AMUSE-ing winds in binary stars

A study of the impact of stellar winds on the orbital evolution of double stars

Martha Irene Saladino Rosas
Propositions associated with the thesis

AMUSE-ing winds in binary stars
A study of the impact of stellar winds on the orbital evolution of double stars

1. The angular momentum loss of wind-interacting binaries can be robustly predicted by SPH simulations, independent of the assumed numerical resolution. The same does not hold for the amount of mass accreted. (Chapter 3)

2. The angular momentum lost by a binary system and the mass accretion of the companion star are primarily determined by the wind-terminal-velocity-to-orbital-velocity ratio and the mass ratio of the binary. (Chapters 3 and 4)

3. Wind mass transfer can lead to a shrinkage of the orbit which opens a new window in our understanding of the interaction of low- and intermediate-mass binary stars. (Chapters 3 and 4)

4. A donor star in corotation with the binary can lead to a similar morphology to wind Roche lobe overflow. (Chapter 4)

5. Wind mass transfer can in principle increase the eccentricity of a binary system. (Chapter 5)

6. Keeping a clean computer screen could prevent spurious data points in figures that are not easy to explain.

7. Similar to wedding planners, PhD planners SHOULD exist.

8. Awareness about depression and anxiety in PhD students should be raised and taken seriously by all academics.

9. A bit of humanity from huisartsen would not hurt any patient.

10. Sometimes a wedding dress opens more doors than having a PhD.
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A mis abuelos
¿Cuix oc nelli’n tlaca?
Ye yuh ca ayoc nelli in tocuic.
¿Tlen o zo ihca?
¿Tle hual quiza?

¿Acaso son verdad los hombres?
Porque si no, ya no es verdadero nuestro canto.
¿Qué está por ventura en pie?
¿Qué es lo que viene a salir bien?

Does man possess any truth?
If not, our song is no longer true.
Is anything stable and lasting?
What reaches its aim?

-Cantar mexicano (Mexican song)
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Stellar winds are outflows of material that are ejected from the surface of stars. Nearly all stars lose mass through stellar winds at some point of their lives (see e.g. Owocki, 2013). The characteristics of these winds, such as their mass-loss rate, velocity, composition and driving mechanism are determined by the mass and evolutionary stage of the star.

Stars are divided into low-, intermediate- and high-mass objects. Regardless of their mass, stars spend most of their lives fighting against gravity which tries to cause the stars to collapse. The source of the outward force which balances gravity is pressure which can originate from collisions between gas particles, radiation or degeneracy\(^1\). As long as the pressure gradient inside the star balances the inward gravitational force, the star is said to be in hydrostatic equilibrium. Heat and radiation cause an energy loss from the stellar surface which needs to be compensated by an energy source. During most of their lives, this source is provided by nuclear reactions in the star’s interior. This state is called thermal equilibrium, since energy is radiated away at the star’s surface, but is produced at the same rate by nuclear reactions in the core of the star. The longest phase of nuclear fusion in the star’s interior is the fusion of hydrogen into helium, which is called the main sequence phase. Low- and intermediate-mass stars have masses less than 8 M\(_\odot\)\(^2\). These stars use their hydrogen fuel very slowly, resulting in long lifetimes (of the order of \(\approx 50\) million years if the mass of the stars is 8 M\(_\odot\) and up to billions of years if the mass of the star < 2 M\(_\odot\)). The end product of the evolution of these stars is a white dwarf. Our Sun belongs to this group. At the moment the Sun is in the main sequence phase and it will continue in this state for another \(\approx 4.5\) billion years. As you read these lines, the Sun is also losing mass in the form of a stellar wind from its upper atmosphere, which is called the corona. Due to the high temperatures in the corona (\(\approx 10^6\))

\(^1\)White dwarf stars are compact objects which are supported by electron degeneracy pressure. The Pauli exclusion principle states that no two electrons can occupy identical quantum states in the same volume. Therefore, electrons are forced to occupy states of high kinetic energy when the spatial volume is decreased, which creates a degeneracy pressure that prevents the star from collapsing.

\(^2\)1 M\(_\odot\) is equal to \(1.98 \times 10^{30}\) kg and corresponds to the mass of the Sun.
K) particles in this region are very energetic, which is the reason they can escape the gravity of the Sun in the form of a wind. The solar wind, which is composed mainly of electrons, protons and alpha particles, is travelling at velocities of around 250 to 750 km s$^{-1}$ and can be observed from Earth in the form of an aurora when the particles collide with the Earth’s upper atmosphere. The rate at which the Sun loses mass is very small compared to other stars. It only releases $10^{-14} \text{ M}_\odot$ ($\approx 2 \times 10^{16} \text{ kg}$) in one year.

Low- and intermediate-mass stars follow a similar evolutionary path. Once hydrogen is exhausted in the centre of the star, hydrogen is ignited in a shell surrounding the core which adds mass to the core. The energy generated by this hydrogen burning shell is deposited in the envelope of the star, which as result expands, and the luminosity of the star increases. At this point we are left with a large, cool, red luminous star. This stage of the evolution is called the red giant phase. During this phase, our Sun will reach a radius of almost its distance to Earth and it will become thousands of times brighter. As the luminosity and radius of the star increase, the envelope becomes less strongly bound, making it easier for photons to remove mass from the star’s surface. The mechanism driving mass loss during this phase is not well understood, but it is possible that this wind is driven by the pressure from magnetohydrodynamic waves (see e.g. Schröder & Cuntz, 2005). A relation for the mass-loss rates during this phase has been derived empirically from observations (Reimers, 1975; Miller Bertolami, 2016). A star like the Sun will lose mass at a rate of $\approx 10^{-8} \text{ M}_\odot$ per year. Compared to the solar wind described above, these winds are much slower (10–50 km s$^{-1}$; Lamers & Cassinelli, 1999) and they are composed mainly of molecules.

During the red giant phase, the core contracts and heats up allowing a new nuclear reaction to start. Now, helium is being fused in the core of the star via the triple-alpha process, producing carbon and oxygen. In low-mass stars ($0.95 \text{ M}_\odot \lesssim M \lesssim 2 \text{ M}_\odot$), the core is degenerate and the helium burning process occurs in an unstable form resulting in a runaway process called the helium flash. During the helium flash, the core expands and the degeneracy is lifted. Subsequently the further fusion of He occurs in a stable way, as in the intermediate-mass stars.

Once helium is exhausted in the core, the star will move towards what is called the asymptotic giant branch (AGB). During this phase, the star has a degenerate carbon-oxygen core and is fusing helium in the shell around the core. At the same time the star is undergoing stellar pulsations which contribute to the mass loss of the star by lifting material from the stellar surface. The mass-loss rates of AGB stars are much higher than during the red giant phase, ranging between $10^{-7} - 10^{-5} \text{ M}_\odot \text{ yr}^{-1}$ and the velocities of the winds range between 5-30 km s$^{-1}$ (Höfner, 2015). Once most of the envelope of the star is lost, the hot core develops a fast wind which is driven by radiation pressure in UV absorption lines, at the same time the UV flux ionises the already ejected envelope. This is the planetary nebula phase, which is the last stage before the core of the star cools down to what is called a white dwarf.

High-mass stars, on the other hand, are hot and luminous stars with masses greater than 8 M$_\odot$. Because of their large luminosities, these stars consume their fuel more quickly resulting in shorter lifetimes (of the order of millions of years). The evolutionary paths of these stars
are very different from those of low- and intermediate-mass stars. Depending on their initial mass, the end products of such massive stars are either neutron stars or stellar-mass black holes. Massive stars are characterised by having fast winds (hundreds to thousands of km s$^{-1}$). Typical mass-loss rates for these stars range between $10^{-7} - 10^{-5}$ M$_\odot$ yr$^{-1}$ (Puls et al., 2009). We should note that for these stars mass loss due to stellar winds can already be important during the main sequence phase if the stars have masses $\gtrsim 20$ M$_\odot$. The winds of massive stars are called line-driven winds since they are driven by absorption in spectral lines (see e.g. Lamers & Cassinelli, 1999, for a detailed description).

If the star evolves in isolation, the material driven away by stellar winds will just be lost in the interstellar medium. However, unlike our Sun, most stars in the universe with masses $\gtrsim 1$ M$_\odot$ are in binary systems (Moe & Di Stefano, 2016), i.e. two stars orbiting each other under their mutual gravitational influence. This means that if one of the stars is losing a large amount of mass in the form of a strong stellar wind (we call this star the donor star), the material the star loses will likely interact with its companion star. It can also occur that both stars are losing mass via winds at the same time. In this case the stellar winds of the stars will interact and produce what is known as colliding winds (e.g. Stevens et al., 1992). In this thesis, however, we will focus on the specific case where only one star is losing mass in the form of a stellar wind because it has reached the AGB phase. Our aim is to study the impact of the wind interaction with the companion star on the orbit of binary. In the following sections we will describe in more detail the physical processes driving the winds away from the stellar surface, why it is important to study their effect on the orbit, how these effects are studied, and the possible consequences for binary evolution.

### 1.1. The AGB phase

The AGB phase is a very short phase in the evolution of low- and intermediate-mass stars. A star like our Sun will spend only around 20 million years in this phase (Sackmann et al., 1993). This phase begins once the He in the core of the star has been exhausted. During this stage, different chemical elements are produced in the interior of the star, some of these elements are essential material for life on Earth. Examples of the elements formed in the interior of AGB stars are carbon, nitrogen and elements heavier than iron via the s-process$^3$, such as Zr, Sr, Ba, Tc, and Pb.

The interior of an AGB star is formed by an electron-degenerate core, mainly composed of carbon and oxygen, which is surrounded by thin shells where helium (inner shell) and hydrogen (outer shell) are fusing. These shells are separated by a region mainly composed by helium called the intershell region. Above the hydrogen shell an envelope rich in hydrogen.

---

$^3$ s-process elements are elements formed by slow neutron capture reactions on iron nuclei. These reactions are called slow because the time between consecutive neutron captures is slow compared to the $\beta$-decay timescale of unstable neutron-rich isotopes. The free neutrons necessary for this process are produced in the He-rich intershell region.
Figure 1.1: Schematic view of an AGB star (not to scale). The star is formed by a CO core surrounded by thin shells where He and H are burning. An intershell region formed mainly by He separates them. Above them a highly convective envelope lies. Pulsations in the outer envelope trigger shock waves, which lift material to larger radii. In these regions gas is condensed into dust grains because of the low temperatures. Then radiation pressure on dust grains accelerates these grains. Momentum is transferred from the dust to the surrounding gas via collisions dragging the gas away from the AGB star in a stellar wind.
makes up the outer layers of the star (see Figure 1.1). This envelope is convective, i.e. the energy in the envelope is transported by convection. There are two stages in the evolution of an AGB star, the early AGB (E-AGB) and the thermally pulsing AGB (TP-AGB).

In the E-AGB phase the main source of energy comes from the He-burning shell surrounding the core. The products of this nuclear reaction become part of the CO core, which becomes more degenerate. As a result, the envelope of the star expands and cools down which switches off the H burning. As the star expands and its temperature decreases, its opacity increases and in relatively massive stars (\(\gtrsim 4 \, \text{M}_\odot\)) the convective envelope can penetrate into the He-rich layers, mixing He and other products of the H fusion, bringing them to the surface of the star. This is called the second dredge-up\(^4\). For lower-mass stars there is no second dredge-up because the H-burning shell remains active, preventing the convective zone to reach deeper into the star.

As the He-burning shell approaches the H-burning shell, the stellar luminosity decreases since it runs out of fuel. The envelope contracts in response and it reignites the H shell. Now there are two burning shells providing energy to the star. However, at this stage the He-burning shell is very thin which makes it thermally unstable. This marks the beginning of the TP-AGB phase. For most of the time, the He-burning shell is inactive and the H-burning shell adds mass to the intershell region. When the mass of the intershell passes a critical threshold, He is ignited in an unstable way, creating a thermonuclear runaway called a thermal pulse or helium shell flash. At this stage luminosities of \(\approx 10^8 \, \text{L}_\odot\) are reached on a timescale of one year. The large energy release drives convection in the intershell region, mixing carbon and other elements produced by He burning into the intershell region. In addition, this energy release heats up the layers above the He-burning shell and causes them to expand. As the flash dies out, the temperature decreases and the H-burning shells switches off.

The expansion and drop in temperature of the intershell can lead to a penetration of the convective envelope beyond the H-burning shell. As a consequence, material from the intershell is mixed with the envelope, bringing He and He-burning products, as well as s-process elements to the surface of the star. This is called the third dredge-up. After the third dredge-up, the H-burning shell is reignited and the helium-shell extinguishes again temporarily. The TP cycle repeats until almost all of the envelope is lost.

The time a star spends in the TP-AGB phase is determined by the rate at which it loses mass. We know that AGB stars are losing mass because observations of these objects show molecular emission lines with a geometrical extent that is much larger than the size of the star. In addition, cool stars emit an excess of radiation at long wavelengths due to the radiation from the dust in their winds (see below) which is observed at infrared and millimetric wavelengths (see e.g. Lamers & Cassinelli, 1999) However, the mechanism driving mass loss in these stars is not well understood. A combination of dynamical pulsations of the star and radiation pressure on dust particles is the most likely scenario to drive material out of the star (Höfner,\(^4\) The first dredge-up occurs when the star ascends the red giant branch phase.)
Before discussing how this mechanism removes mass from the star’s envelope we will briefly explain what we mean by dynamical pulsations of the star, which should not be confused with the thermal pulses. The thermal pulses going on in the interior of the star due to nuclear process occur on timescales of $\approx 10^5$ yr. However, observations of AGB stars show short period variations in the luminosity of the star between $\approx 10$-1000 day (Wood, 2010), which are caused by fluctuations in the radius of the star as it tries to keep itself in equilibrium. Stars of which the brightness varies are also called variable stars, and if the star is in the red giant phase or the AGB phase they are also referred as Miras or long-period variables. The variations in radius and luminosity of the star are called radial stellar pulsations, but their origin is not well understood. The $\kappa$-mechanism could explain the stellar pulsations of long-period variables. It works in the following way: if we have a region that is partly ionised, then if by some physical process the layers of the star are compressed, gas will heat up and it will become more opaque to radiation, i.e. its opacity, $\kappa$, will increase. Under this condition radiation will be trapped in the layer increasing the pressure which will push the layer outwards. As a result, the layer expands and cools; in consequence the opacity decreases. Therefore, energy is able to escape and the pressure below the layer drops. As the layer falls back the cycle repeats.

The stellar pulsations in the envelope of an AGB star combined with convection, induce shock waves in the atmosphere, which lift material to large radii and increase the gas density in the outer atmosphere. If the kinetic energy imparted to the gas is not enough to reach the escape velocity, the gas will reach a maximum distance and then fall back towards the star. However, if the initial kinetic energy is large enough, gas will move towards larger distances in the stellar atmosphere. At about 2-3 stellar radii, the temperature decreases to $\approx 1500$ K, which allows the gas that reaches these distances to condense into dust. Since dust particles are very opaque and the AGB star is very luminous, radiation pressure will accelerate dust particles, which will collide with gas nearby transferring momentum. Therefore, dust particles will drag gas away from the star in a stellar wind. The final velocity the wind reaches is called the terminal velocity. The velocities and mass loss rates of AGB winds are observationally well constrained (see e.g. Vassiliadis & Wood, 1993). Typical velocities range from 5-30 km s$^{-1}$, which compared to winds from other stars are very low, and mass-loss rates vary between $10^{-7} - 10^{-4}$ M$_\odot$ yr$^{-1}$. An empirical relation is derived by Vassiliadis & Wood (1993) which correlates the the mass-loss rates to the stellar pulsation period. As a star evolves towards larger radii on the AGB, the pulsation period increases and the mass-loss rate increases. The maximum mass-loss rate that a star undergoes, of the order of $10^{-5} - 10^{-4}$ M$_\odot$ yr$^{-1}$, is referred to as the superwind phase. During this stage, the hydrogen-rich envelope is rapidly removed which marks the end of the AGB phase.

---

5 In a partly ionised zone, part of the energy released during the compression of a layer is used for ionisation, rather than for raising the gas temperature. For Mira variables hydrogen ionisation is the most likely cause for the stellar pulsations.
1.2 AGB stars in binary systems

A large fraction of stars in the universe are found in binary systems. The two components of such binary stars are likely to interact at some point during their evolution, especially if their orbital separation is relatively small. This interaction can occur in several ways, including tidal interaction, gravitational wave emission, mass transfer, and binary mergers. In this thesis we will focus on the mass transfer process since, as we will see below, when one of the stars undergoes high mass-loss rates, the exchange and loss of material from the binary can potentially influence the orbital dynamics of the system.

1.2.1. Mass transfer in binary stars

There are two main channels through which mass transfer can take place: Roche lobe overflow (RLOF) and wind mass transfer.

Interaction via RLOF occurs for close binary systems, if one the stars starts filling its Roche lobe. Then, material is able to flow towards the companion star through the inner Lagrangian point. For donor stars with a convective envelope, such as AGB stars, mass transfer will likely be unstable. Usually this occurs because in stars with convective envelopes, the envelope expands as a result of the loss of mass, and the Roche-lobe radius shrinks as mass is transferred from a more massive star to a less massive star, which causes the donor star to overfill its Roche lobe by a large amount. If the companion star is not able to accrete all the material coming from the most evolved star, material will start to pile up onto the companion star overfilling its Roche lobe. This leads to a phase which is known as the common-envelope, where both stars are engulfed in the envelope of the donor star (Iben & Livio, 1993). During the common-envelope phase, friction between the stars and the envelope causes the two stars to spiral inwards, decreasing the orbit. Furthermore, due to the large sizes of the stars during the AGB phase and the relatively close orbital separations at which RLOF takes place, tidal interaction is likely to circularise the orbit if it was originally eccentric (see e.g. Ivanova et al., 2013, for a review on common envelope).

On the other hand, wind mass transfer can occur if one of the stars loses material through strong stellar winds. In particular, if the orbital velocity of the binary is comparable to the wind terminal velocity strong interaction between the stellar wind and the companion star can occur (e.g. Jahanara et al., 2005; Chen et al., 2017; Saladino et al., 2018). In this case, the companion star will gravitationally accrete part of the wind coming from the donor star, potentially leading to chemical pollution if the donor star has a chemically enriched atmosphere. By accreting material, the companion star is also expected to gain angular momentum, chang-

\(^6\) The Roche potential is the effective potential in the corotating frame of a binary system which includes the gravitational potential of the stars and the centrifugal potential. Surfaces of equal potential centred in each star that are connected via the inner Lagrangian point (which marks the point where forces cancel out) are the Roche lobes of the stars. Their sizes are determined by the orbital separation of the stars and its mass ratio.
ing its rotational velocity and spinning up (Boffin, 2014). A similar effect occurs when stars interact via RLOF. In order to quantify the orbital evolution of the system, binary evolution models usually assume that the wind escapes isotropically from the donor star, which leads to a widening of the orbit (Section 1.2.3).

By assuming that mass transfer occurs either via unstable RLOF or isotropic stellar winds, binary population synthesis codes\(^7\) predict a gap in the orbital period distribution of the offspring of binaries that interact during the AGB phase between a few hundred and a few thousand days (Pols et al., 2003; Izzard et al., 2010; Abate et al., 2018). Additionally they also predict that binary stars with initial orbital periods lower than \(\approx 3000\) day, tidal interaction is very strong resulting in a circularisation of the orbit if the system was initially eccentric. For these reasons, observations of these objects can be used to test theoretical models for mass transfer in low- and intermediate-mass binary stars. However, as we will see in the next section, observations are at odds with these expectations.

### 1.2.2. Observations of the offspring of AGB binary stars

There is a variety of chemically peculiar stars which are thought to be the product of the interaction of low- and intermediate-mass binary stars while the most evolved star was in the AGB phase. Among them we find Barium (Ba) stars and CH stars (Keenan, 1942; Bidelman & Keenan, 1951), carbon enhanced metal poor (CEMP-s) stars (Beers & Christlieb, 2005), extrinsic S stars (Smith & Lambert, 1988) and yellow symbiotic stars (Smith et al., 1996). The characteristics of these objects are shown in Table 1.1. One property all these systems have in common is an enhanced abundance of carbon and s-processes elements, which as discussed in Sect. 1.1, are produced by nucleosynthesis during the TP-AGB phase. However, many of these systems are in an evolutionary stage where such elements are not expected to be present. A possible scenario to explain their abundances is a past episode of mass transfer when the most evolved star (currently a white dwarf) was in the AGB phase, which led to the pollution of the companion star (Boffin & Jorissen, 1988; Smith & Lambert, 1988; Starkenburg et al., 2014). This scenario is supported by the fact that most of these stars have a binary companion (McClure et al., 1980a; McClure, 1984; Jorissen et al., 1998; Lucatello et al., 2005). However, when comparing the orbital periods, eccentricities and abundances of the observed offspring of AGB binary stars with those predicted by binary evolution models some disagreements are found.

Note that in Table 1.1 we also mention binary post-AGB stars which are systems in which one of the components is in the post-AGB phase. These systems are classified as binary interaction products not based on their chemical properties, but because of their evolutionary state, orbital properties, and circumbinary environments. Interestingly, the eccentricities that

---

\(^7\)Binary population synthesis codes are algorithms that permit the rapid calculation of the evolution of a binary system by using appropriate formulations for different physical processes, such as mass transfer, angular-momentum loss, mass-accretion efficiency, tidal evolution, etc.
many of these objects show are quite large (see Figure 1.2). This is at odds with binary theory which predicts that such systems should be circularised because their current Roche lobes are small to fit an AGB star (Oomen et al., 2018).

Table 1.1: Properties of the progeny of AGB binary stars.

<table>
<thead>
<tr>
<th>Star</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barium stars</td>
<td>These are spectral type(^1) G to K giant stars of too low luminosities to be on the AGB phase, which show overabundances of (s)-process elements, in particular singly ionised barium (BaII). They belong to the stellar population I(^2).</td>
</tr>
<tr>
<td>CH stars</td>
<td>These stars are the analogues of Ba stars, but they belong to the stellar population II(^3). These stars show strong molecular bands of CH. Heavy elements such as Sr and Ba are also enhanced.</td>
</tr>
<tr>
<td>extrinsic S stars</td>
<td>S stars are M-type stars which are divided into \textit{intrinsic} and \textit{extrinsic} stars, based on the presence or absence of Tc lines. Intrinsic S stars are actual TP-AGB stars with Tc lines, whereas extrinsic S stars are Tc-poor stars whose chemical peculiarities are explained by a mass-transfer episode. Since the Tc half life ((\approx 2 \times 10^5) yr) is short, by the time the star becomes a giant this isotope must have already decayed. Extrinsic stars also show carbon and other (s)-process elements and unlike intrinsic stars, extrinsic stars are either on the RGB or E-AGB phase.</td>
</tr>
<tr>
<td>Yellow symbiotic stars</td>
<td>Symbiotic stars are mass-losing giants with white dwarf companions which ionises the stellar wind from the cool donor star (Merrill, 1942, 1944, 1948, 1950). Yellow symbiotic stars are G or K-type giant stars which are enriched in (s)-process elements. Since these stars are not luminous enough to have (s)-process elements, the observed enrichments are attributed to a mass transfer episode. These stars can be seen as the the population II analogues of extrinsic S stars (Jorissen, 2003).</td>
</tr>
</tbody>
</table>
CHAPTER 1: INTRODUCTION

CEMP-s

As their name suggests, carbon enhanced metal poor stars are very metal-poor stars which are located in the Galactic halo. They belong to population II, and they are usually classified according to their abundances in barium and europium, which are elements associated to the s- and r-process\(^1\) (Abate et al., 2015a). In this work we focus on the CEMP-s stars which show overabundances of s-process elements and account for about 80% of the observed CEMP stars (Aoki et al., 2007). Their enhancement in carbon, barium and other s-process elements is explained by an episode of mass transfer during the AGB phase of the most evolved star.

Binary post-AGB stars

Post-AGB stars are considered to be the transition stage between the AGB phase and the planetary nebulae. A large fraction of these objects are found in binary systems. However, unlike the other objects mentioned in this Table, most of these objects are poor in s-process elements. This could be a consequence of s-process elements being refractory, so that their abundances are affected by depletion.

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\(^1\)Stars are classified according to their spectral characteristics in O, B, A, F, G, K and M, with the O-type stars being the hottest and the M-type the coolest. Our Sun has a spectral type G.

\(^2\)Stars are divided in populations. Population I stars are "young" stars which are metal-rich. These stars are located in the disks of spiral galaxies and they were formed from recycled gas from previous generations of stars.

\(^3\)Population II stars are "old", stars which are less luminous and cooler than Population I stars. Population II stars are metal poor.

\(^4\)r-process elements are elements heavier than iron which are formed from rapid neutron-capture process. The suggested scenarios for the formation of these elements are core-collapse supernovae or merger of neutron stars.

---

Figure 1.2 shows the \((e, \log P)\) diagram, where \(P\) is the orbital period, for different families of descendants of the interaction of AGB binary stars. This diagram serves different purposes (Jorissen, 2003): 1) it permits to know the range of orbital periods where a family of the offspring of AGB binaries typically lies, 2) by comparing the \((e, \log P)\) diagrams for different families of objects, we can estimate the likelihood of a similar evolutionary path for them, and 3) the distribution of orbital periods and eccentricities provides information about the mass-transfer mechanism they went through. Regarding this last point, a major issue that binary evolution models face is that the observed orbital periods of such objects lie in the range where a gap in the orbital period distribution is predicted (between a few hundred and a few thousand years). Furthermore, for relatively short orbital periods (less than a few thousand days), many objects show large eccentricities, up to values of about 0.9. This means that a mechanism which counteracts tidal circularisation or that pumps the eccentricity after tidal interaction must occur. Another problem that theory has encountered is explaining the observed abundances of some objects, such as CEMP-s stars. Theoretical models usually
assume that accretion onto the companion star occurs via the Bondi-Hoyle-Lyttleton (Hoyle & Lyttleton, 1939; Bondi & Hoyle, 1944; Bondi, 1952) formalism, which predicts abundances of the companion star of about an order of magnitude lower than what observations show (Abate et al., 2015a,b). The question that arises, and has yet to be explained by theoretical models, is how these systems evolved to their observed orbital periods, eccentricities and abundances, which are at odds with the current theoretical understanding of binary stellar evolution.

1.2.3. The orbital evolution of a binary system

In Sect. 1.2.1 we qualitatively described the orbital evolution of a binary when an episode of mass transfer occurs. In this section we derive the equation for the change in the orbital separation and eccentricity of the binary as a function of time\(^8\), and we analyse the consequence for the orbit when mass transfer occurs via winds. We start by assuming that both stars are non-rotating, in which case the total angular momentum, \(J\), of the system is given by the orbital angular momentum:

\[
J^2 = G \frac{M_d^2 M_a^2}{M_d + M_a} a(1 - e^2),
\]

where \(G\) is the gravitational constant, \(M_d\) is the mass of the donor star\(^9\) (in our case the AGB star), \(M_a\) is the mass of the companion star (also referred to as the accretor star), \(a\) is the semi-major axis of the orbit and \(e\) is the eccentricity of the system.

In order to compute the change in the orbit as the donor star loses mass, we take the derivative of Eq. 1.1 with respect to time, which yields:

\[
2 \frac{\dot{J}}{J} = \frac{\dot{a}}{a} + 2 \frac{\dot{M}_d}{M_d} + 2 \frac{\dot{M}_a}{M_a} - \frac{\dot{M}_d + \dot{M}_a}{M_d + M_a} - 2 \frac{\dot{e}}{1 - e^2}.
\]

(1.2)

When mass transfer occurs via winds, a fraction of the mass and angular momentum are lost from the system. Here we assume that only a fraction \(\beta\) of the mass the AGB donor star loses (\(\dot{M}_d < 0\)) is gravitationally accreted by the companion star,

\[
\dot{M}_a = -\beta \dot{M}_d.
\]

(1.3)

We also need to account for the angular momentum that is being lost from the system, which we parametrise as,

\[
\left( \frac{\dot{J}}{M_d + M_a} \right)_{\text{loss}} = \eta \frac{j}{\mu},
\]

(1.4)

\(^8\) This derivation is based on Onno Pols’ lecture notes on binary evolution (http://www.astro.ru.nl/~onnop/education/binaries_utrecht_notes/Binaries_ch6-8.pdf)

\(^9\) We also refer to this star as the primary star because throughout this thesis the donor star is the most massive star, and in consequence evolves faster than its companion.
FIGURE 1.2: Eccentricity vs. orbital period (in logarithmic scale) for different offspring of AGB binary evolution. Most of these systems have orbital periods in the range where theoretical models predict a gap, and large eccentricities for relatively short orbital periods, contrary to theoretical predictions. The figure was made from the data of Van der Swaelmen et al. (2017, for extrinsic S stars), Oomen et al. (2018, for post-AGB stars), Van der Swaelmen et al. (2017, for Ba stars), Jorissen et al. (2016, and references therein for CH stars), Hansen et al. (2016a, for CEMP-s) and Vanture et al. (2003, for yellow symbiotic stars). For the specifics of the observational data we refer the reader to the literature mentioned before.
where \( \eta \) is the specific angular momentum of the matter that is lost from the system in units of the orbital angular momentum of the binary per reduced mass, \( \mu = \frac{M_d M_a}{(M_d + M_a)} \). By defining the mass ratio of the binary as \( q = M_d / M_a \), from equation 1.2 we obtain the change in the semi-major axis and the eccentricity in terms of the parameters \( \beta \) and \( \eta \) as:

\[
\frac{\dot{a}}{a} - 2\frac{e \dot{e}}{1 - e^2} = -2\frac{\dot{M}_d}{M_d} \left[ 1 - \beta q - \eta (1 - \beta) (1 + q) - (1 - \beta) \frac{q}{2(q + 1)} \right].
\]

(1.5)

Notice that the difficulty in solving this equation arises because 1) we need an extra equation to separate \( \dot{a} \) and \( \dot{e} \), and 2) \( \beta \) and \( \eta \) are unspecified parameters. If we assume that the eccentricity of the system is zero (which is expected to be a good assumption in the case of RLOF), the second term on the left-hand side of Eq. 1.5 vanishes and the change in the semi-major axis can be solved if \( \beta \) and \( \eta \) are known. However, only for some idealised physical cases \( \beta \) and \( \eta \) can be approximated analytically. For a realistic approximation, hydrodynamical simulations of mass transfer in binary systems are needed. In the next sections we describe the ideal cases for which \( \beta \) and \( \eta \) can be derived when mass transfer occurs via stellar winds, under the assumption of \( e = 0 \).

### 1.2.3.1. **Angular-momentum loss: isotropic winds**

For simplicity we assume that the stars are in a circular orbit, and we also assume (as is usually done in binary evolution models) that the donor star loses mass in the form of a fast, spherically symmetric wind (this mode is also called isotropic-wind mode). Then the wind only removes the specific angular momentum of the donor star in its relative orbit around the centre of mass, with \( a_d = a M_a / (M_d + M_a) \) the distance of the donor star to the centre of mass, and

\[
\left( \frac{j}{M} \right)_{\text{loss,iso}} = a_d^2 \Omega_{\text{orb}} = \left( \frac{M_a}{M_d + M_a} \right)^2 \sqrt{G(M_d + M_a) a} = \left( \frac{M_a}{M_d + M_a} \right)^2 \frac{J}{\mu},
\]

(1.6)

with \( \Omega_{\text{orb}} \) the angular velocity of the binary. We can write this in terms of the unitless parameter \( \eta \), hence for the isotropic mode,

\[
\eta_{\text{iso}} = \frac{1}{(1 + q)^2}.
\]

(1.7)

Replacing \( \eta = \eta_{\text{iso}} \) in Eq. 1.5, we notice that the sign of \( \dot{a} \) will always be positive as long as \( q > 1 \) and \( \beta < q / (2q^2 + q - 2) \) or while \( q < 1 \) and \( 0 \leq \beta \leq 1 \).

### 1.2.3.2. **Mass-accretion efficiency: Bondi-Hoyle-Lyttleton formalism**

The parameter \( \beta \) can also be approximated analytically. In this case we assume that the accretion process onto the companion star can be described by the Bondi-Hoyle-Lyttleton (BHL) formalism (Hoyle & Lyttleton, 1939; Bondi & Hoyle, 1944; Bondi, 1952). The mass-accretion rate in this formalism arises from the interpolation of the estimated mass-accretion...
rate from two simplified accretion mechanisms. In the mechanism derived by Hoyle & Lyttleton (1939), a star is assumed to move at a constant velocity in a pressureless gas medium with initially uniform density and temperature. The gas is gravitationally deflected behind the star. Material arriving from different sides collides, cancelling its transverse velocity and creating an accretion line behind the star. If material in this line has lower velocities than the escape velocity of the star it will be accreted. In the mechanism derived by Bondi (1952), the stellar motion is neglected, but pressure of the gas is included. Based on these models, Bondi (1952) proposed an interpolation formula for the mass-accretion rate, where the velocity of the star and gas pressure are included. In the framework of a binary system, the interpolation of the mass accretion rates of the aforementioned simplified models yields (Boyle, 1984):

$$\beta_{\text{BHL}} = \frac{\alpha_{\text{BHL}}}{a^2} \left( \frac{GM_a}{v_w^2} \right)^2 \frac{1}{[1 + (v_{\text{orb}}/v_w)^2 + (c/v_w)^2]^{3/2}},$$  

(1.8)

where $\alpha_{\text{BHL}}$ is a constant of the order of unity which physically represents the location of the stagnation point in units of the accretion radius (Boffin, 2014), $v_w$ the wind velocity, $v_{\text{orb}}$ the orbital velocity of the binary and $c$ the sound speed, which is ignored in the rest of this thesis since we assume that $v_w \gg c$.

### 1.2.3.3. Winds in eccentric binaries

Up to now we have assumed that the binaries are on circular orbits, which allows to solve Eq. 1.5 as long as $\beta$ and $\eta$ are known. However, if the eccentricity of the system is non-zero we are left with one equation and two unknowns, $\dot{a}$ and $\dot{e}$. Therefore an extra equation is needed. The change in the semi-major axis can be estimated from the change in the orbital energy of the binary system. The specific orbital energy for a Keplerian orbit is given by:

$$\epsilon = -\frac{G(M_a + M_d)}{2a}.$$  

(1.9)

Hence, if the change in $\dot{a}$ is known, we can substitute it in Eq. 1.5, along with $\eta$ and $\beta$ and we can derive $\dot{e}$. Analytically, this problem has been addressed under different assumptions on the mass transfer mechanism (e.g. Huang, 1956; Sepinsky et al., 2009; Eggleton, 2006; Dosopoulou & Kalogera, 2016). For instance, under the assumption of isotropic winds, Dosopoulou & Kalogera (2016) derive the following equations for the change in $\dot{a}$ and $\dot{e}$:

$$\frac{\dot{a}}{a} = \frac{|M_d|}{M_a + M_d} - \dot{M}_a \frac{1 + e^2}{1 - e^2} \left( \frac{2}{M_a} - \frac{2}{M_d} + \frac{1}{M_a + M_d} \right),$$  

(1.10)

$$\frac{\dot{e}}{\dot{a}} = -\dot{M}_a \left( \frac{2}{M_a} - \frac{2}{M_d} + \frac{1}{M_a + M_d} \right),$$  

(1.11)

where for binary stars in eccentric orbits the average mass-accretion efficiency in the BHL approximation is given as (Boffin & Jorissen, 1988):

$$\beta_{\text{BHL}} = \frac{\alpha_{\text{BHL}}}{a^2 \sqrt{1 - e^2}} \left( \frac{GM_a}{v_w^2} \right)^2 \frac{1}{[1 + (v_{\text{orb}}/v_w)^2]^{3/2}}.$$  

(1.12)
After doing some algebra, we notice that Eqs. 1.10 and 1.12 reduce to Eqs. 1.5 and 1.8 respectively in the case $e = 0$. Also notice that the sign of Eq. 1.11 is determined by the mass ratio of the stars. Therefore we see that under the assumption of fast winds the eccentricity of the system will increase for $0 < q < 0.78$ and $\beta > 0$, and it will remain constant if $\dot{M}_a = 0$.

As we have seen in the previous sections, although the isotropic wind approximation can be used to treat the evolution of binaries interacting via AGB winds, binary population synthesis models which use these prescriptions fail to reproduce the observed orbital characteristics and abundances of such objects. This is perhaps not surprising because the isotropic-wind mode does not properly describe AGB winds. First, AGB winds are very slow compared to the fast winds assumed in the isotropic-wind mode. Second, the BHL formalism is valid when the wind velocity is much higher than the orbital velocity of the companion star, which is not generally the case for AGB binary systems where the wind velocity is of the same order as of the orbital velocity. Third, the BHL formalism assumes a plane-parallel flow at infinity without transverse density nor velocity gradients, which does not hold for AGB winds. Therefore, in order to properly investigate the mass transfer mechanism via winds in AGB binary systems, and how this interaction impacts the orbital evolution of the system detailed numerical simulations where both the hydrodynamics of the wind and the dynamics of the stars are properly modelled are needed.

### 1.2.4. Previous work

Theuns & Jorissen (1993) were the first to perform hydrodynamical models of the interaction via AGB winds in binary stars. They showed that the interaction of a slow wind with a companion star which orbital velocity is of the same order of magnitude differs considerably from the idealised case of fast winds. Several other works have followed, which have focused on understanding the mass accretion process onto the companion star (e.g. Theuns et al., 1996; de Val-Borro et al., 2009; Mohamed & Podsiadlowski, 2007; Huarte-Espinosa et al., 2013; Chen et al., 2017). For instance, Theuns et al. (1996) showed that the gas equation of state used influences the mass-accretion efficiency onto the companion. Mohamed & Podsiadlowski (2007) and de Val-Borro et al. (2009) showed that when interaction via AGB winds takes place, an intermediate mass transfer mechanism between wind mass transfer and RLOF may occur, which they called "gravitational focussing" or wind Roche-lobe overflow (WRLOF). Since the stellar wind leaves the donor star with very low velocities, the outflow is confined to the Roche lobe of the donor star and material is able to flow towards the companion star through the inner Lagrangian point, similar to what occurs in the RLOF case. The accretion efficiencies onto the companion star that are found when mass transfer occurs via WRLOF are much more higher than predicted by the BHL formalism (Mohamed & Podsiadlowski, 2012). Jahanara et al. (2005) performed the first hydrodynamical simulations with the aim of studying the amount of angular-momentum that is lost when stars interact via slow stellar winds. They find that the angular-momentum loss is enhanced compared to the isotropic
value when the wind velocity is comparable to or much lower than the orbital velocity of the binary. A more recent study performed by Chen et al. (2018), which included more realistic physical processes than the simple model assumed by Jahanara et al. (2005) (such as cooling of the gas, radiative transfer and dust formation) revealed a similar trend (see Chapter 3 for a detailed comparison). However, with the exception of these two papers, no hydrodynamical study of binaries interacting via winds has been performed so far in which the evolution of the orbital parameters has been studied. Furthermore, only circular binaries were studied in these papers.

In this thesis we perform a large set of hydrodynamical models in which the mass-accretion onto the companion star and the angular-momentum loss from the system are studied for different binary configurations. We study the consequences for the evolution of the orbital parameters of interacting binary systems in both circular and eccentric orbits. Our results may help explain the puzzling orbits of the descendants of AGB binary stars. Moreover, if the orbital evolution of low- and intermediate-mass binary systems turns out to be quite different from the current theoretical picture, this could change the number of systems entering a common envelope phase. Therefore, our results may not only apply to the previously discussed zoo of the offspring of AGB binary stars, but also have consequences for the expected formation rate of the final product of interaction of low- and intermediate-mass binary systems, such as cataclysmic binaries (stars formed by a white dwarf and a mass transferring donor star), double white dwarf systems, and type Ia supernovae (which are the thermonuclear explosions of a CO white dwarf).

1.3. The AMUSE-ing numerical method

A complete study of wind mass transfer in AGB binary stars would require parallel modelling of many different physical processes. On the one hand a stellar evolution code will be needed in order to compute the stellar parameters of the binary components, such as their mass, radius, effective temperature and mass-loss rate of the donor star. On the other hand, in order to model the dynamics of the wind, hydrodynamical codes are necessary, and in order to properly simulate the wind acceleration, pulsations of the AGB donor star, dust formation, and radiative transfer need to be included. Finally, a gravitational dynamics code is desirable to properly follow the evolution of the orbital parameters of the binary system due to the influence of the stars on the gas and vice versa.

The astrophysical multipurpose software environment (AMUSE\textsuperscript{10}; Portegies Zwart et al., 2009, 2013; Pelupessy et al., 2013) is a PYTHON\textsuperscript{11} framework for astrophysical simulations, in which different existing codes from various domains in astrophysics can be coupled, allowing a complete study of different phenomena. Examples of the domains that can be combined

\textsuperscript{10}http://www.amusecode.org/
\textsuperscript{11}https://www.python.org/
1.3 The AMUSE-ing numerical method

Figure 1.3: Diagram of the coupling of codes in AMUSE. All codes (N-body, SPH, stellar_wind.py and bridge.py) are included in AMUSE and can easily be called. We use stellar_wind.py (van der Helm et al., 2019) to generate wind particles that are given to the SPH code, which takes care of their evolution. The gravitational dynamics of the stars is evolved using an N-body code. The communication between SPH and N-body is done using the module bridge.py.
are stellar dynamics, hydrodynamics, stellar evolution and radiative transfer. The AMUSE framework therefore facilitates a comprehensive simulation of stars interacting via winds.

