results, we expect the stellar spheroid to be more enriched in $\alpha$-elements compared to Fe than the surviving satellites, as we observe in the Milky Way system. However, a quantitative analysis of the detailed chemical evolution will require a more sophisticated model and accurate descriptions of the delay time for SNe type Ia. Furthermore, we are dealing with a stochastic process since we are comparing the spheroids of only six Milky Way-like galaxies, that have accreted components which are dominated by a few objects. This results in some of the Aquarius haloes having a better match with the Milky Way galaxy in terms of overall stellar mass and spheroid metallicity, while others have an accretion history that more closely matches that of the Milky Way. Also, we observe some scatter from system to system in our models of the timescale of star formation in satellite galaxies and the timescale of star formation in the main halo. A prediction of these models is therefore that not all Milky Way-mass systems will show $[\alpha/Fe]$ ratios similar to those in the Milky Way.

Acknowledgements

The authors are indebted to the Virgo Consortium, which was responsible for designing and running the halo simulations of the Aquarius Project and the L-Galaxies team for the development and maintenance of the semi-analytical code. In particular, we are grateful to Gabriella De Lucia and Yang-Shyang Li for the numerous contributions in the development of the code. Furthermore, we would like to thank the anonymous referee for valuable comments that helped to improve this paper. PvO thanks the Netherlands Research School for Astronomy (NOVA) for financial support. ES gratefully acknowledges funding by the Emmy Noether program from the Deutsche Forschungsgemeinschaft (DFG). AH acknowledges financial support from a VICI grant.
Abstract

Aims: The Galactic halo likely grew over time in part by assembling smaller galaxies, the so-called building blocks (BBs). We investigate if the properties of these BBs are reflected in the halo white dwarf (WD) population in the solar neighbourhood. Furthermore, we compute the halo WD luminosity functions (WDLFs for four major BBs of five cosmologically motivated stellar haloes. We compare the sum of these to the observed WDLF of the Galactic halo, derived from selected halo WDs in the SuperCOSMOS Sky Survey, aiming to investigate if they match better than the WDLFs predicted by simpler models.

Methods: We couple the SeBa binary population synthesis model to the Munich-Groningen semi-analytic galaxy formation model applied to the high-resolution Aquarius dark matter simulations. Although the semi-analytic model assumes an instantaneous recycling approximation, we model the evolution of zero-age main sequence stars to WDs, taking age and metallicity variations of the population into account. To be consistent with the observed stellar halo mass density in the solar neighbourhood ($\rho_0$), we simulate the mass in WDs corresponding to this density, assuming a Chabrier initial mass function (IMF) and a binary fraction of 50%. We also normalize our WDLFs to $\rho_0$.

Results: Although the majority of halo stars are old and metal-poor and therefore the WDs in the different BBs have similar properties (including present-day luminosity), we find in our models that the WDs originating from BBs that have young and/or
metal-rich stars can be distinguished from WDs that were born in other BBs. In practice, however, it will be hard to prove that these WDs really originate from different BBs, as the variations in the halo WD population due to binary WD mergers result in similar effects. The five joined stellar halo WD populations that we modelled result in WDLFs that are very similar to each other. We find that simple models with a Kroupa or Salpeter IMF fit the observed luminosity function slightly better, since the Chabrier IMF is more top-heavy, although this result is dependent on our choice of $\rho_0$.

### 3.1 Introduction

When aiming to understand the formation and evolution of our Galaxy, its oldest and most metal-poor component, the Galactic halo, is an excellent place to study. The oldest stars in our Galaxy are thought to have formed within 200 million years after the Big Bang, at redshifts of $\sim 20 - 30$ [Couchman & Rees, 1986]. Being formed in the largest over-densities that grew gravitationally with time, these stars are now expected to be found predominantly in the innermost regions of the Galactic spheroid, the Galactic bulge [Tumlinson, 2010; Salvadori et al., 2010; Howes et al., 2015; Starkenburg et al., 2016], although also a significant fraction will remain in the halo. It is still unclear whether the most metal-poor stars located in the bulge are actually part of the thick disc or halo, or whether they are part of a distinct ‘old spheroid’ bulge population [Ness et al., 2013; Gonzalez et al., 2015; Ness & Freeman, 2016]. Therefore, although the stellar halo and bulge are classically considered to be two distinct components of our Galaxy, it is very practical to study them collectively as the stellar spheroid.

In a recent study on the accretion history of the stellar spheroid of the Milky Way [van Oirschot et al., 2017b], we modelled how this composite component grew over time by assembling smaller galaxies, its so-called building blocks (BBs). Post-processing the cosmological N-body simulations of six Milky-Way-sized dark matter haloes [the Aquarius project; Springel et al., 2008] with a semi-analytic model for galaxy formation [Starkenburg et al., 2013a], we investigated building block properties such as mass, age, and metallicity. In this work, we apply our findings on the build-up of the stellar spheroid to a detailed population study of the halo white dwarfs (WDs). In particular, we investigate if there are still signatures of the spheroid’s BBs reflected in today’s halo WD population that can be observed with the Gaia satellite.

In van Oirschot et al. [2014, hereafter Paper I] we already modelled a halo WD population assuming a simple star formation history of the stellar halo and a single metallicity value ($Z = 0.001$) for all zero-age main sequence (ZAMS) stars in the halo. Using the outputs of our semi-analytic galaxy formation model, we can now use a more detailed and cosmologically motivated star formation history and metallicity values as input parameters for a population study of halo white dwarfs. Apart from investigating if this more carefully modelled WD population has properties reflecting WD origins in different Galactic BBs, we will compute the luminosity function of the halo WD population (WDLF). The WDLF has been known to be a powerful tool for studying the Galactic halo since the pioneering works of Adams & Laughlin [1996], Chabrier et al. [1996], Chabrier [1999], and Isern et al. [1998]. Particularly, the falloff of the number of observed WDs
below a certain luminosity can be used to determine the age of the population.

The setup of the paper is as follows: in Section 3.2 we summarize how we model the accreted spheroid of the Milky Way and what its BBs’ properties are. In this section, we will also explain how we disentangle building block stars that we expect to find in the stellar halo from those that we expect to contribute mainly to the innermost regions of the spheroid (i.e. contribute to the Galactic Bulge). In Section 3.3 we explain how we model binary evolution, WD cooling, and extinction. In Section 3.4 we show how observable differences in halo WDs occur due to their origins in the various BBs that contribute to the stellar halo in the solar neighbourhood. We investigate the halo WDLF of five simulated stellar halo WD populations in Section 3.5. There, we will also discuss how our findings relate to the recent work of Cojocaru et al. [2015]. We conclude in Section 3.6.

### 3.2 Stellar haloes and their building blocks

In this paper we focus on the accreted component of the Galactic spheroid. We do not consider spheroid stars to be formed in situ, since we assume that this only happens during major mergers, but none of our modelled Milky Way galaxies experienced a major merger. Here, a merger is classified as ‘major’ if the mass ratio (mass in stars and cold gas) of the merging galaxies is larger than 0.3.

Stellar spheroids also grow through mass transfer when there are instabilities in the disc. However, these disc instabilities are thought to result in the formation of the Galactic bar [De Lucia & Helmi, 2008], whereas we are mainly interested in the properties of the Galactic spheroid in the solar neighbourhood area. Nonetheless, the accreted spheroid also contains stars that are situated in the Galactic bulge region. We define this region as the innermost 3 kpc of the spheroid, a definition that was also used by Cooper et al. [2010]. In Section 3.2.3, we explain how we separate the bulge part and the halo part of the stellar spheroid to be able to focus on halo WDs in the solar neighbourhood area. But first, we summarize how stellar spheroids evolve in our model in Sections 3.2.1, 3.2.2, and 3.2.3.

#### 3.2.1 The semi-analytic galaxy formation model

The semi-analytic techniques that we use in our galaxy formation model originate in Munich [Kauffmann et al., 1999; Springel et al., 2001; De Lucia et al., 2004] and were subsequently updated by many other authors [Croton et al., 2006; De Lucia & Blaizot, 2007; De Lucia & Helmi, 2008; Li et al., 2009, 2010; Starkenburg et al., 2013a], including some implemented in Groningen. Hence, we refer to this model as the Munich-Groningen semi-analytic galaxy formation model. We note that the ‘ejection model’ described by Li et al. [2010] was also used by De Lucia et al. [2014]. It is beyond the scope of this paper to summarize all the physical prescriptions of this model [as is done, e.g., by Li et al., 2010]. Instead, we will focus on the evolution of the accreted spheroid after we apply our model to five of the six high-resolution dark matter halo simulations of the Aquarius project [Springel et al., 2008].
Chapter 3: White dwarfs in the building blocks of the Galactic spheroid

Table 3.1: Building block contributions to the total stellar mass of the accreted spheroid. Only if a building block has a stellar mass above this threshold is it also shown. Building block 12 is the largest progenitor accreted spheroid in this example. This corresponds to galaxies with a minimum stellar mass of $1.5 \times 10^7 \, M_\odot$. Only if a building block of a building block itself has a stellar mass above this threshold is it also shown. Building block 12 is the largest progenitor accreted spheroid.

<table>
<thead>
<tr>
<th>Building Block</th>
<th>% Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>45%</td>
</tr>
<tr>
<td>5</td>
<td>31%</td>
</tr>
<tr>
<td>10</td>
<td>10%</td>
</tr>
<tr>
<td>6</td>
<td>4.9%</td>
</tr>
</tbody>
</table>

Figure 3.1: Galaxy merger tree of Aquarius halo A-4, showing only those objects that contribute at least 0.1% to the total stellar mass of the accreted spheroid. Only if a building block has a stellar mass above this threshold is it also shown. Building block 12 is the largest progenitor accreted spheroid in this example. This corresponds to galaxies with a minimum stellar mass of $1.5 \times 10^7 \, M_\odot$. Only if a building block of a building block itself has a stellar mass above this threshold is it also shown. Building block 12 is the largest progenitor accreted spheroid.
3.2 Stellar haloes and their building blocks

The Aquarius dark matter haloes were selected from a lower resolution parent simulation because they had roughly Milky Way mass and no massive close neighbour at redshift 0. The five dark matter haloes that we use, labelled A–E\(^1\), were simulated at five different resolution levels. The lowest resolution simulations, in which the particles had a mass of a few million \(M_\odot\), are labelled by the number 5, with lower numbers for increasingly high resolution simulations, up to a few thousand \(M_\odot\) per particle for resolution level 1. Only Aquarius halo A was run at the highest resolution level, but all haloes were simulated at resolution level 2, corresponding to \(\sim 200\) million particles per halo, or \(\sim 10^4 M_\odot\) per particle. This is the resolution level that we use throughout this paper. The \(\Lambda\) Cold Dark Matter (\(\Lambda\)CDM) cosmological parameters in Aquarius are \(\Omega_m = 0.25\), \(\Omega_\Lambda = 0.75\), \(\sigma_8 = 0.9\), \(n_s = 1\), \(h = 0.73\) and \(H_0 = 100h\) km s\(^{-1}\) Mpc\(^{-1}\). The subfind algorithm [Springel et al., 2001] was used on the Aquarius simulations to construct a dark matter merger tree for a Milky-Way-mass galaxy and its substructure, which can be used as a backbone to construct a galaxy merger tree. From this, we can determine if and when galaxies merge with other galaxies, following prescriptions for stellar stripping and tidal disruption of satellite galaxies [Starkenburg et al., 2013a].

The merger tree of the the modelled Milky Way in Aquarius halo A-4 is plotted in Figure 3.1. This is a slightly lower resolution simulation than we use throughout the rest of this paper, but it suits the visualization purpose of this figure. The number of significant BBs and their relative mass contributions to the fully accreted spheroid of Aquarius halo A is almost identical to that in resolution level 2. Time runs downwards in Figure 3.1 and each circle denotes a galaxy in a different time step. The size of the circle indicates the stellar mass of the galaxy. The BBs of the Milky Way are shown as straight lines from the top of the diagram (early times) until they merge with the main branch of the merger tree, which is the only line that does not run vertically straight.\(^2\) Each building block is given a number; this is indicated on the horizontal axis. The four major BBs of the stellar halo in this case collectively contribute more than 90\% of its stellar mass.

In merging with the Milky Way, each building block undergoes three phases. At first, it is a galaxy on its own in a dark matter halo. During this phase, the building block is visualized as a red circle in Figure 3.1. As soon as its dark matter halo becomes a sub-halo of a larger halo, the galaxy is called a satellite galaxy and the circles’ colour changes to yellow. Once the dark matter halo is tidally stripped below the subfind resolution limit of 20 particles, it is no longer possible to identify its dark matter sub-halo. Because they have “lost” their dark matter halo, we call these galaxies orphans, and the corresponding circles are coloured green.

The semi-analytic model assumes that stars above 0.8 \(M_\odot\) die instantaneously and that those below 0.8 \(M_\odot\) live forever. This is also known as the instantaneous recycling approximation (IRA). Throughout this paper, the metallicity values predicted by our model are expressed as \(\log[Z_{\text{stars}}/Z_\odot]\), with \(Z_{\text{stars}}\) the ratio of mass in metals over the total mass in stars, and \(Z_\odot = 0.02\) the solar metallicity.

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1 Aquarius halo F was not used, because it experienced a recent significant merger and is therefore considered to be less similar to the Milky Way than the other five haloes.

2 Although some BBs merged with the main branch less than a few Gyr ago, they stopped forming stars much earlier.
3.2.2 Spheroid star formation

A stellar halo of Milky Way mass is known to have only a few main progenitor galaxies [Helmi et al., 2002, 2003; Font et al., 2006a; Cooper et al., 2010; Gómez et al., 2013, eg.]. We show the star formation rate (SFR) in Aquarius halo B-2 as an example of the BBs’ contribution to the total star formation history of a Milky-Way-mass galaxy in Figure 3.2 (for more details, see van Oirschot et al. [2017b], hereafter Paper II). With a blue solid line, the SFR in the disc is visualized, and the SFR in the discs of building block galaxies is visualized with a black solid line, collectively forming the SFR of the modelled galaxy’s spheroid. The dashed black line is the sum of these two lines. With five different colours, contributions from the SFRs of the five most massive building blocks are visualized. As can be clearly seen from this figure, they collectively constitute almost the entire SFR of the spheroid. In Section 3.4 we assume that the stellar halo in the solar neighbourhood is built up entirely of four BBs. This is in agreement with the simulations of streams in the Aquarius stellar haloes by Gómez et al. [2013], who used a particle tagging technique to investigate the solar neighbourhood sphere of the Aquarius stellar haloes with the GALFORM semi-analytic galaxy formation model [see also Cooper et al., 2010].

3.2.3 The initial mass function

The Munich-Groningen semi-analytic galaxy formation model assumes a Chabrier [2003] initial mass function (IMF). As explained in Appendix 3.A, the IRA applied to this IMF is equivalent to returning immediately 43% of the initial stellar mass to the interstellar medium (ISM). However, as we also show in Appendix 3.A, the return factor is a function of time (and of metallicity, to a lesser extent). The value 0.43 is only reached after 13.5 Gyr, thus, by making the IRA, our semi-analytic model over-estimates the amount of mass that is returned to the ISM at earlier times. We neglect this underestimation of the present-day mass that is locked up in halo stars, but we correct for the fact that stars have finite stellar lifetimes by evolving the initial stellar population with the binary population synthesis code SeBa. The details of our binary population synthesis model are set out in Section 3.3.

It is not known whether the Chabrier IMF is still valid at high redshifts when the progenitors of the oldest WDs were born. Several authors have investigated top-heavy variants of the IMF [eg. Adams & Laughlin, 1996; Chabrier et al., 1996; Komiya et al., 2007; Suda et al., 2013], initially to investigate if white dwarfs could contribute a significant fraction to the dark matter budget of the Galactic spheroid, and later to explain the origin of carbon-enhanced metal-poor stars. In Paper I, the authors explored whether the top-heavy IMF of Suda et al. [2013] could be the high redshift form of the IMF, by comparing simulated halo WD luminosity functions with the observed halo WDLF by Rowell & Hambly [2011, hereafter RH11], derived from selected halo WDs in the SuperCOSMOS Sky Survey. It was found that the number density of halo WDs was too low to assume a top-heavy IMF, and that the Kroupa et al. [1993] or Salpeter [1955] IMF result in halo WD number densities that match the observations better. We show in Appendix 3.A that the Chabrier [2003] IMF is already more top-heavy than the Kroupa IMF, when it is normalized to equal the amount of stars with a mass below 0.8 \( M_\odot \) for the Kroupa IMF. Because of the results of Paper I, we therefore do not investigate further top-heavy alternatives of the Chabrier [2003] IMF [Chabrier et al., 1996; Chabrier, 1999] is this work.
3.2 Stellar haloes and their building blocks

![Graph showing star formation rate over time for the Milky Way mass galaxy in Aquarius halo B-2.]

**Figure 3.2**: Star formation rate of the Milky-Way-mass galaxy in Aquarius halo B-2 (blue solid line) and the star formation rate of its stellar spheroid (black solid line) as a function of time. Contributions from the five most massive BBs are indicated by different colours (see legend). The black dashed line indicates the complete SFH of the simulated galaxy at $z = 0$, that is, the sum of the blue and the black solid line. The corresponding redshift at each time is labelled on the top axis. At early times, that is the first Gyr of star formation which is shown in the zoom-in panel, the star formation in some of the BBs was much higher than that in the disc of the main galaxy.
Chapter 3 : White dwarfs in the building blocks of the Galactic spheroid

3.2.4 Selecting halo stars from the accreted spheroid

As input for our population study of halo WDs, we use so-called age-metallicity maps. These show the SFR distributed over bins of age and metallicity. For the six Aquarius accreted stellar spheroids, the age-metallicity maps are shown in Figure 3 of Paper II. Halo WDs can only be observed in the solar neighbourhood (out to a distance of $\sim 2.5$ kpc with the Gaia satellite, see Paper I). Because we do not follow the trajectories of the individual particles that denote the BBs [as e.g. done by Cooper et al., 2010], we have to decompose the age-metallicity maps of the accreted spheroids into a bulge and a halo part. Making use of the observed metallicity difference between the bulge and the halo, we select “halo” stars from the total accreted spheroid by scaling the metallicity distribution function (MDF) to the observed one.

We impose the single Gaussian fit to the observed photometric MDF of the stellar halo by An et al. [2013]: $\mu_{[\text{Fe/H}]} = -1.55$, $\sigma_{[\text{Fe/H}]} = 0.43$. We decided not to use the two-component fit to the MDF that was determined by An et al. [2013] to explore the possibility that there are two stellar halo populations, because the lowest metallicity population of halo stars is under-represented in our model, as was already concluded from comparing the Aquarius accreted spheroid MDFs to observed MDFs of the stellar halo in Paper II.