Although the AMUSE framework makes possible a detailed simulation involving all the physics described above, such a numerical computation would be computationally expensive. For this reason, in this thesis we have assumed several simplifications when modelling wind mass transfer in AGB stars. For instance, dust formation and radiative transfer have been neglected. We create wind particles around the donor star with different attributes such as temperature, initial velocity and density have typical values for AGB winds. However, we do not model in detail the acceleration mechanism of the wind due to pulsations and radiation pressure on dust particles (see Chapter 2). Once wind particles are created, they are added to a hydrodynamics code which evolves their dynamics. We also model the thermal evolution of the gas using a cooling prescription (see Chapter 3). We couple the hydrodynamics code with a gravitational dynamics code, which evolves the dynamics of the stellar orbits. The communication between the hydrodynamics and the gravitational dynamics codes is performed using the BRIDGE.PY module, also available in AMUSE. Figure 1.3 shows a diagram of how the different codes used in this thesis are coupled.

1.3.1. The hydrodynamics code: SPH

A number of techniques have been developed to solve the fluid equations in hydrodynamics, among which we find grid-based codes, which follow the Eulerian method, and particle-based codes, which follow the Lagrangian method. Eulerian methods use geometric grids and evaluate the fluid parameters over the grid cells. On the other hand, Lagrangian methods evolve the fluid equations in a co-moving frame. Each of these methods have advantages and disadvantages over the other. In this thesis we use a Lagrangian method to model the wind dynamics because we are interested in computing the angular-momentum loss from the binary, and Lagrangian methods are better at conserving angular momentum than Eulerian codes (Price, 2012).

Smoothed particle hydrodynamics (SPH) codes is a particle-based method in which the fluid is approximated by discrete points or particles (Lucy, 1977; Gingold & Monaghan, 1977). Each particle has mass \( m_j \), position \( r_j \), and velocity \( v_j \). Computing the density of each particle \( \rho_j \) is the fundamental operation in SPH. It is estimated by using a weighted summation over nearby particles:

\[
\rho(r) = \sum_j m_j W(|r - r_j|, h),
\]

(1.13)

here \( W(r, h) \) is the smoothing kernel which determines the weight of each particle; it is a function of the relative separation of the particles and the smoothing length \( h \), which determines the rate of fall-off of \( W \) as a function of the particle spacing. To guarantee computational efficiency, it is desirable that the kernel has compact support, i.e. that instead of requiring interactions with all the particles in the simulation, the number of particles contributing to
the sum is restricted only to a certain number of particles. This is usually done by choosing \( h \) such that the number of interacting particles for each particle is roughly constant, i.e. the smoothing length of each particle is variable. Since the mass of the SPH particles is fixed, from Eq. 1.13 is clear that high densities will be found where there is an increase in the local concentration of particles.

The equations of motion of the particles can be derived from the discretised Lagrangian (Springel & Hernquist, 2002),

\[
\mathcal{L} = \sum_{j=1}^{N} m_j \left[ \frac{1}{2} v_j^2 - u_j(\rho_j, s_j) \right],
\]

i.e. the kinetic energy minus the thermal energy, where \( v_j \) is the velocity of the particle \( j \) and \( u_j \) its specific internal energy, which is a function of its density and entropy. For the case of an ideal gas, the evolution of \( u \) is derived from the entropic function \( A(s) \) (which is a measure of the entropy of the gas) via \( P_j = A_j(s) \rho_j^\gamma \), where \( P_j \) is the pressure and \( \gamma \) is the adiabatic index. Therefore, \( u_j = A_j(\gamma - 1)^{-1} \rho_j^{\gamma - 1} \). Implicit in the Lagrangian formulation of Eq. 1.14 is that the velocities and densities are smoothly varying functions in time. In reality the Euler equation for gas dynamics can develop discontinuities, e.g. shocks. Therefore, a dissipation term needs to be added to the equations of motion (e.g. Gingold & Monaghan, 1983). We call these artificial dissipative terms the artificial viscosity, and it basically transforms kinetic energy of the gas into internal energy.

Within AMUSE two SPH codes are available: Fi (Hernquist & Katz, 1989; Gerritsen & Icke, 1997; Pelupessy et al., 2004) and GADGET2 (Springel et al., 2001; Springel, 2005a). In this thesis, we use the SPH code Fi, however in principle this code could be easily exchanged with GADGET2. Comparing the differences in the results using either code is beyond of the scope of this work.

### 1.3.2. The gravitational dynamics code

The N-body problem states the problem of computing at any time the position and velocity of a system formed by \( N \) bodies interacting under their mutual gravity. Each body in the system has mass (\( m_i \)), position (\( \mathbf{r}_i \)) and velocity (\( \mathbf{v}_i \)). Because of their gravity, each body experiences an acceleration given by:

\[
\mathbf{a}_i = G \sum_{j \neq i}^{N} m_j \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3},
\]

where \( G \) is the gravitational constant, \( m_j \) the mass of particle \( j \) and \( \mathbf{r}_j \) its position. Eq. 1.15 is a set of non-linear second-order ordinary differential equations, the solution of which gives the position and velocities of the particles at any time. However, only for a few cases (\( N = 2 \) and some special cases with \( N = 3 \)) the N-body problem can be solved analytically. For a larger
number of particles, the solution can only be found numerically and gravitational dynamics codes (also called N-body codes) need to be invoked.

In the case of two stars interacting with each other under the influence of their mutual gravity, the N-body problem is reduced to the two-body problem, which as stated before can be solved analytically. The reason why we use an N-body code to evolve the stellar dynamics, is that we want to follow in detail the dynamical evolution of the stars, namely their position and velocity step by step as mass and angular momentum are removed by gas and mass is transferred between the stars. In this way we can know the orbital angular momentum and orbital energy at any time in our simulations and therefore we can estimate the rate of change in the semi-major axis and eccentricity (see Chapters 4 and 5).

In this work, we use the gravitational dynamics code Huayno (Pelupessy et al., 2012; Jänes et al., 2014) to evolve the binary components in our models. Huayno is a symplectic code (it preserves the phase-space area) which conserves momentum to machine precision. A number of different integration methods are available within this code. For the purposes of this work, we use the Kepler solver, which uses the universal variable formulation\(^\text{12}\) (Bate et al., 1971) to solve the two-body Kepler problem. If our code would be applied to higher order multiple systems, the same setup could be used by only changing the integrator option.

1.3.3. A bridge between SPH and N-body

The SPH and N-body codes are coupled using the bridge.py (Fujii et al., 2007; Pelupessy et al., 2013) module available in amuse. Bridge.py was originally designed to couple different gravitational integrators, it uses Hamiltonian splitting and a second-order leapfrog integration scheme, i.e. the position and velocity are updated at interleaved times (see e.g. Portegies Zwart & McMillan, 2018). Within amuse, bridge.py can also be used to couple different codes (Pelupessy et al., 2013). The basic idea of bridge.py is that two (or more) systems interacting gravitationally are able to feel each other via periodic velocity kicks. The evolution over a time interval \( t \) of a system consisting of two components, \( S \) and \( G \), can be computed by mutually kicking both components: first we compute the force exerted by \( S \) on \( G \) and the force exerted by \( G \) on \( S \) and advance the momenta for a time interval \( t/2 \). Then we evolve individually both systems for a time \( t \), and the evolution is completed by another mutual kick between both components.

1.4. This thesis

As we have argued in the previous sections, theoretical models for binary evolution fail to explain the observed orbital periods and eccentricities of the offspring of low- and intermediate-mass binary stars interacting during the AGB phase. The aim of this thesis is to perform a

\(^{12}\)The universal variable formulation is a generalised form of Kepler’s equations to orbits with eccentricities larger than 1. See Everhart & Pitkin (1983) for a derivation.
1.4 This thesis

hydrodynamical study of wind mass transfer and at the same time investigate its effect on the orbit of the binary system. To achieve this we perform several hydrodynamical simulations of different binary configurations and we compute the mass-accretion efficiency and angular-momentum loss which are the main parameters determining the orbital evolution of the binary.

In Chapter 2, we present STELLAR_WIND.py, a module incorporated in AMUSE the aim of which is to generate wind particles that can be used in any SPH code. This module includes three modes to generate stellar winds: simple wind, accelerating wind and heating wind. Throughout this thesis the chosen mode is the accelerating wind which allows modelling low-velocity winds around stellar objects. However, in this chapter we also present a few examples which show the different astrophysical phenomena that can be studied with the different stellar-wind modes.

In Chapter 3 we present a number of hydrodynamical models, in which we study the angular momentum-loss and the mass-accretion efficiency of binary stars in circular orbits interacting via stellar winds. To validate our numerical models, we reproduce the numerical simulations of Theuns & Jorissen (1993) for an adiabatic and an isothermal equation of state. We compare these results with a more realistic model, where cooling of the gas is included. We also perform a convergence test which shows that while the angular-momentum loss is independent of the chosen resolution, the mass-accretion efficiency decreases with decreasing resolutions. Finally, we perform different numerical simulations in which we vary the terminal velocity of the wind. We find two regimes of interaction depending on the wind-terminal-velocity-to-orbital-velocity ratio \( v_\infty / v_{\text{orb}} \). When \( v_\infty < v_{\text{orb}} \) strong interaction between the wind and the companion star occurs which is observed as a complex morphology of the outflow. In this case, the angular-momentum loss and the mass-accretion efficiency have large values which lead to shrinking of the orbit. On the other hand, when \( v_\infty > v_{\text{orb}} \), the wind approximates the isotropic-wind mode and the mass-accretion efficiency can be described by the BHL formalism. In this case the orbit of the system widens.

In Chapter 4 we perform a large number of hydrodynamical simulations for different circular binary configurations. We vary the mass ratio of the system, its initial orbital separation, the initial wind velocity, and we explore the possibility of corotation of the donor star. The initial configurations of the stars are chosen such that they match the possible progenitors of CEMP-s stars. We find that the strength of interaction depends not only on the ratio \( v_\infty / v_{\text{orb}} \), but also on the mass ratio of the binary. The strongest interaction occurs when the mass ratios are comparable. We find that for initial orbital separations of the binary up to 7-10 AU (depending on the mass ratio), tidal effects are important and corotation of the donor star occurs during the AGB phase. If the initial wind velocity is low (1-5 km s\(^{-1}\)), corotation of the donor star modifies the geometry of the outflow which resembles WRLOF. This also impacts the angular-momentum loss and the mass-accretion efficiency which differ from the non-rotating donor case. We provide a relation for both the angular-momentum loss and the mass-accretion efficiency as a function of the wind-terminal-velocity-to-orbital-velocity ratio.
ratio and the mass ratio of the stars, which can be implemented easily in binary population synthesis codes.

In Chapter 5 we present simulations showing the effect of wind mass transfer on the orbits of initially eccentric binaries. The stellar parameters of the binary systems are similar to those presented in Chapter 2 and Chapter 3 for a mass ratio equal to 2. The eccentricities of our models are varied between 0 and 0.8 in such a way that the distance to pericentre is constant. As a function of increasing eccentricity, the angular-momentum loss and the mass-accretion efficiency decrease. For most of the models we find that the effect of wind mass transfer is to widen the orbit and decrease the eccentricity. Only for one model we find that the orbit shrinks as result of the wind interaction and its eccentricity increases on comparable timescales as the tidal circularisation timescale.
Simulating stellar winds in AMUSE

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Abstract

We present STELLAR_WIND.PY, a module that provides multiple methods of simulating stellar winds using smoothed particle hydrodynamics codes (SPH) within the astrophysical multipurpose software environment (AMUSE) framework. The module currently includes three ways of simulating stellar winds: with the simple wind mode, we create SPH wind particles in a spherically symmetric shell for which the inner boundary is located at the radius of the star. We inject the wind particles with a velocity equal to their terminal velocity. The accelerating wind mode is similar, but with this method particles can be injected with a lower initial velocity than the terminal velocity and they are accelerated away from the star according to an acceleration function. With the heating wind mode, SPH particles are created with zero initial velocity with respect to the star, but instead wind particles are given an internal energy based on the integrated mechanical luminosity of the star. This mode is designed to be used on longer timescales and larger spatial scales compared to the other two modes and assumes that the star is embedded in a gas cloud. We present a number of tests and compare the results and performance of the different methods. For fast winds, we find that both the simple and accelerating mode can reproduce the desired velocity, density and temperature profiles. For slow winds, the simple wind mode is insufficient due to dominant hydrodynamical effects that change the wind velocities. The accelerating mode, with additional options to account for these hydrodynamical effects,
can still reproduce the desired wind profiles. We test the heating mode by simulating both a normal wind and a supernova explosion of a single star in a uniform density medium. The stellar wind simulation results matches the analytical solution for an expanding wind bubble. The supernova simulation gives qualitatively correct results, but the simulated bubble expands faster than the analytical solution predicts. We conclude with an example of a triple star system which includes the colliding winds of all three stars.

2.1. Introduction

Stars lose mass through stellar winds during various stages of their evolution (e.g. Meyer-Vernet, 2007; Owocki, 2013; Puls et al., 2015). These winds can affect the gas near the star, creating lower density bubbles (Castor et al., 1975) and regulating star formation (Oey & Clarke, 2009). If a binary companion is present, accretion of the stellar wind material can also affect the evolution of that companion (Boffin, 2014).

The two most important parameters of the stellar wind are the mass-loss rate, $\dot{M}$, and the terminal wind velocity, $v_\infty$, which determine the effect of the wind on the environment. Based on these parameters, stellar winds can be broadly divided into three categories (Owocki, 2013): 1) Cool main-sequence stars like the Sun have winds with very low mass-loss rates ($\dot{M} \sim 10^{-14} \, M_\odot/\text{yr}$) that are thermally or gas pressure driven. 2) Cool giants and super giants have slow ($v_\infty \sim 5–30 \, \text{km/s}$) high mass-loss rate ($\dot{M} \sim 10^{-7} – 10^{-5} \, M_\odot/\text{yr}$) winds driven mainly by radiation pressure on dust (Höfner, 2015). 3) Hot luminous stars have fast ($v_\infty \sim 200–3000 \, \text{km/s}$) high mass-loss rate ($\dot{M} \sim 10^{-7} – 10^{-5} \, M_\odot/\text{yr}$) line driven winds (Puls et al., 2009). The second and third category have the highest kinetic output and therefore have the most pronounced effect on the stellar environment (not including cumulative effects).

To simulate stellar winds in detail, a combination of hydrodynamics, radiative transfer, dust formation and chemical abundances is required. Such simulations have been done for many years although they are extremely computationally expensive. In most cases simulations are limited to 1D or 2D models (e.g. Owocki et al., 1988; Blondin et al., 1990; Kudritzki & Puls, 2000; Boffin, 2014). To investigate the net effect of the stellar wind on the environment, it is often sufficient to simulate the stellar wind using only hydrodynamics (Theuns & Jorissen, 1993; Cuadra et al., 2006; Mohamed et al., 2012). For larger scale simulations, stellar wind feedback is often included using a sub-grid model as it can influence star formation and launch galactic winds (e.g. Agertz et al., 2013; Muratov et al., 2015).

For all these simulations, the astrophysical multipurpose software environment (AMUSE\textsuperscript{1}; Portegies Zwart et al., 2013; Pelupessy et al., 2013; van Elteren et al., 2014; Portegies Zwart & McMillan, 2018; Portegies Zwart et al., 2018) can be useful. It provides a uniform interface

\textsuperscript{1}amusecode.org
for many types of simulations with a large and growing set of simulation codes. The consistent python interface makes it possible to quickly set up a scientific simulation and easily exchange different parts of these simulations. While stellar winds have been simulated before using AMUSE (e.g. Pelupessy & Portegies Zwart, 2012), a consistent and properly tested module was still missing.

The STELLAR_WIND.PY code presented in this paper can be combined with the SPH (smoothed particle hydrodynamics), N-body, stellar evolution and (with some additional work) radiative transfer codes that are already available. We describe the STELLAR_WIND.PY code and explain the different modes in which it can be used (Section 2.2). In Section 2.3 we present a series of tests in which we compare the results from the different modes with theoretical expectations and previous work. We conclude in Section 2.4 with an exposition of some ongoing research projects using this code and ideas for further use.

### 2.2. Methods

The goal of STELLAR_WIND.PY is to create gas particles that represent the stellar wind from one or more stars. The code requires a number of stars, represented by AMUSE particles\(^2\), with stellar properties that can be derived from observations or stellar evolution simulations. Using this, SPH particles are created with the appropriate wind properties in an initially spherically symmetric shell with inner boundary at the radius of the star. The number of SPH particles is computed according to the mass-loss rate associated with the star undergoing mass loss and the predefined SPH particle mass, \(M_{\text{SPH}}\). These particles can be added to any SPH code in AMUSE which simulates the hydrodynamics of the wind.

Creating the SPH particles is only one step in the simulations for which STELLAR_WIND.PY is used. Following the goal of the AMUSE framework, the other parts of the simulations are handled by specialized interchangeable codes. For the hydrodynamics, SPH codes such as FI (Pelupessy, 2005) and GADGET2 (Springel, 2005b) can be used. In many applications, the stars move, for which a large number of N-body codes are available. To couple the stellar dynamics to the hydrodynamics gravitationally, BRIDGE (Fujii et al., 2007) is available. The stellar properties on which the wind is based will generally be calculated using a stellar evolution code. Both parametrized (e.g. Hurley et al., 2000) and Henyey type (e.g. Paxton et al., 2011) stellar evolution codes are available in AMUSE. Any or all of these codes can be combined with STELLAR_WIND.PY to set up a wide variety of simulations (see Section 2.3). For more information about the codes available within AMUSE and examples of how to couple them, we refer the reader to Portegies Zwart & McMillan (2018).

---

\(^2\)A particle set is the fundamental data structure in AMUSE. It is an array of particles (stars, SPH particles etc) which contain information to control the data. Each element (particle) of the particle set has certain attributes, such as mass, position, velocity, etc.
2.2.1. Calculating stellar wind properties

To simulate the stellar winds, the stellar parameters (mass, radius, temperature and position) and wind parameters (mass-loss rate, initial and terminal wind velocity) are required. All these parameters can simply be set directly, however some of them can be derived directly from stellar evolution codes available in AMUSE.

The stellarWind.py module includes user-friendly routines to derive some of the stellar parameters such as stellar mass, mass-loss rate, stellar radius and effective temperature from one of the stellar evolution codes within AMUSE. However, the terminal wind velocity, \( v_\infty \), is not calculated by any code currently in AMUSE. Determining \( v_\infty \) requires detailed and computationally expensive stellar wind simulations which include radiative transfer. For this reason, in order to compute the terminal velocities of hot stars, we provide within stellarWind.py a formula that has been fitted to observations of hot stars (Kudritzki & Puls, 2000), and which, they claim, is valid for these stars within 20%. The terminal velocity of the wind is given by:

\[
v_\infty = C(T_\star) v_{\text{phesc}},
\]

where,

\[
C(T_\star) = \begin{cases} 
1 & T_\star \leq 10000 \text{ K}, \\
1.4 & 10000 \text{ K} < T_\star < 21000 \text{ K}, \\
2.65 & T_\star \geq 21000 \text{ K},
\end{cases}
\]

\[
v_{\text{phesc}} = \sqrt{2g_\star R_\star (1 - \Gamma)} ,
\]

\[
g_\star = \frac{GM_\star}{R_\star^2},
\]

\[
\Gamma = 7.66 \cdot 10^{-5} \sigma_\epsilon \frac{L_\star/L_\odot}{M_\star/M_\odot},
\]

\[
\sigma_\epsilon = 0.398 \frac{1 + I_{\text{He}}Y}{1 + 4Y},
\]

where \( v_{\text{phesc}} \) is the photospheric escape velocity (similar to the escape velocity \( v_{\text{esc}} \) with a correction term for Thomson scattering), \( G \) is the gravitational constant, \( M_\star, R_\star, L_\star \) and \( T_\star \) are the mass, radius, luminosity and effective temperature of the star respectively, \( \Gamma \) is the ratio of radiative Thomson acceleration to gravitational acceleration, \( \sigma_\epsilon \) is the Thomson absorption coefficient, \( Y \) is the Helium fraction and \( I_{\text{He}} \) is the number of electrons per Helium nucleus (in this paper we use default values of \( I_{\text{He}} = 2 \) and \( Y = 0.25 \)). For cooler stars, \( v_\infty \approx v_{\text{phesc}} \) and this formula is still applicable (Kudritzki, private communication).

2.2.2. Simple wind

Within stellarWind.py, there are currently three wind modes available. The simplest mode creates a spherical shell of particles around the star with radial velocity, \( v(r) = v_\infty \) and initial temperature equal to the effective temperature of the star. While this may sound
simplistic, a similar setup has been used effectively for a number of scientific problems (e.g. Mohamed et al., 2012) and it serves as a starting point for the two other modes described in Sections 2.2.3 and 2.2.4. When the gravitational attraction of the star on the wind is included in the simulation, however, this will not result in the desired terminal wind velocity. We therefore release the wind with a larger velocity $v(r) = \sqrt{v_\infty^2 + v_{esc}(r)^2}$ where $v_{esc}(r) = \sqrt{2GM/r}$ is the local escape velocity at the initial particle radius, $r$. We calculate this new velocity for each particle because $v_{esc}(r)$ can vary within the thin shell in which we create the particles.

We set the outer radius ($r_{\text{max}}$) of the shell of new particles at the radius that the innermost part of the previously released shell ($R_\ast$) should have reached in the elapsed simulation time $\delta t$ (see Appendix 2.A.1). We scale the particle positions within the shell to follow the density profile matching the velocity profile as described in Appendix 2.A.2.

2.2.2.1. SPH and initial distributions

SPH is a method to solve the dynamics of a fluid by approximating it with a set of discrete particles (Monaghan, 1992). Each particle has both a mass and a density, where the density is calculated using the distance to, and mass of, other particles that are nearby. To determine which other particles are taken into account (how nearby they have to be) a kernel function\(^3\) and a particle smoothing length ($h$) are used. In all modes of stellar_wind.py, the SPH particle mass is required to be fixed and the same for each particle. The smoothing length, $h$, is set adaptively by fixing the number of neighbouring particles that fall within one $h$ (e.g. see Pelupessy, 2005).

Creating an initial distribution of SPH particle positions is not trivial (e.g. Diehl et al., 2015). Randomly distributed positions are clumpy which can introduce shot noise that can affect the entire simulation. A better alternative is to have more regular spaces between particles, for instance a distribution that follows a grid. However, a regular grid tends to introduce preferred directions in the simulation that can affect the overall results. To solve this, it is common to start with either a random or grid distribution and let the system evolve (relax) to a steady state where the positions are regularly spaced without preferred directions (for example a ‘glass’ initial condition, White, 1996; Wang & White, 2007). While some form of relaxation is preferred for simulations where all particles are created at once, for continuous particle creation like we describe here, this is not generally required.

In stellar_wind.py we implement two methods in which wind particles can be initially distributed. One is a random distribution and the second follows a uniform grid. We present an example of both in Figure 2.1. The random initial distribution (top panel) is available so that users can investigate if it has advantages for their simulations. In this case, a shell with uniform density is created and then the radii are scaled to ensure the correct density profile. In the other option we have included (bottom panel), each new shell is created by cutting it

\(^3\)In the SPH code used in this paper we use the spline kernel.
Figure 2.1: An example of the initial positions of newly created particles using a random (top) and grid (bottom) distribution. A shell of particles was created between 1 and 3 stellar radii ($R_*$) and the x and y positions of a thin slice ($|z| < 0.05 R_*$) are shown. The positions are scaled to match the given density profile (see Appendix 2.A). Note that this is merely an illustration of the difference between random and grid initial distributions. In real simulations, the shells would generally be much thinner.
out from a cube with positions following a uniform grid. The number of particles in this shell is generally not exactly the desired number of particles, $N_{\text{desired}} = \delta t \cdot \dot{M} / M_{\text{SPH}}$. We therefore remove a number of randomly selected particles from the grid (typically $\sim 30\%$) to ensure the correct number of SPH particles. The grid can be randomly rotated each time a new shell is generated to avoid introducing preferred directions into the resulting wind. The positions of the particles are also radially scaled to ensure that the desired density profile is achieved. This is the cause of the curved appearance in the grid in Figure 2.1.

There are many more ways to create initial particle distributions. A good overview of the different methods and their advantages can be found in Diehl et al. (2015). Our method is a mix between the ‘stretched lattice’ and the ‘shell’ methods described there. The reason we do not use the more advanced methods described there is that they would require some form of computationally expensive relaxation for every new shell. This is a common issue with continuous particle creation methods. If the current methods are found to be unsatisfactory for a specific simulation, the code is set up in a modular way so adding a new particle distribution method is relatively easy. The uniform grid with random rotation is the default option used throughout this paper. However, due to the small number of particles in a single shell, the difference between this option and a random distribution is negligible for all the tests in Section 2.3.

### 2.2.3. Accelerating wind

Near the surface of the star, usually within a few stellar radii, the wind is accelerated to the terminal wind velocity. In the accelerating wind mode, the wind particles are created in the same way as in the simple wind mode, but with a lower velocity, $v < v_\infty$. All particles near the star are artificially accelerated in such a way that the wind follows a predefined velocity profile.

The artificial acceleration is implemented using BRIDGE (Fujii et al., 2007). Originally, BRIDGE was designed to couple multiple gravitational codes. In this method, each code is evolved separately for a short, predefined timestep\(^4\). The mutual gravitational effect is included by BRIDGE using a kick-drift-kick scheme (see e.g. Portegies Zwart & McMillan, 2018). This method can also be used to gravitationally couple a pure N-body code with an SPH code, or to apply a gravitational potential to the particles in one or more codes. In STELLAR_WIND.PY, we use BRIDGE by including an artificial potential near the star, and then use the same kick-drift-kick scheme to ensure a smooth acceleration of the wind particles.

In Figure 2.2 and Table 2.1 we present the acceleration functions (sometimes referred to as acceleration laws) currently implemented in STELLAR_WIND.PY. For a given velocity (or acceleration) profile, all required quantities are calculated following the equations in Appendix 2.A. The constant velocity function is similar to the simple wind mode in that when

\(^4\) This should not be confused with the internal timesteps for each code which may be variable, meaning particles have different timesteps depending on conditions such as local particle density.
Figure 2.2: Radial velocity profiles for the wind acceleration functions currently available in Stellar_Wind.py. The formulae for these functions can be found in Table 2.1. For the beta_law we show two curves that are good fits for hot massive stars ($\beta = 0.8$) and cool giants ($\beta = 2.0$) (Lamers & Cassinelli, 1999). To illustrate the shape of the acceleration curves, we have chosen $v_0 = 0.2v_\infty$, $r_{acc\_start} = 2R_*$ (for the delayed_rsquared function), and $r_{mid} = 3R_*$ and $s = \alpha = 10$ (for the logistic and agb function).
### 2.2 Methods

**Table 2.1**: An overview of the acceleration functions currently available in `stellar_wind.py`. Either the acceleration \((a)\) or the velocity \((v)\) is given depending on which is simpler. The corresponding \(v\) or \(a\) function can be derived using \(a(r) = v(r) \frac{dv}{dr}\) and the known boundary conditions. Some functions allow user defined parameters to affect the functions (e.g. \(r_{\text{acc\_start}}\), \(r_{\text{mid}}\), \(\beta\), etc.) The first three functions are rough approximations to the last three functions. Their advantage is that they are computationally faster.

<table>
<thead>
<tr>
<th>Name</th>
<th>Equation</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant_velocity</td>
<td>(v(r) = v_{\infty})</td>
<td>Wide binaries</td>
</tr>
<tr>
<td>rsquared</td>
<td>(a(r) \propto \frac{1}{r^2})</td>
<td>Hot stars</td>
</tr>
</tbody>
</table>
| delayed\_rsquared | \(a(r) \propto \begin{cases} 
0 & r < r_{\text{acc\_start}} \\
\frac{1}{r^2} & r \geq r_{\text{acc\_start}} 
\end{cases}\) | Cool stars    |
| logistic         | \(v(r) = v_0 + \frac{v_{\infty} - v_0}{1 + e^{-\frac{r-r_{\text{mid}}}{r_{\text{mid}}}}}\) | AGB winds     |
| agb              | \(v(r) = v_0 + \frac{v_{\infty} - v_0}{1 + \left(\frac{r_{\text{mid}}}{R_{\ast}}\right)^{\frac{a}{\alpha}} - \alpha}\) | AGB winds     |
| beta\_law        | \(v(r) = v_0 + (v_{\infty} - v_0) \left(1 - \frac{R_{\ast}}{r}\right)^{\beta}\) | Hot/cool stars |
the wind particles are created, they already have the terminal velocity. However, as noted below, when used in the accelerating mode we can add extra terms to compensate for the gravity of the star, as well as the pressure of the gas on the wind. These extra accelerating terms are added after the particles have been created, which is not possible with the simple wind mode. In this way, we guarantee the desired constant velocity profile. The logistic and agb functions provide a fit to the time-averaged behaviour of dynamical models of asymptotic giant branch (AGB) winds from Nowotny et al. (2010). These winds exhibit a specific acceleration zone, the location of which can be chosen with the parameters $r_{\text{mid}}$ and either $s$ or $\alpha$ (for the logistic and agb function, respectively). These parameters determine the center and the width of the acceleration zone. The default values $r_{\text{mid}} = 3$ and $s = \alpha = 10$ are chosen to fit the dynamic models. The beta_law function, which was derived using a combination of observations and theoretical wind models, was taken from Lamers & Cassinelli (1999) and Maciel (2014). The $\beta$ parameter indicates the steepness of the acceleration curve and is often derived from observations. The example values $\beta = 0.8$ and $\beta = 2$ are typical for hot and cool stars respectively. The rsquared and delayed_rsquared functions can be used as rough approximations to the wind profiles of hot stars and cool giants, respectively. They have the advantage of being computationally faster than the beta-law, agb and logistic functions. In the delayed_rsquared model the parameter $r_{\text{acc, start}}$ (with a default value of 2) sets the lower boundary of the acceleration zone. The initial velocity $v_0$, which is used in all functions except for constant_velocity, is of the order of a few km/s due to microturbulence in the stellar atmosphere where the material is launched. We note that low values of $v_0$ result in high densities which lead to slow simulations, so in many cases a higher value of $v_0$ can be used as an approximation. In addition to these predefined functions, new user defined velocity functions can easily be incorporated.

When the gravity of the star is included, an additional acceleration term can be added to compensate for it and ensure that the wind particles follow the chosen velocity profile. The gas pressure can also exert an acceleration on the wind. We therefore provide the option to subtract the expected gas pressure acceleration (see Appendix 2.A.3) from the applied artificial acceleration. If we do not include a hydrodynamical simulation of the stellar interior, unphysical boundary effects near the surface of the star can influence the wind evolution or even prevent the wind from being launched. We therefore provide the option to create a “staging shell” near the star, generally at least twice the thickness of the newly created shells. Within this shell, the accelerations are chosen in such a way that the desired velocities are enforced regardless of the gas dynamics. This shell then provides a better boundary condition for the rest of the simulation.

For many simulations using the simple and accelerating wind we can start the simulation with a vacuum around the star into which the wind particles are released. However, this can lead to an extra acceleration at the outer radius as the vacuum does not exert any pressure on the outermost particles. This can be problematic, especially for slow winds, where this spurious acceleration can significantly increase the velocities. We therefore include a
function in `stellar_wind.py` that creates an initial set of SPH particles following the desired temperature, density and velocity profiles up to a given radius. This function uses the same initial grid distribution described in Section 2.2.2.1. Since the whole grid is created at once, without random rotations between different shells, this can introduce preferential directions. We advise that any scientific measurements are started after all these particles have left the area of interest.

When particles are created, we ensure that they follow the desired density profile by solving equation 2.12 for each particle. For most acceleration functions, this equation has to be solved numerically, which can severely slow down the simulation (see Section 2.3.1). We therefore include the option to define a critical timestep, \( t_c \). When new wind particles are created, \( \delta t \), which determines the thickness of the shell of new particles, is compared to \( t_c \). If \( \delta t < t_c \) it means that the new wind particles have not reached the accelerating region yet. For this reason, the acceleration function is approximated by the constant velocity function, for which equation 2.12 is solved analytically. This approximation is only valid for acceleration functions for which the velocity near the star is close to constant, like the logistic function, not for acceleration functions with a large acceleration near the stellar surface, like the beta_law function (see Figure 2.2).

### 2.2.4. Heating wind

The third wind mode is based on the method used in Pelupessy & Portegies Zwart (2012) and is designed for use in large scale simulations, e.g. embedded star clusters. For these simulations, the main effect of the wind is that it adds mass and energy to the surrounding gas, therefore this mode cannot be used for a star in a vacuum. Studying the detailed kinematics of the wind near the star is not the goal of these simulations and therefore a simpler approximation of the wind interaction is used. The advantage of this approximate approach is that it can be used at far lower resolution (longer timesteps and higher SPH particle mass) which greatly speeds up the simulations. If particles were created with a high velocity, small timesteps would be required to completely sample the particle trajectory and interactions with other particles.

The basic idea of the heating wind mode is that new wind particles do not have an initial velocity relative to the star. Instead they have an internal energy, \( u \), which corresponds to the mechanical energy \( (E_{\text{mech}}) \) of the accumulated wind, defined as,

\[
E_{\text{mech}} = \int_{t_0}^{t_1} L_{\text{mech}}(t) \, dt,
\]

\[
L_{\text{mech}}(t) = \frac{1}{2} \dot{M}(t)v_\infty(t)^2,
\]

(2.2)

where \( L_{\text{mech}} \) is the instantaneous mechanical luminosity and \( t_0 \) and \( t_1 \) are the previous and current wind release time respectively. The integral is numerically approximated in `stellar_wind.py`.
CHAPTER 2: SIMULATING STELLAR WINDS IN AMUSE

\[ u = f_{fb} \frac{E_{\text{mech}}}{\Delta M_*} \]  

(2.3)

where \( \Delta M_* \) is the mass lost and \( f_{fb} \) is the feedback efficiency parameter that accounts for radiative losses. Typical values for these parameters can be found in the examples shown in Sections 2.3.3 and 2.3.4.

As discussed in Pelupessy & Portegies Zwart (2012), this method of creating particles with appropriate internal energy can also be used to simulate a supernova. If a star goes supernova, the calculated mechanical energy is ignored, and instead \( 10^{51} \) erg of energy is divided over the newly created particles. It should be noted that the injection of so much energy in the surrounding gas will cause the gas to evolve very rapidly, which can lead to time-stepping artefacts (e.g. Pelupessy & Portegies Zwart, 2012). One way to prevent this is to use a very small timestep (preliminary tests suggest \( \sim 10 \) yr) shortly after a supernova.

2.3. Application

To ensure that \texttt{stellar\_wind.py} performs as expected, we run test simulations with different initial conditions and wind modes. In this section we present the results of these tests. The tests in Sections 2.3.1 and 2.3.2 are simulations of processes that happen close to the star. Therefore only the simple and accelerating wind modes are applicable. The tests in Section 2.3.3 and 2.3.4 are large scale simulations of the interaction between the stellar wind and the gas in which the star is embedded. For this type of simulation the heating wind mode
is applicable. The final test in Section 2.3.5, where we couple stellar dynamics, hydrodynamics and stellar winds, shows the power of STELLAR_WIND.PY within AMUSE by simulating the colliding winds from a stable triple star system. The initial distribution of the wind particles is the one based on a grid for all the models in these tests. The particular parameters used are described accordingly in the following subsections.

The STELLAR_WIND.PY module is designed to couple different parts of a simulation in AMUSE. Testing the code requires the use of other simulation codes, which have their own parameters to be set. Here we describe the general method and general parameters used in our test models. In Figure 2.3 we present a flow diagram to illustrate the codes and the relationships between them. Note that the codes and coupling strategies used here are merely an example and should be modified in order to be suitable for any specific application. To simulate the gas we use the SPH code FI with an adiabatic equation of state and artificial viscosity parameters $\alpha = 0.5$ and $\beta = 1.0$ (following Lombardi et al., 1999). Self-gravity of the gas is only included in the tests which do not require periodic boundary conditions, i.e. those tests where the simple or accelerating mode are used. For simulating the gravitational attraction of the star on the gas we need BRIDGE, which is also used in the accelerating wind mode (Section 2.2.3). The BRIDGE code requires an additional code for calculating the gravitational force. When we simulate only a single star that does not evolve dynamically, we use FASTKICK\textsuperscript{5}. When we simulate multiple stars that evolve dynamically (Section 2.3.5), we use the N-body code HUAYNO (Pelupessy et al., 2012). For each simulation, we also need to define a number of integration timescales, such as the BRIDGE timestep ($t_{br}$), the (maximum) internal timestep ($t_{N-body}$) of the SPH and N-body codes and the wind release timestep ($t_{wind}$), as well as the end time ($t_{end}$) of the simulation. The choice of these timesteps depends on the problem and the type of simulation. For the bridge leap-frog algorithm to work, we should set $t_{br} \geq t_{N-body}$ and the wind code requires $t_{wind} \geq t_{br}$. It is also a good idea to ensure that larger timescales are integer multiples of smaller timescales. For the simulations in this paper, we only define $t_{wind}$ and choose $t_{wind} = 2t_{br} = 4t_{N-body}$. For the hydro simulation we also need to set the SPH particle mass ($M_{SPH}$).

\subsection*{2.3.1. Fast winds}

In this section we present the results of a set of simulations using the simple and accelerating wind modes. We simulate the wind from a single, hot, massive, luminous star, for which we present the parameters in Table 2.2. Note that these values were not chosen using a specific stellar model and this test should only be considered as an example of the use of STELLAR_WIND.PY. The initial wind velocity, $v_0 = 100 \text{ km/s}$, is based on numerical considera-

\textsuperscript{5}The FASTKICK code, developed by N. de Vries, is an unpublished gpu-enabled code in AMUSE that can calculate the gravitational force of one set of particles on another set of particles. It is ideal for the gravitational coupling between particles in different codes via BRIDGE.
CHAPTER 2: SIMULATING STELLAR WINDS IN AMUSE

Figure 2.4: The analytical and simulated velocity ($v$), density ($\rho$) and temperature ($T$) as a function of radius ($r$) for the fast wind test. The results are from simulations using the simple wind mode (top) and the accelerating wind mode (bottom). The stellar and wind parameters can be found in Table 2.2. To calculate the analytical temperature profile, we assume adiabatic expansion.
2.3 Application

Figure 2.5: The same as Figure 2.4, but only for a simulation using the accelerating wind mode with the logistic acceleration function. We have varied the resolution by changing the SPH particle mass ($M_{\text{SPH}}$) and through that the number of particles in the simulation. We have added the smoothing length, $h$, as a function of radius for each simulation, which is a measure of the local spatial resolution.
CHAPTER 2: SIMULATING STELLAR WINDS IN AMUSE

Figure 2.6: The part of the simulation time used for the `stellar_wind.py` code while creating new particles (circles) and accelerating them (stars) compared to the time used by the SPH code (diagonal lines). The three panels show results for simulations with the simple wind mode (top), the accelerating wind mode with the logistic acceleration function without a critical timestep (middle) and the accelerating wind mode with a critical time step (bottom). Marks along each line denote separate simulation runs. At the top axis we give an estimate of the number of SPH particles \(N\) actually used in the simulation with the corresponding particle mass \(M_{\text{sph}}\). The remaining simulation time (white space) was mostly spent on unoptimized administrative tasks like saving snapshots and removing escaping particles.

\[N \approx 10^2 \quad 10^3 \quad 10^4 \quad 10^5\]

\[\text{time code/tot}\]

simple wind

accelerating wind

accelerating wind with critical timestep

\[M_{\text{sph}} (M_\odot)\]
Table 2.2: Stellar and wind parameters used in the fast wind test. Derived parameters are indicated with an arrow (→). Since the smoothing length is highly variable with extreme outliers, we include the median value of all gas particles shown in Figure 2.4.

<table>
<thead>
<tr>
<th>name</th>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass-loss rate</td>
<td>$M$</td>
<td>$10^{-6}$ M$_\odot$/yr</td>
</tr>
<tr>
<td>terminal wind velocity</td>
<td>$v_\infty$</td>
<td>700 km/s</td>
</tr>
<tr>
<td>initial wind velocity</td>
<td>$v_0$</td>
<td>100 km/s</td>
</tr>
<tr>
<td>stellar mass</td>
<td>$M_*$</td>
<td>20 M$_\odot$</td>
</tr>
<tr>
<td>stellar radius</td>
<td>$R_*$</td>
<td>30 R$_\odot$</td>
</tr>
<tr>
<td>stellar luminosity</td>
<td>$L_*$</td>
<td>100,000 L$_\odot$</td>
</tr>
<tr>
<td>stellar surface temperature</td>
<td>$T_*$</td>
<td>20,000 K</td>
</tr>
<tr>
<td>escape velocity at $R_*$</td>
<td>$v_{esc}(R_*)$ → 504.5 km/s</td>
<td></td>
</tr>
<tr>
<td>wind timestep</td>
<td>$t_{wind}$</td>
<td>0.02 days</td>
</tr>
<tr>
<td>end time</td>
<td>$t_{end}$</td>
<td>5 days</td>
</tr>
<tr>
<td>SPH particle mass</td>
<td>$M_{SPH}$</td>
<td>$10^{-11}$ M$_\odot$</td>
</tr>
<tr>
<td>particles per shell</td>
<td>$N_{shell}$→ ~5</td>
<td></td>
</tr>
<tr>
<td>particles in simulation</td>
<td>$N_{tot}$ → ~1378</td>
<td></td>
</tr>
<tr>
<td>median smoothing length</td>
<td>$h$</td>
<td>→ ~32 R$_\odot$</td>
</tr>
</tbody>
</table>

As we shall see in Section 2.3.2, slow (subsonic\footnote{If the wind speed near the star is lower than the local sound speed, the wind is called subsonic. On the other hand if the wind speed is higher than the local sound speed, i.e. past the 'sonic point', the wind is called supersonic.}) winds are more complex to simulate and for many applications using a higher $v_0$ is sufficient.

In Figure 2.4 we show the velocity, density and temperature profiles for the simple wind mode and two accelerating functions: the beta law and logistic function. In all cases the velocity profiles follow the desired analytical velocity curve. For the simple wind mode, the density and temperature profiles also follow the desired curve, but with more scatter. For the accelerating wind curves, we see that in regions with high acceleration the densities and temperatures in the simulation are too high. This is a result of the low resolution in combination with the way densities are calculated in SPH, using a kernel function that 'smears out' these variables. We show in Figure 2.5 that for a higher resolution (smaller $M_{gas}$), the desired density and temperature curve are recovered. Note that the logistic acceleration function is not a good representation for the velocity profile of a hot massive luminous star. This example was chosen to illustrate the discrepancies that can potentially occur. For any scientific application of this code, a detailed convergence test for the selected setup will still be required.
Table 2.3: Stellar and wind parameters used in the slow wind test. Derived parameters are indicated with an arrow (→). Since the smoothing length is highly variable with extreme outliers, we include the median value of all gas particles shown in Figure 2.8.

<table>
<thead>
<tr>
<th>name</th>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass-loss rate</td>
<td>( \dot{M} )</td>
<td>( 5\times10^{-7} ) ( \text{M}_\odot/\text{yr} )</td>
</tr>
<tr>
<td>terminal wind velocity</td>
<td>( v_\infty )</td>
<td>25 km/s</td>
</tr>
<tr>
<td>initial wind velocity</td>
<td>( v_0 )</td>
<td>2 km/s</td>
</tr>
<tr>
<td>stellar mass</td>
<td>( M_* )</td>
<td>2 ( \text{M}_\odot )</td>
</tr>
<tr>
<td>stellar radius</td>
<td>( R_* )</td>
<td>300 ( \text{R}_\odot )</td>
</tr>
<tr>
<td>stellar luminosity</td>
<td>( L_* )</td>
<td>8000 ( \text{L}_\odot )</td>
</tr>
<tr>
<td>stellar surface temper</td>
<td>( T_* )</td>
<td>3000 K</td>
</tr>
<tr>
<td>escape velocity at</td>
<td>( v_{\text{esc}}(R_*) )</td>
<td>( 50.45 ) km/s</td>
</tr>
<tr>
<td>( R_* )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>wind timestep</td>
<td>( t_{\text{wind}} )</td>
<td>2 days</td>
</tr>
<tr>
<td>end time</td>
<td>( t_{\text{end}} )</td>
<td>2000 days</td>
</tr>
<tr>
<td>SPH particle mass</td>
<td>( M_{\text{SPH}} )</td>
<td>( 10^{-9} ) ( \text{M}_\odot )</td>
</tr>
<tr>
<td>particles per shell</td>
<td>( N_{\text{shell}} )</td>
<td>( \sim 5 )</td>
</tr>
<tr>
<td>particles in simulation</td>
<td>( N_{\text{tot}} )</td>
<td>( \sim 8476 )</td>
</tr>
<tr>
<td>median smoothing length</td>
<td>( h )</td>
<td>( \sim 159 ) ( \text{R}_\odot )</td>
</tr>
</tbody>
</table>

In addition to being accurate, it is also important that a simulation code is fast. In Figure 2.6 we present the time spent in different parts of the simulation code (\( t_{\text{code}} \)) divided by the total cpu (or wall-clock) time (\( t_{\text{tot}} \)) as a function of resolution.\(^7\) In the top panel we see that when using the simple wind mode, the time spent in the STELLAR_WIND.PY code is less than 1% when using more than \( \sim 10^4 \) particles. Most of the simulation time is therefore spent in the SPH code itself, which is what we want. When we use the accelerating wind mode however (middle panel), the particle creation becomes a major bottleneck because numerically solving equation 2.12 is slow. To speed up the simulation, we have included the option to approximate the acceleration function with a constant velocity when particles are created near the star by defining a critical timestep (\( t_c \), see Section 2.2.3). In the bottom panel we see that by using this approximation, the time spent in STELLAR_WIND.PY reduces to < 1% for > \( 10^4 \) particles.

### 2.3.2. Slow wind

For the slow wind test, we simulate the wind from a single, cool, giant star, for which we present the parameters in Table 2.3. The values of these parameters are not computed with a stellar evolution code, but they correspond to typical values for AGB stars. Part of the stellar

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\(^7\)These simulations where all performed on the same desktop computer using a 4-core Intel Xeon E5507 CPU.
2.3 Application

Figure 2.7: Same as Figure 2.4 but for the slow wind simulations using the simple wind mode without gravity where we vary the wind velocity. We have added the expected local sound speed (dotted line) for comparison.
Figure 2.8: Same as Figure 2.7 but for the accelerating wind mode with four different acceleration functions.
Figure 2.9: Same as Figure 2.5 but for the slow wind test.
wind is subsonic and therefore hydrodynamical effects are no longer negligible, unlike in the fast wind test.

In Figure 2.7 we present the velocity profiles of simulations using the simple wind mode and varying $v_\infty$ at $t = 5$ days. We have not included the stellar gravity in these simulations, therefore the escape speed is not a relevant factor. However, when the wind speed is near or below the sound speed, the gas pressure gradient dominates and affects the terminal wind velocity. For very low wind speeds, this can cause wind particles to move inside the star, which is unphysical. We therefore conclude that the simple wind mode is not reliable for slow winds.

In Figure 2.8 we present the results of simulations using the accelerating wind mode. In these simulations we have used the staging shell option (Section 2.2.3) that enforces the correct particle velocity for all SPH particles with radius $r < 1.1R_*$. We have also used the option to subtract the expected gas pressure acceleration from the acceleration to ensure that the particles follow the desired velocity profile. To avoid spurious acceleration of the outer particles, we start the simulation after initially creating a sphere of particles following the desired velocity and density profiles throughout the simulation area (up to $r = 1500R_\odot$).

For the constant, rsquared and beta_law velocity profiles, the particles follow the desired velocity profiles. For the logistic velocity profile, where particles are subsonic for a longer time, the simulated velocity profile deviates from the desired velocity profile in the subsonic region. However, the corrections described above ensure that the resulting velocities after particles pass the sonic point follow the desired velocity profile. To see how this deviation will affect the results of simulations where the subsonic region is of interest, we have compared this velocity profile with detailed velocity profiles of AGB stars (Nowotny et al., 2010). In these profiles, the velocities in the subsonic region oscillate due to stellar pulsations and do not follow the simple acceleration functions we have used here. We therefore advise caution when interpreting results of simulations in the subsonic region.

Similar discrepancies in density and temperature as seen for the fast wind test in Figure 2.4 are also present for the slow wind test in Figure 2.8. In Figure 2.9 we present the results of a resolution test for the slow wind test. We see that the discrepancies in density and temperature decrease with higher resolution, as expected. The deviations in the velocity profile in the subsonic region also decrease, although some differences are still present even at high resolution.

### 2.3.3. Embedded star

For the embedded star test (Table 2.4), we take the hot, massive, luminous star from Section 2.3.1 and embed it in a constant density medium. The stellar wind will heat the gas and create a cavity around the star. This situation is quite common in embedded star clusters and it is what the heating wind mode is designed for. The initial gas is distributed
Figure 2.10: The gas density in the \( x - y \) plane after 0.2 Myr (top) and after 0.6 Myr (bottom) for the embedded star simulation with \( M_{\text{SPH}} = 0.1 \ M_\odot \) and \( \rho_{\text{gas}} = 100 \ M_\odot/\text{pc}^3 \). The embedded star is positioned at the origin (yellow dot) and the red dashed circle shows the radius with the largest mean density.
Figure 2.11: The radius with the highest mean gas density ($r_{\rho_{\text{max}}}$) as a function of time, $t$, for the embedded star simulations. Different colors correspond to different resolutions resulting from different SPH particle masses ($M_{\text{SPH}}$). In the top panel, we show $r_{\rho_{\text{max}}}$ for all simulations as a function of $t$. Lines with open circles correspond to simulations where the gas density, $\rho_{\text{gas}} = 10 \, M_\odot/\text{pc}^3$ and lines with filled circles to simulations with a gas density, $\rho_{\text{gas}} = 100 \, M_\odot/\text{pc}^3$. In the bottom panel, we only show simulations with $\rho_{\text{gas}} = 100 \, M_\odot/\text{pc}^3$ and subtract $\dot{M}/M_{\text{SPH}}$ (the time of the first SPH particle creation) from $t$. The black solid line shows the analytical solution for the shell radius of an energy driven flow in a constant density medium (Dyson, 1984).
2.3 Application

Table 2.4: The parameters used in the embedded star test. The stellar and wind parameters are the same as in Table 2.2.

<table>
<thead>
<tr>
<th>name</th>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>gas density</td>
<td>( \rho_{\text{gas}} )</td>
<td>10 and 100 M(_{\odot})/pc(^3)</td>
</tr>
<tr>
<td>wind timestep</td>
<td>( t_{\text{wind}} )</td>
<td>( 2 \cdot 10^4 ) yr</td>
</tr>
<tr>
<td>end time</td>
<td>( t_{\text{end}} )</td>
<td>( 10^6 ) yr</td>
</tr>
<tr>
<td>SPH particle mass</td>
<td>( M_{\text{SPH}} )</td>
<td>0.05 to 1 M(_{\odot})</td>
</tr>
<tr>
<td>wind release radius</td>
<td>( r_{\text{wind}} )</td>
<td>0.01 pc</td>
</tr>
<tr>
<td>feedback efficiency</td>
<td>( f_{\text{fb}} )</td>
<td>0.01</td>
</tr>
<tr>
<td>particles per shell</td>
<td>( N_{\text{shell}} )</td>
<td>( \rightarrow 0 ) or 1</td>
</tr>
<tr>
<td>new particles in simulation</td>
<td>( N_{\text{new}} )</td>
<td>( \rightarrow 5 ) to 100</td>
</tr>
<tr>
<td>median smoothing length</td>
<td>( h )</td>
<td>( \rightarrow \sim 0.2 ) pc</td>
</tr>
</tbody>
</table>

along a grid\(^8\) to ensure a constant density and a divergence-free random Gaussian velocity field following Bonnell et al. (2003). To ensure that the medium is stable, we use periodic boundary conditions and stop the simulation when the wind-blown bubble covers more than half the simulation box. For the heating wind mode, the outer radius for new wind particles (\( r_{\text{wind}} \)) is set manually to \( r_{\text{wind}} = 0.01 \) pc and the feedback efficiency is set to \( f_{\text{fb}} = 0.01 \) following Pelupessy & Portegies Zwart (2012).

In Figure 2.10 we show the gas density when the bubble has just started to form (\( t = 0.2 \) Myr) and when it has had time to grow (\( t = 0.6 \) Myr). The stellar wind creates an approximately spherical bubble of lower density as the gas is swept up in a high density shell around it. To understand why the bubble is not perfectly spherical, we note that the finite number of gas particles cause small numerical fluctuations in the initial gas density. When we then introduce a small number of wind particles with higher energy than the surrounding gas, these small fluctuations grow into a larger asymmetry in the wind bubble. This growth of small initial asymmetries was observed in SPH simulations of supernovae explosions (Rimoldi et al., 2016) where they found that if the injected energy is spread out over more particles, the asymmetric effects diminish. If the asymmetry in the wind bubbles would become a problem for specific simulations, then the wind energy could be spread out in a similar way.

In Figure 2.10 we have drawn a dashed line that shows the radius where the mean density is highest (\( r_{\rho_{\text{max}}} \)), see Appendix 2.A.4 for details). At the start of the simulation, this radius is undefined, because the gas has a constant density. As the bubble grows and gas is swept up in an approximately spherical shell, the radius of maximum density matches the shell radius, which is what we plot as a function of time in Figure 2.11. Note that \( r_{\rho_{\text{max}}} \) is slightly larger than the shell radius because of the asymmetry of the wind bubble. We present this expansion for different values of \( M_{\text{SPH}} \) (different resolutions) and two different gas densities.

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\(^{8}\)As mentioned in Section 2.2.2.1, this can introduce preferential directions and a glass or other relaxed system should be considered for most applications.
Table 2.5: The parameters used in the supernova test.

<table>
<thead>
<tr>
<th>name</th>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial stellar mass</td>
<td>$M_{\text{ZAMS}}$</td>
<td>20 $M_\odot$</td>
</tr>
<tr>
<td>stellar age</td>
<td>$T_*$</td>
<td>9.78 Myr</td>
</tr>
<tr>
<td>gas density</td>
<td>$\rho_{\text{gas}}$</td>
<td>10 $M_\odot$/pc$^3$</td>
</tr>
<tr>
<td>end time</td>
<td>$t_{\text{end}}$</td>
<td>2000 yr</td>
</tr>
<tr>
<td>SPH particle mass</td>
<td>$M_{\text{SPH}}$</td>
<td>0.01 – 1 $M_\odot$</td>
</tr>
<tr>
<td>wind timestep</td>
<td>$t_{\text{wind}}$</td>
<td>20 yr</td>
</tr>
<tr>
<td>wind release radius</td>
<td>$r_{\text{wind}}$</td>
<td>0.01 pc</td>
</tr>
<tr>
<td>supernova energy</td>
<td>$E_{\text{SN}}$</td>
<td>$10^{51}$ erg</td>
</tr>
<tr>
<td>mass-loss</td>
<td>$\Delta M_*$</td>
<td>12.95 $M_\odot$</td>
</tr>
<tr>
<td>feedback efficiency</td>
<td>$f_{\text{fb}}$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Even when the resolution is very low ($M_{\text{SPH}} = 1.0 M_\odot$), the heating wind method still results in a dispersion of the gas cloud. The expansion is faster for lower gas density, which is in agreement with analytical solutions for the shell radius of an energy driven flow in a constant density medium (Dyson, 1984). However, the bubble expansion starts later for simulations with a lower resolution. This delay corresponds to the time it takes for the star to lose enough mass to create the first wind particle. For example, if $M_{\text{SPH}} = 1.0 M_\odot$ and $\dot{M} = 1 M_\odot$/Myr, this delay is 1 Myr. In the bottom panel of Figure 2.11, we show the shell expansion starting at the moment of the first wind injection. We see that lower resolution results in a faster expansion, caused by the larger energy injected in a single SPH particle. The expansion profile approaches the analytical solution for high resolution (small $M_{\text{SPH}}$). We therefore advise that the choice of $M_{\text{SPH}}$ be based on the stellar mass-loss rate and the delay and expansion profile that would be acceptable in the desired simulations.

2.3.4. Supernova

As discussed in Section 2.2.4, the heating wind mode can also be used to simulate the effect of a supernova on the surrounding gas. The supernova test (Table 2.5) is similar to the embedded star test (Section 2.3.3). We start the simulation by evolving the star using the stellar evolution code SEBA (Portegies Zwart & Verbunt, 2012) to a few timesteps (~40 yr) before the star goes supernova (~9.78 Myr). We place the star inside the uniform density gas medium and use the option to derive the stellar wind parameters from the output of a stellar evolution calculation (see Section 2.2.1). When this option is set, STELLAR_WIND.PY detects the supernova and creates the particles with a combined mass of 12.95 $M_\odot$ and an energy of $10^{51}$ erg. After the supernova feedback is generated we trace the resulting blast wave. Similar to the embedded star test, we get a sphere of high density material moving away from the star, however, due to the higher energy input, the radial velocity is higher.
Figure 2.12: The mean gas density as a function of radius at time $t = 1000 \text{ yr}$ for the supernova test at four resolutions (dashed lines). The analytical solution (solid black line), is the Sedov-Taylor solution for a self-similar blast wave in a uniform medium (Taylor, 1950; Sedov, 1959).
Using test simulations, we have found that a timestep of $t_{\text{wind}} = 20$ yr is required to avoid time-stepping artefacts with this high velocity (also see: Pelupessy & Portegies Zwart, 2012).

In Figure 2.12 we present the radial mean density profile for simulations with four different particle masses. We find that for the low resolution simulation ($M_{\text{SPH}} = 1 M_\odot$), the gas has moved away from the star, but the shape of the shockfront, where the density is highest, is only loosely defined. For the higher resolution simulations, we do see the shape of the main shockfront clearly, and the simulations agree on the radius of highest density at $t = 1000$ yr. However, the radius of the shockfront does not converge to the analytical solution. There is an increased density at $r = 0$ for the $M_{\text{SPH}} = 0.01 M_\odot$ simulation. This feature is present at some point for all high resolution simulations and is the result of a reverse density wave within the outgoing shockwave. These waves are an artefact of the hydrodynamical simulation method used, but they are not an accurate representation of the true physical process. They should not be confused with the reverse shock that takes place in real supernova remnants. While these density waves do subside after a few thousand years, the density inside the supernova bubble shortly after the explosion should not be considered correct.

The time evolution of the expansion of gas from a supernova explosion is usually modelled in separate phases. The first phase is a free expansion, where the ejected gas moves at an approximately constant velocity, sweeping up the gas in the interstellar medium. After the mass of swept up gas is equal to the mass of the originally expelled gas, the expansion
can be approximated as a pure adiabatic expansion, which is described by the self-similar Sedov-Taylor solution. Only this last phase can be simulated with the heating wind mode of \texttt{stellar\_wind.py}, because the particles are given a high internal energy instead of an initial velocity.

In Figure 2.13 we present the time evolution of $r_{\rho\text{max}}$, which we calculate in the same way as for the embedded star test. We now compare it to the analytical solution for the two phases of a supernova blast wave, the free expansion and the Sedov-Taylor solution. We find that the simulations do approach the analytical solution and roughly follow the same shape, but even the highest resolution simulation expands faster than the analytical solution. We do not model the initial free expansion phase and shockwaves are in general difficult to simulate using SPH (e.g. Hubber et al., 2013). Differences with the analytical solution are therefore to be expected and this type of simulation should be interpreted with care.

Given these caveats, both the use of SPH and the chosen approximations may not seem to be the ideal choice for simulating a supernova explosion in a gaseous medium. Indeed, depending on the goal of the simulations, other available methods could be more suitable, for example using a grid based simulation code (e.g. Rogers & Pittard, 2013) or including magnetic fields (e.g. Körtgen et al., 2016). However, the method presented here has two main advantages: 1) It is simple and scales well to very low SPH resolution, making it computationally faster than more detailed simulation techniques. 2) The use of SPH combined with \texttt{bridge} allows easy gravitational coupling between the gas and the stars. We can therefore use this code to run large scale simulations of multiple supernova explosions in a gaseous medium also containing many dynamic stars. These advantages allow us to model a very turbulent stage in the evolution of embedded star clusters.

### 2.3.5. Colliding wind triple

The previous tests were for single stars and therefore the geometry of the outflow was not modified by the environment. In this test (Table 2.6), we simulate a triple star system where all three stars have a strong stellar wind. The system we simulate is loosely based on WR48 ($\theta$ Muscae), which is a triple system (Sugawara et al., 2008) consisting of a WC5/WC6 + O6/O7V binary with a short period (~19 days, Hill et al., 2002) and an O9.5/B0Iab star in a longer orbit (> 130 yrs, Dougherty & Williams, 2000) around that binary. For this simulation we have used similar numerical parameters to the fast wind test in Section 2.3.1.

In Figure 2.14 we show the gas density, temperature and velocity at the end of the simulation. Due to the large difference between the inner and outer orbital periods, the system appears similar to a normal colliding wind binary, which was assumed in previous models of WR48 (Hill et al., 2002). However, the orbital motion of the inner binary creates a spiral

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9 We are aware that the chosen values do not match the most up-to-date observations of the WR48 system. However, the goal of this investigation is to demonstrate the use of the \texttt{stellar\_wind.py} code, not to explain the observed system.
FIGURE 2.14: The gas density (top), temperature (center) and velocity (bottom) in the orbital plane of the colliding wind triple simulation for the inner (left) and outer (right) binary. The sizes of the stars (yellow circles) in the plots on the right hand panels were multiplied by 10 to make them visible. In the left hand panels, the Wolf-Rayet star (star 1, see Table 2.6) can be seen on the right and star 2 on the left. In the right hand panels, the short period binary (star 1 and 2), can be seen on the left and the O9.5 supergiant (star 3) on the right. In the bottom plots, the arrows indicate the wind direction and larger arrows correspond to higher wind velocities, however, the colors provide a more precise indication of the velocities. Note that the two density plots have separate color bars, while the temperature and velocity plots each share a single color bar.
pattern in the density and temperature distribution, which is very different from the wind from a single star. This spiral pattern creates high and low temperature regions in the shockfront where the wind from the inner binary collides with the wind from the third star. The observations of this shockfront can therefore be quite different from observations of a normal colliding wind binary.

It is important to note that this simulation is just an example of what is possible with the STELLAR_WIND.PY module and not an in-depth investigation into wind interactions in a triple star systems. For example, in the middle panels of Figure 2.14 we can see that the temperature of the wind from the inner binary is extremely high ($> 10^8$ K). These high temperatures are unrealistic because in reality the gas would cool, which is not taken into account in this simulation. When using this code for simulations that are to be compared with observations, gas cooling and a convergence test of the shockfront regions should be performed.

2.4. Discussion and Conclusion

We have presented and tested the STELLAR_WIND.PY module, which can be used to simulate stellar winds within the AMUSE framework by creating (and accelerating) SPH particles. The code includes three different modes: the simple mode, the accelerating mode and the heating mode (Table 2.7). We have tested the code for single stars with fast and slow winds, as well as an embedded star with both wind and a supernova explosion. For both fast and slow winds, the simple and accelerating wind modes perform well, although subsonic winds must be simulated with the latter. For the embedded star, the heating wind mode creates a
wind bubble, even at low resolution; with higher resolution the expansion profile approaches the analytical solution. After a supernova, the heating wind mode creates an expanding shell with velocities similar to the analytical solution if small enough timesteps are used. Finally we have shown an example of how this module can be used to tackle new problems, by simulating a colliding wind triple system.

The `stellar_wind.py` module can be used for many different simulations that involve stellar winds and several projects are already in progress. The simple wind mode has been used to simulate the accretion of gas from the winds of the S-stars onto the super-massive black hole Sgr A* (Lützgendorf et al., 2016). The accelerating wind mode is used to simulate the accretion of the wind from a red giant onto a close binary companion (Saladino et al., 2018). The heating wind mode is part of a larger simulation to investigate the evolution of the Arches cluster (van der Helm et al. in prep.). In Table 2.7 we give an overview of the application of the different modes. The code is publicly available in the AMUSE framework.

There are many other types of simulations involving stellar winds that could be done with the AMUSE framework and corresponding modes could be added to `stellar_wind.py`. It would be possible to add the mass and corresponding energy lost by stars to existing SPH particles. This would make it possible to run simulations of embedded stars with even lower resolution (higher SPH particle mass). However, this would result in unequal mass particles, which requires advanced treatment in the SPH codes. In the other extreme, since radiative transfer codes are available in AMUSE, it would be possible to add a mode that solves the radiative hydrodynamics of the wind and this would make detailed stellar wind simulations possible. In fact, such coupled simulations have been performed with AMUSE already (Wall et al., 2017, N. Clementel, private communication).

**Acknowledgements**

We thank N. Lützgendorf for testing and improving the simple wind mode, R. P. Kudritzki for his advice on the $v_\infty$ scaling law and F. I. Pelupessi for providing his code for the heating wind mode. This work was supported by the Netherlands Research Council NWO.
Table 2.7: An overview of the modes in `stellar_wind.py` and their suggested application domains.

<table>
<thead>
<tr>
<th>mode</th>
<th>section</th>
<th>description</th>
<th>application</th>
</tr>
</thead>
<tbody>
<tr>
<td>simple</td>
<td>2.2.2</td>
<td>Creates particles with a radial velocity given by the desired terminal wind velocity.</td>
<td>Detailed wind interaction simulations well outside the acceleration zone and past the sonic point.</td>
</tr>
<tr>
<td>accelerating</td>
<td>2.2.3</td>
<td>Similar to simple wind, but also accelerate particles near the star.</td>
<td>Detailed wind simulation near or inside the acceleration zone and near the sonic point.</td>
</tr>
<tr>
<td>heating</td>
<td>2.2.4</td>
<td>Does not give new particles a radial velocity, but instead adds internal energy to the particles.</td>
<td>Large scale, low resolution simulations of wind from embedded stars, including the effect of a supernova.</td>
</tr>
</tbody>
</table>
2.A. Appendix: Equations

In this appendix, we calculate the analytical predictions for a stationary, spherically symmetric wind which are used in stellar_wind.py. For these calculations, we assume that the mass-loss rate ($\dot{M}$) and the velocity as a function of radius ($v(r)$) are known and we define the acceleration

$$a(r) = \frac{dv}{dt} = \frac{dv}{dr} \frac{dr}{dt} = v(r) \frac{dv}{dr}.$$  \hspace{1cm} (2.4)

2.A.1. Radius as a function of time

To calculate the outer radius of a new wind shell, we need to know the radius as a function of time ($r(t)$) where the wind starts at the stellar surface, so $r(0) = R_*$. Since $v(r)$ is known, we can write

$$v(r(t)) = \frac{dr(t)}{dt},$$

$$dt = \frac{dr}{v(r)},$$

which is solved by,

$$t = \int_{R_*}^{r(t)} \frac{1}{v(r)} \, dr.$$  \hspace{1cm} (2.6)

In general this equation has to be solved numerically\(^{10}\) for $r(t)$, although for some velocity functions we can solve it analytically, for example if $v(r) = v_\infty$ then

$$t = \frac{1}{v_\infty} \int_{R_*}^{r(t)} dr = \frac{r(t) - R_*}{v_\infty},$$

$$r(t) = R_* + t \times v_\infty.$$  \hspace{1cm} (2.7)

2.A.2. New particle radii

When we create a new shell of particles, we want the density profile in the shell to match the density profile corresponding to the chosen velocity profile. To calculate that density profile, we first note that the mass-loss rate, $\dot{M}$ is related to the density and the velocity at any point of the wind via the equation of mass continuity,

$$\dot{M} = 4\pi r^2 \rho(r) v(r),$$  \hspace{1cm} (2.8)

where $\rho$ is the density of the wind. Because we assume that $\dot{M}$ and $v(r)$ are known, we can rewrite this as

$$\rho(r) = \frac{\dot{M}}{4\pi r^2 v(r)}.$$  \hspace{1cm} (2.9)

\(^{10}\) When solving the equations mentioned here numerically, we use the python library scipy (scipy.org). For integrals we use scipy.integrate.quad and for finding a root we use scipy.optimize.brentq. See docs.scipy.org for the details of these methods.
To generate the positions of new particles, we start with a cube filled with particle positions with a uniform density. In our code, this can be a simple grid or randomly generated positions. From that cube, we remove all particles that are not inside the desired shell to get a shell of particles with uniform density. After that, we shift the particle positions in the radial direction to get the desired density profile.

To find the new particle radius, we define the relative enclosed mass, $x$ as

$$x = \frac{\int_{R_p}^{r(t)} \pi r^2 \rho(r) dr}{\int_{R_s}^{r(t)} \pi r^2 \rho(r) dr},$$

where $r_p$ is the radius of the particle and $R_s$ and $r(t)$ are the inner and outer radius of the shell respectively. For the uniform density shell that was generated, this reduces to

$$x_u = \frac{\int_{R_p}^{r(t)} r^2 dr}{\int_{R_s}^{r(t)} r^2 dr} = \frac{r_p^3 - R_s^3}{r(t)^3 - R_s^3}.$$  

For the desired density profile based on a given velocity profile, we rewrite equation 2.10 in terms of $v$ using equation 2.9

$$x_v = \frac{\int_{R_p}^{r(t)} \frac{M}{\bar{v}(r)} dr}{\int_{R_s}^{r(t)} \frac{M}{\bar{v}(r)} dr} = \frac{\int_{R_p}^{r(t)} \frac{1}{\bar{v}(r)} dr}{\int_{R_s}^{r(t)} \frac{1}{\bar{v}(r)} dr}.$$  

We then set $x_u = x_v$ where $x_u$ is calculated with the old particle radius (of the generated uniform density shell). The last step is to solve equation 2.12 to get the new particle radius $r_p$. In general this equation has to be solved numerically, although for some velocity functions we can solve it analytically, for example if $v(r) = v_\infty$ then

$$x = \frac{\int_{R_p}^{r(t)} \frac{1}{v_\infty} dr}{\int_{R_s}^{r(t)} \frac{1}{v_\infty} dr} = \frac{\int_{R_p}^{r(t)} dr}{\int_{R_s}^{r(t)} dr} = \frac{r_p - R_s}{r(t) - R_s},$$

$$r_p = R_s + x(r(t) - R_s).$$

2.A.3. Gas pressure

To calculate the expected acceleration, $a_P(r)$ caused by the gradient of the gas pressure, $P(r)$ we assume a polytropic equation of state,

$$P = K \rho(r)^\gamma,$$

where $K$ is the polytropic constant and $\gamma = 5/3$ is the adiabatic index for a monoatomic ideal gas. Because $K$ is constant we can calculate it at the surface of the star and use that value for the entire wind. To calculate $P(R_s)$ we use

$$P(r) = (\gamma - 1) \rho(r) u,$$
where $u$ is the internal energy of the gas particles defined by
\[ u = \frac{3}{2} \frac{k_B T_s}{\mu}, \]  
(2.16)

where $k_B$ is the Boltzmann constant, $T_s$ is the temperature at the photosphere of the star and $\mu$ is the mean molecular weight of the gas particles. Combining equations 2.14 and 2.15 we get
\[ K = u(\gamma - 1)\rho(R_s)^{1-\gamma}. \]  
(2.17)

The acceleration caused by the gradient of the gas pressure is
\[ a_p(r) = -\frac{1}{\rho(r)} \frac{\partial P(r)}{\partial r}, \]  
(2.18)

which we can rewrite using equations 2.14 and 2.9
\[
\begin{align*}
    a_p(r) &= -\frac{K}{\rho(r)} \frac{\partial \rho^\gamma}{\partial r} \\
    &= -\frac{K}{\rho(r)} \gamma \rho(r)^{\gamma-1} \frac{\partial}{\partial r} \frac{M}{4\pi r^2 v(r)} \\
    &= K \gamma \rho(r)^{\gamma-1} \left( \frac{2}{r} + \frac{1}{v(r)} \frac{dv(r)}{dr} \right). 
\end{align*}
\]  
(2.19)

### 2.A.4. Density as a function of radius

In Sections 2.3.3 and 2.3.4 we calculate the density as a function of radius. For each radius $r$, we take six points in six directions ($+r$ and $-r$ along each axis $x$, $y$ and $z$) and calculate the SPH density at those points. Note that there does not need to be an SPH particle at that point to calculate the density. We then take the mean of these 6 densities to be the density at that radius. To calculate the radius with maximum density $r_{\rho_{\text{max}}}$, we calculate this for a grid of radii and select the radius with the largest density.
Gone with the wind: the impact of wind mass transfer on the orbital evolution of AGB binary systems

M. I. Saladino, O. R. Pols, E. van der Helm, I. Pelupessy, S. Portegies-Zwart

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**Abstract**

In low-mass binary systems, mass transfer is likely to occur via a slow and dense stellar wind when one of the stars is in the asymptotic giant branch (AGB) phase. Observations show that many binaries that have undergone AGB mass transfer have orbital periods of 1-10 yr, at odds with the predictions of binary population synthesis models. In this paper we investigate the mass-accretion efficiency and angular-momentum loss via wind mass transfer in AGB binary systems and we use these quantities to predict the evolution of the orbit. To do so, we perform 3D hydrodynamical simulations of the stellar wind lost by an AGB star in the time-dependent gravitational potential of a binary system, using the AMUSE framework. We approximate the thermal evolution of the gas by imposing a simple effective cooling balance and we vary the orbital separation and the velocity of the stellar wind. We find that for wind velocities higher than the relative orbital velocity of the system the flow is described by the Bondi-Hoyle-Lyttleton approximation and the angular-momentum loss is modest, which leads to an expansion of the orbit. On the other hand, for low wind velocities an accretion disk is formed around the companion and the accretion efficiency as well as the
angular-momentum loss are enhanced, implying that the orbit will shrink. We find that the transfer of angular momentum from the binary orbit to the outflowing gas occurs within a few orbital separations from the centre of mass of the binary. Our results suggest that the orbital evolution of AGB binaries can be predicted as a function of the ratio of the terminal wind velocity to the relative orbital velocity of the system, $v_\infty/v_{\text{orb}}$. Our results can provide insight into the puzzling orbital periods of post-AGB binaries. The results also suggest that the number of stars entering into the common-envelope phase will increase, which can have significant implications for the expected formation rates of the end products of low-mass binary evolution, such as cataclysmic binaries, type Ia supernovae, and double white-dwarf mergers.

### 3.1. Introduction

Depending on their evolutionary state and orbital separation, stars in binary systems can interact in various ways. One of the main interaction processes is the exchange of mass between the stars, which strongly affects the stellar masses, spins, and chemical compositions. Furthermore, as a consequence of mass transfer and the loss of mass and angular momentum from the system, significant changes in the orbital parameters can occur (e.g. van den Heuvel, 1994; Postnov & Yungelson, 2014). In relatively close binaries mass transfer usually occurs via Roche-lobe overflow (RLOF), while in wide binaries mass transfer by stellar winds can take place. Low- and intermediate-mass binary stars usually interact when the most evolved star reaches the red-giant phase or the asymptotic giant branch (AGB). This is a consequence of (1) the large stellar radius expansion during these evolution phases, resulting in a wide range of orbital periods for RLOF to occur, and (2) the strong stellar-wind mass loss of luminous red giants. During the final AGB phase the mass-loss rate is of the order of $10^{-7} - 10^{-4} \text{M}_\odot/\text{yr}$ (Höfner, 2015) and the velocities of the winds are very low (5–30 km/s), which allows efficient wind mass transfer even in very wide orbits.

Several classes of relatively unevolved stars show signs of having undergone such mass transfer in the form of enhanced abundances of carbon and $s$-process elements, which are the nucleosynthesis products of AGB stars. These include barium (Ba) and CH stars (Keenan, 1942; Bidelman & Keenan, 1951), extrinsic S stars (those in which the radioactive element technetium is absent, Smith & Lambert, 1988), and carbon-enhanced metal-poor stars enriched in $s$-process elements (CEMP-$s$ stars, Beers & Christlieb, 2005). These stars are thought to be the result of the evolution of low- and intermediate-mass binaries that have undergone mass transfer during the AGB phase of the more massive star onto the low-mass companion, which is the star we are currently observing. The erstwhile AGB star is now a cooling white dwarf (WD), which is in most cases invisible, but which reveals itself by inducing radial-velocity variations of the companion (McClure et al., 1980b; McClure, 1984; Lucatello et al.,
These systems have orbital periods that lie mostly in the range $10^2 - 10^4$ days and often show modest eccentricities (Jorissen et al., 1998, 2016; Hansen et al., 2016b). They have these orbital properties in common with many other types of binary systems that have interacted during a red-giant phase, such as post-AGB stars in binaries (van Winckel, 2003a), symbiotic binaries (Mikołajewska, 2012) and blue stragglers in old open clusters (Mathieu & Geller, 2015).

These orbital properties are puzzling when we consider the expected consequences of binary interaction during the AGB phase. Interaction via RLOF from a red giant or AGB star in many cases leads to unstable mass transfer (Hjellming & Webbink, 1987; Chen & Han, 2008), resulting in a common envelope (CE) phase that strongly decreases the size of the orbit (Paczynski, 1976). In addition, the orbit is expected to circularise due to tidal effects even before RLOF occurs (Zahn, 1977a; Verbunt & Phinney, 1995). In the case of wind interaction, if the gas escapes isotropically from the donor as is commonly assumed, a fraction of the material will be accreted via the Bondi-Hoyle-Lyttleton (Hoyle & Lyttleton, 1939; Bondi & Hoyle, 1944, hereinafter BHL) process and the rest of the material escapes, removing some angular momentum and widening the orbit. Taking into account these two main mass-transfer mechanisms, binary population synthesis models predict a gap in the orbital period distribution of post-mass-transfer binaries in the range 1–10 yr, and circular orbits below this gap (Pols et al., 2003; Izzard et al., 2010; Nie et al., 2012). However, the observed period distributions of the various types of post-AGB binaries discussed above show no sign of such a gap; in fact their periods appear to concentrate in the predicted gap.

This discrepancy points to shortcomings in our understanding of the evolution of binary systems containing red giants and AGB stars. Some of these shortcomings are related to the inadequacy of the BHL accretion scenario to describe the wind interaction process. The BHL description is a good approximation for wind accretion only when the outflow is fast compared to the orbital velocity. These conditions are not met for AGB winds, which are slow compared to the typical orbital velocities and have substantial density gradients. Furthermore, the slow wind velocities allow the escaping gas to interact with the binary and can produce non-isotropic outflows that carry away more angular momentum than in the isotropic case. The complicated interaction between the slow wind and the binary cannot easily be described by an analytical model, and hydrodynamical simulations are needed to study this process.

Several such hydrodynamical studies of the interaction between a red giant undergoing mass transfer via its wind with a companion star have been performed over the last two decades, covering a wide range of phenomena (Theuns & Jorissen, 1993; Theuns et al., 1996; Mastrodemos & Morris, 1998; Nagae et al., 2004; Mohamed & Podsiałowski, 2007; de Val-Borro et al., 2009, 2017; Huarte-Espinosa et al., 2013; Liu et al., 2017; Chen et al., 2017). Theuns & Jorissen (1993) and Theuns et al. (1996) were the first to carry out 3D smoothed-particle hydrodynamics (SPH) simulations of wind mass transfer, in which they showed that the morphology of the flow differs from BHL expectations. They also showed that the equa-
tion of state (EoS) used is important to determine the formation of an accretion disk around the companion. Mohamed & Podsiadlowski (2007) and Mohamed (2010) studied the problem of AGB mass transfer in very wide binaries using SPH simulations, in which they took into account the acceleration of the wind by radiation pressure on dust particles. They proposed a new mode of mass transfer, intermediate between RLOF and wind accretion, which they called wind Roche-lobe overflow (WRLOF). They showed that if the dust is formed outside the Roche lobe of the AGB star, wind material can be transferred efficiently through the inner Langrangian point, leading to much higher accretion rates than predicted by the BHL prescription.

The studies described above have helped to understand the physical processes involved in AGB wind interaction. However, only a few studies have focused on the implications for the orbital evolution of binary systems undergoing wind interaction. Given the discrepancy between the timescales that are accessible in simulations and binary evolution timescales, the computation of angular momentum loss is an important way to predict the change in the orbit. In the context of X-ray binaries, Brookshaw & Tavani (1993) developed the first numerical simulations to study the loss of angular momentum and its influence on the binary orbit, by means of ballistic calculations to model wind particles ejected by the mass-losing star. Similar calculations are presented in Hachisu et al. (1999) in the context of wide symbiotic binaries with a mass-losing red giant as potential progenitors of Type Ia supernovae. Both studies find that the specific angular-momentum loss depends on the orbital parameters and the wind ejection velocity. Although the ballistic treatment and its neglect of any wind acceleration mechanism appears to be adequate for fast winds, for low-velocity mass outflows a hydrodynamical treatment is needed. Jahanara et al. (2005) used a three-dimensional grid-based code to model binary systems with one star undergoing wind mass loss. They study the amount of angular momentum removed as a function of the wind speed at the Roche-lobe surface for different assumed mass-loss mechanisms. Their calculations show that for low wind velocities the specific angular-momentum loss is large (although less so than implied by the ballistic calculations mentioned above), whereas for high velocities the specific angular-momentum loss decreases to the value expected for a non-interacting, isotropic wind.

Recently, Chen et al. (2018) performed grid-based hydrodynamics simulations which also include radiative transfer in order to determine the orbital evolution of binary systems interacting via AGB wind mass transfer. For relatively short orbital periods they find that mass transfer will occur via WRLOF, leading to a shrinking of the orbit and to a possible merger of the components, whereas for wider separations the mass transfer process resembles the BHL scenario and the orbit tends to expand.

In this paper we try to bridge the previously discussed gap in the orbital period diagram by computing the angular-momentum loss and its effect on the orbit of a binary containing a mass-losing AGB star. To do so, we perform smoothed-particle hydrodynamical simulations including cooling of the gas to model the AGB wind. We present our results as a function of the ratio of the terminal wind velocity to the orbital velocity ($v_\infty/v_{\text{orb}}$). In section 3.2, we
briefly review the equations governing the orbital evolution of a binary system. In section 3.3 we describe the model we used and the numerical set-up for the cases we studied, and in section 3.4 we show the results. In section 3.5 we discuss the implications of our findings for the orbital evolution of binary systems. Finally in section 3.6 we conclude.

3.2. Angular momentum loss and mass accretion rate

The total orbital angular momentum of a binary system in a circular orbit is given by:

\[ J = \mu a^2 \Omega, \] (3.1)

where \( \mu = M_a M_d / (M_a + M_d) \) is the reduced mass of the system, \( M_a \) and \( M_d \) are the masses of the accretor and the donor star respectively, \( a \) the orbital separation of the system, and \( \Omega \) the angular velocity of the binary.