We use the MDF that was constructed from observations in the co-added catalogue in Sloan Digital Sky Survey (SDSS) Stripe 82 [Amis et al., 2014]. The stars that were selected from SDSS Stripe 82 by An et al. [2013] are at heliocentric distances of 5–8 kpc, thus this observed MDF is not necessarily the same as the halo MDF in the $\sim 2.5$ kpc radius sphere around the Sun that we refer to as the solar neighbourhood. However, we consider this observed MDF sufficient to use as a proxy to distinguish the halo part of our accreted spheroids’ MDFs from the bulge part in our models. The single Gaussian that we used was expressed in terms of [Fe/H], whereas the metallicity values in our model can better be thought of as predictions of $[\alpha/H]$, because of the IRA. Using an average $[\alpha/\text{Fe}]$ value of 0.3 dex for the $\alpha$-rich (canonical) halo [Hawkins et al., 2015], we added this to the single Gaussian MDF to arrive at $\mu_{[\alpha/H]} = -1.25$ ($\sigma_{[\alpha/H]} = \sigma_{[\text{Fe/H}]} = 0.43$).

The MDFs of the accreted stellar spheroids in Aquarius haloes are shown with dashed red lines in the left-hand side panels of Figure 3.3 for haloes A–E from top to bottom. In each panel, the green solid line indicates the number of stars in each metallicity bin according to the (shifted) single Gaussian fit to the observed MDF by An et al. [2013], where the observations were normalized to the number of stars in the $-1.5 \leq \log(Z_{\text{stars}}/Z_\odot) \leq -0.7$ bin. The numbers written on top of each bin of the observed MDF indicate how much the red dashed line should be scaled up (when $>1$) or down (when $<1$) in that bin to match it with the green solid line.

Although we underestimate the number of halo stars with the lowest metallicities ($\log(Z_{\text{stars}}/Z_\odot) \lesssim -2$) in our model, we cannot increase this number, because that would imply creating extra stars. We can, however, reduce the number of high metallicity halo stars, by “putting them away” in the bulge. We thus interpret all low metallicity accreted spheroid stars as halo stars, and a large fraction of the high metallicity stars as bulge stars. When we lower the number of stars in a

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3We cannot use the publicly available results of Lowing et al. [2015], because they did not model binary stars and did not make WD tags.

4Since in haloes A–D, the accreted spheroids were not found to have any stars with $\log(Z_{\text{stars}}/Z_\odot)$ $([\alpha/H])$ values above 0.15, these bins are labelled with the $\infty$-sign.
3.3 Binary population synthesis

To model the evolution of binary WDs, we use the population synthesis code SeBa [Portegies Zwart & Verbunt, 1996; Nelemans et al., 2001; Toonen et al., 2012; Toonen & Nelemans, 2013], which was also used in Paper I. In SeBa, ZAMS single and binary stars are generated with a Monte Carlo-method. On most of the initial distributions, we make the same assumptions as were made in Paper I:

- Binary primaries are drawn from the same IMF as single stars;
- Flat mass ratio distribution over the full range between 0 and 1, thus for secondaries \( m_{\text{low}} = 0 \) and \( m_{\text{high}} = m_{\text{primary}} \);
- Initial separation \( (a) \): flat in \( \log a \) (Öpik’s law) between 1 R\(_\odot\) and \( 10^6 \) R\(_\odot\) [Abt, 1983], provided that the stars do not fill their Roche lobe;
- Initial eccentricity \( (e) \): chosen from the thermal distribution \( \Xi(e) = 2e \) between 0 and 1 as proposed by Heggie [1975] and Duquennoy & Mayor [1991].

However, instead of using Kroupa et al. [1993] IMF as standard, we choose the Chabrier [2003] IMF in this paper to match the initial conditions of our population of binary stars as much as possible to those in the Munich-Groningen semi-analytic galaxy formation model (see also Section 3.2.3).

We evolve a population of halo stars in a region of \(~3\) kpc around the Sun (see Paper I for more details).\(^5\) This population is modelled with five different metallicities: \( Z = 0.02 \), \( Z = 0.01 \), \( Z = 0.004 \), \( Z = 0.001 \), and \( Z = 0.0001 \). The choice for these five metallicity values was motivated by our aim to cover as much as possible the effect of metallicity on the initial-to-final-mass relation for WDs (IFMR) (see Figure 3.4). These metallicities correspond with the bins we use in the semi-analytic galaxy formation model (Figure 3.3) when correcting for the fact that the semi-analytic model gives \([\alpha/H]\) that are 0.3 dex higher than \([\text{Fe/H}]\).\(^6\)

The evolution of the stars is followed to the point where they become WDs, neutron stars, or black holes. A binary system is followed until the end-time of the simulation, considering conservative mass transfer, mass transfer through stellar winds, or dynamically unstable mass transfer in a common envelope in each time step with approximate recipes [see Toonen & Nelemans, 2013, and references therein]. Also angular momentum loss due to gravitational radiation, non-conservative mass transfer, or magnetic braking is taken into account. To follow the cooling of the WDs, we use a separate method, explained in the next subsection.

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\(^5\)The boundary condition given in Equation A.11 of Paper I contains a small error: \( \pi/2 \should be \pi \).  
\(^6\)The lowest metallicity bin is chosen to extend to \(-\infty\) in order to also include stars with zero metallicity. These (still) exist in our model because we neglect any kind of pre-enrichment from Population III stars.
3.3 Binary population synthesis

Figure 3.3: Left-hand side panels: MDF of the stellar halo in the solar neighbourhood based on a single Gaussian fit to the observed photometric metallicity distribution (green solid lines) subtracted from the co-added catalogue in SDSS Stripe 82 [An et al., 2013] compared with the spheroid MDFs in our semi-analytical model of galaxy formation combined with the Aquarius dark matter simulations (red dashed lines), for haloes A–E from top to bottom. Here, 0.3 dex was added to the [Fe/H] values of the observed MDF to compare them with our model’s log(Z_{stars}/Z_{\odot}) values [based on an estimation of the [\alpha/Fe] value for the \alpha-rich (canonical) halo by Hawkins et al., 2015], since the metallicity values of our model can better be compared with [\alpha/H] than with [Fe/H]. The numbers written on top of each bin of the observed MDF indicate the discrepancy between our model and the observed value (see text for details). The bin with log(Z_{stars}/Z_{\odot}) between −1.5 and −0.7 was used for the normalization of the observed MDF. The shaded area indicates the model MDF that we use as input for this population synthesis study of halo stars. The text in this shaded area indicates the halo ID, the total stellar halo mass, and the percentage of the total accreted stellar spheroid mass that we assume to be in halo stars. Right-hand side panels: Age-metallicity maps (log(Z_{stars}/Z_{\odot})) corresponding to the assumed stellar halo MDFs in the left-hand side panels, again for haloes A–E from top to bottom. The colour map represents the stellar mass (M_{\odot}) per bin, on a logarithmic scale. The non-linear horizontal axis corresponds to the different sizes of the metallicity bins. The choice for this binning is explained in Section 3.3.

3.3.1 White dwarf cooling and Gaia magnitudes

We use the recent work on the cooling of carbon-oxygen (CO) WDs with low metallicity progenitor stars [Renedo et al., 2010; Althaus et al., 2015; Romero et al., 2015] to calculate the present day luminosities and temperatures of our simulated halo WDs with sub-solar metallicity. For those with solar metallicity, we use the cooling tracks that were made publicly available by Salaris et al. [2010]. As in Paper I, we interpolate and extrapolate the available cooling tracks in mass and/or cooling time to cover the whole parameter space that is sampled by our population synthesis code. The resulting cooling tracks for two different WD masses at five different metallicities are compared in Figure 3.5. Although the effect of a different progenitor metallicity on WD cooling is small, we still take it into account for WDs with a CO core.

Unfortunately, there were no cooling tracks for helium (He) core and oxygen-neon (ONe) core WDs with progenitors that have a range of low metallicity values available to us for this study. For WDs with these core types, we therefore used the same cooling tracks for all metallicities [Althaus et al., 2007, 2013]. As in Paper I, the extrapolation in mass is done such that for the WDs with masses lower than the least massive WD for which a cooling track is still available in the literature, the same cooling is assumed as for the lowest mass WD that is still available. The same extrapolation is chosen on the high mass end. The available mass ranges, as well as those in our simulation, are listed in Table 3.1. We extrapolate any cooling tracks that do not span the full age of the Universe. At the faint end of the cooling track, we do this by assuming Mestel [1952] cooling. At the bright end, we keep the earliest given value constant to zero cooling time.

We found that the Gaia magnitude can be directly determined from the luminosity and temperature of the WD for CO and ONe WDs, rather than from synthetic colours and a colour
Figure 3.4: Initial-to-final-mass relation (IFMR) for WDs with the five different metallicities used in this work. Based on the analytic formulae in Hurley et al. [2000] and similar to their Figure 18. With this choice of metallicity values there is an approximately equal distance between the five lines, so by simulating a stellar population in which the stars have one of these five metallicity values, the effect of metallicity on the IFMR is fully covered.
3.3 Binary population synthesis

Figure 3.5: White dwarf luminosity as a function of age for WDs which have progenitor stars with five different metallicities, for two different masses. Interpolation was used on cooling tracks calculated by several authors: Salaris et al. [2010] for $Z=0.02$, Renedo et al. [2010] for $Z=0.01$, Romero et al. [2015] for $Z=0.004$ and $Z=0.001$, and Althaus et al. [2015] for $Z=0.0001$. 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3_5.png}
\end{figure}
Table 3.1: White dwarf mass ranges in our simulation (S) and those for which cooling tracks are available in the literature (L).

<table>
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<th>Element</th>
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<th>Z = 0.001</th>
<th>Z = 0.004</th>
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<th>Z = 0.02</th>
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</thead>
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<td>0.146</td>
<td>0.142</td>
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<td>1.20</td>
</tr>
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</table>

Notes: The mass range for He core WD cooling tracks that are available in the literature is 0.155–0.435 (only available for metallicity $Z = 0.01$). Magnitudes in the $V$ and $I$ band as a function of cooling time for He core WDs are only available for WDs in the mass range $0.220–0.521$, whose progenitors have metallicity $Z = 0.03$. The mass range for ONe WD cooling tracks that is available in the literature is 1.06–1.28 (only available for metallicity $Z = 0.02$). The simulations yield ONe WDs in the mass range 1.10–1.38 (this simulated mass range is the same for all metallicities).

transformation as done in Paper I (see Figure 3.6). For He core WDs, such a relation does not hold. For those, we apply the same method as in Paper I.

To estimate by which amount the light coming from the WDs gets absorbed and reddened by interstellar dust before it reaches the Gaia satellite or an observer on Earth, we assume that the dust follows the distribution

$$ P(z) \propto \text{sech}^2\left(\frac{z}{z_h}\right), $$

where $z_h$ is the scale height of the Galactic dust (assumed to be 120 pc) and $z$ the cartesian coordinate in the $z$-direction. As in Paper I, we assume that the interstellar extinction between the observer and a star at a distance $d = \infty$ is given by the formula for $A_V(\infty)$ from Sandage [1972], from which it follows that the $V$-band extinction between Gaia and a star at a distance $d$ with Galactic latitude $b = \arcsin(z/d)$, is

$$ A_V(d) = A_V(\infty) \tanh\left(\frac{d \sin b}{z_h}\right). $$

### 3.4 Halo WDs in the solar neighbourhood

In this section, we investigate whether the cosmological building block to building block variation is reflected in the present-day halo WD population, and if it is still possible to observationally distinguish halo WDs originating from different BBs of the Galactic halo. Selecting four BBs from each of the Aquarius stellar spheroids, scaled down in mass to disentangle stellar halo from bulge stars (as explained in Section 3.2.4), we present masses, luminosities, and binary period distributions of five cosmologically motivated stellar halo WD populations in the solar neighbourhood.

Dividing the mass in each bin of a building block’s age-metallicity map by the lock-up fraction of the semi-analytic model ($\alpha = 0.57$, see Appendix 3.C) gives us the total initial mass in stars
3.4 Halo WDs in the solar neighbourhood

Figure 3.6: Gaia magnitude as a function of the product luminosity $\times$ temperature. The red point is the simulation data of Paper I. The black line is a polynomial fit to the data of degree 9, that is, $G = a_0 x^9 + a_1 x^8 + \ldots + a_k x + a_0$ with $x = \log(L/L_\odot) \times \log(T_{\text{eff}}/K)$ and function parameters $a_0 = -8.197 \cdot 10^{-11}$, $a_1 = -6.837 \cdot 10^{-10}$, $a_2 = 8.456 \cdot 10^{-8}$, $a_3 = 7.256 \cdot 10^{-7}$, $a_4 = -2.347 \cdot 10^{-5}$, $a_5 = -1.370 \cdot 10^{-4}$, $a_6 = 2.451 \cdot 10^{-3}$, $a_7 = 1.109 \cdot 10^{-2}$, $a_8 = 2.866 \cdot 10^{-1}$ and $a_9 = 8.701$. 
Chapter 3: White dwarfs in the building blocks of the Galactic spheroid

that was formed. The IMF dictates that 37.2% of these stars will not evolve in 13.5 Gyr (see Appendix 3.A). We thus know how much mass is contained in these so-called unevolved stars for each building block of our five simulated stellar haloes. For each of the Aquarius haloes, we then choose four BBs (after the modification of the accreted spheroid age-metallicity maps to stellar halo age-metallicity maps visualized in Figure 3.3) to represent the BBs that contribute to the stellar halo in the solar neighbourhood. The stars in these selected BBs span multiple bins of the age-metallicity map, although the majority of stars are in the old and metal-poor bins.

The four BBs of the stellar halo in the solar neighbourhood are selected such that they collectively have a MDF that follows the one we used in Figure 3.3 in order to scale down the accreted spheroids’ age-metallicity map to one that only contains stars that contribute to the stellar halo. However, we do have some freedom in selecting which age bins contribute in the solar neighbourhood. We expect that the most massive BBs of the stellar halo cover a volume that is larger than that of our simulation box; thus, if such a building block is selected, we assume that only a certain fraction of its total stellar mass contributes to the solar neighbourhood. The same fraction of stars is taken from all bins of this building block’s age-metallicity map, to avoid changing the age versus metallicity distribution of its stars. The total mass in unevolved stars in our simulation box is set to equal the amount estimated from the observed mass density in unevolved halo stars in the solar neighbourhood by Fuchs & Jahreiß [1998] (see Appendix A of paper I).

By investigating the variety of BBs of the Aquarius stellar spheroids, we found that the least massive BBs have stars only in one or two bins of the age-metallicity map. Most of them are in the lower-left corner of the age-metallicity map, where old and metal-poor stars are situated. To end up with only four BBs contributing to the solar neighbourhood and a MDF that follows the one we used in Figure 3.3, we thus expect a selection of more massive BBs. Here, we aim to verify if it is possible to identify differences in the properties of halo WDs due to their origin in different Galactic BBs. Therefore, we select the BBs to contribute to the solar neighbourhood such that their overlap in the different bins of the age-metallicity map is as small as possible. One should keep this in mind when reading the remainder of this section. This is an optimistic scenario for finding halo white dwarfs in the solar neighbourhood with different properties due to their origin in different Galactic BBs in our model.

In Figure 3.7 we show the age-metallicity maps of the four selected BBs, for Aquarius haloes A–E from top to bottom. The sum of the age-metallicity maps of the four BBs is shown in the leftmost panels. When compared with the total age-metallicity maps of our stellar spheroids (right-hand side panels of Figure 3.3), we see that most features of the total age-metallicity maps are covered by these solar neighbourhood ones. The percentage of the total mass of that building block that we chose to be present in our simulation box is shown in the upper-left corner of each building block panel. In this corner the total mass of that age-metallicity map is also shown (also in the leftmost panels).

With these four BBs as input parameters for our binary population synthesis model, we made mass versus luminosity diagrams for the single halo WDs with $G < 20$ and period versus mass of the brightest WD of unresolved binary WDs with $G < 20$ in our simulations. These diagrams are shown in Figure 3.8. The WDs of each building block are plotted with a separate colour and
Figure 3.7: Age-metallicity maps of four selected BBs from each halo. We have normalized the total mass in our solar neighbourhood volume on the estimated mass density (in unevolved stars) by Fuchs & Jahreiß [1998]. This results in a present day total stellar mass in halo stars in our selected volume of $\sim 5.6 \cdot 10^7 M_\odot$, as indicated in the upper left corner of the leftmost panels, which show the summed age-metallicity maps of the four selected BBs. The mass that each of the BBs contributes to the solar neighbourhood volumes is also indicated in the upper left corner of each of their panels, as is a percentage showing the fraction of the total stellar halo building block (after our modifications to match it to the green lines in Figure 3.3) to which this mass corresponds.
Chapter 3: White dwarfs in the building blocks of the Galactic spheroid

Figure 3.8: Top panels: Mass versus luminosity diagrams for the single WDs in the five Aquarius stellar halo populations built using the age-metallicity maps of the four BBs of each halo presented in Figure 3.7. In the upper-left corner of each panel, we zoomed in on metallicity maps of the four BBs of each halo presented in Figure 3.7. In the upper-left corner of each panel, we zoomed in on metallicity maps of the four BBs of each halo presented in Figure 3.7. In the upper-left corner of each panel, we zoomed in on metallicity maps of the four BBs of each halo presented in Figure 3.7. Bottom panels: Period versus mass of the brightest WD of the unresolved binary WDs in these same simulated stellar haloes with masses between 0.52 and 0.63 $M_\odot$ and $\log(L/L_\odot) ≥ -4$.
marker. The numbers in between brackets in the legend indicate how many WDs (top panels) or unresolved binaries (bottom panels) have $G < 20$ and are plotted in the diagram. For building block C4 this equals 0 and also BBs A4 and E4 contribute less than ten single WDs to the stellar halo in the solar neighbourhood. This is because the masses of these BBs are so small that all WDs that are present in that building block at the present day have $G \geq 20$.

The bottom panels of Figure 3.8 show that there are no large differences between the simulated haloes, including their distinct BBs, in the period versus mass of the brightest WD in unresolved binary WD. All five diagrams look more or less the same, and all BBs cover the same areas in the diagram, although some naturally have more binary WDs (with $G < 20$) than others.

The top panels of Figure 3.8 reveal that the mass versus luminosity diagrams of single halo WDs show slightly larger differences between the simulated haloes and BBs. The nine WDs originating from building block A4 have clearly higher masses than those that were born in the other three selected BBs of Aquarius halo A, which can be understood from its age-metallicity map (the top-right panel of Figure 3.7). A large fraction of the stars in A4 is young and metal-poor, thus based upon Figure 3.13 we expect that many halo WDs from A4 are located to the right of the main curve in this diagram. The same explanation holds for some WDs from BBs B4 and D4. There are no large differences between the simulated single halo WDs from the BBs of Aquarius halo C in this diagram. The selected BBs of Aquarius stellar halo E result in a single halo WD population with a wide mass range in these panels, that is, approximately two times the width of the mass range of the single halo WD population in halo C. This is due to the many young stars in building block E3.

With standard spectroscopic techniques, WD masses can be determined with an accuracy of $\sim 0.04 M_\odot$ [Kleinman et al., 2013], which would make it hard, though not impossible, to identify some of the signatures described in the previous paragraph. With high-resolution spectroscopy, accuracies of $\sim 0.005 M_\odot$ can be obtained [Kalirai, 2012], which would make it much easier to identify these signatures. However, there are two main issues that prevent us from drawing strong conclusions on this. Firstly, it is unclear whether the stellar halo of the Milky Way in the solar neighbourhood is indeed composed out of BBs which are as distinct from each other as those that we selected in this work. We are comparing the haloes of only five Milky Way-like galaxies, that are dominated by a few objects, which makes this a stochastic result. Even for the optimistic scenario studied here, we do not find distinct groups of single halo WDs in the mass versus luminosity diagram in all five haloes. Halo C, for example, does not show this and for halo E there is no gap in the mass range spanned by the four BBs, which makes it observationally impossible to disentangle contributions from the four BBs. Secondly, it was shown in paper I that a WD that is the result of a merger between two WDs in a binary can end up in the mass versus luminosity diagram easily $0.1 M_\odot$ left and right of the main curve, which (in the latter case) makes it indistinguishable from a single WD that was born in a separate building block.

We conclude that there are rather small differences between WDs in realistic cosmological BBs. In Appendix 3.B we show what the maximum differences could be for haloes built from BBs that have wildly different ages and metallicities.
3.5 The halo white dwarf luminosity function

In this section, we will present the WDLFs for the five selected Aquarius stellar haloes from the previous section. We will compare them to the observed halo WDLF by RH11 and also to the three best fit models of Paper I.

In a recent paper, Cojocaru et al. [2015] also investigated the halo WDLF. Although their work focuses on single halo WDs, they also draw conclusions on the contributions from unresolved binaries. There are large differences between their study and ours, the most important one being that they do not follow the binary evolution in detail, whereas we do. Therefore, our simulated WDs have different properties (mainly the Helium core WDs), which clearly results in a different luminosity function. Cojocaru et al. [2015]’s statement that unresolved binaries are found in the faintest luminosity bins more often than single WDs seems implausible when put with our assumption that residual hydrogen burning in He-core WDs slows their evolutionary rate down to very low luminosities. This was shown to be the case, at least for He-core WDs with high-metallicity progenitors, by Althaus et al. [2009]. The effect of a lower metallicity is expected to affect the lifetime previous to the WD stage and the thickness of the hydrogen envelope. White dwarf stars with lower metallicity progenitors are found to have larger hydrogen envelopes [Iben & MacDonald, 1986; Miller Bertolami et al., 2013; Romero et al., 2015] resulting in more residual H burning, which delays the WD cooling time even further. Overall, we find that the effect of progenitor metallicity on the WD cooling is not very large, at least for CO WDs, for which cooling curves for WDs with different metallicity progenitors were available to us (see Figure 3.5) and are used in this paper.

Paper I showed that unresolved binaries mainly contribute to the halo WDLF at the bright end. In fact, $\sim 50\%$ of the stars contributing to the brightest luminosity bins of the halo WDLF ($M_{\text{bol}} \lesssim 4$) are unresolved binary pairs.

The effective volume technique used by RH11 results in an unbiased luminosity function that can directly be compared to model predictions. Therefore, no series of selection criteria should be applied to any complete mock database of halo WDs before comparing it with their observational sample, although Cojocaru et al. [2015] claim otherwise. However, one should apply a correction for incompleteness in the survey of RH11. As we also explain in appendix 3.B, we apply a correction factor of 0.74 to our model lines to compare them with the RH11 WDLF in this work.

The halo WD populations from the five selected Aquarius stellar halo WDs in the solar neighbourhood result in five halo WDLFs that are very similar to each other. They are plotted as a single red band in Figure 3.9. The thickness of the band indicates the spread in the five models, since the upper and lower boundaries of the band indicate the maximum and minimum value of the WDLF in the corresponding bin. With a black line with errorbars, RH11’s observed halo WDLF is shown. The reduced $\chi^2$ values for the five different Aquarius stellar halo selections are 3.4, 3.5, 3.3, 4.1, and 4.6 for haloes A–E respectively. The fact that these five models are so similar is not surprising given that the stellar haloes from which the four major BBs were selected all were modified to follow the same MDF, and normalized to observed local halo mass density in unevolved stars [Fuchs & Jahreiß, 1998, see Appendix 3.C]. We again stress that it is remarkable that we find such an agreement with the observed WDLF with this normalization, as we also found in the bottom-right panel of Figure 3.16 (see also Figure 4 of Paper I). Most other authors,
including Cojocaru et al. [2015], simply normalize their theoretical WDLF to the observed one.

For comparison, the WDLFs predicted by the three best-fit models of Paper I are shown with a light blue band in Figure 3.9. The blue line in this band corresponds to the 100% binaries model (Kroupa IMF). For most bins, this line is in between the 50% binaries line with Kroupa IMF (upper boundary of the blue band) and Salpeter IMF (lower boundary of the blue band). Since the correction factor for incompleteness that we apply in this work is slightly different from the one that was applied in Paper I (0.74 instead of 0.45), we find that the 100% binaries model of Paper I, with a reduced $\chi^2$ value of 2.2, actually fits the RH11 WDLF slightly better than the standard model in Paper I. For both this standard model (Kroupa IMF, 50% binaries) and the model with a Salpeter IMF, we now find a reduced $\chi^2$ value of 2.4. In Paper I the effect of a different normalization on the reduced $\chi^2$ values was already investigated. Since the corrected correction factor $0.74/0.45 = 1.64$ is close to the optimal multiplication factor to obtain a minimum reduced $\chi^2$ value for the 100% binaries model, it is not surprising that this model comes out best. The small rise of the WDLF in the brightest bin could be due to the contribution from unresolved binaries (see Paper I). However, due to our choice of the normalization the model lines are too low in this bin.
The fact that the lines in the blue band have a lower reduced $\chi^2$ value than the ones in the red band is mainly due to the bad fit at the faint end of the WDLF, that is, in the bins centred at 12.25, 14.25, and 14.75. Since we normalize our model lines to the corresponding present-day mass in unevolved local halo stars, $\rho_0$, and the Chabrier IMF is slightly more top-heavy than the Kroupa IMF, this leads to many more stars that have evolved to WDs at the present day (see Figure 3.11). A similarly bad fit was seen for the top-heavy IMF used in Paper I. However, the estimated $\rho_0$ has a statistical uncertainty that we did not take into account here. Fuchs & Jahreiß [1998] found that the most likely value of $\rho_0$ lies in the range 1.5 to $10^{-4} M_\odot$ pc$^{-3}$, with the latter value, in their view, being a firm lower limit. If we use this latter value of $\rho_0$, we find WDLFs that are two thirds lower than these ones. The red band in Figure 3.9 would be shifted to the current position of the blue band, and its corresponding average reduced $\chi^2$ value would be 2.3. Shifting down the blue band would not increase its fit to the observed data points. Although the model with a Kroupa IMF would then have a reduced $\chi^2$ value of 2.2, the 100% binaries and Salpeter IMF models would respectively have reduced $\chi^2$ values of 2.5 and 2.9.

### 3.6 Conclusions

By combining the Munich-Groningen semi-analytic galaxy formation with the SeBa binary population synthesis code to study the stellar halo WD population, we tried to identify observational features in the halo WD population that arise due to their origin in distinct BBs of the stellar spheroid. In the mass versus luminosity diagram of single halo WDs with $G < 20$, one main curve for the majority of halo WDs can be seen along with some WDs that are offset from this main curve (see the top panels of Figure 3.8). The WDs on this main curve all have approximately the same age, thus if one assumes a main sequence evolutionary lifetime of these WDs, the age of the stellar halo can be derived from the WD mass corresponding to this curve. A similar age-determination of the inner halo was suggested by Kalirai [2012]. We found that single halo WDs originating in a building block with a significant fraction of young halo stars ($\sim 4$ Gyr old) in the solar neighbourhood (e.g. B4 from Figure 3.7) will have positions offset from the main curve in this diagram. Unfortunately, however, WDs that are the result of a binary WD merger in any building block can have the same offset from the main curve. Thus it will not be possible to assign the offset WDs to a building block of the Galactic halo that contains a larger fraction of young halo stars. An offset to the other side of the curve is expected for WDs from BBs with more metal-rich stars (see Appendix 3.B) and again from binary WD mergers, although the former are not expected to contribute a significant number of bright WDs to the stellar halo in the solar neighbourhood.

The predicted diagrams of the unresolved binary WD period versus the mass of the brightest WD in these systems (the bottom panels of Figure 3.8) are very much alike for the five simulated stellar haloes. Therefore, we conclude that the differences between unresolved binary WD populations originating from ZAMS stars in different bins of the age-metallicity map are no longer visible in a realistic population of halo WDs. However, there are significant uncertainties in the binary evolution at low metallicity that can only be resolved once a larger set of binary WDs at low metallicity has been observed.
3. A The return factor

The five Aquarius stellar halo WDLFs that we simulated from the combined WD populations in the four selected BBs of each stellar spheroid do not differ much from each other, mainly because we defined the stellar mass in unevolved stars in our simulation box to equal the expected value from the observed mass density by Fuchs & Jahreiß [1998]. Furthermore, all models assume the same IMF and WD cooling models. It is, however, interesting to compare the WDLF band spanned by these five models with the observed halo WDLF by RH11 and with the best-fit models of Paper I. We saw that models with a Kroupa or Salpeter IMF fit the WDLF better than those with a Chabrier IMF, since the Chabrier IMF can be considered more top-heavy than the Kroupa and Salpeter IMFs after fixing the halo WD mass in unevolved stars (see Figure 3.11), which leads to an over-estimation of the number of WDs in total in our simulation box. Overall, there is, however, still quite a good match to the observed WDLF, especially regarding the fact that we normalized the WDLF independently, that is, we did not fix our theoretical curve to the observed one. Furthermore, if we had taken the lower limit of $\rho_0$, the Aquarius stellar halo WDLFs would fit the observed WDLF just as well as the simpler models when these are normalized using our standard value of $\rho_0$ that is 1.5 times larger.

In paper I we found that Gaia is expected to detect $\sim 1500$ halo WDs. Using cosmologically motivated models of the stellar halo of the Milky Way in the solar neighbourhood, we now find $\sim 2200$ halo WDs with $G < 20$ in our simulation box. Although this new estimate might be too large, since the number of WDs in some bins of the WDLF is much larger than in the one observed by RH11, the total number of known halo WDs will be greatly improved with respect to previous catalogues by observations of the Gaia satellite, which will also greatly improve the constraints on the halo WDLF.

Acknowledgements
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3. A The return factor

In this paper we distinguish between unevolved stars, that is, stars that do not lose any fraction of their mass $M_{\text{unev}}$ to the ISM, and evolved stars, which do lose mass to the ISM, thus for which their final mass $M_{\text{f, ev}}$ does not equal their initial mass $M_{\text{i, ev}}$. We define $R_{\text{ev}}$ as the fraction of their initial mass that evolved stars lose to the ISM: $M_{\text{i, ev}} = (1 - R_{\text{ev}})M_{\text{i, ev}}$. The return factor $R$ is defined as the fraction of the initial mass in all stars that is returned to the ISM, and the lock-up fraction $\alpha = 1 - R$ represents the mass that is locked up in all stars, that is, that which is not lost to the ISM:

$$\alpha = \frac{M_{\text{unev}} + M_{\text{f, ev}}}{M_{\text{unev}} + M_{\text{i, ev}}} = 1 - \frac{R_{\text{ev}}M_{\text{i, ev}}}{M_{\text{unev}} + M_{\text{i, ev}}}.$$  

(3.3)
Chapter 3: White dwarfs in the building blocks of the Galactic spheroid

After 13.5 Gyr, only stars above $0.8 M_\odot$ have evolved, which we define as the boundary mass between evolved and unevolved stars. Their mass ratio follows directly from the IMF. The Chabrier [2003] IMF that is used in this paper is defined as

$$\phi(m) \equiv \frac{dN}{dm} \propto \begin{cases} \frac{1}{m} \exp \left[- \frac{\log_{10}(m/\mu)}{2\sigma^2}\right] & \text{if } 0.1 < m \leq 1.0 \\ A \ m^{-2.35} & \text{if } 1.0 \leq m < 100, \end{cases}$$

(3.4)

where $N$ is the number of stars, $m$ the stellar mass in units of $M_\odot$, $\mu = 0.079$, $\sigma = 0.69$, and the normalization constant

$$A = \exp \left[- \frac{\log_{10}(\mu)}{2\sigma^2}\right] = 0.279.$$  

(3.5)

For this IMF, the initial mass in evolved stars ($m > 0.8$) is

$$M_{i,\text{ev},\text{Chabrier}} \propto \int_{0.8}^{1.0} \exp \left[- \frac{\log_{10}(m/\mu)}{2\sigma^2}\right] dm + \int_{1.0}^{100} A \ m^{-1.35} dm = 0.700,$$

(3.6)

whereas the mass in unevolved stars ($m \leq 0.8$) is

$$M_{\text{unev},\text{Chabrier}} \propto \int_{0.1}^{0.8} \exp \left[- \frac{\log_{10}(m/\mu)}{2\sigma^2}\right] dm = 0.414.$$  

(3.7)

The mass percentage of a single stellar population that is returned to the ISM is of course a function of time that is increasing as the population gets older. We found that its dependence on the binary fraction is negligibly small. The effect of the population’s metallicity is also small, as we show in Figure 3.10. We found that the evolved stars that were born according to a Chabrier IMF lost on average 68% of their mass, after evolving them for 13.5 Gyr with the binary population synthesis code SeBa, that is, $R_{\text{ev}} = 0.68$, although the population with $Z = 0.0001$ lost 1% less mass. This yields

$$R_{\text{Chabrier}} = \frac{0.68 \cdot 0.700}{0.414 + 0.700} = 0.43.$$  

(3.8)

Alternatively, we could have used the Kroupa et al. [1993] IMF, given by

$$\phi(m) \propto \begin{cases} B \ m^{-1.3} & \text{if } 0.1 \leq m < 0.5 \\ m^{-2.2} & \text{if } 0.5 \leq m < 1.0 \\ m^{-2.7} & \text{if } 1.0 \leq m < 100 \end{cases}$$

(3.9)

with normalization constant

$$B = \frac{0.5^{-2.2}}{0.5^{-1.3}} = 1.866.$$  

(3.10)

How this IMF compares to the Chabrier IMF is visualized in Figure 3.11. Here, the initial mass in evolved stars is

$$M_{i,\text{ev},\text{Kroupa}} \propto \int_{0.8}^{1.0} m^{-1.2} dm + \int_{1.0}^{100} m^{-1.7} dm = 1.600,$$

(3.11)
Figure 3.10: Return factor of evolved stars (dashed lines) and all stars (solid lines) as a function of Lookbacktime for the five different metallicities used in this study.
Figure 3.11: Kroupa et al. [1993] versus Chabrier [2003] initial mass functions, normalized such that the integral of the IMF between $m = 0.1$ and $m = 100$ equals one for each IMF (solid lines). The thick blue dashed line visualizes the Chabrier IMF when it is normalized such that the number of unevolved stars equals that number predicted by the Kroupa IMF. The thin black dashed line indicates the boundary between evolved and unevolved stars.

whereas the mass in unevolved stars is

$$M_{i,\text{unev,Kroupa}} \propto \int_{0.1}^{0.5} B \cdot m^{-0.3} \, dm + \int_{0.5}^{0.8} m^{-1.2} \, dm = 1.624.$$  

(3.12)

Furthermore, the mass percentage that is returned by evolved stars to the ISM after 13.5 Gyr with a Kroupa IMF is only 62%, which yields a much lower return factor,

$$R_{\text{Kroupa}} = \frac{0.62 \cdot 1.600}{1.600 + 1.624} = 0.31.$$  

(3.13)

3.B Halo WDs in the different bins of the age-metallicity map

In this appendix we explore how halo WDs that originate from stars born in different bins of the age-metallicity map differ from each other. We make the extreme assumption that all our simulated stars were born in the short timespan of a single age bin of the age-metallicity map
3.B Halo WDs in the different bins of the age-metallicity map

with a uniform SFR and that they all have the corresponding metallicity value. The simulated stellar mass in unevolved stars was set to be $1.5 \times 10^{-4} M_\odot/\text{pc}^3$, based on the observed value of Fuchs & Jahreiß [1998]. Multiplying this with a factor $(1 + 0.700/0.414)$ to obtain the total mass in ZAMS stars (see Appendix 3.A), and dividing by a timespan of 0.9021 Gyr, we implement a SFR of $4.4 \times 10^{-13} M_\odot \text{yr}^{-1} \text{pc}^{-3}$.

As can be seen from Figure 3.3, the age-metallicity maps of our stellar haloes and their BBs have $15 \times 5$ bins. Most BBs span a range of bins, as can be seen in Figure 3.7. The resulting stellar populations therefore do not represent realistic BBs of the stellar halo, but they give an idea of the variations between different BBs due to the different bins of the age-metallicity map that they span. Figure 3.12 shows the bins that we selected to investigate in this section. The arrows in this figure indicate sequences of colours that were used in Figures 3.13, 3.15, and 3.16.

Figures 3.13, 3.15, and 3.16 all contain six panels. We show WDs with three different metallicities in the top three panels of these figures, where five different colours correspond to five different ages. In the bottom three panels we show WDs with three different ages (taken slightly offset from the ones in the top panels to allow for a consistent colouring scheme. See Figure 3.12). Here, five different colours represent five different metallicities. The colours match those in Figure 3.12.

In Figure 3.13 we show the masses versus luminosity of the single halo WDs with Gaia magnitude $< 20$. As already remarked on in Paper I, these WDs are expected to follow a narrow curve in this diagram, due to the fact that most of these brightest WDs have just been formed. Most of them thus have the same mass, which is one to one related to their initial zero-age main sequence mass and their age, because these are selected not to be in binaries. Compared to the Gyrs of evolution on the main sequence, the time these WDs need to cool from luminosities above solar to $\log(L/L_\odot) < -3$ is a short time (see Figure 3.5). Those WDs that are in these diagrams with lower luminosities and higher mass are visible with $G < 20$ because they are close to us in terms of distance.