If the donor star is losing mass at a rate \( \dot{M}_d < 0 \) and mass transfer is non-conservative, the companion star will accrete a fraction \( \beta \) of the material and the rest will be lost, carrying away angular momentum from the system. The change in orbital angular momentum can be parametrised as:

\[ \dot{J} = \kappa_a a^2 \Omega \dot{M}_{\text{bin}}, \] (3.2)

where \( \dot{M}_{\text{bin}} = (1 - \beta) \dot{M}_d \) is the mass-loss rate from the system and \( \eta \) is the specific angular momentum lost in units of \( J/\mu \). Hence, the change in orbital separation for non-conservative mass transfer will be:

\[ \frac{\dot{a}}{a} = -2 \frac{\dot{M}_d}{\dot{M}_{\text{bin}}} \left[ 1 - \beta q - \eta(1 - \beta)(1 + q) - (1 - \beta) \frac{q}{2(1 + q)} \right], \] (3.3)

where \( q = M_d / M_a \) is the mass ratio of the stars. This equation can be solved analytically only for a few limiting cases. In the Jeans or fast wind mode, mass is assumed to leave the donor star in the form of fast and spherically symmetric wind. Since the speed of the wind is much larger than the orbital velocity of the system, the wind does not interact with the companion, escaping and taking away the specific orbital angular momentum of the donor \( \eta_{\text{iso}} = M_a^2 / M_{\text{bin}}^2 \). In the case that \( \beta = 0 \), the change in the orbit is \( \dot{a}/a = -\dot{M}_d/\dot{M}_{\text{bin}} \). This mass transfer mode leads to a widening of the orbit. The mass transfer efficiency \( \beta \) is often described in terms of the BHL analytical model, which can be used as a reference with which to compare the simulation results. In the framework of a binary system, the BHL accretion rate is given by:

\[ \dot{M}_{\text{BHL}} = \alpha_{\text{BHL}} \pi R_{\text{BHL}}^2 v_{\text{rel}} \rho, \] (3.4)

for a high-velocity wind and assuming a supersonic flow. Here, \( R_{\text{BHL}} = 2GM_a / v_{\text{rel}}^2 \) is the BHL accretion radius, \( v_{\text{rel}}^2 = v_w^2 + v_{\text{orb}}^2 \) is the relative wind velocity seen by the accretor (Theuns et al., 1996), \( \rho \) is the density at the position of the companion and \( \alpha_{\text{BHL}} \) is the efficiency parameter, of order unity, that physically represents the location of the stagnation point in
units of the accretion radius (Boffin, 2014). Using that \( \rho = \dot{M}_d / 4\pi a^2 \nu_w \) for a steady-state spherical wind and \( v_{\text{orb}}^2 = G(M_a + M_d)/a \), we can write the accretion efficiency in the BHL approximation as:

\[
\beta_{\text{BHL}} = \frac{\alpha_{\text{BHL}}}{(1 + q)^2} \frac{v_{\text{orb}}^4}{\nu_w(v_{\text{w}}^2 + v_{\text{orb}}^2)^{3/2}}.
\]

Theoretical considerations and numerical simulations of BHL accretion (e.g. see Edgar, 2004; Matsuda et al., 2015) indicate that for a uniform flow, \( \alpha_{\text{BHL}} \) has a value of about 0.8. In applications to wind accretion Eq. 3.5 is sometimes divided by a factor of two (e.g. Boffin & Jorissen, 1988; Hurley et al., 2002; Abate et al., 2013). The corresponding value of \( \alpha_{\text{BHL}} \) is then higher by factor of two (e.g. Abate et al., 2015b, use a standard value of \( \alpha_{\text{BHL}} = 1.5 \) in their models).

In realistic situations, and especially in the case of AGB wind mass transfer in binaries, the values of \( \beta \) and \( \eta \) cannot be expressed in analytical form and have to be derived from, for example, hydrodynamical simulations. This is the purpose of this paper. We use the fast wind mode and the BHL accretion efficiency as a reference with which to compare our simulation results.

### 3.3. Method

#### 3.3.1. Stellar wind

The mechanism driving the slow winds of AGB stars is not well understood. The pulsation-enhanced dust-driven outflow scenario describes it in two stages (Höfner, 2015): in the first stage, pulsations and/or convection cause shock waves which send stellar matter on near-ballistic trajectories, pushing dust-free gas up to a maximum height of a few stellar radii. At this distance, the temperature has dropped enough (~1500 K) to allow condensation of gas into dust. In the second stage, the dust-gas mixture is accelerated beyond the escape velocity due to radiation pressure onto the dust. Simulating this process involves several physical mechanisms for which hydrodynamical and radiative transfer codes have to be invoked. This goes beyond the scope of this work. Instead we use a simple approximation to model the velocity profile of the AGB wind. In order to do so, we use the STELLAR\_WIND.py (van der Helm et al., 2019) routine of the AMUSE\(^2\) framework (Portegies Zwart et al., 2013; Pelupessy et al., 2013; Portegies Zwart et al., 2009). Below we briefly explain how this routine works, for a detailed description we refer the reader to van der Helm et al. (2019).

STELLAR\_WIND.py has three different modes to simulate stellar wind. The mode which best approximates wind coming from an AGB star is the accelerating wind mode and it works

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\(^1\)In this paper we will refer to this equation as the mass accretion rate predicted by BHL, although for cases in which \( v_r \gg c \), where \( c \) is the sound speed, this case corresponds to the Hoyle-Lyttleton approximation.

\(^2\)http://amusecode.org/
3.3 Method

in the following way. The user provides the stellar parameters, such as mass, effective temperature, radius and mass loss rate, which can be derived from one of the stellar evolution codes currently in AMUSE. The initial and terminal velocities of the wind, \( v_{\text{init}} \) and \( v_{\infty} \), are user input parameters. SPH particles are injected into a shell that extends from the radius of the star to a maximum radius defined by the position that previously released particles have reached after a timestep \( \Delta t \). The positions at which these particles are injected correspond to those derived from the density profile of the wind \( \rho(r) \). Thus, it is assumed that the velocity and density profiles of the wind, which are related by the mass conservation equation, are known. The wind acceleration function \( a_w(r) \) is obtained from the equation of motion:

\[
v \frac{dv}{dr} = a_w(r) - \frac{GM_*}{r^2} + c^2 \left( \frac{2}{r} + \frac{1}{v} \frac{dv}{dr} \right),
\]

where \( v = v(r) \) is the predefined velocity profile for the wind, and \( c \) is the adiabatic sound speed in the wind. The second term on the right-hand side describes the deceleration by the gravity of the star and the last term represents the gas-pressure acceleration. By computing \( a_w \) in this way and applying this acceleration to the gas particles, we guarantee that the gas follows the predefined density profile. If \( v_{\text{init}} = v_{\infty} \), then the term on the left hand side will be zero and the acceleration of the wind balances the gravity of the star and the gas pressure gradient. We note that Eq. 3.6 assumes an adiabatic equation of state (EoS) and ignores cooling of the gas. We have verified that ignoring the cooling term in equation 3.6 does not change the physical properties of the wind such as density and velocity. The module STELLAR_WIND.PY offers a variety of functions that resemble different types of accelerating winds. Although the most appropriate prescription would be to choose a function with an accelerating region, for the present work we have chosen a constant velocity profile, i.e. \( v_{\text{init}} = v_{\infty} \), for two reasons: to reduce the computational time and to be able to compare our results to other work where similar assumptions have been made.

3.3.2. Cooling of the gas

The internal energy change of the gas plays an important role when modelling the interaction of a star undergoing mass loss with a companion star. Theuns & Jorissen (1993) found that the formation of an accretion disk around the companion star depends on the EoS. In their study they modelled two cases, one in which the EoS was adiabatic (\( \gamma = 5/3 \)) and one in which the EoS was isothermal (\( \gamma = 1 \)). In the first case, no accretion disk was formed: when material gets close to the companion star, it is compressed by the gravity of the accretor, increasing the temperature and enhancing the pressure, expanding it in the vertical direction. In the isothermal case the pressure does not increase too much, permitting material to stay confined in an accretion disk. Although illustrative, these cases are not realistic and a more physical prescription for cooling of the gas is needed.

Proper modelling of cooling of the gas requires the implementation of radiative transfer codes. For simplicity, in this study, we use a modified approximation for modelling the change
in temperature in the atmospheres of Mira-like stars based on Bowen (1988). In his work, the cooling rate $\dot{Q}$ is given by:

$$\dot{Q} = \frac{3k}{2\mu m_u} \frac{(T - T_{\text{eq}})\rho}{C} + \dot{Q}_{\text{rad}}. \quad (3.7)$$

The first term in Eq. 3.7 assumes that cooling comes from gas radiating away its thermal energy trying to reach the equilibrium temperature given by the Eddington approximation for a grey spherical atmosphere (Chandrasekhar, 1934):

$$T_{\text{eq}}^4 = \frac{1}{2} T_{\text{eff}}^4 \left[ 1 - \left(1 - \frac{R^2}{r^2}\right)^{1/2} \right] + \frac{3}{2} \int_r^{\infty} \frac{R^2}{r^2} (\kappa_g + \kappa_d) \rho dr \quad (3.8)$$

where $T_{\text{eff}}$ is the effective temperature of the star, $\kappa_g$ and $\kappa_d$ the opacities of the gas and dust respectively, $R$ is the radius of the star, $r$ the distance from the star and $\rho$ the density of the gas. The first term, $W = \frac{1}{2} [1 - (1 - R^2/r^2)^{1/2}]$, corresponds to the geometrical dilution factor, and the term in the integral plays the role of the optical depth in plane-parallel geometry. The constant $C$ in equation 3.7 is a parameter reflecting the radiative equilibrium timescale. Following Bowen (1988), we adopt a value of $C = 10^{-5}$ g s cm$^{-3}$.

The second term in equation 3.7 corresponds to radiation losses for temperatures above $\sim 7000$ K, where the excitation of the $n = 2$ level of neutral hydrogen is mainly responsible for the energy loss (Spitzer, 1978). In this work however, we use an updated cooling rate prescription for high temperatures ($\log T \geq 3.8$ K) by Schure et al. (2009). The advantage of this prescription is that contributions to the cooling rate come not only from neutral hydrogen but individual elements are taken into account according to the abundance requirements. We use the abundances for solar metallicity given by Anders & Grevesse (1989). Then, the second term in equation 3.7 becomes:

$$\dot{Q}_{\text{rad}} = \frac{\Lambda_{\text{hd}} n_H^2}{\rho} \quad (3.9)$$

with $\Lambda_{\text{hd}}$ interpolated between the values given in table 2 of Schure et al. (2009) and $n_H = X\rho/m_H$.

For low temperatures the first term of equation 3.7 is the dominant term. If this term is not taken into account the internal energy of the particles reaches very low values leading to unphysical temperatures. We note that in equation 3.8, the optical depth at large distances from the star will be very small regardless of the opacity values, thus for distant regions the equilibrium temperature will be mainly determined by the geometrical dilution factor $W$. For this reason, and since the opacities are only used to calculate the equilibrium temperature at given radius, the opacities in equation 3.8 are taken constant during the simulations with values of $\kappa_g = 2 \times 10^{-4}$ cm$^2$ g$^{-1}$ and $\kappa_d = 5$ cm$^2$ g$^{-1}$ (Bowen, 1988).
3.3 Method

3.3.3. Computational method

To calculate self-consistently the 3D gas dynamics of the wind in the evolving potential of the binary, we used the SPH code fi (Hernquist & Katz, 1989; Gerritsen & Icke, 1997; Pelupessy et al., 2004) with artificial viscosity parameters as shown in Table 3.1. The AGB star was modelled as a point mass, whereas the companion star was modelled as a sink particle with radius corresponding to a fraction of its Roche Lobe radius \( R_{L,a} \). In order to model the dynamics of the stars, we coupled the SPH code with the N-body code HUAYNO (Pelupessy et al., 2012) using the BRIDGE module (Fujii et al., 2007) in AMUSE. The N-body code is used to evolve the stellar orbits, and to determine the time-dependent gravitational potential in which the gas dynamics is calculated. Since the interest in this work is to study the evolution of the gas under the influence of the binary system, the BRIDGE module allows the gas to feel the gravitational field of the stars, however the stars do not feel the gravitational field of the gas; also self-gravity of the gas is neglected. This is a fair assumption given that the total mass of the gas particles in the simulation \( 4 \times 10^5 M_\odot \) is very small compared to the binary mass \( 4.5 M_\odot \). This assumption does not have a significant effect on the evolution of the gas, as was verified in a test simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_d )</td>
<td>3 ( M_\odot )</td>
<td>Mass of donor star</td>
</tr>
<tr>
<td>( M_a )</td>
<td>1.5 ( M_\odot )</td>
<td>Mass of accretor</td>
</tr>
<tr>
<td>( R_d )</td>
<td>200 ( R_\odot )</td>
<td>Inner boundary for SPH particles about donor star</td>
</tr>
<tr>
<td>( \dot{M}_d )</td>
<td>( 10^{-6} M_\odot \text{ yr}^{-1} )</td>
<td>Mass loss rate</td>
</tr>
<tr>
<td>( \alpha_{\text{SPH}} )</td>
<td>0.5</td>
<td>Artificial viscosity parameter</td>
</tr>
<tr>
<td>( \beta_{\text{SPH}} )</td>
<td>1</td>
<td>Artificial viscosity parameter</td>
</tr>
</tbody>
</table>

3.3.3.1. Binary set up

The stellar parameters of our simulated systems are chosen to match those presented in Theuns & Jorissen (1993). This will allow a direct comparison between our results and their work. The mass of the AGB star is \( M_d = 3 M_\odot \) and that of the low-mass companion is \( M_a = 1.5 M_\odot \). The radius of the primary star is \( R_d = 200 R_\odot \) and for the secondary we assume a sink radius equal to \( R_a = 0.1 R_{L,a} \). The orbit is circular and the separation of the stars is \( a = 3 \text{ AU} \) in our test simulations. The mass-loss rate \( \dot{M}_d = 10^{-6} M_\odot \text{ yr}^{-1} \) is constant during the simulation; the terminal velocity of the wind is \( v_\infty = 15 \text{ km s}^{-1} \) and the gas is assumed to be monoatomic \( \gamma = 5/3 \) with constant mean molecular weight \( \mu = 1.29 \) corresponding to an atomic gas with solar chemical composition \( (X = 0.707, Y = 0.274, Z = 0.019) \) (Anders & Grevesse, 1989). The effective temperature of the AGB star, which is also the initial temperature given to the gas
particles, is \( T_{\text{eff}} = 2430 \) K for all models except the isothermal test case, for which \( T_{\text{eff}} = 4050 \) K in order to compare to Theuns et al. (1996). We assume the AGB star to be non-rotating as a result of its history of expansion after the main sequence, combined with angular momentum loss resulting from earlier mass loss on the RGB and AGB. Thus, we ignore the possibility of subsequent spin-up by tidal interaction (see Sect. 3.5.1.4 for a discussion). The stars are placed in such a way that the centre of mass is located at the origin of the system of reference. The setup for the other simulations is the same except for the values of the separations and the terminal velocities of the wind. We also ran a few simulations with a smaller sink radius (see Table 3.2). Each simulation was run for eight orbital periods.

### 3.3.4. Testing the method

In this subsection, we compare our simulations (models T1, T2, T3 in Table 3.2) with those of Theuns & Jorissen (1993). The top panel of Figure 3.1 shows the velocity field of the gas in the \( y = 0 \) plane, perpendicular to the orbital plane, for models T1, T2 and T3 after 7.5 orbits. The colourmap shows the temperature of the gas in the \( y = 0 \) plane. The donor star is located at \( x = 1 \) AU, \( z = 0 \) AU and the companion star at \( x = -2 \) AU, \( z = 0 \) AU. Gas expands radially away from the primary star. Similar to Theuns & Jorissen (1993), in the adiabatic case (model T1) we observe a bow shock close to the companion star where high temperatures are reached. The high-temperature regions are located symmetrically above and below the star in this plane. The gravitational compression of the gas by the companion also enhances the temperature in this region, expanding the gas in the vertical direction and preventing it from creating an accretion disk. On the other hand, when an isothermal equation of state is used (model T2), an accretion disk is formed around the companion. In addition, two spiral arms are observed around both systems similar to those observed by Theuns & Jorissen (1993). These features can be seen in the bottom panel of Figure 3.1, where density in the orbital plane (\( z = 0 \)) is shown. Model T3 shows that using an EoS that includes cooling of the gas (Equation 3.7) gives a similar outflow structure as in the isothermal case, including the formation of an accretion disk. However, in this model there is an increase in temperature where the spiral shocks are formed. These effects are also found in our science simulations, which we describe in detail in section 3.4.2.
Table 3.2: Simulation parameters. The values of $M_a$, $M_d$ and $\dot{M}_d$ are the same for all simulations.

<table>
<thead>
<tr>
<th></th>
<th>#</th>
<th>$a$</th>
<th>$v_\infty$</th>
<th>$\dot{v}<em>\infty/\dot{v}</em>{\text{orb}}$</th>
<th>$R_a$</th>
<th>Cooling</th>
<th>$m_g$</th>
<th>$\bar{h}$</th>
<th>$n$</th>
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<tr>
<td>Test models</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>T1</td>
<td>3</td>
<td>15</td>
<td>0.41</td>
<td>20.8</td>
<td></td>
<td>Adiabatic</td>
<td>$10^{-10}$</td>
<td>0.24 (0.16–0.40)</td>
<td>$2 \times 10^5$</td>
</tr>
<tr>
<td>T2</td>
<td>3</td>
<td>15</td>
<td>0.41</td>
<td>20.8</td>
<td></td>
<td>Isothermal</td>
<td>$10^{-10}$</td>
<td>0.12 (0.01–0.36)</td>
<td>$2 \times 10^5$</td>
</tr>
<tr>
<td>T3</td>
<td>3</td>
<td>15</td>
<td>0.41</td>
<td>20.8</td>
<td></td>
<td>Bowen + Schure</td>
<td>$10^{-10}$</td>
<td>0.13 (0.01–0.37)</td>
<td>$2 \times 10^5$</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>R1</td>
<td>5</td>
<td>15</td>
<td>0.53</td>
<td>34.7</td>
<td></td>
<td>Bowen + Schure</td>
<td>$10^{-11}$</td>
<td>0.13 (0.01–0.25)</td>
<td>$3.3 \times 10^6$</td>
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<tr>
<td>R2</td>
<td>5</td>
<td>15</td>
<td>0.53</td>
<td>34.7</td>
<td></td>
<td>Bowen + Schure</td>
<td>$4 \times 10^{-11}$</td>
<td>0.20 (0.02–0.39)</td>
<td>$1.1 \times 10^6$</td>
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<tr>
<td>R3</td>
<td>5</td>
<td>15</td>
<td>0.53</td>
<td>34.7</td>
<td></td>
<td>Bowen + Schure</td>
<td>$8 \times 10^{-11}$</td>
<td>0.26 (0.03–0.49)</td>
<td>$5.6 \times 10^5$</td>
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<tr>
<td>V10a5</td>
<td>5</td>
<td>10</td>
<td>0.35</td>
<td>34.7</td>
<td></td>
<td>Bowen + Schure</td>
<td>$2 \times 10^{-11}$</td>
<td>0.17 (0.06–0.29)</td>
<td>$2.2 \times 10^6$</td>
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<tr>
<td>V15a5</td>
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<td>0.53</td>
<td>34.7</td>
<td></td>
<td>Bowen + Schure</td>
<td>$2 \times 10^{-11}$</td>
<td>0.16 (0.01–0.32)</td>
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<tr>
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<td>30</td>
<td>1.06</td>
<td>34.7</td>
<td></td>
<td>Bowen + Schure</td>
<td>$2 \times 10^{-11}$</td>
<td>0.29 (0.14–0.45)</td>
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<tr>
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<td>5</td>
<td>150</td>
<td>5.31</td>
<td>34.7</td>
<td></td>
<td>Bowen + Schure</td>
<td>$2 \times 10^{-11}$</td>
<td>0.50 (0.27–0.78)</td>
<td>$2.2 \times 10^6$</td>
</tr>
<tr>
<td>V19a3</td>
<td>3</td>
<td>19.33</td>
<td>0.53</td>
<td>20.8</td>
<td>Bowen + Schure</td>
<td>$2 \times 10^{-11}$</td>
<td>0.07 (0.004–0.22)</td>
<td>$10^6$</td>
<td></td>
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<tr>
<td>Test sink radius</td>
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<td></td>
<td></td>
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<tr>
<td>V10a5s2</td>
<td>5</td>
<td>10</td>
<td>0.35</td>
<td>20.8</td>
<td></td>
<td>Bowen + Schure</td>
<td>$8 \times 10^{-11}$</td>
<td>0.14 (0.01–0.43)</td>
<td>$5.6 \times 10^5$</td>
</tr>
<tr>
<td>V15a5s2</td>
<td>5</td>
<td>15</td>
<td>0.53</td>
<td>20.8</td>
<td></td>
<td>Bowen + Schure</td>
<td>$2 \times 10^{-11}$</td>
<td>0.08 (0.004–0.28)</td>
<td>$2.2 \times 10^6$</td>
</tr>
<tr>
<td>V30a5s2</td>
<td>5</td>
<td>30</td>
<td>1.06</td>
<td>20.8</td>
<td></td>
<td>Bowen + Schure</td>
<td>$2 \times 10^{-11}$</td>
<td>0.28 (0.14–0.45)</td>
<td>$2.2 \times 10^6$</td>
</tr>
</tbody>
</table>

Notes: The second column corresponds to the name of the model. $a$ is the initial orbital separation of the system in AU. $v_\infty$ is the assumed terminal velocity of the wind in km s$^{-1}$. The ratio $\dot{v}_\infty/\dot{v}_{\text{orb}}$ is the ratio of the terminal velocity to the orbital velocity of the system. $R_a$ is the radius of the sink in $R_\odot$. Column 7 specifies the EoS for the model. $m_g$ corresponds to the mass of the SPH particles in units of $M_\odot$. $\bar{h}$ corresponds to the mean smoothing length of the SPH particles within radius $a$ from the centre of mass of the system in units of AU. The numbers brackets correspond to the 95% confidence interval, i.e. 2.5% of the particles have values below the lower limit and 2.5% particles have values above the upper limit of the given interval. The final column gives the total number of particles $n$ generated at the end of the simulation.
3.3.5. Mass accretion rate and angular momentum loss

We measure the mass accretion rate by adding the masses $m_g$ of the gas particles that entered the sink during each timestep $\delta t$ in the simulation. Due to the discreteness of the SPH particles, the resulting accretion rates will be subject to shot noise. This can be seen as fluctuations in the mass accreted as a function of time. In order to suppress statistical errors in the results, we average the mass accretion rate over longer intervals of time.

Measuring the angular momentum loss rate from the system is more complicated. An advantage of SPH codes is that they conserve angular momentum extremely well compared to Eulerian schemes (Price, 2012). One question that arises is at which distance an SPH particle is no longer influenced by the gravitational potential of the binary system. In order to determine this, we construct spherical shells at various radii $r_S$ from the centre of mass of the binary. In the same way as for the accretion rate, we compute the net mass-loss rate through such a surface. Each particle $i$ carries angular momentum $\vec{J}_i$, of which we add up the components $J_{z,i}$ along the $z$-axis, perpendicular to the orbital plane. We compute the specific angular momentum of the outflowing mass as follows,

$$j_{\text{loss}} = \frac{\sum_i^N J_{z,i}}{\Delta N m_g},$$

with $\Delta N$ the number of particles that crossed the shell during a timestep $\Delta t$. Similar to the accretion rate, we average the mass and angular-momentum loss over longer intervals of time to suppress statistical fluctuations. We also verified that, once the simulation has reached a quasi-steady state, the orthogonal ($x$ and $y$) components of the angular momentum-loss are negligible.

In Figure 3.2 we show the resulting average mass and angular momentum-loss per orbit in our standard model V15a5 (see Table 3.2) as a function of time for different shell radii $r_S$. After several orbits, both quantities become approximately constant in time and almost independent of the chosen shell radius. This steady state is reached later for larger radii, corresponding to the longer travel time of the wind particles to reach this distance from the donor star. The near-constancy of the angular-momentum loss rate as a function of $r_S$ indicates that the torque transferring angular momentum from the orbit to the escaping gas operates inside a few times the orbital separation $a$. Beyond $r_S \approx 10$ AU ($\approx 2a$) the transfer of angular momentum to the gas is essentially complete, and angular momentum is simply advected outwards with the flow. We therefore adopt the the mass and angular momentum loss values measured at $r_S = 3a$, which reach a steady state after about 4 orbits in model V15a5. The time taken to reach the quasi-steady state depends on the terminal velocity of the wind, being shorter for higher velocity.

Once a simulation has reached a quasi-steady state, we compute the mass accretion efficiency $\beta = \dot{M}_a/\dot{M}_d$ from the average mass accretion rate over the remaining $N$ orbits in the simulation. Similarly, we average the angular momentum loss (Eq. 3.10) over the last $N$
3.3 Method

**Figure 3.1:** Top: Flow structure in the \( y = 0 \) plane for different EoS after 7.5 orbits. The colourmap shows the temperature at \( y = 0 \). The white arrow on top of the velocity field plots corresponds to the magnitude of a 50 km s\(^{-1}\) velocity vector. In the three images we can observe the wind leaving the donor star radially at \( x = 1 \) AU. The accretor star is located at \( x = 2 \) AU. For model T1, the temperatures reached in the region near the companion star are very high preventing material to settle into an accretion disk. Bottom: The gas density in the orbital plane (\( z = 0 \)) is shown. The accretor is located at \( x = -2, y = 0 \), and the AGB star at \( x = 1, y = 0 \). In the middle panel we see that for the isothermal EoS (T2) gas in the vicinity of the accretor settles into an accretion disk. When cooling is included (T3), an accretion disk around the companion is also formed.
orbits to compute the value of $\eta$, i.e. the specific angular momentum of the lost mass in units of $a^2\Omega$.

### 3.4. Results

We performed 13 different simulations, listed in Table 3.2, for which we discuss the results in the following subsections. The first three simulations (T1-T3) are performed to test the EoS (see Sect. 3.3.4). The following three simulations (R1-R3) correspond to the test of different SPH resolutions, compared to simulation V15a5 which we consider the standard model. For this model we chose an orbital separation of 5 AU and a terminal velocity of the wind of 15 km s$^{-1}$. V10a5, V30a5 and V150a5 correspond to simulations with the same orbital separation but different velocities of the wind, either higher or lower than for the standard case. V10a5s2, V15a5s2 and V30a5s2 correspond to test simulations in which we study the effect of assuming a smaller sink radius ($R_a = 0.06R_{\ast,a}$) on the mass-accretion rate and radius of the accretion disk. It should be noted that simulation V10a5s2 was performed at a lower resolution than the others. Finally in V19a3 we change the orbital separation to 3 AU, keeping the same ratio of the velocity of the wind to the orbital velocity of the system as in V15a5.

#### 3.4.1. Convergence test

Choosing the best resolution for SPH simulations is not simple and depends on the physical process under investigation. To determine the optimal resolution within a reasonable computational time, we performed four simulations of our standard model in which we vary the mass of the SPH particles $m_g$ (R1, R2, R3 and V15a5 in Table 3.2). We note that a different $m_g$ also implies a change in the average smoothing length $\bar{h}$ of the SPH particles. Table 3.2 shows the value of $\bar{h}$ within a radius $a$ of the centre of mass of the binary, and the 95% confidence interval of smoothing-length values within this radius. The total number of particles $n$ generated during one simulation is given by $n = \dot{M}_g t_{\text{end}}/m_g$. Thus, the number of particles generated for the lowest resolution (largest $m_g$) is $n = 5.6 \times 10^5$ and $n = 3.3 \times 10^6$ for the highest resolution (smallest $m_g$). This last simulation ran only for 6 orbital periods.

Since the interest of this study is to obtain numerical estimates for the average mass-accretion efficiency and average angular-momentum loss during the simulation, these were the quantities we used to check for convergence. Figure 3.3 shows the values obtained for these quantities (as explained in Section 3.3.5) as a function of the mass of the SPH particles $m_g$. The error bars in this figure correspond to five times the standard error of the mean $\sigma_m$, which was estimated by means of $\sigma_m^2 = 1/N \sum_{i=1}^{N} \sigma_{m,\text{orb},i}^2$, where $\sigma_{m,\text{orb}}$ is the standard error
3.4 Results

**Figure 3.2:** Top: Average mass loss per orbit for the standard model V15a5 measured at different radii from the centre of mass of the binary as indicated by different colours. Bottom: Corresponding angular-momentum loss. The black thick curve for \( r_S = 15 \) AU corresponds to the radius at which we measure \( J_{\text{bin}} \).
of the mean per orbit\(^3\) and \(N\) the number of orbits during which the quasi-steady state has been reached. Since the number of timesteps over which the average is taken is large, the standard error turns out to be very small. This is the reason why we chose to plot five times the value of \(\sigma_m\) in Figure 3.3. Because the simulation with the highest resolution (smallest \(m_g\)) only ran for 6 orbital periods, and the system reaches the quasi-steady state after 4 orbits, the error in the mean is larger than for the other simulations.

One thing that can be seen from Figure 3.3 is that, even though by only a small amount, the accretion efficiency increases with increasing resolution, contrary to what Theuns et al. (1996) observed. However, we note that in their work the accretion rate was computed by setting an accretion radius proportional to the resolution of the SPH particles (in terms of the smoothing length \(h\)), whereas in our case the radius of the sink has the same constant value for all our models. On the other hand, the angular momentum per unit mass lost by the system is approximately independent of the resolution.

We also find that the time variability in the mass-accretion rate (described in Sect. 3.4.2.3) increases with increasing resolution. It is not clear why this takes place, but we should bear in mind that this variability could also be an artefact of the numerical method. As will be discussed in Sect. 3.4.2.3 there appears to be a correlation between the accretion disk mass and the mass-accretion rate. Given that the results of SPH simulations for accretion disks depend on the formulation of artificial viscosity, we performed a test simulation with the lowest resolution (\(m_g = 8 \times 10^{-11}\)) adopting artificial viscosity parameters in \(f_l\) equal to those used by Wijnen et al. (2016) for their protoplanetary disk models (\(\alpha_{\text{SPH}} = 0.1\) and \(\beta_{\text{SPH}} = 1\)). We find almost the same value for the average mass-accretion rate (\(\beta = 0.223 \pm 0.002\), versus \(\beta = 0.221 \pm 0.002\) for simulation R3). With this alternative prescription of artificial viscosity we find similar variability in the mass-accretion rate.

In order to guarantee reliable results within a reasonable computational time, the chosen resolution for the other simulations is the one in which the mass of the individual SPH particles is \(m_g = 2 \times 10^{-11} \, M_\odot\). For this simulation, the value of \(\beta\) differs by 9% compared to the value obtained for the simulation with highest resolution (R1). This is indicative of the numerical accuracy of the accretion rates obtained in our simulation. By contrast, the angular-momentum loss rates (\(\eta\) values) we measure differ by <0.4% between simulations at different resolution.

\(^3\)The standard error of the mean per orbit is:

\[ \sigma_{m,\text{orb}} = \frac{1}{N_p} \sqrt{\sum_{i=1}^{N_p} (\dot{m}_i - \langle \dot{m} \rangle)^2}, \]

where \(N_p\) is the number of timesteps \((t_1, t_2, ..., t_p)\) into which the orbit is divided, \(\dot{m}_i\) the mass accretion rate at time \(t_i\) and \(\langle \dot{m} \rangle\) the average mass accretion rate per orbit.
3.4 Results

Figure 3.3: Mean values of the $\beta$ and $\eta$ parameters, relating to mass and angular momentum loss, as a function of the mass of the SPH particles used to test different resolutions for the simulations. The error bars correspond to the overall standard error of the mean $\sigma_m$ multiplied by a factor of 5 for better appreciation.
### Table 3.3: Simulation results and orbital evolution parameters

<table>
<thead>
<tr>
<th>Simulation</th>
<th>( N )</th>
<th>( \beta_{\text{sink}} )</th>
<th>( \beta_{0.4R_L} )</th>
<th>( \beta_{\text{BHL}} )</th>
<th>( \eta )</th>
<th>Disk</th>
<th>( R_{\text{disk}} )</th>
<th>( M_{\text{disk}} )</th>
<th>( \dot{a}/a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>4</td>
<td>0.0291</td>
<td>0.0183</td>
<td>0.21</td>
<td>0.452</td>
<td>No</td>
<td>-</td>
<td>-</td>
<td>(-4.65 \times 10^{-7})</td>
</tr>
<tr>
<td>T2</td>
<td>4</td>
<td>0.289</td>
<td>0.348</td>
<td>0.21</td>
<td>0.623</td>
<td>Yes</td>
<td>( \approx 0.37 ) ( \approx 1.3 \times 10^{-6} )</td>
<td>(-7.54 \times 10^{-7})</td>
<td></td>
</tr>
<tr>
<td>T3</td>
<td>4</td>
<td>0.309</td>
<td>0.341</td>
<td>0.21</td>
<td>0.681</td>
<td>Yes</td>
<td>( \approx 0.37 ) ( \approx 1.0 \times 10^{-6} )</td>
<td>(-8.32 \times 10^{-7})</td>
<td></td>
</tr>
<tr>
<td>V10a5</td>
<td>4</td>
<td>0.3879</td>
<td>0.366</td>
<td>0.26</td>
<td>0.641</td>
<td>Yes</td>
<td>( \approx 0.40 ) ( \approx 2.4 \times 10^{-7} )</td>
<td>(-7.70 \times 10^{-7})</td>
<td></td>
</tr>
<tr>
<td>V10a5s2</td>
<td>4</td>
<td>0.344</td>
<td>0.388</td>
<td>0.26</td>
<td>0.609</td>
<td>Yes</td>
<td>( \approx 0.40 ) ( \approx 2.8 \times 10^{-6} )</td>
<td>(-7.33 \times 10^{-7})</td>
<td></td>
</tr>
<tr>
<td>V15a5</td>
<td>4</td>
<td>0.235</td>
<td>0.231</td>
<td>0.14</td>
<td>0.525</td>
<td>Yes</td>
<td>( \approx 0.47 ) ( \approx 3.6 \times 10^{-7} )</td>
<td>(-6.20 \times 10^{-7})</td>
<td></td>
</tr>
<tr>
<td>V15as2</td>
<td>4</td>
<td>0.1563</td>
<td>0.237</td>
<td>0.14</td>
<td>0.5315</td>
<td>Yes</td>
<td>( \approx 0.56 ) ( \approx 3.3 \times 10^{-6} )</td>
<td>(-6.30 \times 10^{-7})</td>
<td></td>
</tr>
<tr>
<td>V19a3</td>
<td>4</td>
<td>0.260</td>
<td>0.316</td>
<td>0.14</td>
<td>0.544</td>
<td>Yes</td>
<td>( \approx 0.30 ) ( \approx 1.3 \times 10^{-6} )</td>
<td>(-6.51 \times 10^{-7})</td>
<td></td>
</tr>
<tr>
<td>V30a5</td>
<td>6</td>
<td>0.039</td>
<td>0.039</td>
<td>0.037</td>
<td>0.1849</td>
<td>No</td>
<td>-</td>
<td>-</td>
<td>(6.06 \times 10^{-8})</td>
</tr>
<tr>
<td>V30a5s2</td>
<td>6</td>
<td>0.0353</td>
<td>0.0353</td>
<td>0.037</td>
<td>0.1862</td>
<td>No</td>
<td>-</td>
<td>-</td>
<td>(4.35 \times 10^{-8})</td>
</tr>
<tr>
<td>V150a5</td>
<td>7</td>
<td>(3.46 \times 10^{-4})</td>
<td>(3.45 \times 10^{-4})</td>
<td>(1.32 \times 10^{-4})</td>
<td>(0.1116)</td>
<td>No</td>
<td>-</td>
<td>-</td>
<td>(2.21 \times 10^{-7})</td>
</tr>
</tbody>
</table>

**Notes:** 
- \( N \) is the number of orbits over which the values are averaged after the quasi-steady state is reached. \( \beta_{\text{sink}} \) is the averaged mass-accretion efficiency measured from the net sink inflow. \( \beta_{0.4R_L} \) is the averaged mass-accretion efficiency measured from the net inflow into a shell of radius 0.4\( R_L \), i.e. it includes the mass inflow of the sink and of the accretion disk, if present. \( \beta_{\text{BHL}} \) corresponds to the mass accretion efficiency as predicted by BHL (Eq. 3.5) with \( \alpha_{\text{BHL}} = 1 \). \( \eta \) is the averaged specific angular momentum of material lost. \( R_{\text{disk}} \) is the approximate radius of the disk in AU after eight orbital periods, except for model V10a5, where the radius corresponds to 2 orbital periods. \( M_{\text{disk}} \) is the mass within \( R_{\text{disk}} \). \( \dot{a}/a \) is the relative rate of change of the orbit as derived from Equation 3.3 in units of yr\(^{-1}\). Positive sign means the orbit is expanding, whereas negative sign means that the orbit is shrinking.
3.4 Results

3.4.2. Summary of model results

3.4.2.1. Morphology of the outflow

Since the velocity of the wind is of the same order as the orbital velocity of the system, Coriolis effects play an important role in shaping the outflow. Figure 3.4 shows the density of the gas in the orbital plane for different simulations after 7.5 orbits. The orbital motion is anticlockwise in this view. We observe that in the simulations with relatively low wind velocities, $v_{\infty}/v_{\text{orb}} < 1$ (V10a5, V15a5, and V19a3), two spiral arms are formed around the system. These spiral arms delimit the accretion wake of the companion star. Similar to Theuns & Jorissen (1993), we find that the inner spiral shock, wrapped around the donor star, is formed by stellar-wind material leaving the AGB star colliding with gas moving away from the companion star with $v_x > 0$. Near the companion star a bow shock is created by the wind coming from the AGB star as it approaches the accretor. The temperature of the gas in this region increases and material is deflected behind the star forming the outer spiral arm. The Mach number of the flow near the accretor has values between 2 and 6. Because of its proximity to the donor star, where the gas density is high, the inner spiral arm has higher densities than the outer spiral arm. In these systems we also observe an accretion disk around the companion (Sect. 3.4.2.2). The disk is asymmetric and is rotating anticlockwise.

Figure 3.4 shows that for model V30a5 the density of the gas in the accretion wake is lower than in models with $v_{\infty} < v_{\text{orb}}$ (we note that a different colour scale is used in this panel). The density of the accretion wake is high close to the companion star, but it decreases along the stream. Two spiral arms delineate the edge of the accretion wake, which converge into what appears to be a single arm close to the companion star. No accretion disk is observed in this model. For model V150a5 a single spiral arm is distinguished. The density along this stream is very low, because for simulation V150a5 the gas density is much lower than for the other simulations and only a small fraction of the stellar wind is focused into the accretion wake. Also notice that given the low density of the gas, the effective resolution of this model is lower. Given the large velocity of the wind compared to the orbital velocity of the system, the wind escapes the system without being affected by the presence of the companion star and remains spherically symmetric.

If we compare simulations V15a5 and V30a5 we see that the angle formed by the inner high-density spiral arm and the axis of the binary changes as a function of the velocity of the wind. For the low-velocity model, this angle is very sharp and as the wind velocity increases the angle of the stream becomes more oblique. This is expected given that the angle of the accretion wake $\theta$ depends on the apparent wind direction seen by the accretor in its orbit, such that $\tan \theta = v_{\text{orb}}/v_{\text{wind}}$. 

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Figure 3.4: Gas density in the orbital plane ($z = 0$) for V10a5, V10a5s2, V15a5, V19a3, V30a5 and V150a5 after 7.5 orbits.
3.4 Results

3.4.2.2. Accretion disk

An accretion disk is formed around the accretor in the simulations with $v_{\infty}/v_{\text{orb}} < 1$ (V10a5, V10a5s2, V15a5, V15as2 and V19a3). Figure 3.5 shows the density of the gas in the orbital plane for the accretion disks formed around the secondaries for simulations V10a5, V15a5 and V19a3. One common feature is that the accretion disks are not symmetric. This asymmetry is associated with wind coming from the donor star colliding with material already present in the disk. Another feature in the flow pattern of the simulations is formed by two streams of gas feeding the disk: as seen from the velocity field in the corotating frame (not shown), one stream arises from material moving along the inner spiral arm with $v_x, v_y < 0$ and the second one comes from material moving along the outer spiral arm with $v_x, v_y > 0$.

Even though the accretion disk is not symmetric, we approximate it as such in order to estimate its size and mass. We construct spheres of different radii centred at the position of the companion star and measure the mass within each sphere as a function of time. The size of the disk is taken as the radius of that sphere outside which the mass is approximately constant as a function of radius. The disk mass is taken as the total gas mass within that sphere. This approach is reasonable because the gas mass inside the spheres is dominated by the mass of the disk. Table 3.3 shows the resulting values of the accretion disk radius and mass for the simulations in which a disk was present after eight orbital periods, except for model V10a5 where the given radius and mass correspond to a time of 2 orbits.

In the right-hand panels of Figure 3.5, we show the mass within the disk as a function of time for the same models. For model V15a5 (middle right), it is clear that the disk mass is variable over time. It is not obvious where this variability comes from, but we note that the outer disk radius is not much larger than the sink radius (34.7 $R_\odot$) which determines the inner edge. To investigate this behaviour we performed a simulation with the same parameters but a smaller sink radius (20.8 $R_\odot$, V15a5s2). In this simulation the disk reaches a larger radius and gradually grows in mass, up to a value at the end of the simulation about 9 times larger than in V15a5 (see Table 3.3). This can be understood as the smaller sink allows gas in the disk to spiral in to closer orbits around the secondary by viscous effects, transferring angular momentum to the outer regions of the disk which thus grows in size. Such behaviour is physically expected, but simulations with even smaller sink radii and extending over a longer time are required to investigate the mass and radius of the disk and their possible variability.