It can clearly be seen from the top panels of Figure 3.13 that if the halo WD population is younger, the WDs with the lowest mass of the population are more massive than those with the lowest mass in an older population. The luminosities of the faintest WDs in a young population are furthermore brighter than the faintest ones in an older population, simply because they had less time to cool. The curves thus shift to the lower left corner of the panels for increasing population age. The curves also become narrower, because the ratio of the timespan of the age-bin ($\sim 0.9$ Gyr) over the main-sequence evolution time is larger for younger WDs. Since the evolution time of higher mass stars is shorter than that of younger stars, a larger mass range is visible at the present day if the population is younger.

The numbers within brackets in the legend of Figure 3.13 indicate the number of single halo WDs with $G < 20$ over the total number of single halo WDs in each selected bin of the age-metallicity map (e.g. including also those with $G \geq 20$). The total number of WDs is obtained by evolving the total number of ZAMS stars in our simulation box (see Appendix 3.C) with SeBa. From these numbers we see that there are less WDs with $G \geq 20$ in the younger and more metal-rich populations.

This can be explained by Figure 3.14. There we plot the percentage of single ZAMS stars with an initial mass $> 0.8 M_\odot$ that have evolved to WDs (the initial population was assumed
Figure 3.12: 5 × 15 bins of the age-metallicity map that is sampled in this study. Patch colours match the colours of the points in Figures 3.13, 3.15, and 3.16. The vertical arrows indicate the sequence of five colours used in the top panels of these figures, and the horizontal arrows indicate this sequence in their bottom panels. The horizontal colour scheme follows the halo MDF (the green line in Figure 3.3), that is, the darkest colour is used for the bin where the MDF peaks (Z=0.001). The age bins for constant metallicity are also set such that the age bins with increasingly realistic ages for halo stars have darker colours (i.e. darker colours for older stars). To avoid confusion, the age values in the vertical sequence are set to be slightly different from those in the horizontal sequence.
3.B Halo WDs in the different bins of the age-metallicity map

Figure 3.13: Luminosity as a function of stellar mass for single halo WDs in the solar neighborhood that can be observed with Gaia (G < 20), assuming a single metallicity value for halo stars, for different age ranges (top panels) and a small age spread for halo stars, for different metallicity values (bottom panels).
Figure 3.14: Main sequence evolution timescales for the five different metallicities used in this study. Colours of the lines are the same as in Figure 3.4. Based on the evolution of ~10^7 single ZAMS stars with initial mass > 0.8M_☉, following a Chabrier IMF, for 13.5 Gyr.
3.3 Halo WDs in the different bins of the age-metallicity map

to follow a Chabrier IMF), as a function of time \( (t) \), for the five different metallicities used in this study. In the younger populations there are less white dwarfs simply because the evolution time of the ZAMS stars was shorter. The fact that a more metal-rich population of a certain age (larger than a few 100 Myr, as is the case in Figure 3.13) has less white dwarfs in total follows from their slower evolution times; for example, the number of ZAMS stars that have evolved to become WDs at that particular age is smaller than for a more metal-poor population. Although there are less WDs in total in younger populations, the number of bright WDs \( (G < 20) \) is larger than in older populations of the same metallicity (top panels of Figure 3.13), because the WDs had less time to cool.

Also in the bottom three panels of Figure 3.13 we see that there are less WDs in total in more metal-rich populations at a particular age. However, here we see that the number of \( G < 20 \) WDs with \( Z = 0.0001 \) is lower than that of \( G < 20 \) WDs with \( Z = 0.001 \). The difference in the evolution time of the ZAMS stars between these two populations is very small, as can be seen from Figure 3.14 and the total number of single WDs in the simulated populations in the bottom three panels of Figure 3.13. It is due to the faster cooling of massive CO WDs with a lower metallicity (as can be seen from the dashed lines above cooling times of \( 10^9 \) years in Figure 3.5) that there are less \( G < 20 \) WDs for the \( Z = 0.0001 \) population than for the \( Z = 0.001 \) population in this case.

In Figure 3.15 we show the period versus the mass of the brightest star in unresolved binary WDs for the same populations as in Figure 3.13. As in Paper I, unresolved binaries are defined as those for which the orbital separation is smaller than 0.3 arcsec, based on the assumption that two stars in a binary should be separated by at least 0.1–0.2 arcsec in order to be spatially resolved by Gaia [Arenou et al., 2005]. The more recent work of de Bruijne et al. [2015] shows that the minimum separation to which Gaia can resolve a close binary probably lies in between 0.23 and 0.70 arcsec, dependent on the orientation angle under which the binary is observed. The different aspects of these period versus the mass diagrams were explained for a standard halo model in Paper I. Here, we are mainly concerned with variations of this figure when modelling populations with a different age or metallicity.

In the top panels of Figure 3.15 we see that younger populations have systems with \( G < 20 \) in which the brightest WD has a higher mass than in older populations, similar to the mass trend with population age for single WDs in Figure 3.13. Also the mass range is again larger. For more metal-rich populations, the period gap (at \( M_{\text{bright}} \sim 0.5 \, M_\odot \)) shifts towards longer periods. Since single stars of higher metallicity evolve more slowly (Figure 3.14), we capture their binary systems with larger periods because they had less time to evolve towards shorter periods at a particular age. An interesting feature of Figure 3.15 is the (partial) disappearance of the narrow line of systems, with \( M_{\text{bright}} \lesssim 0.5 \, M_\odot \) moving into the above-mentioned period gap for populations with higher metallicity. This also happens in the bottom three panels for the populations that are younger. The systems on this line have undergone two mass-transfer phases of which the second one was stable, similar to the Type Ia Supernovae progenitors in the single degenerate scenario where a non-degenerate companion transfers mass to a WD [for a review see e.g. Wang & Han, 2012]. This mechanism does not occur for the most metal-rich and/or young populations. In the bottom-right panel, we see that also the line shifts towards longer periods for more metal-rich
Figure 3.15: Mass of the brightest star in the binary system versus the period of that system in days, for unresolved binary halo WDs in the solar neighbourhood that can be observed with Gaia (G > 20). Colours for age and metallicity values are the same as in Figures 3.12, 3.13, and 3.16.
3.B Halo WDs in the different bins of the age-metallicity map

Figure 3.16: Halo WD LFs based on the assumption that all halo WDs originate from ZAMS stars in a single bin of the age-metallicity map (Figure 3.12). Colours for age and metallicity values are the same as in Figures 3.12, 3.13, and 3.15. The yellow lines with errorbars show the observed halo WDLF derived from selected halo WDs in the SuperCOSMOS Sky Survey [Rowell & Hambly, 2011].

Rowell & Hambly 2011
Chapter 3: White dwarfs in the building blocks of the Galactic spheroid

populations.

Between brackets in the legend of Figure 3.15, the number of unresolved binary WDs with G<20 is written over the total number of unresolved binary WDs in our simulation box for each of the simulated populations. In the bottom panels of Figure 3.15, we clearly see the effect of population age on the number of unresolved binaries with G < 20.

Finally, in Figure 3.16 we show halo WDLFs for the 30 stellar populations that we investigate in this appendix. Each panel of Figure 3.16 also shows the observed halo WDLF by RH11. We applied a correction factor of 0.74 for incompleteness of the observed WDLF to our model lines, based on the estimate of RH11. This correction is a little bit smaller than the one that was applied the model lines in Paper I to compare them with the RH11 data. There, it was incorrectly assumed that this incompleteness is due to the tangential velocity cut RH11 applied. Instead, it should be assigned to their underestimation of the number density of WDs in the solar neighbourhood, as they explain in their section 7.4.

It is remarkable that the five model lines in the bottom-right panel of this figure fit the data so well, given that we did not normalize our model lines to the data, as most other authors do. Instead, we normalize the halo WDLF to the corresponding observed mass density of local halo (low-mass) main sequence stars in the solar neighbourhood [Fuchs & Jahreiß, 1998, see Appendix 3.C]. From the other panels, it is clear that the effect of age on the WDLF is much larger than the effect of metallicity. Also, we derive from this figure that the majority of stars in the stellar halo must be at least 9.92 Gyr old in order to match the observed data below a reduced $\chi^2$ value of five.

3.C Normalization

From the observed stellar mass density in unevolved halo stars in the solar neighbourhood [Fuchs & Jahreiß, 1998], we have determined the stellar mass corresponding to unevolved stars $M_{\text{unev}} = 3.6 \cdot 10^7 M_\odot$ in our simulation box that we use to determine how many stars to simulate (see Appendix A of Paper I). Let $C$ be the normalization constant of the Chabrier IMF, that is, $M_{\text{unev,Chabrier}} = 0.414 \, C$ in Equation 3.7. We have

$$C = \frac{3.6 \cdot 10^7}{0.414} = 8.7 \cdot 10^7$$

(3.14)

and $N_{ev} = N_{ev,\text{lognormal}} + N_{ev,\text{Salpeter}}$, with

$$N_{ev,\text{lognormal}} = C \int_{0.8}^{1.0} \exp \left[ -\frac{\log_{10}(m/\mu)}{2\sigma^2} \right] \frac{dm}{m} = 6.0 \cdot 10^6$$

(3.15)

and

$$N_{ev,\text{Salpeter}} = C \int_{1.0}^{100} A m^{-2.35} dm = 1.8 \cdot 10^7.$$  

(3.16)

These numbers are determined from the assumption that all stars are single. We assume that 50% of the stars are in binaries, however, and that they follow a flat mass ratio distribution, thus the mass of the secondary is on average half the mass of the primary. Therefore, the total number of single stars (which is equal to the total number of binary systems) is equal to the sum of the above mentioned numbers (3.15+3.16) divided by 2.5.
Alternatively, the semi-analytic model predicts how many stars are born in each bin of the age-metallicity map. Instead of using the estimate of the mass in unevolved stars in our simulation box from the observed mass density, we can use the mass (in evolved and unevolved stars) in each bin of the age-metallicity map (initially, that is the present-day mass in each bin divided by \( \alpha = 1 - 0.43 \)). Dividing this mass by \( (3.6+3.7) \) yields a normalization constant of the IMF for each bin of the age-metallicity map, after which the same method is used as above to determine the number of evolved stars in our simulation box.
Neutron star mergers (NSM) are likely to be the main production sites for the rapid (r-) neutron capture process elements. We study the r-process enrichment of the stellar halo of the Milky Way through NSM, by tracing the typical r-process element Eu in the Munich-Groningen semi-analytic galaxy formation model, applied to three high resolution Aquarius dark matter simulations. In particular, we investigate the effect of the kick velocities that neutron star binaries receive upon their formation, in the building block galaxies (BBs) that partly formed the stellar halo by merging with our Galaxy. When this kick is large enough to overcome the escape velocity of the BB, the NSM takes place outside the BB with the consequence that there is no r-process enrichment. We find that a standard distribution of NS kick velocities decreases [Eu/Mg] abundances of halo stars by $\sim 0.5$ dex compared to models where NS do not receive a kick. With low NS kick velocities, our simulations match observed [Eu/Mg] abundances of halo stars reasonably well, for stars with metallicities $[\text{Mg/H}] \geq -1.5$. Only in Aquarius halo B-2 also the lower metallicity stars have [Eu/Mg] values similar to observations. We conclude that our assumption of instantaneous mixing is most likely inaccurate for modelling the r-process enrichment of the Galactic halo, or an additional production site for r-process elements is necessary to explain the presence of low-metallicity halo stars with high Eu abundances.
4.1 Introduction

The rapid (r-) neutron capture process which leads to the formation of roughly half of the elements heavier than iron has been known since the classical paper of Burbidge et al. [1957]. Yet, there is an ongoing debate about its astrophysical production sites. The two main candidates are neutron star mergers (NSM) and core-collapse supernovae (SNe), where for the latter scenario a distinction is made between the prompt explosions of massive stars with masses in the range $8 \sim 10 \, M_\odot$ [eg. Wheeler et al., 1998] and the delayed explosions of very massive stars [$\geq 20 \, M_\odot$, eg. Woosley et al., 1994]. Those with a mass between 10 and 20 $M_\odot$ are not likely candidates because chemical evolution models predict little to no scatter in [Eu/Fe] abundance ratios for the stars born out of their ashes, which clearly contradicts observations [Ishimaru & Wanajo, 1999]. Whereas neutrino-driven winds associated with delayed explosions seem too proton rich to be the production sites of the high-mass r-process elements [Arcones et al., 2007; Arcones & Thielemann, 2013], prompt explosions might not actually exist in reality [Argast et al., 2004]. Other suggested r-process production sites are long-duration gamma-ray bursts [see e.g. Metzger et al., 2008], magnetic proto-neutron star winds [Suzuki & Nagataki, 2005] and supernovae fallback [Fryer et al., 2006] amongst others.

This paper focusses on the NSM scenario for r-process enrichment, which is arguably the most likely r-process production site, because of the following reasons:

- NSM nucleosynthesis models robustly produce both heavy and light r-process nuclei [Goriely et al., 2011; Wanajo et al., 2014].

- The $^{244}$Pu interstellar medium density as a function of time is incompatible with continuous r-process production, as would be the case in the core-collapse SNe production scenario, but instead points towards a low rate / high yield production site, and perfectly matches the NSM scenario [Wallner et al., 2015; Hotokezaka et al., 2015].

- The recent direct detection of gravitational waves from a NSM and associated signal emitted by radioactive (r-process) material [Abbott et al., 2017b,a] favor the NSM scenario for r-process enrichment.

- Ultra faint dwarf galaxies and the Milky Way are likely to have the same production mechanism for r-process elements, triggered by rare events [Beniamini et al., 2016b].

However, a combination of the two different production sites could also explain the origin of r-process elements in our Galaxy [eg. Ji et al., 2016b].

We study the r-process enrichment by using europium (Eu) as its tracer, because this stable r-process element is one of the r-process elements that can be observationally detected without exceptionally high data quality [eg. Frebel, 2010]. In Figure 4.1 we show the scatter in [Eu/Mg] as a function of [Mg/H], for stars in the Galactic halo and in nearby dwarf galaxies, made after querying the VizieR database [Frebel, 2010]. Mg is chosen as a metallicity tracer to compare the Eu-abundance with, instead of the more commonly used Fe, to obtain an abundance distribution that is directly comparable to our model predictions (as we will explain in the next sections). At low metallicities (eg. $[Mg/H]< -2.5$) some halo stars have high Eu abundances, a feature that should be explained by any chemical evolution model of the r-process.
4.1 Introduction

For many years it has been assumed that NSM alone are unable to reproduce the [Eu/Mg] ratios for stars with [Mg/H] $< -2.5$ [Argast et al., 2004; Wehmeyer et al., 2015], i.e. that Eu enrichment through NSM cannot take place at these low metallicities, mainly because NSM would be too rare and their average coalescence times too large. Also Matteucci et al. [2014] and Cescutti et al. [2015] showed that neutron star binaries can only be the dominant production sites of r-process elements if they merge after 1 Myr or in less than 10 Myr, respectively. However, population synthesis studies of neutron star binaries suggest a wide distribution of delay times proportional to $1/t$, with the first NSM occurring after 10 Myr or 100 Myr and $t$ the time since the formation of the neutron star binary [Belczynski et al., 2006]. However, a small fraction of double neutron stars can merge on even smaller timescales of 1-10 Myr [e.g. Beniamini et al., 2016a].

Recently, Tsujimoto & Shigeyama [2014], Ishimaru et al. [2015] and Komiya & Shigeyama [2016] have shown that Eu-enhanced halo stars are expected in a Galactic halo that is formed from merging subhaloes, where the Eu originates from NSM with a coalescence time of $\sim 100$ Myr. There is only a small probability that the subhaloes with small galaxies ($\sim 10^5 M_\odot$) host a NSM at early times, but if this occurs the next generation of stars formed in such a galaxy would be extremely enhanced in Eu (e.g. [Eu/Mg] $\gtrsim +2$), because of the large Eu yields per NSM. Therefore, this can be considered an other argument in favor of the NSM production scenario for r-process elements, as it is now generally accepted that the Galactic halo has in part grown from the mergers of smaller systems, building blocks (BB) as it were, a scenario that is based on the hierarchical growth of structures as explored in e.g. White & Rees [1978]. Observational evidence supporting growth via accretion and mergers has been reported in e.g. Ibata et al. [1994], Helmi et al. [1999] and Belokurov et al. [2006] amongst others.

However, with the exception of the work of Beniamini et al. [2016a, 2018], and Hotokezaka et al. [2018], little attention has been given in the literature to the fact that neutron star binaries receive a kick velocity at the formation time of the two neutron stars, which might expel the system out of its host galaxy at early times, when the escape velocity of the host galaxy is still small. Therefore, we investigate in this paper if it is still possible to enrich the early Milky Way galaxy in r-process elements through the NSM scenario if neutron star kicks are taken into account. We use a semi-analytic galaxy formation model different from those of Tsujimoto & Shigeyama [2014] and Ishimaru et al. [2015].

This paper is structured as follows: we explain our methods in section 4.2, which contains one subsection summarizing the semi-analytic techniques that are used in our galaxy formation model and another subsection explaining how the r-process (Eu) enrichment of the galaxy by NSM is modelled. We show the results for a standard model in section 4.3, expand upon this result and discuss variations of the standard model in section 4.4. In section 4.5, we compare the effect of the assumption that the NS receive a kick at birth between BB galaxies and surviving satellites. We end the paper with a summary and discussion section (section 4.6).
Figure 4.1: $[\text{Eu}/\text{Mg}]$ abundances for halo stars selected from the VizieR database [Frebel, 2010]. The typical errorbar of $\sim 0.3$ dex in abundance analyses is indicated in the upper left corner. Halo stars are shown as blue dots, red diamonds represent stars originating in classical dwarf galaxies and symbols with downward pointing grey arrows indicate upper limits. The r-process enhanced stars in the ultra-faint dwarf galaxy Ret II [Ji et al., 2016a,b] are not included in the VizieR database, but are manually added to this figure as green squares with errorbars.
4.2 Methods

The build-up of the Galactic halo through merging building BBs is modelled using the Munich-Groningen semi-analytic galaxy formation model [Kauffmann et al., 1999; Springel et al., 2001; De Lucia et al., 2004; Croton et al., 2006; De Lucia & Blaizot, 2007; De Lucia & Helmi, 2008; Li et al., 2010; Starkenburg et al., 2013a]. As a backbone of this model, three of the six (A, B and C) high-resolution Milky Way mass Aquarius dark matter halo simulations are used [Springel et al., 2008]. Resolution level 2 is used (∼10^4 M⊙ per particle), because only Aquarius A is available at higher resolution. The semi-analytic model does not trace the evolution of individual elements explicitly, and assumes an instantaneous recycling approximation (IRA). Therefore, it does not accurately predict the abundance of elements that are produced on long timescales, such as Fe. Also, as we explain in section 4.2.2, the abundance of Eu as a function of cosmic time was put into the model explicitly for this study.

4.2.1 The flow of baryons in the semi-analytic model

As described in detail in the literature [eg. Starkenburg et al., 2013a], the semi-analytic galaxy formation model that we use distinguishes between three different types of galaxies. Main galaxies in the centre of a main dark matter halo (type 0), satellite galaxies in the centre of a dark matter subhalo (type 1) and so called orphan galaxies that have lost their dark matter subhalo through tidal disruption (type 2). For type 0 galaxies, the flow of baryons in the model is visualized in Figure 4.2. In the next paragraphs, we will briefly summarize the physical processes that drive the mass exchange between the different boxes baryonic mass can be in, since these also affect the route of Eu. As soon as a galaxy becomes a satellite, the dark filled arrows in Figure 4.2 are no longer allowed. One arrow with the dashed lines represents two possible heating processes, of which one is only allowed for satellites.

In the top left corner of Figure 4.2, the infalling (pristine) gas or outflowing hot gas is shown. The amount of infalling/outflowing gas M_{i/o} is determined by the difference between the assumed (average) baryonic mass \overline{M}_b(M_{\text{vir}}, z) of the galaxy with virial mass M_{\text{vir}} at redshift z, taking reionization into account, and the baryonic mass in the galaxy, i.e. the sum of the mass in the other five elliptically shaped boxes of Figure 4.2, \( M_b = M_\ast + M_{\text{kg}} + M_{\text{hg}} + M_{\text{eg}} + M_{\text{bh}} \).

\[
M_{i/o} = \overline{M}_b(M_{\text{vir}}, z) - \sum_i M_{b,i} \quad (4.1)
\]

where the sum is taken over all (i) galaxies in the halo. \( \overline{M}_b(M_{\text{vir}}, z) \) is used as approximated by Gnedin [2000],

\[
\overline{M}_b(M_{\text{vir}}, z) = \frac{f_b M_{\text{vir}}}{[1 + 0.26 M_F(z)/M_{\text{vir}}]^3} \quad (4.2)
\]

with the filtering mass M_F(z) as described in Appendix B of Kravtsov et al. [2004]. A cosmic baryon fraction f_b of 0.17 is assumed, consistent with the first-year WMAP results [Spergel et al., 2003], and reionization is assumed to take place between redshift 15 and 11.5 [see also Li et al., 2010]. Since only hydrogen is affected by reionization, Eu does not follow this arrow. However, the process is shown for completeness.
Figure 4.2: Flow of baryons in the semi-analytic model. The physical processes that drive the mass exchange are explained in the text. Dark filled arrows are only allowed for main galaxies (type 0). Satellite galaxies only have Cold Gas and Stellar Mass boxes. Two possible heating processes are behind the single hatched arrow from Cold Gas to Hot Gas, although one of these adds baryons to the Hot Gas component of the main galaxy in the dark matter halo, a process that is only allowed for satellite galaxies. Since satellites do not have Hot Gas in our simulation, in case baryons are moved from the Stellar Mass or Cold Gas boxes in satellites to the Hot Gas box, it is the Hot Gas component of the main galaxy in the dark matter halo. The dashed line from Stellar Mass to Hot Gas furthermore indicates that this process is not allowed for galaxies with a viral mass above $5 \cdot 10^{10} M_\odot$. Note that the arrow from Gas Infall/Outflow to Hot Gas points in both directions because a single prescription determines the direction of the arrow. The arrow from Cold Gas to Stellar Mass points in both directions to visualize the adopted instantaneous recycling approximation.
4.2 Methods

Hot gas cools following the prescriptions described by De Lucia et al. [2004], Croton et al. [2006] and Li et al. [2010]. The cooling rate is strongly dependent on the gas temperature and on its metallicity. The collisional ionization cooling curves of Sutherland & Dopita [1993] are used to model these dependencies. Galaxies with a virial temperature \( T_{\text{vir}}(z) \) below the atomic hydrogen cooling limit \( (T \sim 10^4 \text{K}) \) are prevented to cool gas in our model, since it is assumed to be very inefficient at early times. We assume that the amount of Eu to the total metal budget of a galaxy that accounts for cooling is negligible at all times. All metals (including Eu) that are contained in the fraction of the hot gas that is cooling are also cooled onto the cold disk of the galaxy.

From the cold gas box, baryons can flow to three different other boxes of Figure 4.2 (or four if it is a main galaxy). For main galaxies (type 0), the energy injected by SN, as described by Kauffmann & Haehnelt [2000] is deposited into the ejected gas box of Figure 4.2.

\[
\Delta M_{\text{reheated}} = \frac{4}{3} \frac{\eta_{\text{SN}} E_{\text{SN}}}{V_{\text{vir}}^2} \Delta M_\star \tag{4.3}
\]

with \( \Delta M_\star \) is the stellar mass formed, \( V_{\text{vir}} \) the circular velocity of the galaxy at the virial radius \( (R_{\text{vir}}) \), \( E_{\text{SN}} = 10^{51} \) erg the energy released per SN, and \( \epsilon = 0.05 \) and \( \eta_{\text{SN}} = 8 \cdot 10^{-3} M_\odot^{-1} \) are the feedback efficiency and the number of SNe respectively [Li et al., 2010]. We define the virial radius as the radius of a sphere with mass \( M_{\text{vir}} \) and mean interior density is 200 times the critical density for closure of the universe [Navarro et al., 1997]:

\[
M_{\text{vir}} = \frac{4}{3} \pi R_{\text{vir}}^3 200 \frac{3H^2}{8 \pi G} = \frac{100H^2R_{\text{vir}}^3}{G} \tag{4.4}
\]

with \( H \) the Hubble parameter. Smaller galaxies have shallower potential wells and thus a lower \( V_{\text{vir}} \), which makes them more efficient reheaters.

Two different physical processes are behind the arrow from cold gas pointing back to hot gas (highlighted with dashes), describing reheating of the gas. One is related to the radio activity of the active galactic nucleus (AGN) of the galaxy, as described by Croton et al. [2006], and should technically be considered a suppression of the cooling flow. This recipe is applied to all galaxy types. The other heating recipe again corresponds to the energy injected by SN, equation (4.3), but then only applied for satellite galaxies (types 1 and 2). For those, the virial mass is taken to be the number of particles in the simulated galaxy times the particle mass. For both reheating recipes, all metals (including Eu) that are contained in the fraction of the cold gas that is reheated are moved to the hot gas phase of the host galaxy.

The baryons in the ejected gas box, \( M_{\text{eg}} \), can be re-incorporated into the hot gas box, following the prescription of De Lucia et al. [2004]:

\[
\dot{M}_{\text{re-inc.}} = \gamma_{\text{eg}} \cdot M_{\text{eg}} \frac{V_{\text{vir}}}{R_{\text{vir}}} \tag{4.5}
\]

with \( \dot{M}_{\text{re-inc.}} \) the amount of gas that is reincorporated during one timestep, and \( \gamma_{\text{eg}} = 0.5 \) the ejected gas reincorporation efficiency [Croton et al., 2006, table 1]. The ratio \( R_{\text{vir}}/V_{\text{vir}} \) is proportional to the dynamical time of the galaxy [Springel et al., 2001]. As for all the other processes, all metals (including Eu) contained in the fraction of the ejected gas that is re-incorporated are moved to the hot gas box.
Only main galaxies have ejected gas and hot gas components. Satellites transfer their ejected mass to their hosts (the mains). This is indicated through the dashed line from Stellar Mass to Hot Gas in Figure 4.2.

Both cold gas and hot gas can be accreted onto the central black hole (BH). With $M_{bh}$, we denote the baryonic part of the BH mass. The change in this parameter during one timestep is indicated with an overdot. From the hot gas box, quiescent accretion follows the empirical recipe described by Croton et al. [2006], i.e. the ‘radio mode’:

$$\dot{M}_{bh,r} = \kappa_{\text{AGN}} \left( \frac{M_{bh}}{10^8 M_\odot} \right) \left( \frac{M_{hg}}{0.1 M_{\text{vir}}} \right) \left( \frac{V_{\text{vir}}}{200 \text{ km s}^{-1}} \right)^3.$$  \hspace{1cm} (4.6)

Here, $M_{hg}$ is the baryonic mass in the form of hot gas and $\kappa_{\text{AGN}} = 7.5 \cdot 10^{-6} M_\odot \text{yr}^{-1}$ the hot gas BH accretion rate [De Lucia & Blaizot, 2007]. Cold gas accretion happens when a satellite galaxy merges with the central (main) galaxy. This recipe is also described by Croton et al. [2006] and dubbed ‘quasar mode’:

$$\Delta M_{bh,q} = f_{bh} \left( \frac{m_\ast + m_{cg}}{M_\ast + M_{cg}} \right) \frac{M_{cg}}{1 + (280 \text{ km s}^{-1}/V_{\text{vir}})^2}$$  \hspace{1cm} (4.7)

with $M_\ast$ and $M_{cg}$ the stellar mass and mass in cold gas of the main galaxy and $m_\ast$ and $m_{cg}$ the stellar mass and mass in cold gas of the satellite galaxy respectively. $f_{bh} = 0.03$ is the cold gas BH accretion fraction. Again, all metals (including Eu) contained in the fraction of the hot or cold gas that is accreted are moved to the central BH box.

Last, but not least, we describe how stars are formed from cold gas and how metals are added to the hot and cold gas components. We assume cold gas is transformed into stars in an exponential thin disc [by the formalism of Mo et al., 1998], which is assumed to extend to three scalelengths, when its density is above a critical threshold [Kennicutt, 1989; Kauffmann, 1996]:

$$\Sigma_{\text{crit}}(M_\odot \text{pc}^{-2}) = 0.59 \sqrt{V_{\text{vir}}(\text{km s}^{-1})/R_{\text{disc}}(\text{kpc})}$$  \hspace{1cm} (4.8)

The star formation rate is then proportional to the amount of cold gas available. See also De Lucia & Helmi [2008] and Starkenburg et al. [2013a]. Furthermore, during a galaxy merger, a collisional starburst recipe [Somerville et al., 2001] is triggered, where a fraction

$$e_{\text{burst}} = \beta_{\text{burst}} \left( \frac{m_\ast + m_{cg}}{M_\ast + M_{cg}} \right)^{\alpha_{\text{burst}}}$$  \hspace{1cm} (4.9)

of the combined cold gas of the two galaxies ($m_{cg} + M_{cg}$) is turned into stars. The parameters $\alpha_{\text{burst}}$ and $\beta_{\text{burst}}$ are set to be 0.7 and 0.56 respectively [Croton et al., 2006]. Due to our adopted IRA and assumed Chabrier [2003] initial mass function (IMF), 43% of the cold gas is returned immediately to the cold gas component during star formation, hence the two-way pointing arrow in Figure 4.2. Also, 43% of the metals (including Eu) in the cold gas available for star formation are returned immediately to the cold gas, the other 57% ends up in stars. After a starburst and instantaneous recycling, new metals (other than Eu, which is injected through a different channel - see section 4.2.2) become available in the two gas phases. We follow the recipe of Li et al. [2010], who suggested a two-state value of the fraction of metals deposited directly into the hot gas of...
4.2 Methods

the main galaxy in the dark matter halo (which is the galaxy itself in case it is a type 0). For galaxies with a virial mass below $5 \cdot 10^{10} M_\odot$, 95% of the metals are added to this box and 5% to the cold gas box of the galaxy that produced the metals. Higher mass galaxies deposit all their new metals into the cold gas phase of their galaxy. We assume a metal yield of 0.03 [Croton et al., 2006].

4.2.2 Adding the europium

First we run the semi-analytic model once without any enrichment in europium, and record the total stellar mass formed in each galaxy, as well as the maximum circular velocity of a particle in the potential well of the galaxy at each time step. Then, assuming a NSM rate, we know in post-processing for each star formation episode how many NS mergers happen after a delay time picked out of an assumed delay time distribution (DTD). In our standard model, we assume a NSM rate of $10^{-4}$ per $M_\odot$ stars formed. This assumption is based on the NS merger rate inferred from gravitational wave (GW) observations [Côté et al., 2018]. It is about a factor 10 higher than the rate assumed by Tsujimoto & Shigeyama [2014], who deduce a NSM rate of one per $\sim 1000$ core-collapse SNe\(^1\), and about a factor 100 higher than that of Cescutti et al. [2015], who assume a NSM rate of a few times $10^{-6}$ per $M_\odot$ stars formed.

In our standard model, we assume a DTD proportional to $1/t$, with the first NSM occurring after 10 Myr, which we will abbreviate as 10 Myr +1/t in the remainder of this paper. As already mentioned in the introduction, this DTD is compatible with binary population synthesis studies [Belczynski et al., 2006].

Assuming a NFW profile for the density of dark matter haloes at radius $r$ [Navarro et al., 1996],

$$\rho(r) = \frac{\rho_0}{(r/a) (1 + r/a)^2},$$  \hspace{1cm} (4.10)

where $\rho_0$ and $a$ are fit parameters, it can be shown that the escape velocity that a particle needs to have in order to escape from the centre of the potential well is

$$v_{esc}(0) = \sqrt{\frac{2}{0.465}} V_{max},$$  \hspace{1cm} (4.11)

where $V_{max}$ is the maximum of the circular velocity of the galaxy. At the scale radius $a$, the escape velocity is a factor $\sqrt{\ln(2)} = 0.833$ lower.

We assign a kick velocity to each NS binary at the formation time of the two NS, following the kick velocity distribution of Arzoumanian et al. [2002]: a two-component velocity distribution (Maxwellians) with characteristic velocities of 90 and 500 km/s, with 40% of the population being drawn from the distribution with the lowest characteristic velocity. Since we assume that the system remains bound after receiving the kick, we assign half the velocity picked from this distribution as the kick velocity of the binary. We compare this velocity with the escape velocity from the centre of the galaxy hosting the NS binary (equation 4.11). If the kick velocity is smaller than the escape velocity, we assume that the neutron star binary will be inside the galaxy at the time of the merger. Otherwise, the binary is discarded as a potential source of Eu enrichment. We also investigate the effect of applying the low velocity Maxwellian distribution of Arzoumanian

\(^1\)In our semi-analytic model we assume $8 \cdot 10^{-3}$ core-collapse SNe per $M_\odot$ stars formed [Li et al., 2010].
et al. [2002] only, i.e. that with a characteristic velocity of 90 km/s. This assumption is hereafter referred to as the “low kick” model, which is compared to the “no kick” (standard kick model) and “with kick” (assuming the full kick velocity distribution) models. The choice for the low kick model is motivated by the fact that many double neutron stars are found to have small NS kick velocities [Tauris et al., 2017]. Furthermore, Verbunt & Cator [2017] have recently shown that a description with two Maxwellians, with distribution parameters $\sigma_1 = 77$ and $\sigma_2 = 320$ km/s, is significantly better than that of Arzoumanian et al. [2002].

For those binaries that do not leave the host galaxy after receiving their kick, we assume a europium yield of $1.5 \cdot 10^{-5} M_\odot$/NSM in our standard model. This is the upper limit of the Eu yield inferred from GW observations [Côté et al., 2018] and slightly larger than what was assumed by Matteucci et al. [2014, 2015], who assume Eu yields in the range $10^{-5} - 10^{-7} M_\odot$, but a factor 10 smaller than the estimate of Komiya & Shigeyama [2016]. We assume that halo stars are only formed in BBs. If a NSM happens when its hosting BB has already merged with the central galaxy, the gas enriched by this NSM might form new stars in the disk or inner spheroid, but since this paper focusses on the halo component where no new star formation takes place this is not relevant for our research. Since the amount of Eu is negligible compared to the total NSM metal yield, which is already added to its metal budget through the “metals from stars to gas” channel (see section 4.2.1), we also do not add the Eu to the gas metal budget of the main galaxy’s spheroid in this scenario.

Knowing exactly when a NSM happens in each galaxy, we now re-run the semi-analytic model and incorporate the chemical evolution of Eu in the ensemble of galaxies. The europium will be instantaneously mixed into the gas of each galaxy, which is used for the formation of new stars. Similar to the enrichment of the galaxy in other metals (see section 4.2.1), we assume that 95% of the newly produced Eu is mixed with the hot gas in the main galaxy in the dark matter halo if the galaxy that produced the Eu has a virial mass below $5 \cdot 10^{10} M_\odot$, and 5% is added to the cold gas budget of their own galaxy$^2$. All Eu is added to the cold gas budget of the galaxy itself in case it has a virial mass above $5 \cdot 10^{10} M_\odot$.

### 4.2.3 Metallicities

The total metallicity of a stellar population at each time step is traced by our model as the parameter $Z_{\text{stars}}$, the ratio of mass in metals over the total mass in stars. Because of the IRA, this value can thus best be compared with the abundance of elements which are mainly originating in (short-lived) Supernovae type II, such as the $\alpha$-elements. In this paper we show predicted $\log[Z_{\text{stars}}/Z_\odot]$ values of our model, with $Z_\odot = 0.0142$ the reference metallicity [Asplund et al., 2009]. These can thus be thought of as [Mg/H] values. Similarly, $\log[(X_{\text{Eu}}/Z)_{\text{stars}}/(X_{\text{Eu}}/Z)_\odot]$ values, with $X_{\text{Eu},\odot} = 3.6 \cdot 10^{-10}$ [Asplund et al., 2009], can be thought of as [Eu/Mg]. That is why we chose to plot these abundance ratios against each other in Figure 4.1, instead of the more commonly used [Fe/H] and [Eu/Fe].
4.3 Results for a standard model

Figure 4.3: Upper left panel: [Eu/Mg] vs. [Mg/H] map for the accreted spheroid of Aquarius halo A-2. Colors indicate the stellar mass in each bin on a logarithmic scale. The dashed line indicates the solar $X_{\text{Eu}}/Z$ ([Eu/Mg]) abundance, the solid line separates out the stars without any enrichment in Eu. The ones in the bottom leftmost bin also do not have other metals. These stars (still) exist in our model because we neglect any kind of pre-enrichment from Population III stars. Lower left panel: MDF of this accreted stellar spheroid (green line), the single Gaussian fit to the observed IMF from An et al. [2013] (blue line), normalized on the $-1.5 \leq \log[Z_{\text{stars}}/Z_\odot] < -1.0$ bin from the model MDF, and the model MDF of stellar halo stars only (red line). Right panel: [Eu/Mg] vs. [Mg/H] map for the accreted stellar halo stars of Aquarius halo A-2 only, using the same colormap to indicate the stellar masses in each bin as in the left panel. Blue filled circles denote observed halo stars selected from the VizieR database [Frebel, 2010] with standard errors, blue open circles show halo stars selected from this database for which only upper limits in [Eu/Mg] have been determined.
Chapter 4: Semi-analytic modelling of the europium production by neutron star mergers in the halo of the Milky Way

4.3 Results for a standard model

4.3.1 The accreted spheroid

In the upper left-hand panel of Figure 4.3, we show the \([\text{Eu/Mg}]\) versus \([\text{Mg/H}]\) map predicted by our model for the accreted spheroid of Aquarius halo A-2, assuming that the NS do not receive a kick velocity at their formation time (see section 4.2.2). Colors indicate the stellar mass per bin, on a logarithmic scale, with darker colors representing more stars.