For model V10a5 (top right panel of Fig. 3.5), we observe a highly dynamic accretion disk, which disappears after two orbital periods. The moment when the disk vanishes can be seen as a steep decrease in the mass of the accretion disk. The reason for this disappearance is that after two orbital periods the size of the outer radius of the disk becomes smaller than the radius of the sink particle. When performing a similar simulation but with a smaller sink radius, V10a5s2, the accretion disk remains for the rest of the simulation and gradually increases in mass, similar to what was seen in simulation V15a5a2 discussed above. The size of the disk remains smaller than in the models with a wind velocity of 15 km s$^{-1}$. A
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Figure 3.5: Left panel: Density in units of g cm$^{-3}$ in the orbital plane of the region centred on the accreting star for three models with $v_\infty/v_{\text{orb}} < 1$, zooming in on the accretion disks. The size of the box corresponds to the Roche-lobe diameter of the companion star. For models V15a5 and V19a3, the situation after 7.5 orbits is shown, whereas for model V10a5 we show the accretion disk before it disappears, but in the same orbital phase. Right panel: Mass of the accretion disk, i.e. the mass contained in a sphere of radius $R_{\text{disk}}$ (see Table 3.3) as a function of time.
similar pattern is found in model V19a3 (bottom panel of Figure 3.5), in which the disk mass increases with time. A longer simulation will be needed to see if it converges towards a constant value.

For the models in which \( v_\infty / v_{\text{orb}} > 1 \), no accretion disk is formed around the companion. Also in a test simulation with a smaller sink particle (V30a5s2) the accreted matter is swallowed by the sink particle without forming an accretion disk. We conclude that if an accretion disk were to form in this system it will be smaller than the assumed sink radius.

### 3.4.2.3. Mass-accretion rate

Figure 3.6 shows the average mass-accretion rate over intervals of one year for models V10a5, V15a5, V30a5 (solid lines), V10a5s2, V15a5s2 and V30a5s2 (dashed lines).

In models V15a5 (middle panel) and V19a3 (not shown), we find a lot of variation in the accretion rate, which clearly exceeds the noise associated with the finite number of particles accreted. This suggests a correlation between the mass of the accretion disk and the accretion rate. This can be observed if we compare the middle right panel of Figure 3.5 and the middle panel of Figure 3.6, where the peaks and troughs in the mass of the disk coincide in time with those observed in the mass-accretion rate. In the case of model V15a5s2, where the mass in the disk gradually increases with time, the accretion rate as a function of time also shows an increase (dashed line in the middle panel of Fig. 3.6). The average accretion rate for model V15a5s2 is a factor of 1.5 lower than for model V15a5 where the sink radius is larger. For model V10a5 (top panel of Figure 3.6), we find that after the accretion disk becomes smaller than the sink and is swallowed, the variation in the accretion rate decreases. For model V10a5s2 with a smaller sink, although the accretion disk is present for the entire simulation, the variability in the accretion rate is lower than for model V15a5s2 (where the accretion disk is also present over the entire simulation). However, we should note that the resolution for model V10a5s2 is much lower. Similar to models V15a5 and V15a5s2, the mass-accretion efficiency is lower for model V10a5s2 than for model V10a5. In the bottom panel of the same figure, we show the accretion rate for model V30a5. Apart from the Poisson noise associated with the finite resolution of the SPH simulations, the accretion rate is constant in time. For the same simulation but with a smaller sink we find a slightly lower accretion efficiency.

We find that the mass that is not accreted in models V10a5s2 and V15a5s2 is stored in the accretion disk. Figure 3.7 shows the net mass inflow rate into a shell of radius 0.4\( R_{\text{L,2}} \) around the secondary for models V10a5, V15a5 and V30a5 (solid lines) and their corresponding models with a smaller sink radius (dashed lines). The chosen shell radius is larger than the size of the accretion disk in all the simulations, so that the inflow rate corresponds to the total mass-accumulation rate of the sink and the disk, \( \dot{M}_{0.4R_{\text{L,2}}} = \dot{M}_{\text{sink}} + \dot{M}_{\text{disk}} \), since the amount of gas within this volume that is not in the disk is very small. In the steady state, the inflow rate we find in this way is insensitive to the chosen sink radius. For simulations V15a5 and V15a5s2, this combined accretion rate is seen to be very similar and also shows similar...
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Figure 3.6: Average accretion rate over one-year intervals for models V15a5 (top), V10a5 (middle) and V30a5 (bottom) is shown in blue. The dashed red lines correspond to the year-averaged mass accretion rate for the corresponding models with a smaller sink radius.

Figure 3.7: Net inflow rate into a sphere of radius $0.4R_{L,2}$ around the accretor star averaged over intervals of one year for the same models as in Figure 3.6.
variability over time. A similar feature holds for simulation V10a5 and V10a5s2. Since it is likely that the mass in the disk will eventually reach the stellar surface, this may be a better measure of the steady-state accretion rate that is insensitive to the assumed sink radius. In the case of models V30a5, V30a5s2, we observe the same small discrepancy in the inflow rate at $R = 0.4R_\text{L,2}$ that we find in Figure 3.6. The net influx of mass within the shell is the same in both simulations, reflecting the steady-state condition reached. However, when the sink radius is larger it captures material that may otherwise escape. Therefore, for simulations such as V30a5 in which no accretion disk is formed, our results provide only an upper limit to the amount of mass accreted.

In order to determine the long term evolution of the orbit, one of the quantities of interest from these simulations is the average mass-transfer efficiency $\beta$. Given the results discussed above and shown in Figures 3.6 and 3.7, we take the mass-transfer efficiency as $\beta_{0.4R_\text{L}} = \dot{M}_{0.4R_\text{L}}/\dot{M}_d$. In Table 3.3, we provide the mean values of this quantity for the science simulations, as well as the mean values for the material captured by the sink only, i.e. $\beta_{\text{sink}} = \dot{M}_{\text{sink}}/\dot{M}_d$. In Figure 3.8, we show the corresponding values for the accretion efficiency $\beta_{0.4R_\text{L}}$ as a function of the ratio $v_\infty/v_{\text{orb}}$ for the science simulations. The dotted line corresponds to $\beta_{\text{BHL}}$ in equation 3.5 with $\alpha_{\text{BHL}} = 1$. Solid dots in the figure correspond to models V15a5-V150a5 and stars to models T3 and V19a3, both of which have $a = 3$ AU.

Model V150a5, in which we use a wind velocity ten times higher than the typical velocities of AGB winds, was set up to approximate the isotropic wind mode. Figure 3.8 shows that for this model, our numerical result for the accretion efficiency exceeds the expected value for BHL accretion by a factor of 2.5 (see also Table 3.3). However, we note that the BHL accretion radius for this simulation is smaller than the radius of the sink, which will result in a discrepancy with the BHL prediction for the accretion efficiency. The dashed line in the same figure corresponds to the accretion efficiency $\beta_{\text{sink}}$ assuming the geometrical cross section of the sink instead of the BHL cross section. The difference between the simulation result and $\beta_{\text{sink}}$ is reduced to a factor of 1.3.

Fig. 3.8 shows several interesting features. As the ratio $v_\infty/v_{\text{orb}}$ increases, the value of $\beta$ decreases, following a similar trend as expected from BHL accretion. However, for $v_\infty/v_{\text{orb}} < 1$ we find $\beta$ values that exceed the $\beta_{\text{BHL}}$ by up to a factor of 1.5-2.3. For the lowest wind velocity, the mass transfer efficiency is approximately 40%. Finally, for models V15a5 and V19a3, which have the same $v_\infty/v_{\text{orb}}$ ratio, the accretion efficiencies shown in Figure 3.7 differ by a factor of 1.4.

3.4.2.4. Angular-momentum loss

The second quantity of interest resulting from our simulations is the value of the specific angular momentum of the material lost by the system. Figure 3.9 shows the values of this quantity in units of $J/\mu$ as a function of $v_\infty/v_{\text{orb}}$ for the different models. We see that the lower the velocity of the wind, the larger the specific angular momentum of the material lost. This
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Figure 3.8: Fraction of mass accreted by the companion as a function of $v_\infty / v_{\text{orb}}$. The dotted line corresponds to BHL accretion rate with $\alpha_{\text{BHL}} = 1$, and the dashed line to the accretion rate for a geometrical cross section of size $R_{\text{sink}}$. Dots correspond to models in which the orbital separation is 5 AU and stars to models in which $a = 3$ AU. The values shown are measured at a radius of 0.4 $R_{\text{L,2}}$ ($\beta_{0.4R_{\text{L,2}}}$).

is consistent with the expectation that when the velocity of the wind is small compared to the relative orbital velocity of the system, more interaction between the gas and the stars occurs and more angular momentum can be transferred to the gas. This strong interaction is also seen in the high mass-accretion efficiency, as well as in the existence of a large accretion disk and in the structure of the spiral arms of the systems. The simulations with $v_\infty < v_{\text{orb}}$ show spiral arms that are more tightly wound around the system, and with higher gas densities, than the simulations for higher wind velocities (see Sect. 3.4.2.1 and Fig. 3.4). These spiral arms correspond to the accretion wake of gas interacting with the secondary star. We interpret the loss of angular momentum as the result of a torque between the gas in this accretion wake and the binary system. The magnitude of this torque depends on the orientation of the wake and the density of the gas behind the companion star. When the accretion wake is approximately aligned with the binary axis and the density in the wake is relatively low, as in simulations with $v_\infty / v_{\text{orb}} > 1$, the torque between the gas and the binary will be small. The torque exerted on the gas will increase as the wake misaligns with the binary axis, and as the gas density in the accretion wake becomes higher. Both these effects occur for low values of $v_\infty / v_{\text{orb}}$, in particular in the inner spiral arm that has a high density and is oriented almost perpendicular to the binary axis (Fig. 3.4). Therefore, the transfer of angular momentum from the orbit to the outflowing gas increases strongly with decreasing $v_\infty / v_{\text{orb}}$. Furthermore, it implies that most of the angular-momentum transfer occurs at short distances from the binary system, as is confirmed by the test discussed in Section 3.3.5 (see Figure 3.2).
3.5 Discussion

3.5.1 Assumptions and simplifications

A complete study of binary systems interacting via AGB winds requires the simultaneous modelling of hydrodynamics, radiative transfer, dust formation and gravitational dynamics. In this study, several simplifications have been made.

Model V150a5 approximates the isotropic or fast wind mode fairly well. In this case, the ejected matter escapes from the system with very little interaction, only removing the specific angular momentum of the orbit of the donor star. For this simulation, the value we obtained for \( \eta \) is \( \eta_{\text{num}} = 0.112 \pm 0.001 \), very close to the expected value for the mass ratio of the stars in our models, \( \eta_{\text{iso}} = 0.111 \) (dotted line in Fig. 3.9).

An interesting result of our models is that for the same value of \( v_\infty/v_{\text{orb}} \), but different orbital separations, the specific angular momentum of the mass lost is very similar. For models V15a5 \( \eta \approx 0.52 \) and for V19a3, \( \eta \approx 0.54 \). Model V10a5 has the largest loss of angular momentum among the high-resolution simulations with \( \eta = 0.64 \), although the low-resolution simulation T3 has an even large amount of angular-momentum loss, \( \eta = 0.68 \).

Figure 3.9: Specific angular momentum of the mass lost from the system as a function of \( v_\infty/v_{\text{orb}} \). The lower the velocity of the wind compared to the orbital velocity, the higher the specific angular momentum loss. For equal velocity ratios, the specific angular momentum is very similar. The dashed line shows the value \( \eta_{\text{iso}} \) expected for an isotropic wind for a mass ratio \( q = 2 \).
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3.5.1.1. Wind acceleration

Dust formation and radiative transfer are not included in our models (except for the simplified effective cooling model) although these processes play a major role in accelerating the gas away from the star. Instead of explicitly computing the gas acceleration, a constant velocity of the gas is assumed with values typical of the terminal velocities of AGB winds. This method is chosen to simplify the calculations and to guarantee that the velocity of the wind $v_\infty$ at the location of the companion star has a predefined value as we want to study the effect of the velocity of the wind on the interaction of the gas with the stars. On the other hand, observations and detailed wind models both indicate that most of the acceleration of AGB winds occurs in the dust-formation zone located at $R_{\text{dust}} \approx 2 - 3R_*$ (Höfner & Olofsson, 2018). For the orbital parameters assumed in our models, $R_{\text{dust}}$ is similar to or larger than the Roche-lobe radius of the donor star, $R_{L,1}$. As described in Mohamed (2010) and Abate et al. (2013), in this situation mass transfer can occur by WRLOF, i.e. by a dense flow through the inner Lagrangian point into the Roche lobe of the companion. However, given that the wind particles in our simulations are injected with predefined terminal velocities from the radius of the donor star, no WRLOF is observed.

If the gradual acceleration of the wind is taken into account, the density and velocity structure of the gas inside the orbit of the companion will be different which may affect the outflow morphology in our low wind-velocity simulations. To some extent the effects of wind acceleration can be accounted for by assuming an increasing velocity profile in Eq. 3.6, mimicking the velocity profile of an AGB wind. The resulting acceleration in such models would still be spherically symmetric. In addition, the modified outflow in a binary can change the optical depth of the dust and thus affect the acceleration of the wind itself, making it non-isotropic. This can only be studied by explicitly modelling dust formation and radiative transfer in wind mass-transfer simulations (e.g. see Mohamed & Podsiałowski, 2012; Chen et al., 2017) and is beyond the scope of the present study. For this reason, and although the simulations presented here give insight into the dependence of the angular-momentum loss and the accretion efficiency on the $v_\infty/v_{\text{orb}}$ ratio, our results should be interpreted with some caution.

3.5.1.2. Accretion

Given the small radius of a main-sequence or white-dwarf companion star, resolving it spatially would be very computationally demanding. For this reason the accretor was modelled as a sink particle with radius equal to a fraction of the size of its Roche lobe. In the simulations where an accretion disk is formed, the inner disk radius is limited by the sink radius. However, physically there is no reason to expect that the disk will not extend inwards to the stellar radius by viscous spreading, and any gas captured in the disk will be transferred towards the stellar surface. As we have shown in Section 3.4.2.3, the sum of the accretion rate onto the sink and the mass accumulation rate in the disk appears to be independent of
the assumed sink radius, and should give a reliable measure of the mass transfer rate to the companion. On the other hand, we cannot be sure that all matter that approaches the surface of the star through the disk will be accreted, as some fraction of it may be expelled by a wind or jet formed in the inner part of the accretion disk. In simulations with higher wind velocities where we do not find an accretion disk, it is possible that a disk is still formed at a smaller radius within the sink. However, some of the mass captured within the sink may not end up in the disk and reach the surface of the companion. For these reasons, and although our results for the mass accretion efficiency with different prescriptions for the artificial viscosity and sizes of the sink particle seem to converge, the values provided should be taken as upper limits.

3.5.1.3. Angular-momentum transfer

In our simulations the gravitational field of the gas is neglected, i.e. the gas feels the gravitational potential of the stars but the stars do not feel the gravity of the gas. This does not prevent us from inferring the gravitational influence of the gas on the binary system, because conservation of angular momentum dictates that the angular momentum transferred to the gas, which we measure from the simulation, is taken out of the orbit. This is expressed by Equation 3.3 in Sect. 3.2. We have performed test simulations in which the gravitational forces are symmetric, i.e. the stars do feel the gravity of the stars, and find that the results for the gas dynamics and angular-momentum loss are not affected. We also find (see Sect. 3.4.1) that the specific angular momentum of the outflowing gas is very well conserved once it travels beyond a few orbital radii. This is an advantage of using an SPH code for this work, because SPH codes are much better at conserving angular momentum than grid-based hydrodynamical codes. Therefore, we are confident that our results for the angular-momentum loss are robust and insensitive to the numerical assumptions in the simulations.

3.5.1.4. Rotation

The stars in our simulations are assumed to be non-rotating. As mentioned in Section 3.3, given the evolutionary history of expansion and mass loss of the AGB star, we expect the rotation rate of the donor to be negligible compared to the orbital frequency, as long as tidal interaction can be ignored. However, the tidal synchronisation timescale of the donor can be fairly short. Applying the equilibrium tide model as described in Hurley et al. (2002) and using the orbital and stellar parameters of the simulated systems (in particular, a stellar radius of 200 R_☉) we find a synchronisation timescale of several times 10^4 years for an orbital separation of 5 AU, and about 10^3 years for 3 AU. This is much shorter than the expansion timescale of the AGB star, which is of the order of 10^6 years, indicating that the donor is likely to rotate synchronously with the orbit. As a consequence, an additional transfer of angular momentum to the gas will take place at the expense of the rotational angular momentum of
the donor. This occurs at a rate:

\[ J_{\text{rot}} = \frac{2}{3} R_d^2 \Omega \dot{M}_d, \quad (3.11) \]

if we assume the wind decouples from the star at a spherical shell of radius \( R_d \). In the absence of magnetic or hydrodynamic coupling between the outflowing wind and the star, \( R_d \) can be identified with the stellar radius. If the donor remains tidally locked to the orbit, this angular momentum is continually supplied to the gas from the orbit, leading to additional orbital angular-momentum loss compared to the non-rotating case. We choose not to include this in our simulations, because the effects of spin angular-momentum loss and tidal interaction can be taken into account separately, as is often done in binary population synthesis modelling (e.g. Hurley et al., 2002; Abate et al., 2015c). For the systems we have simulated, the rotational angular-momentum loss implied by Eq. 3.11 is only a small fraction of the orbital angular-momentum loss, so the effect on the orbit will be small. By ignoring rotation, however, we also neglect the possibility that the morphology of the outflow itself is modified, and thereby the way it interacts with the companion. However, if the wind decouples from the donor star at a radius much smaller than the orbital separation, the flow in the vicinity of the companion will not be strongly affected.

On the other hand, the companion star will gain not only mass but also angular momentum from the material accreted which will lead to spin-up of the star. Once the star is spun up to its critical rotation, no more accretion can take place (Packet, 1981; Matrozis et al., 2017), imposing a constraint on the amount of accreted material. In our simulations we keep track of the angular momentum accreted by the sink particle. However, this is not representative of the true angular momentum added to the companion because we use a sink radius that is much larger than the expected stellar radius. In cases where an accretion disk is formed, the angular momentum of the accreted gas corresponds to a Kepler orbit at the sink radius, whereas in reality by the time the gas reaches the surface of the star it would have transferred angular momentum to the outer regions of the disk. Therefore our simulations do not allow us to study the effects of angular-momentum accretion on the secondary (Liu et al., 2017).

### 3.5.2. Comparison to other work

#### 3.5.2.1. Angular-momentum loss

Our results for the angular-momentum loss are in approximate agreement with other work which uses different methods. Jahanara et al. (2005) performed 3D hydrodynamical grid simulations of a star undergoing mass loss and interacting with a companion star. They study the amount of angular-momentum loss as a function of the wind speed at the surface of the donor for various mass ratios and different assumed mass-loss mechanisms. The mechanism that best approximates the mass loss from an AGB star is the radiatively driven (RD) wind mechanism. This is roughly equivalent to our assumptions, although Jahanara et al. (2005) use an adiabatic EoS without cooling and they assume a much higher sound speed than in our
3.5 Discussion

Simulations, which leads to substantial gas-pressure acceleration in their models at low wind velocities. Despite these differences, and even though the mass ratio in our simulations, \( q = 2 \), is different from the mass ratios assumed in Jahanara et al. (2005), we can make a rough comparison of our results with their Figure 7, in which \( q = 1 \). In that figure \( V_R = \frac{V_{w,RL}}{v_{orb}} \) corresponds to the average wind velocity at the Roche-lobe surface of the donor, which can be compared to our \( \frac{v_{\infty}}{v_{orb}} \) ratio. The parameter \( \ell_w \) corresponds to the specific angular momentum of the material lost in units of \( J/\mu \) and is equivalent to our \( \eta \), although in Jahanara et al. (2005) the donor is kept in corotation with the orbit, so that \( \ell_w \) includes the spin angular momentum loss from the donor star. We compare these results to our simulations in Figure 3.10, where we have subtracted the small amount of spin angular momentum-loss from the \( \ell_w \) values using Equation 3.11, so that they only represent the orbital angular momentum loss. Jahanara et al. (2005) find that the strongest angular-momentum loss, with \( \ell_w \approx 0.6 \), occurs for the lowest wind velocities, corresponding to \( V_R \approx 0.7 \), which is comparable but slightly larger than our results (see Figure 3.10). For increasing wind velocity the specific angular momentum decreases, converging to values equal to the isotropic case for \( q = 1 \), i.e. \( \eta = 0.25 \), which is consistent with our results for model V150a5.

Chen et al. (2017, 2018) also performed grid code simulations of binary systems interacting via AGB wind mass transfer in order to study the orbital evolution, but including pulsations of the AGB star as well as cooling, dust formation and radiative transfer. They modelled systems consisting of a primary star of 1 \( M_\odot \) with a terminal wind velocity \( v_{\infty} \approx 15 \text{ km s}^{-1} \) and orbital separations between 3 and 10 AU. Most of their models have a secondary mass of 0.5 \( M_\odot \), i.e. the same mass ratio as in our simulations. Their models with \( a > 6 \text{ AU} \) display a similar morphology to our simulations with \( \frac{v_{\infty}}{v_{orb}} \gtrsim 1 \), showing a spiral arm structure corresponding to the BHL accretion wake of the secondary. On the other hand, their models with smaller separation (and \( \frac{v_{\infty}}{v_{orb}} \lesssim 1 \)) show a flow morphology resembling WRLOF and appear to be forming a circumbinary disk, which we do not find in our low-velocity simulations. These differences are likely due to the differences in modelling the wind acceleration process. Chen et al. (2018) express the angular momentum loss from the system in terms of a parameter \( \gamma \), which is the specific angular momentum of the matter lost in units of \( J/M_{\text{bin}} \). This is equivalent to our description in terms of \( \eta \), using the transformation

\[
\eta = \frac{M_{\text{bin}}}{\mu} \gamma = \frac{q}{(1+q)^2} \gamma.
\]  

They find larger angular momentum loss from systems with smaller \( a \), i.e. with smaller \( \frac{v_{\infty}}{v_{orb}} \), similar to our results. For their models with \( q = 2 \) and \( a \geq 6 \text{ AU} \) (which have \( 1.0 \lesssim \frac{v_{\infty}}{v_{orb}} \lesssim 1.3 \)) they find similar values of \( \eta \) to our model V30a5 with \( \frac{v_{\infty}}{v_{orb}} = 1.06 \). However, for the model with the same mass ratio and \( a = 4 \text{ AU} \) (\( \frac{v_{\infty}}{v_{orb}} \approx 0.8 \), intermediate between our models V15a5 and V30a5), they obtain a higher \( \eta \approx 0.6 \) than we find for V15a5 which has lower \( \frac{v_{\infty}}{v_{orb}} \).

Interestingly, their 3 AU model with \( q = 10 \) has almost the same \( \frac{v_{\infty}}{v_{orb}} \) and \( \eta \) value as the 4 AU model with \( q = 2 \), although the equivalent \( \gamma \) value is different.
we have to take into account that in the Chen et al. (2018) simulations the AGB star spin is synchronised with the orbit. The spin angular momentum transferred to the gas in their simulations is substantially larger than given by Eq. 3.11, which they ascribe to subsonic turbulence between the photosphere and the dust formation radius. When we subtract the contribution of the stellar spin (as given in their Table 4) from the total angular-momentum loss, the resulting $\eta$ values (in particular for $a = 4$ AU) are in better quantitative agreement with our results (see open squares in Figure 3.10).

3.5.2.2. Accretion efficiency

Several previous studies have investigated the mass-accretion efficiency during wind mass transfer. Theuns et al. (1996) and Liu et al. (2017) performed SPH simulations of a binary with exactly the same parameters as our test models (T1–T3), using an EoS without cooling. They find accretion efficiencies between 1% and 2.3% when using an adiabatic EoS, which is quite comparable to the low $\beta$ value of our model T1 (and substantially less than expected from BHL accretion, see also Nagae et al., 2004). On the other hand, when they apply an isothermal EoS the accretion efficiency increases to 8% and 11%, respectively. This is about a factor of three smaller than we find in our isothermal test model T2 and in model T3 that includes gas cooling explicitly. The reason for this difference is unclear, but we note that the algorithm for computing the accretion rate in these papers is different than the method we use in this work. Besides the discrepancies in the accretion efficiencies, these studies and ours both indicate that allowing for gas cooling results in substantially higher accretion efficiencies.

As discussed in Section 3.5.1.1, our simulations lie in a regime where WRLOF might be expected if wind acceleration were taken into account. In the WRLOF regime accretion onto the companion star occurs by a combination of material flowing through the inner Lagrangian point and gravitational focusing towards the companion star. Our simulations show the latter effect, but not the former. In the WRLOF simulations by Mohamed (2010) (see also Abate et al., 2013) and Chen et al. (2017), which also include gas cooling, high accretion efficiencies of up to 40–50% are found. It is interesting that we find quite similar $\beta$ values in our lowest wind-velocity models, even though there is no significant flow through the inner Lagrangian point in our models. This suggests that the high accretion efficiencies found in WRLOF simulations may be caused predominantly by gravitational focusing onto the companion star. However, given the uncertainties in reliably determining accretion rates as discussed in Sects. 3.4.2.3 and 3.5.1.2, one should be cautious in making such comparisons.

3.5.3. Implications for binary evolution

A very interesting result of our simulations is the fact that for models with different orbital separations but the same $v_{\infty}/v_{\text{orb}}$, we find that a similar specific angular momentum is removed from the system. This suggests that the angular momentum loss, as expressed in the
Figure 3.10: Comparison of our results (Fig. 3.9) to other work. The data points correspond to the specific orbital angular momentum lost from the system via winds obtained using different methods. The open squares correspond to the results of Chen et al. (2018), triangles show the results of Jahanara et al. (2005) for the radiatively driven wind mode. In the latter case we have interpreted the wind velocity at the Roche-lobe surface as corresponding to $v_\infty$. The results from both papers have been corrected for the spin angular-momentum loss (see text) to compare with our results. The colours correspond to different mass ratios $q$ and the lines to the fit formula described by equation 3.13. The blue dotted line is the fit for $q = 2$, and the orange dashed line for $q = 1$. 
parameter $\eta$, may depend primarily on $v_\infty/v_{\text{orb}}$ and relatively little on other parameters of the system. The comparison made above with the results of Jahanara et al. (2005) and Chen et al. (2018) strengthens this tentative conclusion, and also suggests that $\eta$ may be relatively insensitive to the binary mass ratio.

We find that the results of all three sets of simulations, after correcting $\eta$ for the spin contribution as described in Section 3.5.2, are fairly well described by the following simple function:

$$\eta = \eta_\text{iso} + \frac{1.2 - \eta_\text{iso}}{1 + \left(\frac{2.2 v_\infty}{v_{\text{orb}}}\right)^3}.$$

This relation is shown in Fig. 3.10 for two values of the mass ratio, $q = 2$ (corresponding to our simulations and those of Chen et al., 2018) and $q = 1$ (corresponding to the results of Jahanara et al., 2005). The function is constructed to converge for very small $v_\infty/v_{\text{orb}}$ at $\eta = 1.2$, which is the maximum value found by Jahanara et al. (2005) in their low-velocity, ‘mechanically driven’ wind models and appears to be independent of $q$. Most of the points are reasonably well fitted by this relation, the main exception being the Chen et al. (2018) result for $v_\infty/v_{\text{orb}} \approx 0.8$ which is well above the line. However, the sparse data from hydrodynamical simulations available so far do not warrant a fitting function with a larger number of adjustable parameters. We stress that we consider Equation 3.13 to be very preliminary. A larger grid of simulations with different mass ratios, separations and mass-loss rates will be needed in order to investigate if in general the specific angular-momentum loss of the material can be written simply in terms of the velocity ratio.

Knowing the amount of angular momentum-loss and the fraction of mass accreted by the companion, i.e. the values of $\eta$ and $\beta$, we can predict the evolution of the orbital separation by means of equation 3.3. Figure 3.11 shows the theoretical prediction for $\dot{a}/a$ based on the values obtained for $\beta$ and $\eta$ for a mass ratio $q = 2$. For velocity ratios $v_\infty/v_{\text{orb}} > 1$, we conclude that the orbit should widen, and for high terminal wind velocities relative to the orbital velocity the isotropic-wind regime is approached, with an accretion rate similar to the BHL approximation. On the other hand, when $v_\infty/v_{\text{orb}} < 1$ the orbit will shrink, on a timescale similar to the mass-loss timescale of the AGB star. As an example, by integrating Eq. 3 over time and keeping constant the value of the mass-loss rate and the values of $\beta$ and $\eta$ obtained for model V15a5, we find that by the time the AGB star reaches a typical WD mass of $0.6 \, M_\odot$, the orbital separation will shrink by a factor of $\approx 0.6$, from 5 AU to $\approx 3$ AU.

This has important consequences for the evolution of AGB binaries and may help to explain the puzzling orbital periods of Ba stars, CEMP-s stars and other post-AGB binary systems as discussed in Sect. 3.1. Abate et al. (2015a,b) showed that in order to simultaneously explain the observed abundances and the short orbital periods of individual CEMP-s stars, efficient mass accretion and enhanced angular-momentum loss is needed compared to the predictions of BHL accretion and an isotropic stellar wind. In their models they used a value
for the specific angular momentum of the escaping gas equal to two times the average specific angular momentum of the binary, i.e. $\gamma = 2$. Using Eq. 3.12 and a mass ratio $q \approx 2$, this translates into $\eta$ values compatible with our simulations for $v_\infty/v_{\text{orb}} < 1$. In their population synthesis models of Ba stars Izzard et al. (2010) found that including angular-momentum loss with the same value $\gamma = 2$ helps to reproduce the observed period distribution, while the isotropic-wind model produces periods that are too long. However, the impact of our results, in particular the relation between $\eta$ and the wind velocity ratio, on the period distributions of post-AGB binaries has to be verified by population synthesis modelling.

Our results also suggest that the number of systems entering a CE phase will increase as a result of the shrinking orbits and high accretion efficiency during low-velocity wind interaction. This is important given that many classes of evolved binaries are thought to be the product of a CE phase, such as cataclysmic binaries consisting of an accreting WD and a Roche-lobe filling low-mass main-sequence star (Knigge et al., 2011), binary central stars of planetary nebulae (Miszalski et al., 2009) and close double white-dwarf binaries (Iben & Tutukov, 1984a). All these systems currently have orbital periods ranging from hours to a few days, but must initially have been wide enough to accommodate the red-giant progenitor of a WD, so that drastic orbital shrinkage must have occurred during a CE phase (Paczynski, 1976). Furthermore, the double-degenerate formation scenario for type Ia supernovae (SN Ia) invokes the merger of two sufficiently massive carbon-oxygen WDs, brought together into a close orbit during a CE phase and eventually merging due to gravitational-wave emission (Iben & Tutukov, 1984b; Webbink, 1984). The formation rates of all these systems and events may increase as a result of angular momentum-loss during wind interaction of their wide-orbit progenitors. In addition, enhanced angular-momentum loss and high accretion efficiencies during wind mass transfer may increase the number of potential SN Ia progenitors via the wide symbiotic channel in the single-degenerate scenario (Hachisu et al., 1999).

Finally, we note that although our simulations strictly apply to low-mass binaries with AGB donor stars, the same physical processes are likely to occur in other binaries in which a star loses mass via stellar winds. Whenever the wind velocity is similar to or lower than the orbital velocity, enhanced angular-momentum loss from the orbit may occur. In particular, this may apply to massive binaries under two circumstances: when the mass-losing star is a red supergiant in a very wide binary, or when it is a compact Wolf-Rayet star in a very close binary orbiting a compact object or another Wolf-Rayet star. These binary configurations occur as intermediate stages in several of the progenitor scenarios proposed for the mergers of binary neutron stars and black holes (e.g. Mandel & de Mink, 2016; Tauris et al., 2017; van den Heuvel et al., 2017). Dedicated simulations of stellar-wind interaction in such massive binaries are needed to quantify the effect on the evolution of their orbits and the possible consequences for the detection rates of gravitational waves by compact binary inspirals (Belczynski et al., 2002; Chruslinska et al., 2018).
3.6. Summary and conclusions

We have performed hydrodynamical simulations of low-mass binaries in which one of the components is an AGB star losing mass by a stellar wind at a constant rate and with constant velocity. The companion star is represented by a sink mass with radius equal to a fraction (0.06-0.1) of its Roche lobe and is located at separations of 3-5 AU from the AGB star. We have performed simulations for different ratios of the terminal wind velocity to the relative orbital velocity $v_\infty/v_{\text{orb}}$ in order to study the effect of wind mass loss on the orbits of the binary by determining the specific angular momentum of the mass that is lost and the mass-accretion rate onto the companion.

We find two regimes of interaction in terms of $v_\infty/v_{\text{orb}}$. For cases in which $v_\infty < v_{\text{orb}}$ an accretion disk is formed around the companion star, as well as two spiral arms around the system that merge at larger distances. The inner spiral arm wraps closely around the mass-losing star and consists of high-density gas. In these systems, we also find a high value for the accretion efficiency $\beta$, and a high angular-momentum loss per unit mass of material lost from the system. The values of both quantities increase with decreasing velocity of the wind relative to the orbital velocity. For $v_\infty > v_{\text{orb}}$ the BHL accretion regime is approached and the angular-momentum loss is smaller than for cases with $v_\infty < v_{\text{orb}}$. In these models, only one or two spiral arms are observed with relatively low gas density. No accretion disk is found in any of these models; if a disk forms it must be smaller than the assumed sink radius. Our models indicate that the exchange of angular momentum occurs at close distances from the centre of mass of the binary and is mainly caused by the torque between the binary and the companion.
3.6 Summary and Conclusions

Gas in the accretion wake that forms behind the accretor. The strength of this torque increases with the gas density in the wake and its misalignment angle, both of which are larger for lower values of $v_\infty < v_{\text{orb}}$.

Even though the orbital separations chosen in our simulations are in a regime where WRLOF is expected, we do not find the characteristic flow through the inner Lagrangian point encountered in WRLOF simulations because we impose a constant-velocity outflow from the AGB star. Nevertheless, we find similarly high accretion efficiencies in our low wind-velocity models as in WRLOF simulations, in which the gradual acceleration of the wind is modelled explicitly. Our work suggests that gravitational focusing by the companion, in combination with efficient gas cooling, are the main processes that result in a high accretion efficiency during wind mass transfer. We note, however, that accretion rates are difficult to determine accurately from our simulations, in contrast to the angular-momentum loss rates which are computationally very robust.

Based on the results we obtain for the mass-accretion efficiency and the specific angular momentum of the material lost, we predict the effect on the orbital separation. We find that for $v_\infty < v_{\text{orb}}$ the orbits will shrink and when $v_\infty > v_{\text{orb}}$ the orbits will widen. Furthermore, for the same ratio of wind velocity to orbital velocity we find approximately the same value for the specific angular momentum of the material lost. This suggests that the velocity ratio is the main factor determining the orbital evolution in systems undergoing wind mass transfer. Our results can help explain the puzzling orbits of post-AGB binaries, such as Ba stars and CEMP-s stars, but this has to be verified by binary population synthesis models. Our results also suggest that systems that are initially too wide to undergo Roche-lobe overflow can enter a CE phase, which will have consequences for the expected formation rates of systems such as cataclysmic variables, Type Ia supernovae and white-dwarf mergers producing gravitational waves.

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Slowly, slowly in the wind: 3D hydrodynamical simulations of wind mass transfer and angular-momentum loss in AGB binary systems

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Abstract

Wind mass transfer in binary systems with asymptotic giant branch (AGB) donor stars plays a fundamental role in the formation of a variety of objects, including barium stars and carbon-enhanced metal-poor (CEMP) stars. In an attempt to better understand the properties of these systems, we carry out a comprehensive set of smoothed-particle hydrodynamics (SPH) simulations of wind-losing AGB stars in binaries, for a variety of binary mass ratios, orbital separations, initial wind velocities and rotation rates of the donor star. The initial parameters of the simulated systems are chosen to match the expected progenitors of CEMP stars. We find that the strength of interaction between the wind and the stars depends on both the wind-velocity-to-orbital-velocity ratio ($v_\infty/v_{\text{orb}}$) and the binary mass ratio. Strong interaction occurs for close systems and comparable mass ratios, and gives rise to a complex morphology of the outflow and substantial angular-momentum loss, which leads to a shrinking of the orbit. As the orbital separation increases and the mass of the companion star decreases, the morphology of the...
outflow, as well as the angular-momentum loss, become more similar to the spherically symmetric wind case. We also explore the effects of tidal interaction and find that for orbital separations up to 7-10 AU, depending on mass ratio, spin-orbit coupling of the donor star occurs at some point during the AGB phase. If the initial wind velocity is relatively low, we find that corotation of the donor star results in a modified outflow morphology that resembles wind Roche-lobe overflow. In this case the mass-accretion efficiency and angular-momentum loss differ from those found for a non-rotating donor. Finally, we provide a relation for both the mass-accretion efficiency and angular-momentum loss as a function of $v_\infty/v_{\text{orb}}$ and the binary mass ratio that can be easily implemented in a population synthesis code to study populations of barium stars, CEMP stars and other products of interaction in AGB binaries, such as cataclysmic binaries and supernovae type Ia.

4.1. Introduction

In recent years, major efforts have been made to characterise the properties of the outflow of nearby asymptotic giant branch (AGB) stars (e.g. Decin et al., 2010; Blind et al., 2011; Jeffers et al., 2014; Decin et al., 2015; Kervella et al., 2016). These studies shed unprecedented light on the structure of the winds, their chemical composition and dust-to-gas ratios, wind velocity profiles and mass-loss rates. In many cases, they reveal the presence of a binary companion which interacts with the outflow and modifies its structure (Blind et al., 2011; Jeffers et al., 2014; Decin et al., 2015; Bujarrabal et al., 2018). As many AGB stars are known to be in binary systems, several authors have attempted to model the complex interactions between the outflow and the two stellar components, with the aim of reproducing their observed morphologies (e.g. Mohamed & Podsiadlowski, 2007; Kim et al., 2017; Ramstedt et al., 2017) and to understand how the evolution of the system is affected. To investigate the latter, several authors have studied what fraction of wind material can be transferred onto the low-mass companion (Theuns et al., 1996; Nagae et al., 2004; de Val-Borro et al., 2009; Mohamed & Podsiadlowski, 2007, 2012; Liu et al., 2017; Chen et al., 2017; de Val-Borro et al., 2017) and how much angular momentum is lost by wind ejection (Brookshaw & Tavani, 1993; Jahanara et al., 2005; Chen et al., 2018; Saladino et al., 2018).

A thorough understanding of wind mass transfer in binary systems is of fundamental importance to determine the formation mechanism of a variety of objects, such as symbiotic binaries (Merrill, 1942, 1944, 1948, 1950; Kenyon, 1992), barium (Ba) stars (Bidelman & Keenan, 1951), carbon and s-process enhanced metal-poor (CEMP-s) stars (Beers & Christlieb, 2005), and blue stragglers in old open clusters (Mathieu & Geller, 2015). Binary population-synthesis studies typically assume that the AGB star loses mass in the form of a fast spherically symmetric wind and that the mass accretion process is described by the Bondi-
4.1 Introduction

Hoyle-Lyttleton (BHL) formalism (Hoyle & Lyttleton, 1939; Bondi & Hoyle, 1944; Edgar, 2004). In this simplified model, the orbit of the binary expands in response to mass loss. As a consequence of these assumptions, theoretical studies of the progeny of AGB binary systems, such as CEMP-s and Ba stars, predict a dearth of systems with orbital periods between a few hundred and a few thousand days (Pols et al., 2003; Izzard et al., 2009, 2010; Nie et al., 2012; Abate et al., 2018) which is at odds with the observations (e.g. Jorissen et al., 1998, 2016; Hansen et al., 2016a). In addition, in order to reproduce the chemical abundances of some of the progeny of AGB binary systems, mass-accretion efficiencies much larger than those predicted by the BHL formalism are needed (e.g. see Abate et al., 2015b,c, for CEMP stars).

In Saladino et al. (hereinafter Paper I 2018), we performed smoothed particle hydrodynamic (SPH) simulations including cooling of the gas to model interactions of non-rotating AGB binary systems via their winds. We used a predefined constant wind velocity and adopted the same initial stellar parameters of the binary components in all models. We studied the effect of the terminal velocity of the wind on the mass-accretion efficiency and on the angular-momentum loss. We found that these quantities show a strong dependence on the ratio of the wind velocity to the relative orbital velocity, $v_\infty/v_{\text{orb}}$. Despite differences in the assumptions, by comparing to the hydrodynamical models by Jahanara et al. (2005) and Chen et al. (2017), we found a similar trend for the angular-momentum loss as a function of $v_\infty/v_{\text{orb}}$. In Paper I, we also presented a preliminary analytical expression which describes the angular-momentum loss in terms of $v_\infty/v_{\text{orb}}$.

In the context of CEMP-s stars, Abate et al. (2018) performed binary population synthesis simulations in an attempt to reproduce the observed orbital-period distribution of the CEMP stars in the sample obtained by Hansen et al. (2016a). In the simulations of Abate et al. (2018) mass-accretion efficiency and angular-momentum are treated independently, because hydrodynamical simulations computing simultaneously the mass transfer rates and the angular-momentum loss via winds were not available at the time. In their model sets M6 and M7, Abate et al. (2018) implement the fit we provide in Paper I for angular-momentum loss, and adopt the wind Roche-lobe overflow (WRLOF) model for the mass-accretion efficiency (Mohamed & Podsiadlowski, 2007; Abate et al., 2013). Although with these assumptions they find a slight increase in the proportion of evolved binaries at short orbital periods ($<2500$ day) compared to their default model set assuming isotropic winds, the period distribution of their synthetic population does not reproduce the observations. A self-consistent model in which angular-momentum loss and mass-accretion efficiencies are calculated from simulations where the same set of conditions are adopted may help to improve the results of binary population synthesis codes on the populations of CEMP-s stars. In addition, most hydrodynamical studies have focused on investigating the consequences of modifying one orbital parameter at a time on the mass-accretion efficiency or angular-momentum loss. No hydrodynamical study has yet been performed with the same set of assumptions for a large grid of binary and wind parameters. The aim of this work is to perform such hydrodynam-
ical simulations for a much wider range of binary and wind parameters, in order to have a better understanding of the important quantities driving the interaction of AGB winds with the binary systems.