The accreted spheroid contains bulge stars and halo stars. Their combined metallicity distribution function (MDF), is shown with a green line in the lower left-hand panel of Figure 4.3. Since the stars selected from the VizieR database and presented in Figure 4.1 that we want to compare our model \([\text{Eu/Mg}]\) vs. \([\text{Mg/H}]\) maps with are all halo stars, we want to divide this accreted spheroid into a bulge part and a halo part. To do this, we use the same technique as van Oirschot et al. [2017a]. The semi-analytic model underestimates the number of accreted spheroid stars with metallicities \(-3 \leq \log[Z_{\text{stars}}/Z_{\odot}] < -1\). Therefore, we assume all stars in these and lower metallicity bins to be halo stars, and consequently all accreted bulge stars to have metallicities \(\log[Z_{\text{stars}}/Z_{\odot}] > -1\). The fraction of halo stars in these \(\log[Z_{\text{stars}}/Z_{\odot}] > -1\) bins of the accreted spheroid MDF is set to follow a single Gaussian fit to the observationally determined MDF by An et al. [2013], which is normalized to the \(-1.5 \leq \log[Z_{\text{stars}}/Z_{\odot}] < -1.0\) bin of the accreted spheroid MDF and shown with a blue line in the lower left-hand panel of Figure 4.3. The red line is the resulting stellar halo MDF of the semi-analytic model (SAM).

The downscaling of the stellar mass in the \(\log[Z_{\text{stars}}/Z_{\odot}] > -1\) bins of the \([\text{Eu/Mg}]\) vs. \([\text{Mg/H}]\) map of accreted spheroid Aq-A-2 is done linearly for all age bins, and the resulting map is shown in the right-hand panel of Figure 4.3. The same scale for the colormap is used as for the \([\text{Eu/Mg}]\) vs. \([\text{Mg/H}]\) map in the left panel. The observed halo stars shown in Figure 4.1 are overplotted with blue circles here (i.e. stars known to be in satellite galaxies are left out). Open circles represent stars for which only upper limits are available, filled circles represent stars which have standard errors in the determined \([\text{Eu/Mg}]\) values.

It is a known shortcoming of the Munich-Groningen semi-analytic model that it underestimates the number of low metallicity halo stars. Therefore, it is not surprising that our model cannot explain the large number of observed stars with \([\text{Mg/H}] < -1.5\). More importantly, hardly any of the few low-metallicity stars that are predicted by our model, have \([\text{Eu/Mg}] > 0\) abundances, whereas these are observed in the Galactic halo.

In the next section we give an example of how the Galactic halo is built up from BBs, in order to investigate if we can confirm the findings of Tsujimoto & Shigeyama [2014], Ishimaru et al. [2015] and Komiya & Shigeyama [2016] that Eu-enhanced halo stars naturally occur in a Galactic halo that is formed from merging subhaloes. In the remainder of this paper we will not select out the accreted bulge stars from modelled spheroids \([\text{Eu/Mg}]\) vs. \([\text{Mg/H}]\) maps, because we will focus our comparisons with data solely on the lower metallicities where we expect to have accreted halo stars only. However, the same method could be applied as in Figure 4.3 to end up with \([\text{Eu/Mg}]\) vs. \([\text{Mg/H}]\) maps of accreted halo stars only.

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2In case the galaxy is not a satellite, the main galaxy in the dark matter halo is also the galaxy itself.
Figure 4.4: [Eu/Mg] vs. [Mg/H] maps for the simulated Milky Way galaxy in Aquarius halo A-2 (“building block” 0) and one of its major building blocks (BB 1) and its building block galaxies (numbered 2 - 8). The galaxy merger tree is shown in the lower right-hand panel, where the colors represent the different stages a building can be in, as explained in the beginning of section 4.2.1: red for main galaxies, yellow for satellites and green for orphans. The size of the circles corresponds to the stellar mass as indicated in the legend. A black line indicates a galaxy merger. In the main figure, eight time snapshots are shown, with the corresponding lookback time ($t$) and redshift ($z$) shown on the left. The colormap indicates the logarithm of the stellar mass per bin (note that a different scale is used than in Figure 4.3). The maximum $\log_{10}$ of the stellar mass of the building blocks 0 – 8 from left to right is 0: 11.3 (simulated Milky Way), 1: 9.3, 2: 3.5, 3: 6.5, 4: 3.9, 5: 6.7, 6: 7.9, 7: 4.9 and 8: 5.0, respectively.
4.3.2 Building blocks of the Galactic spheroid

In this paper, we consider the accreted component of the stellar spheroid, which is assumed to be built from a few main progenitor galaxies, in agreement with the findings of many others [eg. Helmi et al., 2002, 2003; Font et al., 2006a; Cooper et al., 2010; Gómez et al., 2013]. In order to study the contribution of one of these large BBs of the stellar spheroid to the $[\text{Eu/Mg}]$ vs. $[\text{Mg/H}]$ map in depth, we show a part of the galaxy merger tree of Aquarius halo A-2 in Figure 4.4 (bottom right). Only the main (simulated Milky Way) galaxy and one major BB are shown, respectively with numbers 0 and 1. BB 1 itself is built from four other BBs (labelled 2, 3, 5 and 6), of which some in turn are also built from building blocks.

In Figure 4.4 we also show the $[\text{Eu/Mg}]$ vs. $[\text{Mg/H}]$ maps of these BB galaxies as a function of cosmic (lookback) time. Each panel contains a $[\text{Eu/Mg}]$ vs. $[\text{Mg/H}]$ map, with successive time snapshots of the same BB vertically ordered. The labels of the axes are left out for clarity, but they are identical to those of Figure 4.3. However, the chosen colormap which again indicates the stellar mass in each bin of a $[\text{Eu/Mg}]$ vs. $[\text{Mg/H}]$ map, is slightly different from the one used in Figure 4.3. This is because Figure 4.4 also contains the stars in the disc of the main (Milky Way) galaxy of Aquarius A-2 in its leftmost panels (hence this colormap extends to $10^{11} \text{M}_\odot$), whereas Figure 4.3 only contains accreted spheroid stars (and its colormap extends only to $10^{9.5} \text{M}_\odot$). The structure of the galaxy merger tree (bottom right) is matched in the main part of the figure. The $[\text{Eu/Mg}]$ vs. $[\text{Mg/H}]$ maps of BBs that have not yet formed any stars, or those of the BBs that merged with other BBs in a particular time step, are not shown.

From this figure, it is clear that all except the lowest mass BB of this major BB (i.e. number 2, which has a maximum stellar mass of $10^{3.5} \text{M}_\odot$) form stars with Eu. This is a result of the assumed NSM rate of $10^{-4}$ per $\text{M}_\odot$ stars formed, since that makes the threshold BB mass for forming stars which are enriched in Eu $\sim 10^4 \text{M}_\odot$. Furthermore, we see from this figure that most of the few accreted spheroid stars with enhanced Eu abundances in this model (shown above the dashed line in each panel), as well as those with low-metallicities ($\log\left[Z_{\text{stars}}/Z_\odot\right] < 1.5$) were not born in (BBs of) this major BB.

Then of course the question arises, how can we explain the large number of halo stars with $[\text{Eu/Mg}] > 0$ abundances? We will investigate the effect of different assumptions on the NS kick, the NSM rate or the DTD in the next section, but it might very well be our assumption of instantaneous mixing that results in these low $[\text{Eu/Mg}]$ abundances. Eu-enhanced stars naturally arise due to the inhomogeneity of gas enriched by NSM, as was also found on the subgrid level of the cosmological magnetohydrodynamical IllustrisTNG simulations recently [Naiman et al., 2018].

4.4 The effect of NS kicks, the NSM rate and the DTD

Although our standard model does predict some halo stars with high Eu-abundances for stars with low $[\text{Mg/H}]$ values, shown above the dashed line in the $[\text{Eu/Mg}]$ vs. $[\text{Mg/H}]$ maps presented in the previous section, compared with observations (right-hand panel of Figure 4.3) the number density of Eu-enhanced stars predicted by the SAM applied to Aquarius halo A-2 is too low. Therefore, we will investigate variations of our standard model in this section. Hereafter, we
4.4 The effect of NS kicks, the NSM rate and the DTD

Define enrichment in Eu to take place if all of the following three conditions are met:

- A NSM takes place in the BB.
- The coalescence time of the NS binary is shorter than the timescale on which the BB merges with the central galaxy.
- The NS kick velocity is smaller than the escape velocity of the BB.

The assumed NSM rate affects the first condition, the DTD affects the second condition and the assumed binary NS kick velocity distribution affects the third condition. We will show the effect of variations in all of these three variables.

4.4.1 NS kick velocities

In our standard model, we assume that NS binaries do not receive a kick, i.e. the third of the above conditions is always met. Therefore, introducing a kick velocity naturally decreases the fraction of Eu-enriched halo stars. To investigate by how much, we show in Figure 4.5 the fraction of BBs with Eu-enrichment by NSM as a function of BB stellar mass, for three different assumptions on the NS kick velocity. We also show the effect of two different assumptions on the DTD for each of these three models.

With our standard model assumptions (no kick) ~ 90% of the highest mass BBs ($m_\ast > 10^{8.5}M_\odot$) and ~ 80% of the lowest mass BBs ($m_\ast < 10^{5.5}M_\odot$) is enriched in Eu. Assuming longer delay times through a 100 Myr +1/t DTD in this case give 5 – 10% lower enrichment fractions,
Figure 4.6: $[\text{Eu}/\text{Mg}]$ vs. $[\text{Mg}/\text{H}]$ maps for stellar spheroids in the Aquarius haloes A, B and C (from left to right). The top, middle and bottom panels correspond to models in which the NS binary received no kick, a low kick or a higher kick, respectively.
because fewer NS binaries merge before the BB merges with the central galaxy in that model. Introducing the 2-peak Arzoumanian et al. [2002] kick velocity distribution, drastically decreases this fraction, to less than 50% of the highest mass BBs and to only 10% of the lowest mass BBs. This result is almost identical if we take a 100 Myr $+1/t$ DTD instead of our standard 10 Myr $+1/t$. The enrichment fractions corresponding to the low kick models (only the low velocity peak of the Arzoumanian et al. [2002] distribution) fall in between the other lines for low mass BBs, and overlap with the lines for no NS kick for the highest mass BBs because these highest mass BB have escape velocities which are always larger than these low NS kick velocities.

In Figure 4.6, the [Eu/Mg] vs. [Mg/H] maps are shown for the full accreted spheroids of Aq-A-2, Aq-B-2 and Aq-C-2 from left to right respectively. The top, middle and bottom panels show the difference between assuming that the NS binary receives no kick, a low kick, or a higher kick. A DTD of 10 Myr $+1/t$ is assumed. Figure 4.6 shows that there is indeed a larger Eu-enrichment in case the NS do not receive a kick. Roughly, the distribution of stars in these [Eu/Mg] vs. [Mg/H] maps shifts 0.5 dex downwards when we assign a NS kick velocity. Furthermore, this figure gives an impression of the stochastic scatter between the Aquarius haloes. One cannot expect to find a galaxy in one of the Aquarius haloes that is an exact replica of our Milky Way, because only six simulations were done (of which we study three in this paper). In particular, our semi-analytic model “no kick” and “with low kick” applied to Aquarius halo B results in an [Eu/Mg] vs. [Mg/H] map that is in very good agreement with the observations. The majority of r-process enriched stars in this halo originate in a single relatively massive ($10^{7.45} M_\odot$) BB, in which a relatively large starburst occurred at late times (10% of its stars were formed in a burst about 9.5 Gyr ago, whereas most halo stars were formed about 11 Gyr ago). Our model thus predicts quite some variation between galaxies in terms of r-process enrichment.

### 4.4.2 Delay time distribution

The effect of assuming a different DTD on the [Eu/Mg] vs. [Mg/H] map of stellar spheroid Aq-A-2 is shown in Figure 4.7. A constant delay time of 1 Myr may be unrealistically short to be compatible with population synthesis studies [Belczynski et al., 2006], but this is what has been assumed before to explain Eu abundances in the Milky Way galaxy with a NSM enrichment scenario [Matteucci et al., 2014]. The models in which a fixed delay time of 1 Myr is assumed yield [Eu/Mg] vs. [Mg/H] maps where most stars have [Eu/Mg]$\sim 0$ with little scatter, since NSM are treated almost like SNe Type II with this assumed DTD and fairly high NSM rate. The bulk of the stars is shifted to higher [Eu/Mg] abundances by about $\sim 0.5$ dex compared to our standard model (10 Myr $+1/t$). Thus the effect is similar to that of assuming a kick velocity for the binary NS.

Assuming a longer DTD on the other hand (100 Myr $+1/t$), results in the majority of stars being spread out over the low [Eu/Mg], high [Mg/H] abundance bins (eg. $0\gtrsim$[Eu/Mg]$\gtrsim -2$ for $0\gtrsim$[Mg/H]$\gtrsim -1$), which is clearly not where the majority of the observed halo stars are (see Figures 4.1 and 4.3).
Figure 4.7: $\text{[Eu/Mg]}$ vs. $\text{[Mg/H]}$ maps of stellar spheroid Aq-A-2, for three different assumptions on the DTD: 1 Myr fixed, 10 Myr $+1/t$ and 100 Myr $+1/t$ from left to right. The three different assumptions on the NS kicks are again “no kick”, “low kick” and “with kick” from top to bottom.
4.4 The effect of NS kicks, the NSM rate and the DTD

Figure 4.8: [Eu/Mg] vs. [Mg/H] maps of stellar spheroid Aq-A-2, for three different assumptions on the NSM rate: low ($10^{-5} M_\odot^{-1}$), standard ($10^{-4} M_\odot^{-1}$) and high ($10^{-3} M_\odot^{-1}$) from left to right, and corresponding assumed Eu yields of $1.5 \cdot 10^{-4} M_\odot$/NSM, $1.5 \cdot 10^{-5} M_\odot$/NSM and $1.5 \cdot 10^{-6} M_\odot$/NSM. The three different assumptions on the NS kicks are again “no kick”, “low kick” and “with kick” from top to bottom. In all these models, the assumed DTD is 10 Myr $+1/t$. 

[Diagram showing the [Eu/Mg] vs. [Mg/H] maps for different NSM rates and NS kicks.]
4.4.3 NSM rate

Three different NSM rates are assumed in Figure 4.8. These are combined with Eu yield assumptions such that the Galactic disc stars have solar Eu abundances at [Mg/H]=0 in our models, i.e. the production of the NSM rate and the Eu yield always equals $1.5 \times 10^{-9}/\text{NSM}$. With high NSM rates ($10^{-3} M_\odot^{-1}$) and low Eu yields ($1.5 \times 10^{-6} M_\odot/\text{NSM}$), the effect of continuous Eu production is studied, similar to how it would be in a supernova enrichment scenario. We also investigate a scenario in which NSM are a factor 10 rarer than in our standard model ($10^{-5} M_\odot^{-1}$), but yield a large amount of Eu ($1.5 \times 10^{-4} M_\odot/\text{NSM}$).

In the left-hand panels of Figure 4.8, we show the models with an increased Eu yield and correspondingly lower NSM rate, compared to our standard model. Here, only the most massive BBs are expected to form Eu-enriched stars. This can also be seen from the number of stars without any Eu, that end up in the lowest [Eu/Mg] bin ($[\text{Eu/Mg}] < -3.5$, below the solid line) of each [Eu/Mg] vs. [Mg/H] map. There are much more of these in the left-hand panels of Figure 4.8 (low NSM rate) than in the right-hand panels (high NSM rate). The difference between the middle and the right-hand panels seems not very large.

4.5 Comparing BBs with surviving satellites

Defining galaxies that contain stars with $[\text{Eu/Mg}] > 0$ to be Eu-rich, and those with some Eu-enriched stars but no stars above the solar [Eu/Mg] abundance as Eu-poor, we show the average number of galaxies in these two categories as a function of galaxy stellar mass in Figure 4.9, for the three different assumptions on the NS kick. The average is taken over the three Aquarius haloes A, B and C. We distinguish BBs (filled gold histograms) and surviving satellites (transparent histograms with thick blue edges) to investigate the effect of the timescale on which the galaxies merge with the central galaxy: surviving satellites per definition have not (yet) merged with the central galaxy in the simulation. If a galaxy is not fully disrupted, it is counted as partially stripped and divided between the BB and surviving satellite categories according to the fraction of the total stellar mass that is stripped. For example, if 70% of its initial mass is stripped, the galaxy is counted as 0.7 BB stripped material and 0.3 surviving satellite galaxy.

We do not trust our model’s prediction for galaxies with a stellar mass below a few times $10^4 M_\odot$ due to the limiting resolution of our simulation. Largely overlapping with these, the galaxies with less than five star formation snapshots are left out of Figure 4.9, because the neglect of pre-enrichment from the very first generation of stars in our model is most clear in these galaxies [Starkenburg et al., 2013a]. This can be thought of as a mass cut since more massive galaxies sustain star formation for more extended periods and show star formation in more than five snapshots. The number five is a consequence of the time resolution of our code, which is redshift dependent but in the order of tens to hundreds of Myr. The ultra-faint dwarf galaxy Reticulum II (Ret II), which was recently discovered to be extremely enriched in r-process elements [Ji et al., 2016a,b], falls below this mass resolution limit [Simon et al., 2015]. Although we assume a NSM rate of only one per $10^4 M_\odot$ stars formed, due to Poisson scatter galaxies with stellar masses as low as that of Ret II can have Eu-rich stars in the NSM r-process enrichment scenario. In the left-hand panels of Figure 4.9, it can be seen that this is most likely to be the case if the NS
4.5 Comparing BBs with surviving satellites

Figure 4.9: Average number of building blocks (filled gold histograms) and surviving satellites (transparent histograms with thick blue edges) with more than 4 star formation snapshots (see text) per mass bin in two different categories: Eu-rich (left-hand panels) and Eu-poor (right-hand panels), for three different assumptions on the NS kick: no kicks (top panels), low velocity kicks (middle panels), or higher kick velocities (bottom panels). See section 4.5 for a definition of how the Eu-rich and Eu-poor categories are defined. The average is taken over the three Aquarius haloes A, B and C. Material stripped from a surviving satellite is partially counted as building block and the fraction of the initial satellite mass that is not stripped is contributes as surviving satellite. The dark shaded filled histogram shows how this material contributes to the building block distribution. The total average number of building blocks and surviving satellites per category is shown in brackets in the legend.
binary receives no kick, or a kick with a low velocity.