Another aspect that may be of importance and that is sometimes overlooked is the effect of the spin of the donor star on the interacting binary. During the AGB phase the rotational velocity of a single star is greatly reduced due to the expansion of its envelope and because angular momentum is carried away by the mass the star is losing. For this reason, the angular velocity of a single AGB star is negligible small. However, when the AGB star is in a binary system tidal effects can trigger spin-orbit coupling. In the equilibrium-tide model the synchro-nisation timescale depends on the ratio \( (R_d/a)^{-6} \), where \( R_d \) is the radius of the donor star and \( a \) the orbital separation of the binary (Zahn, 1977b). This implies that for AGB binaries in which the donor star is close to filling its Roche lobe, tidal effects can be important since the synchronization timescale is usually much shorter than the typical evolution timescale of an AGB star. Most of the previous hydrodynamical studies of AGB binaries are performed in the corotating frame, thus corotation of the AGB star is implicit in the simulations. Only a few works (e.g. Theuns et al., 1996; Saladino et al., 2018) have performed hydrodynamical simulations in the inertial frame of the binary system, assuming the donor star is not rotating. The morphologies found in some of the models in which corotation of the donor is assumed, correspond to the WRLOF geometry, i.e. the dense wind is confined to the Roche lobe of the donor star and flows through the inner Lagrange point towards the companion star (e.g. de Val-Borro et al., 2009; Mohamed & Podsiadlowski, 2012; Chen et al., 2017). In contrast, the morphologies found when considering a non-rotating donor star are somewhat different, mainly showing spiral patterns formed by the accretion wake of the companion star. The models by Mohamed & Podsiadlowski (2012) and Chen et al. (2017) show that when interaction of the binary stars occurs via WRLOF the mass-accretion efficiency onto the companion star is enhanced. Considering the differences in the results found in numerical models, it is necessary to better understand how rotation modifies the morphology of the AGB outflow in binary systems, and to constrain the range of orbital parameters within which it is realistic to assume corotation of the AGB donor.

In this work we present a comprehensive study of how angular-momentum loss and accretion efficiency in low-mass binary stars vary as a function of mass ratio, orbital separation, and wind-to-orbital velocity ratios. We provide two analytical expressions for angular-momentum loss and mass-accretion efficiency that can be easily implemented in binary population synthesis codes. In addition, we explore how the results are changed if we assume that the AGB donor is corotating with the binary system instead of non-rotating and under which circumstances corotation can be expected.
4.2 Method

4.2.1 Numerical method

The numerical method used in this paper for the SPH simulations is similar to that described in Paper I. Here we will only give a brief summary. We use the Astrophysical Multi-purpose Software Environment AMUSE (Portegies Zwart et al., 2009; Pelupessy et al., 2013; Portegies Zwart et al., 2013) to couple the smoothed particle hydrodynamics (SPH) code FI (Hernquist & Katz, 1989; Gerritsen & Icke, 1997; Pelupessy et al., 2004) with the N-body code HUAYNO (Pelupessy et al., 2012) using the BRIDGE module (Fujii et al., 2007). We use the N-body code to evolve the orbits of the stars. We choose an SPH code because these codes are arguably better at conserving angular momentum than grid-based codes (Price, 2012). Unlike in Paper I, we allow the stars to feel the gravitational field of the gas and the self-gravity of the gas is taken into account.

The AGB wind is modelled using the "accelerating wind" mode of the STELLAR_WIND.PY routine (van der Helm et al., 2019, also available in AMUSE). Wind particles are created in a shell around the star losing mass and are injected with initial velocity \( v_{\text{init}} \). The initial temperature of the wind particles is equal to the effective temperature of the donor star \( T_{\text{eff}} \).

The acceleration given to the SPH particles balances the gravity of the donor star:

\[
a_w(r) = \frac{G M_d}{r^2}.
\]

Due to gas pressure, the wind will experience an outward acceleration close to the donor star until it reaches its terminal velocity \( v_\infty > v_{\text{init}} \) at some distance from the surface of the AGB star. This acceleration mechanism differs from Paper I, in which the acceleration was set to compensate for each term in the equation of motion of the gas, such that the velocity of the wind was constant as a function of distance from the star \( (dv/dr = 0) \).

Similar to Paper I, the gas is assumed to be monoatomic, with an adiabatic index \( \gamma = 5/3 \). The equation governing the cooling or heating of the gas is given by:

\[
\dot{Q} = \frac{3 k_B}{2 \mu m_u} \frac{(T - T_{\text{eq}}) \rho}{C} + \dot{Q}_{\text{rad}},
\]

where \( k_B \) is the Boltzmann constant, \( T \) is the gas temperature, \( \rho \) is the gas density, \( \mu \) is the mean molecular weight and \( C \) a constant parameter which value is \( 10^{-5} \) g s cm\(^{-3}\). The first term assumes that cooling comes from gas radiating away or absorbing thermal energy trying to reach the equilibrium temperature at radius \( r \) given by the Eddington approximation for a gray spherical atmosphere (Chandrasekhar, 1934):

\[
T_{\text{eq}}^4 = \frac{1}{2} T_{\text{eff}}^4 \left[ 1 - \left( 1 - \frac{R_d^2}{r^2} \right)^{1/2} \right] + \frac{3}{2} \int_r^\infty \frac{R_d^2}{r^2} \left( \kappa_g + \kappa_d \right) \rho \, dr,
\]

where \( \kappa_g \) is the gas opacity and \( \kappa_d \) is the dust opacity. Unlike Paper I, where we assumed a constant value of the total opacity \( (\kappa_g + \kappa_d) \), in this work we only allow for the dust opacity
to come into play at distances larger than three times the radius of the donor star (3\(R_d\)), where dust is expected to form according to AGB wind models (Höfner & Olofsson, 2018). The values of the opacities are constant in the calculations and equal to \(\kappa_g = 2 \times 10^{-4} \text{ cm}^2 \text{ g}^{-1}\) and \(\kappa_d = 5 \text{ cm}^2 \text{ g}^{-1}\) (Bowen, 1988). The second term in Equation 4.2 corresponds to the cooling rate prescription for high temperatures (\(\log T \geq 3.8 \text{ K}\)) of Schure et al. (2009).

Because the abundances in this paper are different from those in Paper I (see section 4.2.2), we compute a new cooling table using equation 3 of Schure et al. (2009) and the abundances \(n_i\) from Table 4.4.

The resolution in SPH is defined by the smoothing length \(h_i \propto (m_g/\rho_i)^{1/3}\), where \(m_g\) is the SPH gas particle mass and \(\rho_i\) the density of the gas at the position \(r_i\) of the particle \(i\). In Paper I, we found that the angular-momentum loss is independent of the resolution used, while the accretion efficiency is only weakly dependent on the resolution (see Figure 2 of that paper). Because of this, and in order to minimize the computational time, the resolution we use in this work corresponds to that used in model R3 of Paper I. For an orbital separation of 5 AU, and scaling the density to the value of the mass-loss rate, \(\dot{M}_{d}\), used (see Section 4.2.2), the corresponding mass of an SPH particle is \(m_g = 1.2 \times 10^{-9} \text{ M}_\odot\). Furthermore, in order to optimize the numerical computation at large orbital separations, we choose the typical smoothing length to be proportional to the semi-major axis of the binary, \(a\). Since the average gas density is expected to decrease with the inverse square of the distance to the donor star, i.e. to scale with \(a^{-2}\), we can achieve this by choosing the SPH particle mass \(m_g\) to be proportional to \(a\).

As seen in Paper I (see Figure 1 of that paper) the transfer of angular momentum from the binary orbit to the gas occurs within a few times the orbital separation. For this reason, and in order to minimize the computational time, we remove the SPH particles once they have crossed a boundary at 5\(a\) from the centre of mass of the system. Finally, similar to Paper I the values of the artificial viscosity parameters are \(\alpha_{\text{SPH}} = 0.5\) and \(\beta_{\text{SPH}} = 1\). As shown in Paper I, adopting different values for these parameters does not modify the results in a significant way.

### 4.2.2. Binary set up

The motivation of this paper is to study the effect of angular-momentum loss and mass-accretion efficiency for a range of parameters relevant for low-mass binary stars. In this work we explore the low-metallicity case of the donor star, which applies to the progenitors of CEMP-s stars. The low metallicity will impact several properties of the donor star during the AGB phase, such as stellar radius, effective temperature and consequently the mass-loss rate. In order to obtain realistic parameters for the AGB donor, we compute the evolution of a single star of initial mass \(M_{d,\text{init}} = 1.5 \text{ M}_\odot\) and initial metallicity \(Z_{\text{init}} = 10^{-4}\) using the population nucleosynthesis code binary.c\(^1\) (Izzard et al., 2004, 2006, 2009; Abate et al., 2013; Izzard

\(^1\)SVN revision No. 5045
et al., 2018). The initial metal abundances of the star are equal to solar (Asplund et al., 2009) scaled down to $Z_{\text{init}}$. The mass-loss rate of the AGB star is calculated using the method described in Vassiliadis & Wood (1993). As starting conditions for the simulations, we adopt the stellar mass, $M_d$, radius, $R_d$, effective temperature, $T_{\text{eff}}$, mass-loss rate, $\dot{M}_d$, and chemical abundances at the moment the star has reached the superwind phase on the AGB, and the radius is close to its maximum value. This is the phase of evolution when the strongest interaction between the wind and the companion will occur. At this moment the mass is reduced to 1.2 $M_\odot$, its age is 1796 Myr and its surface is strongly enriched in carbon (see Tables 4.1 and 4.4).

**Table 4.1: Constant parameters of the donor star in the hydrodynamical simulations**

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<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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</thead>
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<td>$M_d$</td>
<td>1.2 $M_\odot$</td>
<td>Mass of the AGB star</td>
</tr>
<tr>
<td>$R_d$</td>
<td>330 $R_\odot$</td>
<td>Radius of the AGB star, and inner boundary for release of new SPH particles</td>
</tr>
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<td>$\dot{M}_d$</td>
<td>$1.5 \times 10^{-5} M_\odot$ yr$^{-1}$</td>
<td>mass-loss rate</td>
</tr>
<tr>
<td>$T_{\text{eff}}$</td>
<td>3240</td>
<td>Effective temperature</td>
</tr>
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</table>

Similar to Paper I, the companion star is modelled as a sink particle with radius equal to $0.1 R_{L,2}$, where $R_{L,2}$ is the Roche-lobe radius of the accretor star. For the companion star mass, $M_a$, we take $M_a = 1.2 M_\odot$, 0.9 $M_\odot$, 0.6 $M_\odot$, 0.4 $M_\odot$, and 0.3 $M_\odot$. The mass ratio, $q = M_d/M_a$, varies accordingly between 1 and 4. These mass ratios encompass the mass ratios of CEMP-s progenitors from Abate et al. (2015b,a). We should note that these are not the initial mass ratios, but the mass ratios at the time when AGB mass transfer takes place.

Using the set up described above, we perform 31 simulations (Table 4.2) in which the initial orbit of the binary system is circular and its initial orbital separation ranges between 4 and 20 AU. These orbital separations are chosen so that they correspond to the typical orbital periods of CEMP-s progenitor stars found by Abate et al. (2015b,a). The stars are placed in the inertial frame and the centre of mass of the binary system is positioned at the origin. We simulate models with different initial velocities of the wind $v_{\text{init}} = 1$, 5 and 12 km s$^{-1}$. We note that in the models with the closest orbits (4-5 AU) the donor star fills a substantial fraction of its Roche lobe and its surface layers and atmosphere will be deformed by tides from the companion to some extent. We do not take this tidal deformation into account in the simulations, since the wind particles are always injected from a spherical shell with inner radius $R_d$ (see Section 4.2.1).
4.2.3. Rotation and spin-orbit coupling

In this paper we explore the possibility of rotation of the AGB donor. In order to check for which orbital separations corotation of the AGB donor with the orbit is expected, we perform a set of simulations within binary_c of binary systems interacting via isotropic winds and we compute their tidal evolution. The physics of tidal evolution in binary_c is implemented following the work of Hurley et al. (2002, Section 2.3 and references therein) and the equation for the tidal circularisation and synchronisation timescale is based on Rasio et al. (1996). The equations of tidal evolution are originally reported by Hut (1981). We evolve systems with initial orbital separations between 3 and 30 AU, initial masses as in Section 4.2.2 and with the initial angular velocity of the donor star, $\Omega_{\text{spin}}$, set equal to zero, i.e. the star is initially non-rotating. We check the angular velocity of the donor star at the time the mass of the AGB star and the mass ratios are equal to those assumed in our hydrodynamical simulations. We find that for orbital separations up to $\approx 15-20$ AU, depending on the mass ratio, tidal interactions have spun up the donor star to a non-zero angular velocity. For initial separations smaller than 7-10 AU a state of near-corotation, with $\Omega_{\text{spin}}$ comparable to the angular velocity of the binary, $\Omega_{\text{bin}}$, is reached at some point during the AGB phase. However, except for very close orbital separations ($a \lesssim 4 - 6$ AU), corotation is lost again by the time we start our hydrodynamical simulations as a result of the high mass-loss rate of the AGB star. The ratio $\Omega_{\text{spin}}/\Omega_{\text{bin}}$ at this moment is a decreasing function of separation and mass ratio (see Appendix 4.D for details).

So as to account for spin-orbit coupling, and in order to investigate the effect of rotation on the angular-momentum loss and mass-accretion efficiency, in a subset of our simulations we assume both non-rotation and corotation with the orbit for models with the same mass ratio and the same orbital separation (see Table 4.2; the corotating models are designated by the symbol "$\Omega$"). In the models where corotation of the binary is assumed, we add a tangential component to the velocity of the wind particles as they leave the donor star, $v_T = \dot{\Omega}_{\text{spin}} \times r$, with $\Omega_{\text{spin}} = \Omega_{\text{bin}}$ and $r$ the position of the gas particles with respect to the centre of mass of the donor star. For the other simulations we assume the donor star to be non-rotating regardless the fact that $\Omega_{\text{spin}}$ may be non-zero.

4.3. Results

4.3.1. Terminal velocity of the wind

As mentioned in section 4.2.1, in this work the acceleration of the wind only balances the gravitational deceleration by the AGB star. For this reason, the terminal velocity of the wind will be different from that with which the wind is injected. In Paper I, where we imposed a constant velocity on the wind particles with a predefined terminal velocity, we only explored
### Table 4.2: Parameters of the simulations

<table>
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<th>Model</th>
<th>(M_a)</th>
<th>(a)</th>
<th>(P_{\text{orb}})</th>
<th>(q)</th>
<th>(\Omega_{\text{spin}})</th>
<th>(v_{\text{init}})</th>
<th>(v_{\text{orb}})</th>
<th>(R_{L,1})</th>
<th>(R_{L,2})</th>
<th>(m_{g})</th>
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<td></td>
<td>(M_\odot)</td>
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<td>yr</td>
<td>-</td>
<td>-</td>
<td>(s^{-1})</td>
<td>km (s^{-1})</td>
<td>km (s^{-1})</td>
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<td>AU</td>
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<td>4</td>
<td>0</td>
<td>5</td>
<td>16.32</td>
<td>2.52</td>
<td>1.33</td>
<td>1.2 \times 10^{-9}</td>
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<td>1.07</td>
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<td>1.33</td>
<td>1.2 \times 10^{-9}</td>
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<td>1.2 \times 10^{-9}</td>
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<td>5.04</td>
<td>2.66</td>
<td>2.4 \times 10^{-9}</td>
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</table>

**Notes:** \(M_a\) is the mass of accretor star. \(a\) is the orbital separation of the binary. \(P_{\text{orb}}\) is the orbital period. \(q\) is the mass ratio \(M_d/M_a\). \(\Omega_{\text{spin}}\) is the angular velocity added to the gas particles to mimic the rotation of the donor star. It is the same as the angular velocity of the binary. \(v_{\text{init}}\) is the initial velocity with which particles are injected at the radius of the donor star. \(v_{\text{orb}}\) is the relative orbital velocity of the binary system. \(R_{L,1}\) is the size of the Roche-lobe of the donor star. \(R_{L,2}\) is the size of the Roche-lobe of the companion star. \(m_{g}\) is the mass of the SPH gas particles.
velocities as low as 10 km s\(^{-1}\) for numerical reasons\(^2\). Here we explore lower values of the initial wind velocities. We determine the terminal velocities that can be achieved by performing simulations of single AGB stars. In these models the star is allowed to eject matter for 40 yr in order to reach stable conditions. The average terminal velocities are measured at a distance of 25\(R_d\), and averaged over the last 1.5 yr. Figure 4.1 shows the velocity and density profiles of the wind as a function of distance for a single star with stellar parameters as shown in Table 4.1 and an initial velocity of the wind of \(v_{\text{init}} = 12\) km s\(^{-1}\). Because of gas pressure, the wind accelerates to an average terminal velocity \(v_\infty = 15.1\) km s\(^{-1}\). For \(v_{\text{init}} = 5\) km s\(^{-1}\), we find that the average \(v_\infty = 10.8\) km s\(^{-1}\) and for \(v_{\text{init}} = 1\) km s\(^{-1}\), \(v_\infty = 6.0\) km s\(^{-1}\). The injection of particles with such low initial wind velocities allows the wind to achieve terminal velocities which lie within the observed range as given by Vassiliadis & Wood (1993) for the winds of AGB stars. We describe the results of our binary simulations in terms of the terminal velocity achieved by the wind of a single star with the same initial velocity as assumed for the donor star.

\(^2\) Modelling systems with lower wind velocity is computationally more expensive since it takes longer time to reach the steady state.
4.3 RESULTS

Figure 4.2: Gas density in the orbital plane for models Q1a5, Q1a10 and Q1a15 (the same mass ratio, but different orbital separations). The companion star is located on the left and the AGB donor on the right. Each figure is scaled to display the full simulation, i.e. a sphere with radius equal to 5a, and shows the situation after 9.5 orbital periods.

4.3.2. Morphology of the outflow

In the first part of this section we present the results on the outflow morphologies for the non-rotating models. In the second part, we discuss the differences between the models in which corotation of the AGB donor is assumed and the corresponding models in which rotation is neglected.

4.3.2.1. Non-rotating models

The outflow morphologies in the simulations with a non-rotating donor star are very similar to those found in Paper I and they are in agreement with the results of similar works (e.g. Mastrodemos & Morris, 1998; Liu et al., 2017). In general, for all the models we observe two
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Figure 4.3: Same as Figure 4.2, but for models with the same orbital separation ($a = 5$ AU) and different mass ratios.
4.3 Results

Figure 4.4: Gas density on the orbital plane for models with the same orbital separation
and mass ratio, \((a = 5 \text{ AU}, q = 1)\), but different initial wind velocities (1 km s\(^{-1}\) and 12 km s\(^{-1}\)). Notice that for better appreciation in this figure the density scale is different from the previous figures. Also notice that for model Q1a5 (right) this figure is a close-up version of Fig. 4.3a.

Spiral arms wrapped around the binary system. These spiral arms delimit the accretion wake behind the companion star. In the following we will describe the observed morphologies as a function of the orbital separation (Figure 4.2), the mass ratio (Figure 4.3) and the wind velocity (Figure 4.4).

Figure 4.2 shows the gas density profile in the orbital plane for models Q1a5, Q1a10 and Q1a20, which have the same mass ratio \((q = 1)\) but different orbital separations \((a = 5, 10, \text{ and } 20 \text{ AU})\), at the same orbital phase. We observe that the complexity of the morphology decreases with increasing orbital separations, which suggests that as the orbital separation increases, the interaction between the companion star and the wind becomes less strong. In relatively close systems, the gas density in the spiral arms is higher than in wider systems. Also, the spiral arms which delimit the accretion wake behind the companion star are less tightly wound around the binary for wider systems. One more feature that varies in these models is the opening angle, \(\theta\), of the accretion wake. We find that for close orbital separations, \(\theta\) is large and it decreases when the orbital separation increases. This occurs because \(\theta\) is a function of the Mach number, \(\mathcal{M}\), i.e. \(\theta = \sin^{-1} \mathcal{M}^{-1}\) (see Appendix 4.B). We find that for the model with \(a = 5 \text{ AU}\), \(\mathcal{M} \approx 2\) in the vicinity of the accretor while the Mach number increases to \(\mathcal{M} \approx 4.5\) and \(\mathcal{M} \approx 5\) for the systems with \(a = 10 \text{ AU}\) and \(a = 20\) respectively. In addition, we find that for model Q1a5 an accretion disk is formed, which is not observed in the wider models. However, as is extensively discussed in Paper I, since the radius of the sink particle is large, the disk may not be able to form in these cases.

The same trend in the morphology of the outflow with orbital separation is found for systems with larger mass ratios. For close orbital separations a stronger interaction between the
gas and the stars occurs, as shown by the presence of an accretion disk around the companion star and dense tightly wound spiral patterns around the binary.

In order to illustrate the dependence of the morphology on the mass ratio, Figure 4.3 shows the density profile in the orbital plane for models Q13a5, Q2a5, Q3a5 and Q4a5 (the same orbital separation, but different mass ratio). This can be compared with model Q1a5 (Figure 4.2a), which has the same orbital separation. However, these models do not show an accretion disk around the companion star, for the reasons explained above. Overall the gas densities are similar and all models have two arms tightly wound around the binary. The main differences occur in the accretion wake behind the companion star, which has lower density and a smaller opening angle as the mass ratio increases. Similar to Liu et al. (2017)\(^3\), we find that for higher mass ratios the spiral arm is less tightly wound. These differences in the geometry of the outflow can be explained by the lower gravity of the companion star. When the companion star is more massive, its gravitational attraction is stronger and so is the spiral shock formed in the vicinity of the accretor. At the same time, the opening angle of the spiral shock becomes wider. This is explained by the dependence on the Mach number as discussed above. As the mass ratio increases from \(q = 1\) to \(q = 4\), the Mach number near the companion increases from \(M \approx 2.5\) to \(M \approx 5\). On the other hand, as the mass ratio increases, the density along the inner spiral arm decreases. As will be shown in Section 4.3.3, a lower density in the accretion wake implies a weaker torque on the binary system which affects the amount of angular momentum exchanged between the orbit and the gas.

In general we observe that as the orbital separation increases, the structures in the morphology of the outflow become less prominent and less complex. This suggests that the interaction between the gas and the stars decreases as a function of distance. A similar trend was found in Paper I, where the free parameter in the simulations was the velocity of the wind. In the models discussed above the terminal velocity of the wind is fixed and we vary the orbital separation, which is equivalent to changing the orbital velocity of the system. As a result, the ratio \(v_\infty/v_{\text{orb}}\) of the models changes. We find very similar flow morphologies as in Paper I when comparing models with the same \(v_\infty/v_{\text{orb}}\). Furthermore, we confirm that the relationship found in Paper I on the morphology as a function of \(v_\infty/v_{\text{orb}}\) also holds for different mass ratios.

Finally, in order to study how the wind velocity influences the morphology of the outflow, in Figure 4.4 we compare two models with a mass ratio \(q = 1\), a small orbital separation \((a = 5\ \text{AU})\), and initial wind velocities of 1 km s\(^{-1}\) and 12 km s\(^{-1}\). Since the sound speed close to the donor star is about 6 km/s, in the first model (Q1a5v1) the wind leaves the donor star in the subsonic regime, whereas for higher-velocity model (Q1a5) the wind is supersonic when it leaves the donor star. In contrast to Figures 4.2 and 4.3, only the inner region of the simulation is shown in Figure 4.4. In model Q1a5 (Fig. 4.4b) the wind leaving the donor remains spherically symmetric until it reaches the accretion shock and the inner spiral arm.

\(^3\)Note that their definition of mass ratio is \(Q = 1/q\).
However, when the initial wind velocity is subsonic (model Q1a5v1, Fig. 4.4a), the outflow in the vicinity of the donor star’s surface is clearly asymmetric and partly focussed into a broad stream that is deflected behind the companion’s accretion wake. The deflection is related to the non-rotation of the donor star, as will be discussed in Section 4.3.2.2. We note that due to the small orbital separation in these models, the surface of the donor star and the wind launch region will be somewhat deformed by tides from the companion. This is not taken into account in our simulations, but is likely to enhance the asymmetry in the inner part of the outflow seen in Fig. 4.4b. On larger scales than shown in Fig. 4.4, both models show a spiral pattern similar to that observed in Fig. 4.2a. However, as expected because of the low wind velocity, the density in the spiral arms of model Q1a5v1 is higher than in model Q1a5.
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4.3.2.2. Rotating models

In the models where the AGB donor is in corotation with the binary, we find two different regimes in the geometry of the outflow. This difference appears to be related to the initial velocity of the wind. For models Q2a5Ω and Q4a5Ω, where \( v_{\text{init}} = 12 \text{ km s}^{-1} \), the outflows are very similar to their non-rotating counterparts (Q2a5 and Q4a5; see Figure 4.3). For model Q1a5Ω (not shown), we observe a similar morphology to that of Q1a5 (Figure 4.2a). Both systems have an accretion disk and two spiral arms tightly wound around the binary. However, the inner spiral arm for model Q1a5 is slightly denser than the one for model Q1a5Ω.

On the other hand, when we compare the corotating models with small orbital separations (4 AU and 5 AU) and low initial velocities of the wind (\( v_{\text{init}} = 1 \text{ km s}^{-1} \) and \( v_{\text{init}} = 5 \text{ km s}^{-1} \)) with their non-rotating counterparts, we note clear differences in the morphologies of the outflow. We note that at these orbital separations, the companion star is located at a position where the gas has not yet reached its terminal velocity and that the donor star is very close to filling its Roche lobe (\( R_d/R_{L,1} = 0.86 \) for Q2a4v1Ω and \( R_d/R_{L,1} = 0.69 \) for Q2a5v5Ω). Interestingly, the geometries in these corotating models resemble the WRLOF morphology found in simulations performed in the corotating frame (e.g. de Val-Borro et al., 2009; Mohamed & Podsiadlowski, 2012, 2007; Chen et al., 2017). In Section 4.4.1 we compare the results of our corotating models Q2a4v1Ω and Q2a5v5Ω to the results of simulations performed in the corotating frame.

Figure 4.5 shows the gas density on the planes \( z = 0 \) (left) and \( y = 0 \) (right) for models Q2a4v1 (top) and Q2a4v1Ω (bottom). The velocity field in the inertial frame is plotted in the same figure. In spite of the accretion disk around the companion star which is a common feature in both models, there are many dissimilarities between the models. A clear difference that can be observed in the orbital plane, \( z = 0 \) (Figures 4.5a and 4.5c), is the high gas density along the inner spiral arm for model Q2a4v1 (Figure 4.5a; \( x \approx -2.5 \text{ AU} \), \( 0 \text{ AU} \leq y \leq 0.1 \text{ AU} \)), which is substantially lower in the corotating model Q2a4v1Ω (Fig. 4.5c). Also, the opening angle of the inner arm is larger in the non-rotating model and wraps around the donor star at closer distances than in the corotating model. Another clear difference is the stream of gas flowing from the donor star to the companion star in model Q2a4v1Ω, which can be observed in both planes (\( z = 0 \) and \( y = 0 \)). In the orbital plane, this stream is seen to flow from the inner Lagrangian point, \( L_1 \), at about \( x \approx -0.49 \text{ AU} \) to \( x \approx -2.5 \text{ AU} \), where it encounters the accretion wake of the companion star. The presence of the stream between the two stars in Fig. 4.5c, which resembles RLOF, creates the impression that the effect of the rotating donor is to focus the wind in the direction of the accretor. Conversely, model Q2a4v1 does not show this stream of gas moving from \( L_1 \) towards the companion star, but instead the gas moving from the AGB star towards the companion is deflected behind this star, where it is captured by the accretion wake and then falls onto the accretion disk. However, unlike model Q2a4v1, in model Q2a4v1Ω the gas appears to pass through the accretion wake once
it reaches it. This might be due to the fact that for Q2a4v1 the density in the accretion wake is higher than for the rotating model, and is able to trap the gas more easily when it reaches it. In addition, from the flow in the $y = 0$ plane (Figs. 4.5b and 4.5d), we observe a region in model Q2a4v1 where material is moving away from the binary ($-3 \text{ AU} \lesssim x \lesssim -5 \text{ AU}$ and $-1 \text{ AU} \lesssim z \lesssim 1 \text{ AU}$). Finally, in model Q1a4v1, the material moving away is confined to a narrower band around the $z = 0$ plane, whereas the gas at $z \gtrsim 0.5 \text{ AU}$ and $z \lesssim -0.5 \text{ AU}$ is observed to be moving towards the companion star.

The morphology of model Q2a5v5Ω (not shown) is somewhat similar to that of model Q2a4v1Ω with gas being focused in the direction of the companion star. We find a stream of gas flowing through the inner Lagrangian point and moving towards the inner spiral arm. Similar to Q1a4v1Ω, the inner spiral arm is less dense than its non-rotating analogue. The main difference we find with model Q1a4v1Ω is that the region in the $y = 0$ plane where gas moves away from the binary is narrower in the $z = 0$ plane giving the impression that less material escapes the system.

### 4.3.3. Angular-momentum loss

The loss of angular momentum for non-conservative mass transfer in binaries can be parameterised as:

$$ J = \eta a^2 \Omega_{\text{bin}} (1 - \beta) M_d + J_{\text{spin}}, $$

(4.4)

The first term corresponds to the change in the orbital angular momentum, $J_{\text{orb}}$, where $\beta = -\dot{M}_a / M_d$ is the fraction of ejected mass which is accreted by the companion star, $M_d < 0$ is the mass-loss rate, and $\eta$ is the specific angular momentum of the material lost in units of the orbital angular momentum of the system per reduced mass, $J/\mu = a^2 \Omega_{\text{spin}}$. The second term in Equation 4.4 corresponds to the angular momentum lost due to rotation of the donor star. If the donor star is assumed to be non-rotating, this term is zero.

In a similar fashion to Paper I, we take the values of the mass and angular momentum lost from the system as those taken away by gas particles which cross a boundary of $3a$ from the centre of mass of the system. In order to reduce statistical fluctuations, we average the angular-momentum loss over intervals of one orbital period from the time the simulation reaches the steady state, which occurs after about four orbits (cf. Saladino et al., 2018, Section 3.5).

In the simulations in which rotation is included, the numerical value measured for $J$ is the combination of the angular momentum removed from the orbit and the angular momentum taken from the spin of the donor star, $J_{\text{spin}}$, which we cannot disentangle. In order to apply Equation 4.4 and derive the value of $\eta$, $J_{\text{spin}}$ is assumed to be given by the analytical equation

\[ \text{Note that Eq. 4.4 ignores the rotational velocity of the secondary star, which could accrete angular momentum from the wind and be spun up. However, since we model the accretor star as a sink we cannot account for the real angular momentum that is transferred (see Paper I for a discussion).} \]
### Table 4.3: Results from the simulations

<table>
<thead>
<tr>
<th>Model</th>
<th>$v_\infty$ km s$^{-1}$</th>
<th>$v_{RL,1}$ km s$^{-1}$</th>
<th>$v_\infty/v_{orb}$</th>
<th>$\eta$</th>
<th>$\beta$</th>
<th>$\dot{a}/a_{\text{bin}}$ yr$^{-1}$</th>
<th>$\dot{a}/a_{\text{dyn}}$ yr$^{-1}$</th>
<th>Accretion disk</th>
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<td>Q1a5v1</td>
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<td>0.610</td>
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<td>14.5</td>
<td>1.793</td>
<td>0.073</td>
<td>0.003</td>
<td>0.003</td>
<td>8.17 x 10$^{-6}$</td>
<td>8.48 x 10$^{-6}$</td>
</tr>
<tr>
<td>Q4a4v1</td>
<td>6.0</td>
<td>2.6</td>
<td>0.329</td>
<td>0.152</td>
<td>0.048</td>
<td>0.078</td>
<td>-6.54 x 10$^{-6}$</td>
<td>-4.47 x 10$^{-6}$</td>
</tr>
<tr>
<td>Q4a5v5</td>
<td>10.9</td>
<td>8.7</td>
<td>0.665</td>
<td>0.110</td>
<td>0.032</td>
<td>0.026</td>
<td>-1.20 x 10$^{-6}$</td>
<td>-5.00 x 10$^{-7}$</td>
</tr>
<tr>
<td>Q4a4</td>
<td>15.1</td>
<td>12.4</td>
<td>0.828</td>
<td>0.093</td>
<td>0.022</td>
<td>0.017</td>
<td>1.65 x 10$^{-6}$</td>
<td>2.87 x 10$^{-6}$</td>
</tr>
<tr>
<td>Q4a5Ω</td>
<td>15.1</td>
<td>13.4</td>
<td>0.925</td>
<td>0.084</td>
<td>0.019</td>
<td>0.013</td>
<td>3.02 x 10$^{-6}$</td>
<td>4.39 x 10$^{-6}$</td>
</tr>
<tr>
<td>Q4a5</td>
<td>15.1</td>
<td>13.4</td>
<td>0.925</td>
<td>0.083</td>
<td>0.017</td>
<td>0.013</td>
<td>3.29 x 10$^{-6}$</td>
<td>4.37 x 10$^{-6}$</td>
</tr>
<tr>
<td>Q4a10</td>
<td>15.1</td>
<td>14.4</td>
<td>1.308</td>
<td>0.058</td>
<td>0.005</td>
<td>0.005</td>
<td>7.27 x 10$^{-6}$</td>
<td>7.67 x 10$^{-6}$</td>
</tr>
</tbody>
</table>

**Notes:** $v_\infty$ is the average velocity of the wind at a distance 25$R_d$ from a single star. $v_{RL,1}$ is the average velocity of the wind at the Roche lobe of the donor star. We should note that this average velocity is taken assuming a single star undergoing mass loss via winds. $v_\infty/v_{orb}$ is the velocity ratio of the terminal velocity of the wind to the relative orbital velocity of the binary system. $\eta$ is the average specific angular momentum-loss of the mass that crossed a boundary at 3$a$ from the binary system in units of $a^2\Omega_{\text{bin}}$ as derived from Equation 4.4. $\beta$ is the average mass-accretion efficiency measured from the inflow in a shell at 0.4$R_{L,2}$. $\dot{a}_{\text{BHL}}$ is the mass-accretion efficiency predicted by BHL. It is determined from Equation 4.10 with $\alpha_{\text{BHL}} = 0.75$. $(\dot{a}/a)_{\text{bin}}$ is the rate of change in the orbital separation of the binary system. It is derived from Equation 4.13. $(\dot{a}/a)_{\text{dyn}}$ is the rate of change in the orbital separation of the binary system. It is derived dynamically from the simulation. Note that for the corotating systems we cannot make a direct comparison between $(\dot{a}/a)_{\text{bin}}$ and $(\dot{a}/a)_{\text{dyn}}$ because in the first case the change in the orbital separation is computed from the change in the orbital angular momentum, whereas when measured dynamically the rate of change in $a$ includes the change of angular momentum due to the spin of the donor star.
4.3 Results

for isotropic mass loss from a spherical surface:

\[ J_{\text{spin}} = \frac{2}{3} R_d^2 \dot{M}_d \Omega_{\text{spin}}, \quad (4.5) \]

where \( \Omega_{\text{spin}} = \Omega_{\text{bin}} \) in the case of corotation. In order to verify that the analytical value for \( J_{\text{spin}} \) given by Equation 4.5 accurately represents the spin angular-momentum loss, we perform a numerical simulation of a single star with the properties of Table 4.1, rotating at \( \Omega_{\text{spin}} = 2.76 \times 10^{-8} \text{ s}^{-1} \). We find that the numerical value obtained for \( J_{\text{spin}} \) agrees with Equation 4.5 within 2%. For this reason, for corotating systems in which the flow morphology is not strongly modified by rotation (see Section 4.3.3.2), we expect Equation 4.4 to yield similar \( \eta \) values as for their non-rotating analogues.

In the following we present the results for the angular-momentum loss of our models in terms of the parameter \( \eta \). First we discuss the models in which the donor star is assumed to be non-rotating \( (J_{\text{spin}} = 0) \) and in the second part we present the results for the angular-momentum loss of the corotating models.

4.3.3.1. Non-rotating models

Figure 4.6 shows the specific angular momentum lost, \( \eta \), in units of \( J/\mu \) as a function of the velocity ratio \( v_{\infty}/v_{\text{orb}} \) for the different non-rotating models studied in this paper. Colours represent equivalent mass ratios. This figure only shows those systems with non-rotating donor stars. When compared to the results of Paper I (open stars in Figure 4.6), where angular-momentum loss was studied only for \( q = 2 \), we observe a similar trend of the \( \eta \) values as a function of \( v_{\infty}/v_{\text{orb}} \). We emphasize that in Paper I the parameter we varied was \( v_{\infty} \), whereas here \( a \) is varied. For high velocity ratios, the values of \( \eta \) approach the isotropic wind values given by:

\[ \eta_{\text{iso}} = \frac{1}{(1 + q)^2}, \quad (4.6) \]

which are shown as dashed lines in Figure 4.6. On the other hand, when \( v_{\infty}/v_{\text{orb}} \) decreases, the angular-momentum loss of the models in enhanced. A notable difference with Paper I is that the \( \eta \) values found in this work for low \( v_{\text{init}} \) are smaller than those found in Paper I at the same \( v_{\infty}/v_{\text{orb}} \). However, we should keep in mind that in this work there are a number of differences in the numerical method compared to Paper I. The stellar radius and effective temperature of the donor star are larger in this work. This leads to a different temperature profile in the outflow, which may influence the interaction between the gas and the stars. In addition, the mechanism of injection of the wind particles is different from the one used in Paper I. This may affect the angular momentum transfer, in particular when the companion star is located at a position where the wind is still being accelerated.

This corresponds to the fast or Jeans mode where mass lost from the star occurs in the form of spherical symmetric isotropic wind, and the angular-momentum loss is the specific angular-momentum loss of the donor star in its relative orbit.
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\textbf{Figure 4.6:} Orbital angular-momentum loss in units of $J/\mu$ as a function of $v_\infty/v_{\text{orb}}$ for models assumed to be non-rotating. Colours represent equivalent mass ratios. Dashed lines show the value of $\eta_{\text{iso}}$ for different mass ratios. Filled circles correspond to models in which $v_\infty = 15.1$ km s$^{-1}$, triangles for models where $v_\infty = 10.9$ km s$^{-1}$ and squares for models where $v_\infty = 6$ km s$^{-1}$. The solid lines correspond to Equation 4.7 for different mass ratios.
From Figure 4.6, we also observe that as the mass ratio increases, the angular-momentum loss decreases, for similar values of $v_{\infty}/v_{\text{orb}}$. This behaviour is expected according to the results discussed in Section 4.3.2.1. Given the lower densities behind the accretion wake for larger mass ratios, the torque exerted by the wake on the binary is weaker, resulting in a smaller exchange of angular momentum. An interesting feature we find is that for low $v_{\infty}/v_{\text{orb}}$ and $q \lesssim 2$, $\eta$ appears to level off at a maximum value of about 0.6. However, when $q \gtrsim 3$, $\eta$ never reaches such high values. Finally, although for most of our simulations $\eta$ seems to grow monotonically with decreasing $v_{\infty}/v_{\text{orb}}$ until reaching its maximum for the lowest $v_{\infty}/v_{\text{orb}}$, this does not seem to be the case for the $q = 4/3$ models. For these models the maximum of $\eta$ occurs for $v_{\infty}/v_{\text{orb}} \approx 0.6$ and it decreases for the lowest $v_{\infty}/v_{\text{orb}}$.

In order to apply our results in binary population synthesis simulations, the angular-momentum loss needs to be expressed as a function of the physical parameters of the systems. In Paper I, we provide an analytic relation for the angular-momentum loss as a function of $v_{\infty}/v_{\text{orb}}$ but independent of $q$. However, we find that this relation only matches our current results for high $v_{\infty}/v_{\text{orb}}$ and mass ratios between 1 and 2. For this reason, we provide a new function which describes our results as a function of the mass ratio and the ratio between the terminal velocity of the wind and the system’s orbital velocity:

$$\eta(q, v_{\infty}/v_{\text{orb}}) = \min\left(\frac{1}{c_1 + (c_2 v_{\infty}/v_{\text{orb}})^3} + \eta_{\text{iso}}, 0.6\right),$$

where

$$c_1 = \max(q, 0.6 q^{1.7}),$$

$$c_2 = 1.5 + 0.3q,$$

and $\eta_{\text{iso}}$ is given by Equation 4.6.

This fit is overplotted in Figure 4.6 for several values of the mass ratio, corresponding to our simulations. For comparison, we also plot the results for the angular-momentum loss from Jahanara et al. (2005, blue crosses), where the wind is accelerated in a similar fashion as in this work, and the results of Chen et al. (2018, orange plus signs), who used a different method to model the stellar wind. For both works, we apply Equation 4.4 to the published results to correct for the spin angular momentum due to corotation of the system.