The fact that there is a large difference between the number of stars in the three categories between the three assumptions on the NS kick, is related to our definition of Eu-rich stars, i.e. those with $[\text{Eu}/\text{Mg}]>0$. By introducing a kick velocity, the distribution shifts to lower $[\text{Eu}/\text{Mg}]$ abundances, by about $\sim 0.5$ dex. Those galaxies with $[\text{Eu}/\text{Mg}]$ abundances just above zero move to values just below zero when a kick is introduced, which makes them fall into a different category.

Figure 4.9 also predicts a difference between the Eu enrichment of the stellar halo of the Milky Way and that of its surviving satellite galaxies. Surviving satellites more often are expected to have Eu-rich stars than those that are fully disrupted. An explanation for this can be found in both the time the galaxy takes to merge with the central galaxy, and in the average timespan of star formation, that is longer for surviving satellite galaxies than for disrupted BBs [van Oirschot et al., 2017b]. Galaxies with a longer star formation timespan for a similar final stellar mass have a lower star formation rate, a slower build-up of the metallicity (Mg) budget due to Type-II supernovae, and correspondingly more stars with high $[\text{Eu}/\text{Mg}]$ abundance. A similar argumentation is used to explain the high alpha-over-iron element abundances in the stellar halo compared to that of surviving satellite galaxies [eg. Font et al., 2006a,b; Geisler et al., 2007; Robertson et al., 2005].

In the top left panel of Figure 4.9 we see that in our standard model, 27% of the modelled Milky Way spheroid stars with $[\text{Eu}/\text{Mg}]>0$ comes from BBs that still have a surviving counterpart. For the models “with kick” this ratio is as high as 54%, as can be seen in the bottom left panel of Figure 4.9. However, it is unlikely that many stars with high Eu abundances in the Milky Way halo are stripped from surviving satellites, since the Eu-enhanced are local field halo stars for which no known stream or parent is known. Furthermore, these stars are $[\alpha/\text{Fe}]$ enhanced, which the satellite galaxies that have survived are not.

### 4.6 Summary and discussion

In this paper, we have modelled the europium production by neutron star mergers in the halo of the Milky Way with the Munich-Groningen semi-analytic galaxy formation model. In particular, we have investigated the effect of the kick velocity the NS binary receives upon its formation, which may lead to a NS merger outside the BB galaxy hosting the double NS. Although this may lead to Eu enrichment in the inner spheroid or disk of the main galaxy in the simulation, in our models this situation will not lead to enrichment of the stellar halo, which is assumed to be formed from merging BBs only. The effect is dependent on the NS kick velocity and the escape velocity of the hosting BB. The NS kick velocity influences the amount of enrichment of the Galactic spheroid in Eu: in $[\text{Eu}/\text{Mg}]$ vs. $[\text{Mg}/\text{H}]$ stellar density maps, stars have $\sim 0.5$ dex lower $[\text{Eu}/\text{Mg}]$ abundances if a NS kick drawn from the Arzoumanian et al. [2002] velocity distribution is assumed (Figure 4.6). Of the BBs with a stellar mass below $10^{5.5} \, M_\odot$, only 10% will be enriched in Eu when a NS kick is assumed, compared to $\sim 80\%$ when no NS kick is assumed (Figure 4.5). For more massive BBs the enrichment is always larger, as these have deeper potential wells from which NS binaries have to escape.

We assumed the velocity of the center of mass of the binary to be half the kick velocity, which is slightly oversimplifying the effect of the binary evolution. Neglecting the effect of the kick after
the supernova of the first star, and assuming the star that undergoes the second supernova drags along the other star, which is assumed to be of equal mass, we arrive at the factor 2. Once a complete binary stellar evolution model is combined with the semi-analytical galaxy formation model, a more accurate value for the system velocity after the supernova of the second star can be assumed.

Our simulations indicate that our standard assumption on the DTD (proportional to $1/t$, with the first merger occurring after 10 Myr) results in $[\text{Eu/Mg}]$ vs. $[\text{Mg/H}]$ maps which match that of the observed stellar halo population reasonably well, for stars with metallicities $[\text{Mg/H}] \geq -1.5$. Although we confirm the result of Tsujimoto & Shigeyama [2014], Ishimaru et al. [2015] and Komiya & Shigeyama [2016] that a stellar halo built from merging building block galaxies naturally hosts stars with high Eu abundances at low metallicities, our model predicts too few low-metallicity spheroid stars. The MDF bins with metallicities between $-3 \leq \log([Z_{\text{stars}}/Z_{\odot}]) < -1$ contain too few stars compared to the single-Gaussian fit to the observed MDF of the Galactic halo by An et al. [2013]. It is exactly in this metallicity range, where surprisingly many stars are found to be enriched in Eu [Frebel, 2010]. Contrary to the findings of others, eg. Komiya & Shigeyama [2016] or Naiman et al. [2018], we do not find many Eu-rich stars (i.e. those with $[\text{Eu/Mg}] > 0$) at low metallicity in our model of the Galactic stellar halo, although for Aquarius halo B there are stars with $[\text{Eu/Mg}] > 0.5$ at $-3 \leq \log([Z_{\text{stars}}/Z_{\odot}]) < -1.5$ in our simulations when we assume that NS binaries do not receive a kick velocity or receive a low kick velocity.

A delay time proportional to $1/t$, but with the first merger occurring after 100 Myr (instead of 10 Myr as assumed in our standard model) results in too many stars with high metallicity and low Eu abundance (Figure 4.7). A shorter, fixed delay time of 1 Myr, which Matteucci et al. [2014] concluded to be necessary for a pure NSM-enrichment scenario for the Galactic stellar halo, results in $[\text{Eu/Mg}]$ vs. $[\text{Mg/H}]$ maps with little scatter around $[\text{Eu/Mg}] \sim 0$. Furthermore, population synthesis studies favor longer delay times [Belczynski et al., 2006]. Reducing or neglecting the NS kick velocity has a similar effect as assuming a shorter delay time, with respect to increasing the Eu-abundances of the modelled halo stars. To enhance the r-process enrichment of a simulated galaxy, one could either increase the Eu yield per NSM or assume a higher NSM rate. In Figure 4.8 we resolve this degeneracy in high NSM rate and low r-process yield versus low rate and high yield. We find that assuming a lower NSM rate and higher Eu yield compared to our standard model results in a $[\text{Eu/Mg}]$ vs. $[\text{Mg/H}]$ map that is less in agreement with observations of Eu-enhanced halo stars. A higher NSM rate and lower Eu yield on the other hand, does not solve the issue that there are too few Eu-enhanced halo stars with $\log([Z_{\text{stars}}/Z_{\odot}])([\text{Mg/H}]) < -1.5$ in our standard model.

Although we do not assume instantaneous recycling of Eu, our neglect of the short ($\sim 10$ Myr) but nonzero recycling time of alpha-elements such as Mg could partly explain the discrepancy with the observed stellar halo stars. It is more likely however, that the absence of these halo stars with high Eu abundances at low metallicities indicates that our assumption of instantaneous (i.e. homogeneous) mixing is incorrect. From the chemical signatures of the metal-poor stars in dwarf galaxies there have been hints of inhomogeneous mixing processes, for instance of SNe Type Ia products [see, for example Venn et al., 2012; Starkenburg et al., 2013c; Jablonka et al., 2015b], although the energies and outflow of new material of SN Ia are quite different from the energies...
Chapter 4: Semi-analytic modelling of the europium production by neutron star mergers in the halo of the Milky Way

and outflow of NSM products. This paper provides another line of evidence that the gas in dwarf galaxies and BBs is probably inhomogeneously mixed at early times. Furthermore, there are several works that suggest that the IMF in ultra-faint dwarf galaxies can be very different from the standard one [e.g. Geha et al., 2013]. Since a very different IMF can change the neutron star density in such a stellar population, this leads to even more inhomogeneity in r-process enrichment.

An alternative conclusion one could draw, is that an additional production mechanism for r-process elements is necessary to explain the discrepancy between our models and the observations. This mechanism could enhance halo stars in r-process elements already at early times, when the metallicity of the halo stars is low and many building blocks of the stellar halo still have shallow potential wells from which NS binaries easily escape after receiving a standard kick velocity. This scenario is supported by the recent observation of a only mildly r-process enhanced star in the ultra-faint dwarf galaxy Tucana III [Hansen et al., 2017], contrary to the highly r-process enhanced stars observed in Ret II.

Finally, the star formation history of (the BBs of) the stellar halo could play a role, since we found that Aquarius halo B is more in line with observations than haloes A and C (see Figure 4.6, which gives an impression of the stochastic scatter between the Aquarius haloes). In a comparison between the Eu-enrichment in our modelled BB galaxies of the Galactic spheroid and the surviving satellite population, we also find that the surviving satellites on average more often have stars with [Eu/Mg]>0 abundances than the fully disrupted BBs and that low metallicity spheroid stars with high Eu abundances are often stripped from a satellite galaxy with a still surviving counterpart (Figure 4.9). However, this is unlikely to be the case in the Milky Way halo, since there is no observational evidence supporting it.

van de Voort et al. [2015] found that stars with low [Fe/H] and high [Eu/Fe] are at large galactocentric radius in a cosmological zoom-in simulations of a Milky Way-mass galaxy, but they are formed in-situ at high redshift and moved there through dynamical effects. Furthermore, Naiman et al. [2018] find no correlation between europium abundances and assembly history, in Milky Way-like galaxies in the IllustrisTNG simulations.

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Bibliography


Bibliography

Chapter 1

In the first chapter of this thesis, we predict the shape of the halo white dwarf luminosity function (WDLF) based on simple assumptions on the star formation history (SFH) of the Galactic halo and on binary star evolution calculations of the population synthesis code SeBa [Portegies Zwart & Verbunt, 1996; Toonen & Nelemans, 2013]. We normalise the WDLF to the observed mass density of local halo (low-mass) main sequence stars in the solar neighbourhood $\rho_0$ [Fuchs & Jahreiß, 1998], and find that the assumed initial mass function (IMF) has a large effect on this normalised WDLF. Assuming a top-heavy IMF [Suda et al., 2013] results in a WDLF that is inconsistent with the observed halo WDLF from the SuperCOSMOS Sky survey [Rowell & Hambly, 2011]. However, there are several options to change the normalisation, for example changing the assumed slope of the IMF on the low mass end, or taking an upper or lower limit of $\rho_0$. Both the Salpeter [1955] IMF and the Kroupa IMF [Kroupa et al., 1993] are (roughly) consistent with the observational data, and within approximately a factor of 2, also the assumed top-heavy IMF is.

As we rectified in chapter 3, of all simulations presented in chapter 1, the model with

- 100% binary stars in the stellar halo;
- formed in a single starburst 13 Gyr ago;
- following a Kroupa IMF;
- using the Althaus white dwarf (WD) cooling models [Althaus et al., 2007, 2013; Renedo et al., 2010]

matches the observed stellar halo WDLF the best, slightly better than the model with a binary fraction of 50% and for the rest the same assumptions.

When the mass of youngest single WDs\(^3\) in the stellar halo is determined accurately, the age of the stellar halo can be estimated [Kalirai, 2012]. WDs of a given mass correspond to progenitors with a specific mass, which had a fixed stellar lifetime. This lifetime was longer if the WD mass

\(^3\)The brightest single WDs are necessarily the youngest ones, because WDs only fade with time.
is lower. Furthermore, high mass WDs cool faster, thus the brightest single WDs with the lowest mass correspond to the oldest stars. From this lifetime, one can thus estimate the age of the stellar halo. One has an anchor point on the stellar lifetime versus WD mass function, from the observationally determined age and stellar mass of WDs in globular clusters. However, as we point out in the first chapter of this thesis, at this point only preliminary conclusions can be drawn since the observational uncertainties are large and the effect of binarity and/or metallicity can be underestimated.

Chapter 2

In the second chapter we use the Munich-Groningen semi-analytic model (SAM) for galaxy formation [Kauffmann et al., 1999; Starkenburg et al., 2013a] and the Aquarius N-body dark matter simulations [Springel et al., 2008] to study the formation of Milky Way (MW) mass stellar haloes. We show that there is a large difference, in terms of stellar mass, between the full spheroid of a simulated MW mass galaxy (including the galactic bar) and the accreted component of the spheroid, which for some Aquarius haloes contains only about 10% of all spheroid stars. As we estimate in chapter 3, our simulated stellar haloes on their turn contain only about 10% of the stars of the accreted stellar spheroids (the other ∼ 90% of the stars of an accreted spheroid correspond to the bulge).

In particular we study in this chapter the possible differences between those galaxies that were accreted by the progenitor MW galaxy (the so called building blocks (BBs) of the stellar halo of the MW) and the satellite galaxies of the MW that are still surviving at the present day. Confirming the results of others [eg. Helmi et al., 2002; Cooper et al., 2010], we find that the accreted spheroid is mainly built from a few massive BBs, ranging between 1 and 6 for the six Aquarius haloes. We find that the stellar mass - metallicity relation of the BBs is comparable to that of surviving satellite galaxies, but the time-averaged star formation rate is higher in the former. We show how this is related to the higher virial mass of the parent haloes in which these BBs reside compared to the virial masses of the haloes of the satellites.

Enrichment of the stellar halo in the α-elements\(^4\) happens almost immediately after the formation of the first stars, due to the core collapse supernovae (SN) occurring at the end of the lifetime of the short-living massive stars. Enrichment in iron (Fe) on the other hand, occurs later due to SN Type Ia. Our model predicts that in a timespan\(^5\) of ∼ 1 Gyr almost 90% of the BBs formed half their stars, whereas only 60% of the surviving satellite galaxies formed half their stars in this timespan. In general, BBs have a shorter episode of star formation compared to the surviving satellites. This thus results in a higher [$\alpha$/Fe] abundance in the stellar halo, which is in agreement with observational data of the MW [eg. Tolstoy et al., 2009].

\(^4\)Those that are produced from multiple α-particles (helium nuclei), for example magnesium or calcium.

\(^5\)SN Type Ia commutatively can change a galactic stellar population from predominantly α-rich to α-poor in a timespan of 1 Gyr [eg. Matteucci & Recchi, 2001].
Chapter 3

In the third chapter of this thesis, we combine the two models that were used in the first two chapters, to study again, in more detail, the halo white dwarf population. In particular, we investigate if the properties of the BBs the WDs originate in are reflected in the halo WD population. Also, we investigate if a cosmologically motivated assumption on the SFH of the stellar halo significantly affects the WDLF, compared to the simple assumptions we made on the SFH of the stellar halo in chapter 1.

As we learned from our study that resulted in chapter 1, if the WDLF is normalised to the unevolved low mass halo stars, the assumed IMF has a larger effect on our model’s prediction than the assumed SFH. We also find in chapter 3 that the effect of assuming the Chabrier [2003] IMF instead of the Kroupa et al. [1993] IMF in the population synthesis code SeBa on the predicted WDLF is larger than the effect of the cosmologically motivated SFH. Since the Chabrier IMF is more top-heavy, a value close to the lower limit of $\rho_0$ estimated by Fuchs & Jahreiß [1998] is needed to match the predicted WDLF equally well to the observations as the WDLF predicted when the Kroupa IMF and the most likely value of $\rho_0$ are assumed.

WDs born in BBs containing young and/or metal rich stars stand out from the bulk of the halo WDs, that are remnants of old and metal poor stars, i.e. in the mass versus luminosity diagram. However, WDs that are the result of a WD merger similarly stand out, thus observationally it will be challenging, if not impossible, to infer what BB galaxy a halo WD originated in.

Chapter 4

In the fourth chapter, we again look at a stellar population in the cosmological context of the SAM of galaxy formation. This time we simulate the evolution of binary neutron stars (NS), which are the most likely production sites of r-process elements. The effect of the natal kick velocity and the delay time until the NS merge is investigated in particular, because it is often neglected that binary NS can receive a kick upon formation with a velocity higher than the escape velocity of the galaxy they reside in, especially at early times (i.e. when halo stars were formed) when the escape velocities are still small. For the three MW mass galaxies that we model in this chapter, we find that assuming a Arzoumanian et al. [2002] kick velocity distribution causes the stellar halo populations to have $\sim 0.5$ dex smaller $\text{[Eu/Mg]}$ abundances, compared to models without natal kicks.

We also find large differences between the three stellar haloes we simulated with respect to the number of r-process enriched metal-poor stars. A larger number of these stars was found in a halo with a relatively late starburst, in a high mass BB which however not yet had an extensive SFH before the starburst. Many r-process enhanced metal-poor stars are observed in the halo of the MW [Frebel, 2010]. This could indicate that our assumption of instantaneous mixing is inaccurate, that a late starburst occurred, or that an additional production mechanism for r-process elements is necessary to explain the discrepancy between our models and the observations.

By comparing the BBs with the surviving satellites in terms of r-process enhanced stars, we find that surviving satellites more often have r-process enhanced stars, probably because of their
longer SFH. However, it is unlikely that the r-process enhanced Galactic halo stars are stripped from surviving satellites, since no streams or parent galaxies are known for these observed stars.

Future work

Due to the large number of stars that are currently being observed with the Gaia satellite [Perryman et al., 2001], we will learn a lot more about our Galaxy in the near future. For example, it was recently discovered that the nearby halo is dominated by a single (very massive) BB, that was accreted about 10 Gyr ago [Helmi et al., 2018]. This object has metal-rich stars, so we learned that there are many stars with halo-like orbits that have a higher metallicity than what is usually assumed for a halo star\(^6\). This also slightly affects our conclusions of Chapter 3, since we wrote there that metal-rich stars are not expected to contribute a significant number of bright WDs to the stellar halo in the solar neighbourhood.