### 4.3.3.2. Rotating models

Figure 4.7 shows the orbital angular-momentum loss, as measured by the parameter $\eta$, as a function of the ratio $v_{\infty}/v_{\text{orb}}$ for the corotating models (open shapes) with their non-rotating counterparts (filled shapes). We observe that for $v_{\infty}/v_{\text{orb}} \gtrsim 0.8$ the orbital-angular-momentum loss is equal for the rotating and non-rotating models. This means that the extra angular-momentum loss measured in the corotating models indeed comes from the spin of the donor star and is correctly described by Equation 4.5. However for low $v_{\infty}/v_{\text{orb}}$, the orbital
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Figure 4.7: Orbital angular-momentum loss in terms of $\eta$ for models in corotation with the binary and their non-rotating analogues. Colours represent equivalent mass ratios. Circles correspond to models where the initial velocity was set as $v_{\text{init}} = 12$ km s$^{-1}$ ($v_\infty = 15.1$ km s$^{-1}$) and squares to models where $v_{\text{init}} = 1$ km s$^{-1}$ ($v_\infty = 6$ km s$^{-1}$). Open shapes are the corresponding corotating models.
angular-momentum loss in the corotating models is lower than that measured for the non-rotating models. The difference is small for model Q1a5Ω, and somewhat larger for Q2a4v1Ω (≈ 8%) and Q2a5v5Ω (≈ 13%). We note that \( \eta \) measures the exchange of angular momentum between the escaping gas and the binary orbit. The fact that \( \eta \) is independent of rotation for larger \( \nu_\infty/\nu_{\text{orb}} \) is consistent with the fact that we find no differences in the morphology between rotating and non-rotating models at these velocities (see Section 4.3.2.2). The lower angular momentum exchange in models Q2a4v1Ω and Q2a5v5Ω compared to Q2a4v1 and Q2a5v5 respectively, could be due to a smaller torque exerted by the accretion wake on the binary in the rotating models, given the lower density in the inner spiral arms for these models compared to their non-rotating analogues.

### 4.3.4. Mass-accretion efficiency

The BHL approximation provides an estimate of the mass-accretion efficiency onto a body moving at given speed in a high-velocity wind assuming supersonic flow. Under this approximation, in the case of a binary system in which one of the stars is losing mass via winds, the companion star will accrete a fraction of the mass lost by the donor given by:

\[
\beta_{\text{BHL}} = \frac{\alpha_{\text{BHL}} \left( \frac{v^4_{\text{orb}}}{v^4_w + v^4_{\text{orb}}} \right)^{3/2}}{(1 + q)^2 v^2_w (v^2_w + v^2_{\text{orb}})^{3/2}},
\]

where \( \alpha_{\text{BHL}} \) is the efficiency parameter, which has a value between 0.5 and 1, and \( (v^2_w + v^2_{\text{orb}})^{1/2} \) is the relative wind velocity seen by the accretor (Theuns et al., 1996).

Considering the assumptions made in the canonical BHL approximation and the fact that AGB winds are very slow, a more suitable way to estimate the mass-accretion efficiencies is to measure them directly from detailed hydrodynamical simulations. However, we emphasize that since we cannot resolve the surface of the companion star computationally, our numerical results for the mass-accretion efficiency should be taken as upper limits. As discussed in Paper I, rather than measuring the mass flux that crosses the boundary of the sink particle, a more reliable measurement for the accretion rate onto the companion star is derived from the sum of the mass accretion rate on the sink particle and the rate at which mass is added to the accretion disk. For this reason, and following the method used in Paper I, we adopt as the mass-accretion efficiency the net flux crossing a shell centred on the companion star with radius equal to 0.4 \( R_{L,2} \). Similar to the angular-momentum loss, to avoid statistical fluctuations, we measure the average mass-accretion efficiency over several orbital periods once the system has reached the steady state.

#### 4.3.4.1. Non-rotating models

Figure 4.8 shows the fraction of mass accreted \( \beta = \dot{M}_{0,4R_{L,2}}/|\dot{M}_{\text{d}}| \) as a function of \( \nu_\infty/\nu_{\text{orb}} \). The dotted lines correspond to the predicted BHL value (Equation 4.8) with \( \alpha_{\text{BHL}} = 0.75 \) (as Abate et al., 2013). Similar to Paper I, for individual mass ratios, we find that the smaller the
FIGURE 4.8: Mass-accretion efficiency for the non-rotating models. Colours represent equivalent mass ratios. Circles correspond to models where $v_\infty = 15.1$ km s$^{-1}$, triangles to models where $v_\infty = 10.9$ km s$^{-1}$ and squares to models where $v_\infty = 6$ km s$^{-1}$. The dotted lines correspond to the values predicted by BHL as given by Equation 4.10 assuming $\alpha_{BHL} = 0.75$. Continuous lines correspond to the fit given by Equation 4.11.
Figure 4.9: Similar to Figure 4.8 but for the rotating models.
\( \frac{v_{\infty}}{v_{\text{orb}}} \) ratio, the higher the mass-accretion efficiency we measure. In general we notice that for different mass ratios and \( \frac{v_{\infty}}{v_{\text{orb}}} > 0.5 \), the accretion efficiency measured in our models is always larger than the BHL prediction (see Table 4.3). The maximum difference between the BHL approximation and our results occurs for models Q1a5 and Q13a5v5, where we find that the value of \( \beta \) from the hydrodynamical models is about 1.6 times larger than the BHL prediction. Only when \( \frac{v_{\infty}}{v_{\text{orb}}} \) is very high the accretion efficiency approaches the BHL approximation. In the cases where this occurs (see Section 4.3.2.1) the morphologies of the outflow are less complex suggesting less interaction between the wind and the companion star. On the other hand, for models with \( v_{\text{init}} = 1 \) km s\(^{-1} \) there is no clear trend in the accretion efficiencies we measure, which can be either higher or lower than the \( \beta \) values predicted by the BHL approximation. The maximum accretion efficiencies we obtain never exceed about 30\%, in contrast with the accretion efficiencies of Paper I, where for low velocity ratios, we found values up to about 40\%. We should note that the stellar parameters used in this study differ from Paper I, and that the current numerical resolution is much lower than the resolution used in Paper I, which could be influencing our results (see Sect. 4.4.3).

In order to implement our results in a binary population synthesis code, we construct a function that describes the behaviour found from our hydrodynamical simulations for the mass-accretion efficiency. Similar to the angular-momentum loss, we fit our results as a function of mass ratio and the ratio \( \frac{v_{\infty}}{v_{\text{orb}}} \). A relation that describes our numerical results is given by:

\[
\beta(q, \frac{v_{\infty}}{v_{\text{orb}}}) = \min(\alpha \beta_{\text{BHL}}, \beta_{\text{max}}),
\]

(4.11)

where:

\[
\alpha = 0.75 + \frac{1}{k_1 + (k_2 \frac{v_{\infty}}{v_{\text{orb}}})^5}
\]

(4.12)

and

- \( k_1 = 1.7 + 0.3q \),
- \( k_2 = 0.5 + 0.2q \),
- \( \beta_{\text{max}} = \min(0.3, 1.4 \; q^{-2}) \),

and \( \beta_{\text{BHL}} \) is given by Equation 4.10 with \( \alpha_{\text{BHL}} = 1 \). This relation is shown in Figure 4.8 for different mass ratios.

### 4.3.4.2. Rotating models

Figure 4.9 shows the mass accretion efficiencies for the models assumed to be in corotation. We find that the mass-accretion efficiency for models Q2a5Ω and Q4a5Ω is slightly larger (by a factor of about 1.1) than for their non-rotating counterparts, while for Q1a5Ω and Q2a5v5Ω it is slightly smaller. However, for model Q2a4v1Ω, where the injection velocity of the wind is \( v_{\text{init}} = 1 \) km s\(^{-1} \) and the geometry of the outflow resembles WRLOF, we find
that the accretion efficiency is less than half the value of its non-rotating analogue (see Table 4.3). Although we may attribute this low mass-accretion efficiency to material crossing the low density accretion wake of the companion star (see Section 4.3.2.2), this appears to be in contrast to the SPH simulations performed by Mohamed & Podsiadlowski (2007) in which they find high accretion efficiencies when mass transfer occurs via WRLOF. For comparison, in Figure 4.9, we plot the mass accretion efficiencies from Abate et al. (2013) based on the hydrodynamical simulations of Mohamed (2010) as a function of \( \nu_\infty / \nu_{\text{orb}} \) (brown Y shapes). We compute \( \nu_{\text{orb}} \) from the orbital periods provided in Abate et al. (2013) and we take \( \nu_\infty = 4 \) km s\(^{-1}\) as in Mohamed & Podsiadlowski (2012). In the same figure, we also show the mass-accretion efficiencies from the hydrodynamical models of Chen et al. (2017) (orange plus signs). We observe that in both works the trend of the mass-accretion efficiency as a function of \( \nu_\infty / \nu_{\text{orb}} \) is similar to the one we find, i.e. \( \beta \) decreases with increasing \( \nu_\infty / \nu_{\text{orb}} \). However, for \( \nu_\infty / \nu_{\text{orb}} \) in the range between 0.5 and 1, both Mohamed & Podsiadlowski (2012) and Chen et al. (2017) find much larger values than predicted by BHL. This differs from the values we find in the same velocity range, where our \( \beta \) values are only slightly larger than BHL (see Section 4.4 for a discussion). On the other hand, for \( \nu_\infty / \nu_{\text{orb}} < 0.5 \), the \( \beta \) values found by Mohamed & Podsiadlowski (2012) decrease again and, similar to our results, for the lowest velocity ratio \( \beta \) is much lower than the BHL expected value, even though WRLOF occurs.

### 4.3.5. Change in orbital separation

Due to the loss of angular momentum via stellar winds, the orbital separation of the binary will be affected. The rate of change of \( a \) can be derived from angular momentum conservation (see Paper I):

\[
\frac{\dot{a}}{a} = \frac{2 \dot{M}_d}{M_d} \left[ 1 - \beta q - \eta(1 - \beta)(1 + q) - (1 - \beta) \frac{q}{2(1 + q)} \right],
\]

(4.13)

where the parameters \( \eta \) and \( \beta \) are computed from detailed hydrodynamical simulations. We should note that when there is efficient spin-orbit coupling by tidal interactions, another term should be added to Equation 4.13 to take this into account.

Another method to compute the change in \( a \) is to measure it dynamically from our numerical simulations. To compute it in this way, we use the Newtonian description of the motion of two masses under their mutual gravitational field (see Appendix 4.C). From our numerical simulations we know the position \( \mathbf{r} \) and the relative velocity of the stars \( \mathbf{v} \) at any time. From these we compute the orbital energy and angular momentum per reduced mass, and by approximating the orbits as Keplerian, we compute instantaneous values of \( a \) and \( e \). We then calculate \( \dot{a} \) as the average rate of change of \( a \) over the last orbits. We find that \( e \) is not exactly zero, but its value shows a sinusoidal variation for all models. However this variation is so small (\( \sim 10^{-5} \)) that it can be neglected.

Figure 4.10 shows the change in orbital separation calculated in the two different ways described above for the non-rotating models. On the x-axis we plot the values of \( \dot{a} / a \) mea-
Figure 4.10: Evolution of the orbital separation, $\dot{a}/a$, measured with two different methods. The values on the x-axis are measured directly from the evolution of the orbits of the stars. The values on the y-axis are obtained by substituting the parameters $\eta$ and $\beta$ measured from the gas dynamics in equation 4.13.
4.4 Discussion

4.4.1 On the morphology of the outflow: rotating vs non-rotating systems

Observations of low-mass binary systems interacting via winds show spiral patterns wrapped around the stars (e.g. Kim et al., 2017; Ramstedt et al., 2017). Numerical simulations also measured directly from the simulations (Eqs. 4.14 and 4.16) and on the y-axis we show the values computed by substituting the measured values of $\eta$ and $\beta$ (Table 4.3) in Equation 4.13. For most systems in which the orbit expands both values agree quite well. In these binaries, the wind leaves the system in an almost spherically symmetric way. However, in systems with shrinking orbits we find a substantial difference between the two $\dot{a}$ values. The discrepancy appears to be correlated with the speed at which the orbit shrinks, and in most cases the systems shrink slower if measured directly from the simulations. We note that the cases where the discrepancy is large correspond to models where the interaction between the gas and the stars is strong (squares and triangles in Figure 4.10), i.e. where more angular momentum is transferred from the binary orbit to the gas. This means that in these cases, if angular momentum is not conserved precisely, an error in this quantity may build up.

For a TreeSPH code like F1 angular momentum is not conserved exactly (Wijnen et al., 2016). For this reason, we compute how well angular momentum is conserved for our models. In order to do so, we compute the total initial angular momentum, $J_{\text{tot,init}} = J_{\text{stars,init}} + J_{\text{gas,init}}$, where $J_{\text{stars,init}}$ is the initial orbital angular momentum of the binary and $J_{\text{gas,init}}$ is the angular momentum of the gas which is initially in the simulation. We also compute the total angular momentum at the moment we stop the simulations, $J_{\text{tot,end}} = J_{\text{stars,end}} + J_{\text{gas,end}} + J_{\text{acc}} + J_{\text{esc}}$, where $J_{\text{stars,end}}$ is the orbital angular momentum at the last timestep of our simulation, $J_{\text{gas,end}}$ is the angular momentum of the gas at the end of the simulation, $J_{\text{acc}}$ is the cumulative angular momentum of the gas accreted by the sink particle and $J_{\text{esc}}$ the cumulative angular momentum of the gas we removed during the simulation. We express the error, $\delta J$, as the difference $J_{\text{tot,end}} - J_{\text{tot,init}}$ in units of the total initial angular momentum per unit of time.

We find that in most of our models angular momentum is conserved fairly well. For model Q3a20, where the difference between the two methods to compute $\dot{a}/a$ is only 4%, we find that $\delta J = 4.46 \times 10^{-7}J_{\text{tot,init}} \text{ yr}^{-1}$. However, for model Q2a4v1 where the discrepancy in $\dot{a}/a$ is $\approx 37\%$, angular momentum is conserved up to $2.47 \times 10^{-5}J_{\text{tot,init}} \text{ per year}$. Since the error in the angular momentum of the stars and the gas is a fraction of the total error computed and we cannot separate the errors, it is not possible to precisely correct the $\dot{a}/a$ values computed either dynamically or from the parameters $\beta$ and $\eta$. However, we have verified by error propagation that both $(\dot{a}/a)_{\text{bin}}$ and $(\dot{a}/a)_{\text{dyn}}$ are consistent within the total error in the angular momentum budget for each system.
produce these features in the outflow (e.g. Mastrodemos & Morris, 1998; Liu et al., 2017; Saladino et al., 2018). In addition, some planetary nebulae, which are thought to be the final evolutionary stage of low- and intermediate-stars, show morphologies which cannot be explained by single-star evolution. For these reasons, as is discussed in Paper I, understanding how the morphologies are influenced by the interacting binary can provide an insight into the mechanisms driving mass transfer between the stars.

In this work we find that the spin of the star plays a significant role in shaping the morph-ology of the outflow. For cases in which the angular velocity of the donor star is zero, we find similar morphologies to those in Paper I: two spiral arms surrounding the binary system and in some cases an accretion disk. Similar to Paper I, a strong interaction of the wind with the stars can be observed as an increasingly complex morphology in the outflow, that is, the morphology of the wind differs significantly from the spherical symmetric case. In addition, we find that the strength of the interaction depends on the orbital separation and mass ratio of the binary system. With increasing orbital separation the morphology of the outflow becomes less complex, and for large orbital separations the wind escapes the binary in a nearly isotropic way. Varying the orbital separation of the system results in similar geometries as a function of \( \frac{v_\infty}{v_{\text{orb}}} \) as when \( v_\infty \) is varied for a fixed separation (as has been done in Paper I). In terms of the mass ratio, we find spiral patterns that become less prominent as the mass ratio is further from unity. Similar structures of which the complexity decreases as a function of the mass ratio are also found by Liu et al. (2017), who used a different EoS. These results on the morphology can help to constrain the orbital separations and mass ratios of binary observations.

Similar to the zero-spin models, in the rotating models we find that the geometry of the outflow becomes less complex with increasing \( \frac{v_\infty}{v_{\text{orb}}} \), and more closely resembles the spherical symmetric wind case. A similar result is observed in the hydrodynamical models in the corotating frame of Chen et al. (2017), where although a spiral outflow morphology is found for wide binaries, this becomes less prominent with increasing orbital separation (i.e. with increasing \( \frac{v_\infty}{v_{\text{orb}}} \)). We find that the effect of rotation on the morphology of the wind is negligible when \( \frac{v_\infty}{v_{\text{orb}}} \gtrsim 0.7 \) and that for these models the geometries of the outflow are similar to those observed when the star is not rotating. However, when the wind velocity is low (\( v_{\text{init}} = 1 \text{ km s}^{-1} \) and \( v_{\text{init}} = 5 \text{ km s}^{-1} \)) and the system is relatively close, we find structures in the outflow which resemble WRLOF. The morphologies we find are similar to those observed by de Val-Borro et al. (2009), Mohamed & Podsiadlowski (2012) and Chen et al. (2017) in the corotating frame. By comparing our results with these works, we notice that these geometries are generally found when the velocity of the wind is much lower than the orbital velocity of the binary. For instance, in their more recent hydrodynamical models of binary stars interacting via winds, de Val-Borro et al. (2017) do not find the prominent stream flowing between the stars discussed in de Val-Borro et al. (2009), where the adopted wind velocities were lower.

In the physical interpretation of Mohamed & Podsiadlowski (2007), WRLOF is expected
to occur when the dust formation region is larger than the Roche lobe of the star, confining material to the gravitational potential of the donor star and filling the Roche lobe. Material is then transferred to the companion star via the inner Lagrangian point. Our models do not include dust formation and the acceleration of the wind simply balances the gravity of the star. However, given that some acceleration via gas pressure still occurs, the velocity of the wind within the Roche lobe is very small (see Table 4.3) allowing the wind material to fill the Roche lobe and part of the material to be transferred in a way that resembles RLOF. Similar to the models by de Val-Borro et al. (2009) and Mohamed & Podsiadlowski (2012), we observe that the stream of gas is not exactly focused towards the companion star, but slightly deflected towards the accretion wake. Morphologies like those described here are also observed. For instance, recent observations by Bujarrabal et al. (2018) of the symbiotic binary R Aqr show an arc of gas joining the two stars, which is attributed to an episode of mass transfer taking place in the system.

When we compare our models on spatial scales larger than about 5AU (as shown in Figure 4.5) to figure 4 of Chen et al. (2017), we observe that generally the outflow geometries are very similar, with two differences. In the inner region of the binary system our models show the flow to be very steady compared to theirs. A likely explanation for this difference is that in their models pulsations of the AGB star are included which make this region more dynamic. The second difference we note is that in some of their models, especially where the morphologies are similar to WRLOF, a circumbinary disk is found. In our simulations we do not see such a structure. However, we should keep in mind that in our models we remove the escaping material at a relatively short distance from the binary and that we do not include dust formation nor radiative transfer, which may be relevant for the formation of this disk. Longer numerical simulations which include this physics would be needed to check whether a circumbinary disk is formed.

Finally, we note that the peculiarities in the geometries of the outflow discussed here are likely to have an impact on the evolution of the binary via differences in the mass-accretion efficiency and angular-momentum loss. For instance, a denser accretion wake exerts a higher torque on the binary resulting in a larger exchange of angular momentum between the orbit of the stars and the gas.

### 4.4.2. Angular-momentum loss and the impact on the orbit

Recent hydrodynamical studies of angular-momentum loss during wind mass transfer have explored its dependence on only a few parameters. Jahanara et al. (2005) performed grid-based hydrodynamical models of binary systems undergoing mass loss via radiatively-driven winds, i.e., similar to this work the acceleration of the wind only balances the gravitational force of the donor star. In their models, the EoS used is adiabatic and the mass ratio is $q = 1$, they varied the injection velocity of the wind. Chen et al. (2018) also performed grid-based hydrodynamical simulations which include pulsations of the AGB donor
star, dust formation, cooling of the gas and radiative transfer. Except for one model, the mass ratio ($q = 2$) was fixed and they studied the dependence of the angular-momentum loss as a function of the orbital separation of the binary. In Paper I we performed a few exploratory simulations which did not include acceleration of the wind, but included cooling of the gas. The works of Jahanara et al. (2005), Chen et al. (2018) and Paper I show a very similar dependence of the angular-momentum loss on $v_\infty/v_{\text{orb}}$. In this work, we have explored a larger grid of binary parameters and wind velocities and we include the possibility of corotation of the donor star with the binary. In the following we extend the comparison to our present work.

4.4.2.1. Non-rotating models

Figure 4.6 shows the orbital angular-momentum loss, expressed in terms of $\eta$, as a function of $v_\infty/v_{\text{orb}}$ for the non-rotating models. When we compare this to the results of Jahanara et al. (2005, their radiatively driven wind case), Chen et al. (2018) and Paper I, we see that all models show a good agreement for $v_\infty/v_{\text{orb}} > 1$, where $\eta$ tends towards the isotropic-wind value (see Equation 4.6). For the cases where $v_\infty/v_{\text{orb}} < 1$, there is a clear trend of $\eta$ increasing with decreasing $v_\infty/v_{\text{orb}}$. This behaviour is expected because the velocity of the wind is much lower than the relative orbital velocity of the stars, thus the wind has a stronger interaction with the companion star, which allows it to draw more angular momentum from the orbit. However, we notice that despite following the same trend, there is a slight discrepancy between Paper I and this work in the $\eta$ values found as a function of $v_\infty/v_{\text{orb}}$. We associate this difference to the different assumptions made for the acceleration of the wind, as well as the different stellar parameters of the donor star. The different acceleration mechanism can lead to close binaries interacting in a region where the wind has not reached the terminal velocity. The difference in stellar masses and metallicity can influence the temperature profile of the gas, since in this work the effective temperature of the donor star is larger. These two effects influence the interaction between the wind and the binary. Furthermore, unlike what we anticipated in Paper I, our current models show that the values of $\eta$ converge to a maximum of $\eta \approx 0.6$ for low $v_\infty/v_{\text{orb}}$ and $q \leq 2$.

4.4.2.2. Rotating models

For models in which rotation is included, we find two regimes for the measured $\eta$ values. For models with $v_\infty/v_{\text{orb}} \gtrsim 0.7$, where the spin of the star does not affect the geometry of the gas (see Section 4.3.2.2), we find that the orbital angular momentum carried away by the escaping gas is independent of the spin of the donor star, i.e. the values we obtain for $\eta$ as derived from Equation 4.4 are the same as in their non-rotating counterparts. This implies that the terms $\dot{J}_{\text{orb}}$ and $\dot{J}_{\text{spin}}$ act independently and that the orbital evolution of the system can be predicted by considering separately the change in angular momentum due to mass lost from the system and the change in angular momentum due to tides, as usually assumed in
4.4 Discussion

binary population synthesis codes. However, for \( \frac{v_{\text{eq}}}{v_{\text{orb}}} \lesssim 0.7 \), the spin of the star modifies the outflow which also changes the interaction between the gas and the stars compared to the non-rotating models. Consequently, we find different \( \eta \) values for the corotating and non-rotating models. This implies that for cases in which the outflow is strongly modified by the spin of the star, detailed simulations including rotation of the donor are needed to correctly predict the amount of orbital angular-momentum loss and therefore the evolution of the orbit.

In the numerical models performed by Chen et al. (2018) in the corotating frame the parameter that is varied is the orbital separation of the system. They explain why synchronisation of the donor star with the binary is expected only for some of the orbital separations they explore. In order to investigate the cases in which the donor star is not rotating, they provide a prescription in which they simply subtract the contribution of the spin of the star from the total angular-momentum loss they measure (similar to what we do in this work). However, because of the results discussed above, the prescription they use to study the zero-spin cases for the AGB star can only be applied when the rotation of the donor does not modify the outflow of the gas. In addition, due to the nature of their numerical models, they find that gas gains angular momentum as it moves radially away from the mass-losing star, which they attribute to numerical errors due to the intrinsic viscosity of Eulerian codes. This is not observed in our numerical models since SPH codes are better at conserving angular momentum than grid-based codes.

As mentioned in Section 4.3.5, from Equation 4.4 we can predict the rate of change of the orbital separation of the binary system due to orbital angular-momentum loss. However, if tidal interactions are effective in the binary, additional terms need to be taken into account in that equation. As shown in Appendix 4.D, for orbital separations up to \( \approx 10 \) AU, tidal friction is so strong that spin-orbit coupling is expected at some point during the AGB phase of the donor star. Furthermore, even when tidal interactions are not strong enough for synchronisation they can effectively spin up the donor star (see Figure 4.12b) for orbital separations between 10-20 AU. In order to keep the donor star in corotation with the binary via tidal effects, angular momentum needs to be taken from the orbit of the binary. This means that for some of the orbital separations considered in this work tidal evolution plays an important role, implying that additional shrinking of the orbit will take place.

4.4.3. Mass-accretion efficiency

Observations of the progeny of AGB binary systems show enhanced chemical abundances in AGB nucleosynthesis elements, which sometimes cannot be explained by the standard BHL formalism (e.g. the chemical abundances of observed CEMP-s stars Abate et al., 2015b,a). Since the assumptions made for the flow in the BHL formalism usually do not hold for the winds of AGB stars, hydrodynamical models are needed to obtain better estimates of the mass-accretion efficiency onto the companion star. Several previous hydrodynamical studies have
focused on computing this quantity (e.g. Theuns et al. (1996); de Val-Borro et al. (2009); Mohamed & Podsiadlowski (2007); Liu et al. (2017); de Val-Borro et al. (2017)). However, from the numerical point of view this is not an easy task. Since the radius of the companion star cannot be resolved numerically, these works have used a variety of methods to model the accretion process onto the surface of the star. In addition, some works have shown that the mass-accretion efficiency depends on the EoS used and on the numerical resolution of the simulation (Theuns et al., 1996; Saladino et al., 2018). For these reasons comparing results from different studies is not straightforward. However, given some resemblances in the trends of mass-accretion efficiency as a function of mass ratio and $v_\infty/v_{\text{orb}}$, we can attempt to do so.

Figure 4.8 shows that in agreement with Paper I for the non-rotating models, the mass accretion efficiency can be described as a function of $v_\infty/v_{\text{orb}}$, where for large $v_\infty/v_{\text{orb}}$ values $\beta$ approaches the BHL approximation. Similar to Liu et al. (2017), we find that $\beta$ depends on the mass ratio, with higher accretion efficiencies for binary stars with more comparable masses. Liu et al. (2017) provide an analytical expression that describes their numerical results for the mass-accretion efficiency as a function of the mass ratio. Since in their models a constant orbital separation of 3 AU is adopted and the stellar parameters used were different than in our models, we cannot compare our $\beta$ results directly with their analytical expression. However, even for our models with the closest separation ($a = 5$ AU), we find $\beta$ values that are much larger than predicted by their analytical expression. In their models they use an adiabatic EoS to model the thermodynamics of the gas. As shown by Theuns et al. (1996) and Saladino et al. (2018) for their models with $q = 2$ and $a = 3$ AU, an adiabatic EoS usually leads to lower accretion efficiencies than predicted by BHL. For these reasons, the fit we provide in Equation 4.11 is more appropriate for binaries interacting via winds.

We can compare the $\beta$ values of our corotating models with Mohamed & Podsiadlowski (2012) and Chen et al. (2017), who made similar assumptions for the spin of the donor star. Similar to the non-rotating models, these models show a dependence on the ratio $v_\infty/v_{\text{orb}}$. However, in our models $\beta$ is only a factor of $\sim 1.6$ larger than in the standard BHL model, whereas in both Mohamed & Podsiadlowski (2012) and Chen et al. (2017) the difference is much larger (a factor of 2 or more). Although each of these works uses a different numerical method to measure the accretion rate onto the companion star, this is unlikely to explain the differences between the results. For instance, the accretion region in Chen et al. (2017) is smaller than the sink particle assumed in this work, which should result in a lower accretion rate. Mohamed (2010) finds that the smooth method used in Mohamed & Podsiadlowski (2012) to model the accretion process produces lower accretion efficiencies than immediately removing particles from the simulation after they enter the sink region, as we do. On the other hand, we note that the SPH resolution chosen in our models is quite low in order to make the simulations computationally efficient. As shown in Paper I, a resolution like the one adopted in our models can underestimate the accretion efficiency by a small factor compared to a higher resolution. It seems more likely that the different accretion efficiency found in our work compared to Mohamed & Podsiadlowski (2012) and Chen et al. (2017) originates from
the differences in the assumed input physics. Both Mohamed & Podsiadlowski (2012) and Chen et al. (2017) include pulsations of the AGB star which make the outflow very dynamic in the vicinity of the stars compared to the steady outflow in our simulations. Moreover, in their works the acceleration of the wind is driven by the pulsations of the AGB donor star and radiation pressure on dust grains, which leads to a different wind velocity profile than in Figure 4.1. These physical mechanisms may influence the amount of material that is able to reach the accretion region in their models.

In our two models in which the outflow of the gas resembles WRLOF we find that the mass accretion efficiencies are relatively low, either close to the BHL value (model Q2a5v5Ω) or below it (model Q2a4v1Ω; see Table 4.3). From Figure 4.9, we observe that for \( v_{\infty}/v_{\text{orb}} < 0.3 \) the results of the WRLOF simulations of Mohamed & Podsiadlowski (2012), as reported by Abate et al. (2013), also show a much lower value than the one expected from the BHL approximation. This may have important consequences for the results based on this model, which is used to study the population of CEMP-s stars (Abate et al., 2015c, 2018). From our understanding, not only the mass ratio and the orbital separation are important parameters that influence binary interaction, but as pointed out in Paper I and confirmed in the present study, the velocity of the wind plays a major role too. Therefore, the fit for the mass-accretion efficiency by Abate et al. (2013) may only be valid for the low wind velocities studied by Mohamed & Podsiadlowski (2012), \( v_{\infty} \approx 4 \text{ km s}^{-1} \), rather than the 15 km s\(^{-1}\) they used in their population synthesis study. This implies that Abate et al. (2013) over-estimate the mass-accretion efficiency, which may change not only the number of CEMP stars they find, but also their final orbital-period distribution.

### 4.4.4. Possible implications for low-mass binary evolution

In order to quantify the impact of our results for the mass-accretion efficiency and angular-momentum loss on the evolution of low- and intermediate-mass binary stars, the relations for these quantities given in Section 4.3.3.1 and Section 4.3.4.1 need to be implemented in binary population synthesis codes. In the following we discuss how our results may impact the evolution of these binary systems.

Abate et al. (2018) apply the fit for angular-momentum loss as a function of \( v_{\infty}/v_{\text{orb}} \) that we provide in Paper I in combination with the analytical formula of Abate et al. (2013) for the WRLOF mass-accretion efficiency to a population of progenitors of CEMP stars. They find that this model is not able to reproduce the observed orbital periods of CEMP binary stars. A self-consistent model where both the angular-momentum loss and mass-accretion efficiency are derived under the same conditions may help to improve their results. However, we can already foresee some problems that may be encountered if we were to apply our current results. With the canonical BHL model, population synthesis codes usually underestimate the observed frequency of CEMP stars among very metal poor stars. Abate et al. (2013) showed that when mass transfer occurs via WRLOF over a wide range of separations, the
large associated accretion efficiencies increase the frequency of CEMP systems. However, as discussed in Section 4.3.4, our results yield mass-accretion efficiencies that are at most a factor of \( \approx 1.6 \) larger than the BHL estimates and much lower than in the WRLOF model of Abate et al. (2013). As discussed by Abate et al. (2013, 2018), the main effect of a lower accretion efficiency is that CEMP stars are produced in a narrower range of orbital periods and masses. Consequently, the resulting CEMP fraction will be lower with our model than in the WRLOF prescription and will likely underestimate the observations. Another aspect that may influence the results of current binary population synthesis studies is that these typically do not include a velocity profile of the wind, but assume a constant wind velocity. As has been shown in Paper I and in this work, the velocity of the wind plays a major role in driving the interaction of the binary system.

We find that the angular-momentum loss is relatively more enhanced above the isotropic-wind value for systems with lower mass ratios (i.e. more equal mass components). With the angular-momentum loss derived from our hydrodynamical models, systems with \( q = 1 \) and \( v_{\infty} / v_{\text{orb}} \lesssim 1.4 \) will shrink instead of widening as would be predicted if the wind were spherically symmetric. However, as \( q \) increases, the maximum value of \( v_{\infty} / v_{\text{orb}} \) for which the orbit shrinks decreases. For instance, for \( q = 4 \), only systems with \( v_{\infty} / v_{\text{orb}} \lesssim 0.82 \) will shrink. We find that 15 out of our 26 models will shrink, whereas the isotropic wind mode usually adopted in binary population synthesis codes would have predicted a widening of their orbit. Compared to the isotropic-wind mode, such systems will evolve towards shorter orbital periods, implying that equal-mass systems are likely to undergo a common envelope (CE) phase (see e.g. Ivanova et al., 2013, for a review on CE) for a wider range of initial orbital separations. In a test of our fits for the angular-momentum loss and mass-accretion efficiencies in the binary population synthesis code binary_c we find that the maximum initial orbital separation at which systems will enter a common envelope increases by a factor between 1.25 and 1.5 AU compared to the isotropic wind case, depending on the mass ratio of the binary. The smallest increase from 4.38 AU to 5.46 AU, occurs for \( q = 4 \), and the largest shift, from 4.32 AU to 6.58 AU for \( q = 4/3 \). This effect may be important in determining the formation rate of the progeny of low- and intermediate-mass binary systems, such as type Ia supernovae, cataclysmic binaries and double white dwarfs.

4.5. Conclusions

We find that the two main parameters that determine how mass transfer proceeds in low-mass binary stars interacting via AGB winds are the mass ratio of the binary system and the ratio \( v_{\infty} / v_{\text{orb}} \). The morphology of the outflow depends on these parameters and determines the amount of mass accreted by the companion of the AGB star and the specific angular momentum that is carried away by the ejected material. Furthermore, because \( v_{\text{orb}} \) depends on the orbital separation, we find that modifying the orbital separation of the system produces
similar results as in Paper I, where the orbital separation was fixed and the wind velocity was varied.

For any mass ratio, a low value of \( \frac{v_\infty}{v_{\text{orb}}} \) produces high mass-accretion efficiencies and large angular-momentum loss, whereas for large \( \frac{v_\infty}{v_{\text{orb}}} \) the angular-momentum loss approaches the isotropic-wind value and the mass-accretion efficiency approaches the BHL approximation. When the masses of the stars are similar, the interaction between the wind and the companion star is stronger resulting in a larger angular-momentum loss and high accretion efficiency.

Using \texttt{binary_c} we model the evolution of binary systems for a large grid of orbital separations that span our hydrodynamical models. We find that for binary stars with initial orbital separations up to 7-10 AU, depending on the mass ratio, tidal forces are efficient and spin-orbit coupling is expected at some point during the AGB phase. In particular, for systems with stellar parameters similar to those in our hydrodynamical models and orbital separations smaller than \( \approx 4-6 \) AU the donor star is nearly in corotation with the orbit. Our hydrodynamical simulations show that for systems with \( \frac{v_\infty}{v_{\text{orb}}} \gtrsim 0.7 \) rotation of the donor star has little effect on the morphology of the outflow, the mass-accretion efficiency and the orbital angular-momentum loss. On the other hand, for smaller velocity ratios, corotation of the AGB donor star plays an important role in the way the stars interact. The outflow morphologies for the two corotating models in which \( \frac{v_\infty}{v_{\text{orb}}} \lesssim 0.7 \) resemble the WRLOF geometry and their orbital angular-momentum loss as well as the mass-accretion efficiencies are different from their non-rotating counterparts. This implies that for \( \frac{v_\infty}{v_{\text{orb}}} \lesssim 0.7 \), the orbital evolution of the system cannot be predicted by treating independently the tidal interactions and the angular-momentum lost due to material escaping the binary system, as is usually done in binary population synthesis codes.

Finally, we fit an analytical relation to our results for the angular-momentum loss and mass-accretion efficiency as a function of the mass ratio and \( \frac{v_\infty}{v_{\text{orb}}} \). In order to test if our results help to reproduce the orbital-period distribution of the progeny of low-and-intermediate-mass binaries interacting via AGB winds, our fits for angular-momentum loss and mass-accretion efficiency need to be applied in binary population synthesis studies.

\textbf{Acknowledgements}

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Chapter 4: Slowly, slowly in the wind

4.A. Appendix: CEMP abundances

Table 4.4 shows the surface abundances for an AGB star with an initial metallicity of $Z = 10^{-4}$, as obtained with the method described in Section 4.2. These abundances are used in the implementation of the cooling of the gas.

<table>
<thead>
<tr>
<th>Element</th>
<th>log($n/n_H$)</th>
<th>Element</th>
<th>log($n/n_H$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
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<td>Al</td>
<td>-7.63</td>
</tr>
<tr>
<td>C</td>
<td>-2.79</td>
<td>Si</td>
<td>-6.56</td>
</tr>
<tr>
<td>N</td>
<td>-5.33</td>
<td>S</td>
<td>-6.96</td>
</tr>
<tr>
<td>O</td>
<td>-4.35</td>
<td>Ar</td>
<td>-7.69</td>
</tr>
<tr>
<td>Ne</td>
<td>-4.54</td>
<td>Ca</td>
<td>-7.84</td>
</tr>
<tr>
<td>Na</td>
<td>-6.82</td>
<td>Fe</td>
<td>-6.59</td>
</tr>
<tr>
<td>Mg</td>
<td>-6.27</td>
<td>Ni</td>
<td>-7.91</td>
</tr>
</tbody>
</table>

4.B. Appendix: Opening angle

An object moving in a fluid causes small disturbances that propagate as sound waves with speed $c$. In the supersonic case ($v > c$) the sound waves propagate within a cone delimited by tangents to the sound wave spheres. The angle between the line delimiting the cone and the flow moving at velocity $v$ forms what is known as the opening angle $\theta$. From Figure 4.11 we see that from trigonometry this angle is:

$$\theta = \sin^{-1}\left(\frac{ct}{vt}\right) = \sin^{-1}\left(\frac{c}{v}\right).$$

The Mach number, $\mathcal{M}$, is defined as the ratio $v/c$.

4.C. Appendix: Dynamical change in $a$

In the Newtonian approximation, the total energy of the relative orbit of two point masses moving under their mutual gravity is given by the sum of the kinetic and potential energies. The energy per reduced mass is defined as:

$$\epsilon \equiv \frac{v^2}{2} - \frac{G(M_d + M_a)}{r},$$

where $r$ is the distance between the bodies and $v$ their relative velocity. The angular momentum of the binary per reduced mass is:

Adapted from Prof. Jerry M. Seitzman lecture notes (http://www.seitzman.gatech.edu/classes/ae2010/machanglenumber.pdf)
Figure 4.11: Schematic representation of the opening angle. The green lines represent the cone formed by the sound waves (light blue) propagating at different times.\(^6\)

\[
\ell = |r \times v|.
\] (4.15)

Since the change in the orbit is small over one orbital period, we approximate the orbits as Keplerian. In this approximation, the relative orbit of the two masses is always a conic section. In order to check whether the orbits of our models remain circular, we assume that the orbit of the system is an ellipse for which the orbital energy per reduced mass is:

\[
\epsilon = -\frac{G(M_d + M_a)}{2a},
\] (4.16)

and the angular momentum per reduced mass:

\[
\ell^2 = \frac{[G(M_d + M_a)]^2}{2\epsilon}(e^2 - 1).
\] (4.17)

At any time in our models, we estimate \(e\) from Equations 4.14, 4.17 and 4.15. \(a\) can be determined at any time from the orbital energy using Equations 4.14 and 4.16, or if the orbit remains circular, \(a\) can be computed from the angular momentum using Equations 4.17, 4.16 and 4.15. We estimate \(\dot{a}\) as the average rate of change of \(a\) over the last 5 completed orbital periods.

### 4.D. Appendix: Spin of the AGB star

In this section we present the results obtained for spin-orbit coupling for the binary models evolved with binary_c as described in Section 4.2.3. We assume the stars to be initially non-
Figure 4.12: (a) $\Omega_{\text{spin}}/\Omega_{\text{bin}}$ as a function of the semi-major axis at the time the AGB star has stellar parameters as in Table 4.1. Colour lines represent different mass ratios. The region where the vertical lines start corresponds to orbital separations for which the binary system had gone into a common envelope (CE) at the time we computed the binary parameters. (b) Maximum $\Omega_{\text{spin}}/\Omega_{\text{bin}}$ as a function of the initial orbital separation of the system. Colours represent equivalent mass ratios.
rotating and we follow the evolution of the spin and orbital angular velocity ($\Omega_{\text{spin}}$ and $\Omega_{\text{bin}}$) under the influence of tidal interactions.

Figure 4.12a shows the ratio $\Omega_{\text{spin}}/\Omega_{\text{bin}}$ as a function of the orbital separation at the moment the AGB donor star has the stellar parameters used as input for our hydrodynamical simulations (Table 4.1). The lines of different colour correspond to different mass ratios. We see that $\Omega_{\text{spin}}/\Omega_{\text{bin}}$ never reaches a value equal to one. However, for small orbital separations ($a \approx 4 \rightarrow 6$, depending on mass ratio) $\Omega_{\text{spin}}/\Omega_{\text{bin}} \approx 0.95$, which indicates that the AGB donor is nearly in corotation with the orbit of the binary. We also observe that for smaller mass ratios, the tidal effects are strong at relatively larger orbital separations. For instance, for $a \approx 10$ AU, and $q = 1$, the donor star rotates at $\approx 60\%$ of the angular velocity of the binary at the time when the mass of the donor is $M_d = 1.2 \, M_\odot$ (Figure 4.12a). For $q = 4$ at the same orbital separation, the donor star rotates only at $\approx 30\%$ of the orbital angular velocity. It is important to note that even for large orbital separations, tidal interaction can trigger rotation of the AGB star. This effect drops considerably with orbital separation and becomes negligible for $a \gtrsim 20$ AU.