Furthermore, in a recent article from Kilic et al. [2019] 142 halo WDs have already been identified and an age estimate for the inner halo is derived from this sample. It will not take long before a large sample of halo WDs will be used to determine the halo WDLF, in which the bright side (with the corresponding low spatial density of halo WDs) will for the first time be probed. Here, contributions from unresolved binary WDs and bright helium (He) core WDs are considerable, as was shown in chapter 1. Therefore, Gaia might teach us about the binary fraction or the cooling of He WDs. The billion-star surveyor Gaia shall also make it possible to make a new estimate of \(\rho_0\), which has an effect on the normalisation of the WDLF, and the WDLF itself will get smaller errorbars once all halo WDs observed with Gaia are included. With these new observations, the shape of the IMF can be constrained further.

WD cooling model assumptions largely affect the faint end of the WDLF, which will not yet be probed by Gaia. However, the Large Synoptic Survey Telescope (LSST) will teach us about this faint end, since over \(4 \cdot 10^5\) halo WDs to \(r < 24.5\) will be probed [LSST Science Collaboration et al., 2009]. These observations could thus lead to new insights to constrain WD cooling models.

A current shortcoming of the Munich-Groningen SAM for galaxy formation is that it does not reproduce the metallicity distribution function of the stellar halo of the MW. It seems that star formation should be made less efficient somehow in galaxies with intermediate masses, \((10^8 - 10^{10} M_\odot)\) the range of masses of the largest BBs of the stellar halo of the MW. This is work that needs to be done in the future in order to make accurate predictions with this SAM. Furthermore, the model could be combined with a detailed chemical evolution model, in line with the work of Romano & Starkenburg [2013]. Once the instantaneous recycling approximation is no longer in the heart of this SAM, it becomes very interesting to combine it a (binary) population synthesis model, which was also done in two ways in this thesis (chapters 3 and 4). One could then investigate, for example, the origin of the carbon-enhanced metal poor halo stars [Beers et al., 1992], similar to the recent work of Sharma et al. [2018].

\(^6\)This makes the categorization of stars in population classes, i.e. Population II for halo stars, even more old-fashioned than already mentioned in the Introduction.
Een ster is lichtbron aan de hemel, die massa omzet in energie. Afhankelijk van zijn massa, kan een ster aan het einde van zijn leven één van drie verschillende vormen aannemen. Meer dan 90% van de sterren verandert na een evolutie van zo’n tien miljard jaar (ook dit is afhankelijk van de massa), in een zogenaamde witte dwerg. Witte dwergen stralen nog steeds energie (licht en warmte) uit, maar worden in de loop van de tijd steeds zwakker, omdat er geen nieuwe massa meer in energie wordt omgezet. De andere twee, meer zeldzame, vormen die een ster kan aannemen aan het einde van zijn leven zijn die van neutronenster of zwart gat.

Sterren kunnen ook in groepjes van twee of zelfs meer dan twee sterren heel dicht bij elkaar geboren worden. In het geval van een groepje van twee spreken we van een dubbelster. Als de twee sterren in een dubbelster dicht genoeg bij elkaar staan, kan er massa-overdracht plaats vinden van de ene ster naar de andere. Ook kunnen de sterren in deze zogenaamde nauwe dubbelsterren na verloop van tijd met elkaar samensmelten tot een enkele ster. In het geval van twee witte dwergen in een dubbelster spreken we van een dubbele witte dwerg, als er twee neutronensterren samen een dubbelster vormen noemen we dat een dubbele neutronenster.

Een sterrenstelsel is een verzameling van typisch honderd miljard sterren die door hun eigen zwaartekracht bij elkaar gehouden worden. De Melkweg is het sterrenstelsel waar ook de Zon onderdeel van uit maakt. Ook sterrenstelsels kunnen na verloop van tijd met elkaar samensmelten tot een groter sterrenstelsel. Zo waren er ongeveer 13 miljard jaar geleden verschillende sterrenstelsels met een paar honderd miljoen sterren die met elkaar zijn samengesmolten, waarvan de restanten nu in de zogenaamde halo van de Melkweg te vinden zijn. Over een paar miljard jaar verwachten we dat de Melkweg zal samensmelten met de Grote Magelhaanse Wolk, en een paar miljard jaar daarna met het Andromeda sterrenstelsel (figuur 1).

Als een groep sterren geboren wordt, zal ongeveer de helft van alle sterren in die groep er meer dan 14 miljard jaar over doen om een witte dwerg te worden. Omdat het universum nog geen 14 miljard jaar oud is, is dus meer dan de helft van alle sterren in het universum nog geen witte dwerg. Het hangt van de populatie sterren die waargenomen wordt af, welk percentage van die sterren tot witte dwerg geëvolueerd is. In de halo van de Melkweg, waarin voornamelijk oude sterren leven, ligt het percentage witte dwergen rond de 40%. In de omgeving van de Zon, is het percentage witte dwergen ongeveer 5%. 

SAMENVATTING
De afgelopen decennia zijn er computerprogramma’s geschreven door sterrenkundigen in Nederland en in het buitenland, waarmee de verandering van sterren en sterrenstelsels kan worden gemodelleerd. Zo is het onder andere door onderzoekers uit Utrecht en Nijmegen mogelijk gemaakt om in detail de evolutie van miljoenen dubbelsterren uit te rekenen in enkele minuten. Ook kan tegenwoordig de evolutie van een sterrenstelsel in korte tijd worden uitgerekend, onder andere door onderzoek dat in München en Groningen gedaan is. In dit proefschrift zijn deze beide modellen gebruikt, om het beste van twee onderzoeksgebieden met elkaar te combineren en zo meer te weten te komen over de evolutie van dubbelsterren in de halo van de Melkweg. In het bijzonder gaat dit proefschrift over dubbele witte dwergen en samensmelting dubbele neutronensterren in de halo van de Melkweg, die wordt gesimuleerd in een kosmologische setting. Met dat laatste wordt bedoeld dat in deze simulaties ook de veranderingen in het hele Heelal worden meegenomen. Tevens wordt er rekening worden gehouden met de aanwezigheid van donkere materie in de halo van de Melkweg. Deze vorm van materie wordt geassocieerd met zwaartekrachtseffecten die enkel kunnen worden verklaard als er meer materie aanwezig is dan kan worden waargenomen.

In het eerste hoofdstuk van dit proefschrift worden de witte dwergen in de halo van de Melkweg in de omgeving van de Zon gesimuleerd. Er worden verschillende aannames gedaan over de evolutie van zowel enkele sterren als dubbelsterren, het tijdstip waarop de sterren geboren zijn, met welke initiële massaverdeling en de mate van afkoeling van verschillende typen witte dwergen. Vervolgens wordt de verdeling van helderheden van witte dwergen berekend, de zogenaamde luminosity function. De simulaties komen opvallend dicht in de buurt van de waarnemingen, wat aangeeft dat we kennelijk op de goede weg zitten wat onze kennis over de modelvariabelen betreft.

Bijzondere aandacht wordt besteed aan voorspellingen over de waarnemingen van ruimtesatelliet Gaia, die van een miljard sterren in onze Melkweg posities en snelheden bepaalt. Gaia zal ook voor een deel van de witte dwergen in de halo de helderheid kunnen bepalen. Het totale aantal bekende halo witte dwergen zal hierdoor toenemen. Verwacht wordt, dat ook een paar zeldzame heldere witte dwergen in de halo kunnen worden gedetecteerd. Aangezien de meest heldere witte dwergen vaker voor lijken te komen als onderdeel van dubbelstersysteem, zouden de waarnemingen van Gaia ons dus iets meer kunnen leren over het aantal sterren dat onderdeel is van een dubbelstersysteem.

In het tweede hoofdstuk van dit proefschrift zijn middels een simulatie van de vorming van de Melkweg de zogenaamde bouwblokken van de halo van de Melkweg bestudeerd. Deze sterrenstelsels zijn typisch 10 tot 13 miljard jaar geleden samengesmolten met de toen nog veel kleinere Melkweg. Hun eigenschappen worden vergeleken met die van de satellietstelsels van de Melkweg, die in massa vergelijkbaar zijn met de bouwblokken, maar tot de dag van vandaag (nog) niet samengesmolten zijn met de Melkweg. Het blijkt dat de stervorming in de satellietstelsels met een lagere snelheid gaat dan in bouwblokken met een zelfde hoeveelheid sterren. Het blijkt dat op het moment dat de meeste sterren die nu onderdeel zijn van de halo van de Melkweg gevormd werden, de bouwblokken wel meer massa in donkere materie hadden dan de satellietstelsels, wat zou kunnen verklaren waarom de snelheid waarmee sterren gevormd werden groter was in de bouwblokken.

Tevens worden in dit hoofdstuk uitspraken gedaan over de verwachte hoeveelheid ijzer ten opzichte van de zogenaamde alpha-elementen in halosterren en in satellietstelsels. Alpha-elementen
Samenvatting

Zijn elementen met een atoomkern die bestaat uit een veelvoud van alpha-deeltjes (helium kernen), zoals magnesium en calcium. Zowel ijzer als alpha-elementen worden gevormd in zware sterren, en worden gerecycled voor een volgende generatie sterren, nadat ze vrij komen via een zogenaamde supernova explosie aan het einde van het leven van een zware ster. Het meerendeel van het ijzer komt echter vrij bij een bepaald type supernova (type Ia), die gepaard gaat met de samensmelting van een dubbelstersysteem waarvan tenminste één van de sterren een witte dwerg is. Hoewel de hoeveelheid ijzer in sterren niet direct wordt gesimuleerd, kunnen er aannames over de verhouding tussen ijzer en alpha-elementen worden gedaan op basis van de snelheid waarmee sterren gevormd worden en de verschillende tijdschalen waarop verrijking in ijzer plaats vindt ten opzichte van verrijking in alpha-elementen. Een voorspelling van het gebruikte model is dat niet alle sterrenstelsels met Melkweg-achtige massa, dezelfde verhouding tussen ijzer en alpha-elementen laten zien als we in de Melkweg waarnemen.

In het derde hoofdstuk van dit proefschrift zijn opnieuw de witte dwergen in de halo van de Melkweg in de omgeving van de Zon gesimuleerd, maar deze keer worden resultaten van de simulaties van de vorming van de Melkweg gebruikt als input voor de (dubbel)stersimulaties. Er is onderzocht of de sterren uit de verschillende bouwblokken van de halo van de Melkweg witte dwerg populaties opleveren die waarneembaar van elkaar te onderscheiden zijn, gezien onder andere de verschillende momenten waarop de sterren in verschillende bouwblokken geboren zijn. Dit blijkt inderdaad het geval te zijn, als de massa en helderheid van de witte dwergen in de halo met elkaar worden vergeleken, hoewel slechts kleine verschillen in de witte dwerg populatie zijn te verwachten bij grote verschillen tussen de bouwblokken. Bovendien wordt een vergelijkbaar verschil veroorzaakt door het samensmelten van twee witte dwergen in een dubbelstersysteem.

Daarnaast wordt opnieuw de luminosity function bepaald voor de witte dwerg populatie in de halo, die in deze simulaties voortvloeit uit de kosmologisch gemotiveerde geschiedenis van stervorming in de Melkweg. Er worden nauwelijks verschillen gevonden tussen de witte dwergen in vijf gesimuleerde halo’s van de Melkweg in de omgeving van de Zon, omdat de populatie sterren waaruit zij ontstaan in al deze simulaties dezelfde eigenschappen hebben, die waargenomen zijn in de sterren van de halo van de Melkweg. Bovendien zijn in al deze simulaties de halosterren geboren volgens dezelfde initiele massaverdeling.

In het laatste hoofdstuk van dit proefschrift worden samensmeltingsdubbelstersysteem van twee neutronensterren gesimuleerd. Verwacht wordt, dat bij zo’n samensmelting zware elementen, zoals europium worden geproduceerd. Er worden in dit hoofdstuk aannames gemaakt over hoe vaak zo’n samensmelting plaats vindt en hoeveel europium op zo’n moment ontstaat. Vervolgens wordt dit meegenomen in de simulaties van de vorming van de Melkweg, om te bestuderen wat het effect is van deze aannames op de gesimuleerde populatie halosterren. In het bijzonder wordt gekeken naar de verwachte hoeveelheid europium ten opzichte van alpha-elementen, in halosterren met een bepaalde hoeveelheid alpha-elementen ten opzichte van waterstof.

Ook wordt er gekeken naar het effect van een zogenaamde kick die de het dubbelstersysteem ondergaat bij de vorming van de tweede neutronenster. Door de vrijgekomen hoeveelheid energie kan het systeem uit het sterrenstelsel geslingerd worden als de kick-snelheid groter is dan de zogenaamde ontsnappingssnelheid van dit sterrenstelsel. De europium die daarna vrijkomt bij de samensmelting van de neutronensterren, wordt dan niet meer opgenomen in sterren, omdat er
buiten een sterrenstelsel geen stervorming plaatsvindt. Het lijkt erop dat een lage kick-snelheid waarschijnlijker is dan een hoge, om de grote hoeveelheid europium ten opzichte van alpha-elementen in halosterren te kunnen verklaren.
Curriculum Vitae

Pim Petrus Cornelius van Oirschot
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2016–2019 Data scientist at Orikami, personalized healthcare
Responsible for the daily operations of MS sherpa, a medical device
and smartphone application for persons with Multiple Sclerosis.

2011–2016 PhD student at the Netherlands Research School for Astronomy (NOVA)
Subject: White dwarf binaries and neutron star mergers in the
stellar halo of the Milky Way in a cosmological context.
Promotors: Prof. dr. G.A. Nelemans (Radboud University Nijmegen)
Prof. dr. A. Helmi (University of Groningen)

2008–2011 Master of Science in Physics & Astronomy, Radboud University Nijmegen
Supervisor: Prof. dr. G.F. Lewis (University of Sydney, Australia)
cum laude (with honor).

2007–2014 Teaching assistant at Radboud University Nijmegen
Tutoring in exercise classes in Stellar evolution (2012-2014),
General Relativity (2011-2014), Mechanical waves (2011),

2005–2008 Bachelor of Science in Physics & Astronomy, Radboud University Nijmegen
Bachelor Thesis (2008) subject: Physical arguments for local gauge invariance
Supervisor: Prof. dr. W.J.P. Beenakker
LIST OF PUBLICATIONS

Refereed publications in this thesis

2019 **P. van Oirschot**, G. Nelemans, O. Pols, E. Starkenburg
Semi-analytic modelling of the europium production by neutron star mergers in the halo of the Milky Way, *MNRAS* 483, 4, p. 4397-4410


Conference proceedings related to the topic of this thesis


List of publications

Other publications


Acknowledgments

During my PhD, I have been fortunate to work under the supervision of more than one advisor. I would like to start thanking all of them. Having known me from my Bachelor’s and Master’s degree, Gijs Nelemans has put his faith in the successful completion of this doctorate thesis already since my application, for which I am very grateful. Also thanks to Onno Pols, who agreed to offer me the position as a NOVA\textsuperscript{7} PhD student after an interview with me at the University of Utrecht in the spring of 2011. In the vacancy description, five formal supervisors were mentioned. Besides Gijs and Onno, this were Amina Helmi and Eline Tolstoy from the University of Groningen and Anthony Brown from Leiden University. It soon became clear however, that I would collaborate much more with Eline’s former PhD student Else Starkenburg than with Eline. Also Anthony Brown was mainly involved at a distance, as an expert on the soon to be launched Gaia satellite. But, with so many brilliant people to advise me, this has been a fantastic project to work on since day one.

My base office was in Nijmegen. In the first few years of my PhD this was ‘the green office’, at the third floor of the Huygens building of the Radboud University. This was a very pleasant office to work. Not only because of the many plants, but especially because of the people I shared the office with: Silvia, Kars, Sjoert, Sarka, Mikkel, Serena, David, Christiaan, Emilio and Payaswini. On Wednesdays, I went to Utrecht, where I was warmly welcomed by Carlo, Joke, Alexandros, Sander, Sjors, Tim and Adriaan, amongst all the other members of the institute. It was very sad that Utrecht University decided to close the Astronomical Institute. This decision was already made when I started my PhD. After a few months, it became clear that my copromotor Onno would be offered a new position at the Radboud University, together with Bram Achterberg, Frank Verbunt and Søren Larsen and all their PhD students. The other former members of the Astronomical Institute in Utrecht started new positions in Amsterdam and Leiden. This was a relief for all of Dutch Astronomy, including myself.

In the second year of my PhD, I moved to the University of Groningen. Also here, I was warmly welcomed by the research group of Amina and all other employees of the Kapteyn Institute. Special thanks to Hans, Tjitske, Robyn, Shoko, Giacomo, Carlos, Teresa, Maarten and Hao. For a few months, I lived in the Schildersbuurt in Groningen during weekdays. In the weekends, I travelled

\textsuperscript{7}Nederlandse Onderzoekschool Voor Astronomie, i.e., the Netherlands Research School for Astronomy
back to Nijmegen where I lived with my former girlfriend Anneliek.

I am very grateful for the many travelling opportunities that were given to me during my PhD, by the financial support of NOVA and the LKBF\textsuperscript{8} and by my Gijs, who formally approved all these travels. Besides national conferences, I have visited conferences in Münich (Germany), Stockholm (Sweden), Lijiang (China) and Honolulu (Hawaii). Furthermore, in the spring of 2014, I had the great pleasure to visit my copromotor Else at the University of Victoria (Canada).

There are many more people I would like to thank. Jan Kuijpers and Paul Groot for inspiring me to choose astronomy after my undergraduate. Many thanks to Martin, Pim, and Daan, for their help with computer problems, and to the secretaries Cisca, Esther and Marja for all their help with administration. Besides national conferences, I have visited conferences in Münich (Germany), Stockholm (Sweden), Lijiang (China) and Honolulu (Hawaii). Furthermore, in the spring of 2014, I had the great pleasure to visit my copromotor Else at the University of Victoria (Canada).

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Sports and exercise have been very important for me during my PhD. I especially enjoyed playing football, with astronomy colleagues and with the Dragons and friends. Furthermore, I participated in sprint triathlons, and would like to thank everyone at Trion for the unforgettable time. I really enjoyed the runs I had with astronomy colleagues and with friends. In particular, I would like to mention Andrei here, but of course also thanks to Serena, Joost, Thomas and everyone else I ran with, amongst other events at Radboud Sports 2015.

Dear Serena, you became one of my closest friends. Thank you for the wonderful moments and for the support during difficult times. Also thanks to my other friends: Sonny, Jeroen, Dennis, Bert, Joost, Renke, Noud, Bastiaan, Jaap, Koen, Roderik, Giel, Adriaan, Noud, Suzanne, Lex, Rik, Anne and Tom. To my family: Anja, Edwin, Joni, Tim, Thieu, Jetty, Betsie, Nelly, Ria, Feike, Jelle and Manon. Dear Femke, Sef and Ben, thanks for all the love and support.