Since at the time we measure $\Omega_{\text{spin}}/\Omega_{\text{bin}}$, the donor star is at the superwind phase and has already gone through previous episodes of mass loss and interaction with its less evolved companion before and during the AGB phase, we also compute the maximum angular velocity that the donor star reaches during the AGB phase. Figure 4.12b shows the maximum $\Omega_{\text{spin}}/\Omega_{\text{bin}}$ as a function of the initial orbital separation of the binary for different mass ratios. We see that spin-orbit coupling can lead to near-corotation at some point during the AGB phase even for relatively wide binary stars (up to $\approx 10$ AU), and that the orbital separation at which it occurs decreases with increasing mass ratio. By the time the superwind stage is reached (Fig 4.12a) the star has spun down as a result of strong mass loss and corotation is lost again, except in the closest orbits.

Finally, we also explore the case in which the primary star is initially rotating. We arbitrarily set the initial rotational velocity of the donor star to be $200$ km s$^{-1}$. We find that at the moment the donor star has the stellar parameters used as initial set up of our SPH simulations (see Table 4.1), the angular velocity of the AGB star has a similar trend as shown in Figure 4.12a as a function of the orbital separation for a given mass ratio, up to about $20$ AU. However, for orbital separations larger than $20$ AU the spin of the AGB star does not tend to zero, but the star is rotating slowly at about $\approx 0.1\Omega_{\text{bin}}$. This shows that for $a \lesssim 20$ AU the spin of the star is dominated during the AGB phase by tidal evolution and is independent of the initial spin of the donor star.
The eccentric behaviour of windy binary stars

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Abstract

Carbon-enhanced metal-poor stars, barium stars, CH stars, and extrinsic S stars, among other classes of chemically peculiar stars, are thought to be the products of the interaction of low- and intermediate-mass binaries which occurred when the most evolved star was in the asymptotic giant branch (AGB) phase. Binary evolution models predict that because of the large size of the AGB star, if the initial orbital period of such systems was shorter than a few thousand years, their orbits should have circularised due to tidal effects. However, observations of the progeny of AGB binary stars show that some of these objects have eccentricities up to \( \approx 0.9 \). In this work we explore the impact of wind mass transfer on the orbital parameters of AGB binary stars by performing numerical simulations in which the AGB wind is modelled using a hydrodynamical code and the dynamics of the stars is evolved using an N-body code. We find that in most models the effect of wind mass transfer will contribute to the circularisation of the orbit, but on longer timescales than tidal circularisation if \( e \lesssim 0.4 \). We also find that for relatively low initial wind velocities and pseudo-synchronisation of the donor star a structure which resembles wind Roche-lobe overflow is observed as the stars approach periastron. In this case, the interaction between the gas and the star is stronger than when the initial wind velocity is high and the orbit shrinks...
while the eccentricity decreases. In one of our models wind interaction is found
to pump the eccentricity of the orbit in a similar timescale as tidal circularisation
timescale. However, since the orbit of this model is shrinking this implies that
tidal effects will become stronger during the evolution of the system. Although
the current study is based on a small sample of models, it offers some insight into
the orbital evolution of eccentric binary stars interacting via winds. A large grid
of numerical models where different binary parameters are studied is needed to
test if a regime exists where hydrodynamical eccentricity pumping can effectively
counteract tidal circularisation, and if this can explain the puzzling eccentricities
of the descendants of AGB binaries.

5.1. Introduction

A wide variety of objects are thought to result from interaction in asymptotic giant branch
(AGB) binary systems. These include barium stars and CH stars (Keenan, 1942), extrin-
sic S stars (Smith & Lambert, 1988), carbon-enhanced metal poor (CEMP) stars (Beers &
Christlieb, 2005), and post-AGB binary stars (van Winckel, 2003b). Observations of these
objects show that they have large eccentricities (up to $e \approx 0.9$) for relatively short orbital
periods (between 100-1000 days; Jorissen et al., 1998, 2016; Hansen et al., 2016a; Van der
Swaelmen et al., 2017; Oomen et al., 2018). However, because of the large sizes of AGB stars,
binary evolution models predict that such systems should have circularised due to tidal forces
if their orbital periods were initially lower than a few thousand days (Pols et al., 2003; Izzard
et al., 2010). Therefore, a mechanism that counteracts tidal interaction or that enhances the
eccentricity of the binary after tidal circularisation is needed to explain the observed orbits
of these systems.

Several mechanisms that can pump the eccentricity during the evolution of the binary
system have been proposed, such as the interaction of the binary star with a circumbinary
disk (Artymowicz et al., 1991; Artymowicz & Lubow, 1994; Dermine et al., 2013), phase-
dependent mass loss (e.g. Soker, 2000; Bonačič Marinović et al., 2008) or grazing envelope
evolution (Soker, 2015). The eccentricity pumping due to interaction with a circumbinary
disk is observationally supported by the fact that most post-AGB systems show traces of cir-
cumstellar matter (de Ruyter et al., 2006). However, Rafikov (2016) argues that in order
for this mechanism to efficiently increase the eccentricity of the binary, the circumbinary
disk must be very massive and longer lived compared to the inferred estimates for such pa-
rameters. Vos et al. (2015) tests different eccentricity pumping mechanisms, such as phase-
dependent mass loss, and interaction with a circumbinary disk in an attempt to explain the
eccentricities of hot subdwarf stars, which are post-main sequence objects. However, they
find that these proposed mechanisms are insufficient to reproduce their observed eccentrici-
ties. In a recent study, Kashi & Soker (2018) show that the grazing envelope evolution could
efficiently counteract tidal circularisation. However, this study is performed only for a single set of binary parameters.

In addition to these proposed eccentricity pumping mechanisms, thorough analytical studies of the orbital evolution of eccentric binary systems have been performed by Sepinsky et al. (2007b), Sepinsky et al. (2009) Eggleton (2006), and Dosopoulou & Kalogera (2016). For instance, Eggleton (2006) and Dosopoulou & Kalogera (2016) derive the secular orbital evolution of eccentric binary stars when interaction occurs via isotropic winds. However, several hydrodynamical studies have shown that wind interaction in AGB binaries can be quite different from the isotropic-wind mode (e.g. Theuns & Jorissen, 1993; Mohamed & Podsiadlowski, 2007; Saladino et al., 2018; Chen et al., 2018). Therefore, in order to understand how wind mass transfer interaction in eccentric AGB binary systems impacts the orbital evolution of the system hydrodynamical models are needed.

Most of the current hydrodynamical studies of interacting binary systems have been performed for systems in circular orbits (e.g. Theuns & Jorissen, 1993; Theuns et al., 1996; de Val-Borro et al., 2009; Mohamed & Podsiadlowski, 2012; Liu et al., 2017; Chen et al., 2017; Saladino et al., 2018), while only a handful of studies have investigated hydrodynamical models for eccentric binary stars (e.g. Church et al., 2009; Mohamed, 2010; Lajoie & Sills, 2011; van der Helm et al., 2016; Kim et al., 2017). However, with the exception of van der Helm et al. (2016), most of the studies on eccentric binaries have focussed on understanding the mass transfer mechanism, while little attention has been devoted to the effect of mass transfer on the orbital evolution of the binary. The complexity in performing such studies arises from the fact that in order to derive the change in the orbital parameters of the binary, the change in the orbital angular momentum and the orbital energy need to be known. Additionally, in order to determine the change in the orbital angular momentum, the angular-momentum loss from the orbit, as well as the mass-accretion efficiency onto the companion star are needed. Such parameters can be estimated from hydrodynamical simulations (for circular orbits, see e.g. Theuns et al., 1996; Mohamed & Podsiadlowski, 2012; Chen et al., 2018; Saladino et al., 2019). However, in order to study the change in the semi-major axis and eccentricity simultaneously, numerical models in which the dynamics of the stars is modelled in detail are needed because they permit to estimate the change in the orbital energy as the binary interacts. Furthermore, the hydrodynamical models by Kim et al. (2017) show that in the case of eccentric binary stars interacting via winds, the morphology of the outflow can differ considerably from the non-eccentric case. As shown in Saladino et al. (2018, hereafter Paper I) and Saladino et al. (2019, hereafter Paper II), the evolution of the orbital parameters is strongly influenced by the morphology of the outflow. In addition, in Paper II we show that within the numerical uncertainties we can estimate the change in the semi-major axis by measuring it dynamically from numerical simulations.

In order to understand if the puzzling eccentricities of the descendants of AGB binary systems can be explained by an episode of wind mass transfer, in this paper we perform an exploratory numerical study of low- and intermediate mass eccentric binaries interacting via
AGB winds. To achieve this, we perform simulations in which we couple a hydrodynamical code with a gravitational code to follow the evolution of the orbit. This allows us to measure simultaneously not only the amount of angular momentum-loss and mass-accretion efficiency, but also the change in the semi-major axis and eccentricity of the system.
**Table 5.1: Parameters of the models**

| Name  | \( M_d \) | \( M_a \) | \( q \) | \( P \) | \( a \) | \( e \) | \( J_{\text{orb}} \) | \( v_{pe} \) | \( v_{ap} \) | \( v_{\infty}/v_{pe} \) | \( v_{\infty}/v_{ap} \) | \( \Omega_{\text{spin}} \) | \( R_{L,1|\text{pe}} \) | \( R_d \) | \( R_{\text{sink}} \) | \( T_{\text{eff}} \) | \( \dot{M}_d \) | \( m_g \) |
|-------|----------|----------|------|------|-----|----|-------------|--------|--------|-------------|-------------|-------------|-------------|----|----|------|-------|-------|
| Q2e0  | 1.2  | 0.6  | 2   | 5.96 | 4.00 | 0  | 9.51 \( \times 10^{65} \) | 19.98  | 19.98  | 0.76        | 0.76      | 3.34 \( \times 10^{-8} \) | 378.57      | 330 | 27.60 | 3240  | 1.5 \( \times 10^{-5} \) | 9.6 \( \times 10^{-10} \) |
| Q2e02 | 1.2  | 0.6  | 2   | 8.33 | 5.00 | 0.2 | 1.04 \( \times 10^{66} \) | 21.89  | 14.59  | 0.69        | 1.03      | 3.66 \( \times 10^{-8} \) | 376.31      | 330 | 27.22 | 3240  | 1.5 \( \times 10^{-5} \) | 1.2 \( \times 10^{-9} \) |
| Q2e04 | 1.2  | 0.6  | 2   | 12.83| 6.66 | 0.4 | 1.13 \( \times 10^{66} \) | 23.64  | 10.13  | 0.64        | 1.49      | 3.95 \( \times 10^{-8} \) | 371.71      | 330 | 26.90 | 3240  | 1.5 \( \times 10^{-5} \) | 1.6 \( \times 10^{-9} \) |
| Q2e06 | 1.2  | 0.6  | 2   | 23.57| 10.00| 0.6 | 1.20 \( \times 10^{66} \) | 25.28  | 6.32    | 0.60        | 2.39      | 4.22 \( \times 10^{-8} \) | 365.86      | 330 | 26.79 | 3240  | 1.5 \( \times 10^{-5} \) | 2.4 \( \times 10^{-9} \) |
| Q2e08 | 1.2  | 0.6  | 2   | 66.66| 20.00| 0.8 | 1.28 \( \times 10^{66} \) | 26.81  | 2.98    | 0.56        | 5.07      | 4.48 \( \times 10^{-8} \) | 362.47      | 330 | 138.85| 3240  | 1.5 \( \times 10^{-5} \) | 4.8 \( \times 10^{-9} \) |
| Q2e06v1| 1.2 | 0.6  | 2   | 23.57| 10.00| 0.6 | 1.20 \( \times 10^{66} \) | 25.28  | 6.32    | 0.60        | 2.39      | 4.22 \( \times 10^{-8} \) | 365.86      | 330 | 26.79 | 3240  | 1.5 \( \times 10^{-5} \) | 2.4 \( \times 10^{-9} \) |
| Me05  | 3.0  | 1.5  | 2   | 5.27 | 5.00 | 0.5 | 3.64 \( \times 10^{66} \) | 48.95  | 16.32  | 0.31        | 0.93      | 0         | 231.29      | 200 | 16.74 | 2430  | 1.5 \( \times 10^{-5} \) | 1.6 \( \times 10^{-9} \) |

**Notes:** The first column corresponds to the name of the model. The second row indicates the units of the different parameters. \( M_d \) is the mass of the donor star. \( M_a \) is the mass of the accretor. \( q \) is the mass ratio. \( P \) is the orbital period. \( a \) is the semi-major axis. \( J_{\text{orb}} \) is the initial orbital angular momentum of the binary. \( v_{pe} \) is the orbital velocity at periastron. \( v_{ap} \) is the orbital velocity at apastron. \( v_{\infty}/v_{pe} \) is the wind terminal velocity (\( v_{\infty} \approx 15 \text{ km s}^{-1} \)) to the instantaneous orbital velocity at the periastron. \( v_{\infty}/v_{ap} \) is the wind terminal velocity (\( v_{\infty} \approx 15 \text{ km s}^{-1} \)) to the instantaneous orbital velocity at the apastron. \( R_{L,1|\text{pe}} \) is the Roche lobe of the primary star. For \( e = 0 \) it is computed as in Eggleton (1983) and for \( e > 0 \) as in Sepinsky et al. (2007a). \( R_d \) is the radius of the donor star and it determines the inner boundary of the SPH particles. \( R_{\text{sink}} \) is the radius of the sink particle representing the companion star. Except for model Q2e08 it corresponds to 10% of the radius of the Roche lobe of the companion star at periastron. For model Q2e08 it corresponds to 10% of the Roche lobe of a circular orbit for an orbital separation \( a \). \( T_{\text{eff}} \) the effective temperature of the primary star which also determines the initial temperature of the wind particles. \( \dot{M}_d \) the mass-loss rate of the donor star and \( m_g \) is the gas mass.
5.2. Method

The numerical method employed in this paper is similar to that used in Paper I and Paper II. Here we briefly describe the set-up of the simulations. Using the AMUSE\(^1\) framework (Portegies Zwart et al., 2013; Pelupessy et al., 2013; Portegies Zwart et al., 2009) we couple the smoothed-particle hydrodynamics (SPH) code \(\text{FI}\) (Hernquist & Katz, 1989; Gerritsen & Icke, 1997; Pelupessy et al., 2004) with the N-body code \(\text{HUAYNO}\) (Pelupessy et al., 2012) using the \text{BRIDGE} module (Fujii et al., 2007). The SPH code is used to model the gas dynamics of the wind while the N-body code is used to compute the dynamics of the stars. Our models also include the prescription for cooling or heating of the gas as described in Paper II.

The stars are modelled as point masses and the wind particles are created using the \text{STELLAR\_WIND.PY} module (van der Helm et al., 2019) available in AMUSE. Wind particles are injected with initial velocities \(v_{\text{init}} = 12 \text{ km s}^{-1}\) or \(v_{\text{init}} = 1 \text{ km s}^{-1}\) at a spherical surface with the radius of the donor star, \(R_d\), and their initial temperature is equal to the effective temperature of the star (\(T_{\text{eff}}\)). Similar to Paper II, the equation of motion of the wind in all models contains a term that exactly balances the gravity of the donor star, thus wind particles feel an extra acceleration due to gas pressure which drives them to an average terminal velocity \(v_{\infty} \approx 15 \text{ km s}^{-1}\) or \(v_{\infty} \approx 6 \text{ km s}^{-1}\), depending on the initial wind velocity. These velocities correspond to the typical terminal velocities for AGB stars (Höfner, 2015).

To allow for comparison with our previous work, the stellar parameters for most of our models correspond to those described in Paper II for a mass ratio \(q = M_d/M_a = 2\). Only the stellar parameters of model MMe05 are similar to those used in Paper I. In this model the stars are more massive, the radius and effective temperature of the star are smaller and the donor metallicity is solar. In all models the donor star loses mass at a rate \(\dot{M}_d = 1.5 \times 10^{-5} \text{ M}_\odot \text{ yr}^{-1}\) (see Table 5.1 for an overview of all parameters used).

In Paper II, we find that the strength of interaction between the companion star and the wind depends on the wind-to-orbit velocity ratio, \(v_{\infty}/v_{\text{orb}}\). For large \(v_{\infty}/v_{\text{orb}}\) little interaction occurs and the outflow approximates the spherically symmetric wind mode, while the strongest interaction between the wind and the binary occurs for small \(v_{\infty}/v_{\text{orb}}\) and hence small separations. However, for eccentric binaries the relative orbital velocity and the relative distance of the stars are time-variable. Therefore, in order to guarantee a strong interaction at the point of closest approach, while ensuring that the donor star is within its Roche lobe at periastron\(^2\), we set the semi-major axis of our models by keeping the periastron distance constant, \(a_{pe} = 4 \text{ AU}\), for eccentricities between 0 to 0.8.

For a binary with the characteristics of our models, tidal effects are likely to pseudosynchronise the donor star, i.e. the angular velocity of the star is similar to the angular velocity of the binary at periastron. To this end, in a similar fashion to Paper II, we add a

\(^1\)http://www.amusecode.org/
\(^2\) We compute the Roche lobe radius at periastron using the equation given by Sepinsky et al. (2007a) for a binary system in an eccentric orbit.
tangential velocity, $v_T = \dot{\Omega}_{\text{orb},pe} \times r_{g,d}$, to wind particles as we inject them at the surface of the star, where

$$\dot{\Omega}_{\text{orb},pe} = \frac{2\pi (1 + e)^{1/2}}{P (1 - e)^{3/2}},$$

is the orbital angular velocity at periastron, $P$ the binary orbital period and $r_{g,d}$ the position of the gas particles with respect to the centre of mass of the donor star. Only in model MMe05 the donor is non-rotating.

Similar to Paper I and Paper II, we model the companion star as a sink particle with constant radius. In the majority of the models the sink radius is equal to $0.1 R_{L,2|pe}$, where $R_{L,2|pe}$ is the Roche lobe radius of the companion star at periastron. Only for model Q2e08, where $e = 0.8$, we set the sink radius equal to $0.5 R_{L,2|pe}$ The latter setup is chosen to prevent numerical artefacts in the simulation because the resolution of our models is small and for $e = 0.8$ the typical smoothing length in the vicinity of the companion star is much larger than a sink with radius $0.1 R_{L,2|pe}$.

To optimise the numerical computation, we choose the typical smoothing length of the particles to be proportional to the semi-major axis of the binary (see Paper II). In Table 5.1 we show the corresponding masses of the gas particles. In addition, to minimise computational time we remove particles once they cross a boundary of $5a$. The values for the artificial viscosity parameters are set as $\alpha_{\text{SPH}} = 0.5$ and $\beta_{\text{SPH}} = 1$.

5.3 Results

5.3.1. Morphology

In the following we describe the observed morphologies of the outflow for the eccentric models and we compare them with the outflows observed in their circular counterparts. We note that although we only describe the behaviour of the outflow for one orbit, the same structures repeat over the evolution of the systems.

Figure 5.1 shows the density in the orbital plane at four orbital phases (left to right) for models Q2e0 to Q2e08. The eccentricity of the models increases from top to bottom. For small eccentricities ($e = 0.2$ and $e = 0.4$) the geometries of the outflow look very similar to the circular binary case with two spiral arms tightly wound around the binary delimiting the accretion wake behind the companion star. Similar to the circular case, the inner spiral arm is denser than the outer one due to its proximity to the AGB donor star. In addition, the opening angle of the accretion wake varies as a function of the orbital phase and as a function of the eccentricity.

As the eccentricity increases ($e = 0.6$ and $e = 0.8$) a gradual change in the morphology of the outflow is observed. The accretion wake behind the companion star which for small eccentricities was wrapped around the binary in the form of spiral arms becomes a disrupted ring. This ring forms after the companion star has passed through periastron, as dense wind
FIGURE 5.1: Gas density on the orbital plane for different orbital phases for models Q2e0 to Q2e08 during the ninth simulated orbit. Notice that the eccentricity increases from top to bottom. Time is indicated along the top, relative to the orbital period. The first column corresponds to the phase where the stars are at periastron, the second column to the phase $t/P = 8.2$, the third column to the time where stars are at apastron and the fourth column correspond to $t/P = 8.8$. 
leaves the donor star it compresses the accretion wake into a ring. As the companion star moves in its orbit towards apastron, the ring moves away from the binary. In model Q2e08, because of the large wind-velocity-to-instantaneous-orbital-velocity ratio at apastron, little interaction between the wind and the companion star occurs at this distance, i.e. the outflow remains almost spherically symmetric and the accretion wake resembles the Bondi-Hoyle-Lyttleton case (Bondi & Hoyle, 1944; Hoyle & Lyttleton, 1939).

Models Q2e04 and Q2e06 show an accretion disk around the companion star which builds up after the stars have passed periastron, but it disappears as the companion star approaches apastron. Two numerical effects may be contributing to this behaviour: on the one hand, similar to the circular model in Paper I, the radius of the accretion disk varies over time. Although we set the sink radius to be small, it can happen that if the radius of the disk becomes smaller than the sink, the disk is engulfed by the sink and its mass is added to the accretor star. On the other hand, because of the eccentricity of the systems, when the stars are at apastron little interaction between the gas and the stars takes place and the gas density remains low. With the low resolutions chosen, this implies that the typical smoothing lengths of the gas particles at apastron become slightly larger than the sink radius, which can cause numerical artefacts near the accretor.

Model Q2e06v1 (top row of Figure 5.2) shows that when the initial wind velocity is low \( (v_{\text{init}} = 1 \text{ km s}^{-1}) \) the geometry of the outflow becomes more complex, implying a stronger interaction between the wind and the companion star. In this model we observe a dense ring tightly wound around the binary, which builds up as the companion star moves through periastron. Although at the scale displayed in Figure 5.2 it cannot be observed, this model
shows a stream of gas flowing from the donor star towards the companion star which resembles wind Roche-lobe overflow (de Val-Borro et al., 2009; Mohamed & Podsiadlowski, 2007, 2012), which is not observed in model Q2e06. As the stars move towards their closest distance the stream of gas is formed and it vanishes as the companion star makes it way towards apastron. A similar mass transfer mechanism was found in Paper II for models in which the donor star was in corotation and $v_{\text{init}} \lesssim 5 \text{ km s}^{-1}$. Model Q2e06v1 also shows an accretion disk that forms at the passage through periastron, but as explained above it is engulfed by the sink after the passage through apastron.

Model MMe05, in which the mass ratio is similar to models Q2e0 to Q2e08, but the stars are more massive, the donor star has a smaller radius, and is non-rotating, shows the most complex structure among our models (see the bottom panels of Figure 5.2). An accretion disk is also formed, but contrary to the previously discussed models it is not engulfed by the sink although its size decreases as the companion star moves through periastron. In addition, contrary to the large eccentricity models in which the whole accretion wake is compressed in a ring, in this model two rings are observed. A complete ring which surrounds the donor star is formed by the inner part of the accretion wake which builds up as the companion star moves through periastron. This inner ring, is surrounded by an incomplete ring formed by the outer part of the accretion wake. The presence of both rings is probably related to the high density in the accretion wake in this model, which does not allow the wind to be compressed it into a single ring. Another feature to notice in this model is that the external part of the wake is not clearly defined making the outflow very unsteady. In its circular counterpart (model V15a5, Paper I), we also observed a not-so-smooth accretion wake. However, compared to that model, the opening angle of the wake in model MMe05 is smaller and the accretion wake appears to be more misaligned with respect to the binary axis than in model V15a5. We should note, however, that a direct comparison between models MMe05 and V15a5 is hampered by the different assumptions in them. One the one hand, in model V15a5, the velocity of the wind particles was forced to be constant, and in the same model the mass-loss rate was a factor of 15 smaller than in this work. On the other hand, the SPH resolution of the particles is much lower in this study, as well as the distance at which particles are removed from the simulation.

Finally, we can attempt to compare models MMe05 and Q2e06v1 because their mass ratios are equal and their eccentricities are similar. Furthermore, both models have a low wind-to-orbital-velocity ratio at periastron and a similar wind-to-orbital-velocity ratio at apastron (see Table 5.1). From Figure 5.2 we observe that although both systems show a complex geometry, model MMe05 shows more features in the outflow. When the stars are at periastron, the accretion wake of model MMe05 shows a much wider opening angle than in model Q2e06v1. However, the opposite occurs when the stars are at apastron, i.e. at this distance the accretion wake of model Q2e06v1 is wider than in model MMe05. Additionally, at apastron the accretion wake of model MMe05 is much denser than for Q2e02v1 and very misaligned with respect to the binary axis. Model MMe05 creates the impression that the material the ac-
cretor star collects during its passage through periastron interacts with the accretor star once it reaches apastron. On the other hand, it appears that the strongest interaction between the gas and the stars in model Q2e06v1, which is observed as a dense accretion wake, occurs at periastron. We note that given the stellar parameters chosen for model MMe05, the temperature profile is somewhat different to that of model Q2e06v1. In order to check how this could affect the morphology of the outflow, we performed a test in which the effective temperature of the donor star in model MMe05 and the metallicity were similar to model Q2e06v1. However, no clear differences were found.
Figure 5.3: *Top:* Mass-accretion rate as a function of the orbital phase, \( t/P \) (solid dark blue line), for models Q2a02 to Q2e08, (the eccentricity increases from left to right). The green dashed line corresponds to the BHL accretion rate as computed with Eq. 5.3 and \( \alpha_{BHL} = 0.75 \). The dotted light blue line corresponds to the distance between the stars. *Middle:* Mass-loss rate as a function of time (solid red line) for the same models, measured as the flux crossing a sphere of radius 3\( a \). The dotted pink line corresponds to the distance between the stars. *Bottom:* Corresponding angular-momentum loss expressed by the parameter \( \eta \) as a function of time (solid yellow line). The distance between the star is shown with the dotted brown line. The dashed gray line corresponds to the isotropic-wind value \( \eta_{iso} = (1 + q)^{-2} \), and the black dashed dotted line corresponds to the expected angular-momentum loss as computed by applying the fit for angular-momentum loss from Paper II to the average orbital velocity.
5.3 RESULTS

Figure 5.4: Similar to Fig. 5.3, but for models Q2e06v1 (left) and MMe05 (right). For model Q2e06v1 there is no circular analogue to which it can be compared, so we compare it to model Q2e06, where the only parameter that changes is the wind terminal velocity. For model MMe05 the angular-momentum loss is compared to the circular model V15a5 from Paper I.


5.3.2. Mass-accretion rates

The Bondi-Hoyle-Lyttleton (BHL) (Hoyle & Lyttleton, 1939; Bondi & Hoyle, 1944; Edgar, 2004) formalism gives an estimate for the mass-accretion rate expected by a body moving in a gas medium. This model is often applied to wind accretion in binary systems, although the assumption of a uniform density and velocity field does not hold, especially for AGB winds. For binary stars in eccentric orbits the average mass-accretion rate is usually taken as (Boffin & Jorissen, 1988):

\[
\dot{M}_a = -\alpha_{\text{BHL}} \frac{\dot{M}_d}{\sqrt{1-e^2}} \left( \frac{GM_a}{a v_w^2} \right)^2 \left[ 1 + \left( \frac{v_{\text{orb}}}{v_w} \right)^2 \right]^{-3/2},
\]

(5.2)

where \( \alpha_{\text{BHL}} = 0.75 \) is a constant\(^3\), \( \dot{M}_d < 0 \) is the rate at which the donor star is losing mass, \( G \) the gravitational constant, \( a \) the semi-major axis of the system, \( v_w \) the local wind velocity, and \( v_{\text{orb}}^2 = G(M_d + M_a)/a \) is the relative orbital velocity in a circular orbit with the same \( a \). In an eccentric system the relative orbital velocity and the relative separation of the stars are time-variable parameters. For this reason, in order to get a better estimate for the average mass-accretion rate, we substitute \( a \) and \( v_{\text{orb}} \) in Eq. 5.2 by the instantaneous orbital separation, \( r \), and instantaneous relative velocity of the stars, \( v \), so that (Mohamed, 2010):

\[
\dot{M}_a = -\alpha_{\text{BHL}} \frac{\dot{M}_d}{r^2} \left( \frac{GM_a}{v_w^2} \right)^2 \left[ 1 + \left( \frac{v}{v_w} \right)^2 \right]^{-3/2}.
\]

(5.3)

The top panels of Figures 5.3 and 5.4 show the mass-accretion rate onto the companion star as measured from the masses of the gas particles which cross the sink boundary per timestep. Given the discreteness of the SPH model, the mass-accretion rates show an associated shot noise. In order to suppress statistical fluctuations, we average the accreted mass over long time intervals (c.f. Paper I, section 3.5). For better appreciation, we only show the mass-accretion rate for two orbits of each model. The dotted lines in the figures show the distance between the stars for better recognition of the orbital phases. The green dashed lines overplotted in each figure show the BHL analytical estimate as computed from Eq. 5.3.

For models Q2e02 to Q2e08 the BHL prescription predicts an enhancement in the mass-accretion rate when the stars are at their closest distance and a minimum in the mass-accretion rate at apastron. This occurs because in Eq. 5.3 the term containing the distance of the stars \( r^{-2} \) dominates over the factor containing \( v/v_w \). Likewise the BHL formalism predicts a decrease in the mass accretion for large eccentricities. We observe a similar behaviour in our models. Models Q2e04 and Q2e06 show an extra peak in the mass-accretion rate before the stars reach their maximum distance. As discussed in the previous section, an accretion disk builds up after the passage through periastron in these systems. Since the size of the disk is not constant, when the radius of the disk becomes smaller than the sink radius, the material in the disk is swallowed by the sink which is seen as an increase in the mass-accretion

\(^3\) Note that in Boffin & Jorissen (1988) \( \alpha_{\text{BHL}} = \alpha/2 \), with \( \alpha \) a constant between 1 and 2.
rate. Table 5.2 shows the average mass-accretion rate per orbit normalised by the donor star mass-loss rate for our models. For comparison we also compute the average mass-accretion rate as estimated by Eq. 5.2. The mass-accretion efficiencies we find are up to a factor between $\approx 1.1$ to $\approx 1.7$ higher than predicted by the BHL formalism for eccentricities between 0 and 0.6, with the largest difference occurring for the model with a circular orbit. As the eccentricity increases to 0.8, the average accretion rate approaches the BHL approximation.

Model Q2e06v1 (top panel of Fig. 5.4) shows two peaks in the mass-accretion rate, one occurring at periastron and the second occurring near apastron. Similar to model Q2e06, in this model we also observe an accretion disk that builds up after the passage of the stars through periastron and is swallowed by the sink when the stars approach apastron. However, we note that for a system with the characteristics of model Q2e06v1 the BHL formalism predicts that the maximum in the accretion rate occurs when the stars are at their maximum separation, rather than at periastron. The reason for this is that due to the low wind velocity in this model the factor containing the term $v/v_w$ dominates over the factor $r^{-2}$ in Eq. 5.3. Thus, even though the maximum in the mass-accretion efficiency in our simulation occurs because at the same time the accretion disk is engulfed by the disk, a maximum at apastron is likely to occur. For the same model we observe an enhancement in the mass-accretion rate when the stars are at periastron, which peaks nearly at the same value as the accretion rate at apastron. However, we note that the mass-accretion efficiency we find for this model is a factor $\approx 2.7$ lower than in the BHL prescription. A similar trend was observed in Paper II, where for very low initial wind velocities mass-accretion efficiencies below the BHL formalism were found.

Model MMe05 (right panel of Fig. 5.4) only shows a large peak at apastron. As seen from Fig. 5.4 (green dotted line) and similar to model Q2e06v1, the theoretical BHL model predicts that for a system with the characteristics of model MMe05 the maximum in the mass-accretion rate occurs when the star is at apastron. However, the peak in the mass-accretion rate at apastron found in our numerical models is a factor of $\approx 4$ larger than that predicted by the BHL approximation. This is interesting because in model Q2e06v1 the largest peak in the accretion rate, as predicted by the BHL formalism, also occurs at apastron, but our numerical models find accretion rates a factor of $\approx 2$ lower at this phase. If we compare model MMe05 to its circular counterpart, model V15a5 in Paper I, the average mass-accretion rate is a factor $\approx 1.3$ larger.

### 5.3.3. Angular-momentum loss

The middle panels of Figs. 5.3 and 5.4 show the rate at which mass is lost from the binary system per orbit. We measure this quantity as the flux of mass crossing a sphere of radius $3a$. We choose this radius because for circular orbits we have shown that beyond this distance no further exchange of angular momentum between the wind and the orbit takes place (see Paper I).
### Table 5.2: Mass accretion

<table>
<thead>
<tr>
<th>Model</th>
<th>$\langle \beta \rangle_{\text{BHL}}$</th>
<th>$\langle \beta \rangle_{\text{hydro}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q2e0</td>
<td>0.056</td>
<td>0.094</td>
</tr>
<tr>
<td>Q2e02</td>
<td>0.045</td>
<td>0.071</td>
</tr>
<tr>
<td>Q2e04</td>
<td>0.034</td>
<td>0.046</td>
</tr>
<tr>
<td>Q2e06</td>
<td>0.023</td>
<td>0.026</td>
</tr>
<tr>
<td>Q2e08</td>
<td>0.011</td>
<td>0.010</td>
</tr>
<tr>
<td>Q2e06v1</td>
<td>0.162</td>
<td>0.059</td>
</tr>
<tr>
<td>MMe05</td>
<td>0.136</td>
<td>0.293</td>
</tr>
</tbody>
</table>

**Notes:** $\langle \beta \rangle_{\text{BHL}}$ corresponds to the average mass-accretion efficiency computed from Eq. 5.2 divided by $M_d$. $\langle \beta \rangle_{\text{hydro}}$ is the average mass-accretion efficiency per orbit obtained from the hydrodynamical models.

The maximum in the mass-loss rate occurs at the time when most of the material in the ring (for highly eccentric systems) or of the spiral wake (for low-eccentricity systems) crosses the $3a$ boundary. As gas moves away from the binary it removes angular momentum from the orbit, which was exchanged during the strong interaction at close orbital separations. At the same time, gas also removes angular momentum due to the rotation of the donor star. Similar to Paper II, we parametrise the angular momentum lost as:

$$
\dot{J} = \eta_{\text{orb}}J_{\text{orb}}\mu^{-1}(1 - \beta)\dot{M}_d + \dot{J}_{\text{spin}}
$$

(5.4)

where the first term on the right-hand side corresponds to the change in the orbital angular momentum, where $J_{\text{orb}}$ is the orbital angular momentum of the binary, $\mu = M_dM_a/M_d + M_a$ the reduced mass of the binary, $\beta$ is the average mass-accretion efficiency per orbit as computed from Eq. 5.2. The second term in Eq. 5.4 is the contribution from the loss of spin angular momentum, $\dot{J}_{\text{spin}} = 2/3R_d^2\dot{M}_d\Omega_{\text{orb},\text{pe}}$. As shown in Paper II, this accurately describes the angular-momentum loss of a single AGB star in our simulations. The bottom panels of Figs. 5.3 and 5.4 show the specific angular momentum that the wind takes away from the orbit in terms of the parameter $\eta_{\text{orb}}$. It is measured at the time the SPH particles cross the $3a$ boundary. We only take the perpendicular component to the orbital plane of the angular momentum, $J_z$, since we have verified that the other two components, $J_x$ and $J_y$, are very small, i.e. the flow is symmetric with respect to the orbital plane. Note that due to the eccentricity of the systems the angular-momentum lost over one orbit is not constant. We compare the angular-momentum loss of our eccentric models to the isotropic-wind mode case, $\eta_{\text{iso}} = (1 + q)^{-2}$, and to the fit for the specific-angular momentum loss, $\eta_{\text{fit}} = \eta_{\text{orb}}(q, v_{\text{w}}/v_{\text{orb}})$, obtained in Paper II. Since $v_{\text{orb}}$ varies over the orbit, in order to apply the formula for $\eta_{\text{fit}}$, we take $v_{\text{orb}} = \langle v \rangle_{\text{orb}}$, where $\langle v \rangle_{\text{orb}}$ is the time average velocity over the orbit, which is dominated by the long time the stars spend at apastron. Table 5.3 shows the values for $\eta_{\text{iso}}$, $\eta_{\text{fit}}$, and the
average specific angular momentum lost per orbit, \( \langle \eta \rangle_{\text{orb}} \), which in accordance with Eq. 5.4 is computed as,

\[
\langle \eta \rangle_{\text{orb}} = \left( \frac{J_{\text{orb}}}{\mu} \right)^{-1} \left[ \left( \frac{\sum_{i} J_{z,i} J_{\ast,i}^{2}}{N m_{g}} \right)_{\text{orb}} - \frac{2 R_{a}^{2} \Omega_{\text{orb,pe}}}{3 (1 - \beta)} \right],
\]

where \( J_{z,i} \) is the perpendicular component of the angular momentum of the \( i \)’th particle which crosses the 3a boundary, \( N \) the number of particles that cross the 3a boundary in one orbit, and \( m_{g} \) the mass of the SPH particles.

For models with similar stellar parameters but different eccentricities (Q2e02 to Q2e08), we observe that as the eccentricity increases the angular-momentum loss decreases. For model Q2e02, where the eccentricity is very small, the loss in angular momentum is almost the same as in the circular case. However, for the most eccentric case Q2e08 the angular-momentum loss is smaller by a factor \( \approx 1.5 \) and approaches the isotropic-wind mode. This is not surprising since the companion star spends most of its time at apastron, where the outflow is not strongly modified and has an almost spherically symmetric geometry, as shown by the plot of the gas density in the orbital plane for this model (fourth panel of Fig. 5.1). Model Q2e06v1 shows an \( \eta \) value a factor \( \approx 1.3 \) larger than its counterpart with larger wind velocity, which reflects the stronger interaction between the wind and the companion star occurring in this model.

If we compare models MMe05 and Q2e06, where the eccentricities are nearly the same and the mass ratio is equal, we notice that in model MMe05 a much larger amount of angular momentum is lost. We recall that in model MMe05, the geometry of the outflow is strongly modified compared to model Q2e06 (see Sect. 5.3.1). In model MMe05 a very dense accretion wake is observed behind the companion star during the whole orbit at an angle that is considerably misaligned with the binary axis. An accretion wake with these characteristics (high density and misalignment with the binary axis) will exert a stronger torque on the companion star allowing a larger exchange of angular momentum between the wind and the orbit of the stars. Finally and although it cannot be observed in Figure 5.4, the angular-momentum loss in model MMe05 is increasing as a function of time. It is not clear why this increase occurs, but it suggests that at each passage of the stars through periastron a stronger interaction takes place.

Finally, we find that \( \langle \eta \rangle_{\text{orb}} \) agrees within \( \approx 20\% \) to the fit given in Paper II for \( \eta(q, v_{w}/\langle v \rangle_{\text{orb}}) \) for models Q2e0-Q2e08 and MMe05. The best agreement occurs for large eccentricities (\( e = 0.6 \) and \( e = 0.8 \)). However, for model Q2e06v1 we find that the orbital angular momentum lost from the system is a factor \( \approx 2 \) lower than what our fit predicts.
# Chapter 5: The eccentric behaviour of windy binary stars

## Table 5.3: Angular-momentum loss

<table>
<thead>
<tr>
<th>Model</th>
<th>$\eta_{iso}$</th>
<th>$\langle \eta \rangle_{fit}$</th>
<th>$\langle \eta \rangle_{orb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q2e0</td>
<td>0.111</td>
<td>0.278</td>
<td>0.217</td>
</tr>
<tr>
<td>Q2e02</td>
<td>0.111</td>
<td>0.240</td>
<td>0.199</td>
</tr>
<tr>
<td>Q2e04</td>
<td>0.111</td>
<td>0.197</td>
<td>0.175</td>
</tr>
<tr>
<td>Q2e06</td>
<td>0.111</td>
<td>0.154</td>
<td>0.151</td>
</tr>
<tr>
<td>Q2e08</td>
<td>0.111</td>
<td>0.123</td>
<td>0.130</td>
</tr>
<tr>
<td>Q2e06v1</td>
<td>0.111</td>
<td>0.411</td>
<td>0.198</td>
</tr>
<tr>
<td>MMe05</td>
<td>0.111</td>
<td>0.402</td>
<td>0.494</td>
</tr>
</tbody>
</table>

**Notes:** $\eta_{iso}$ corresponds to the specific angular-momentum loss in units of $J/\mu$ for the isotropic-wind case. $\langle \eta \rangle_{fit}$ corresponds to the average angular-momentum loss as derived by computing the average orbital velocity during one orbit and applying the fit for $\eta(v_w/v, q)$ obtained in Paper II. $\langle \eta \rangle_{orb}$ is the average angular-momentum loss per orbit for the numerical models presented in this paper.
5.3 RESULTS

**Figure 5.5:** *Top:* Semi-major axis as a function of time, relative to the orbital period, for models Q2e02 to Q2e08. Notice that since the change in the orbit is very small (between $10^{-4}$ and $10^{-2}$ AU), for better visualisation the order of magnitude of the quantities along the y axis is given on the top left corner. *Bottom:* The eccentricity as a function of $t/P$ for models Q2e02 to Q2e08. Similar to $a$, the order of magnitude of the quantities along the y axis is given on the top left corner. We only show the evolution of $a$ and $e$ for the last 5 orbits of our simulations. The dotted gray lines correspond to the distance between the stars, with the minimum corresponding to periastron and the maximum to apastron.
Figure 5.6: Similar to Fig. 5.5, but for models Q2e06v1 (left) and MMe05 (right).
5.3.4. Changes in the orbital elements

Because we use an N-body code to compute the dynamics of the stars, it is possible to measure the change in the semi-major axis and eccentricity of the orbit directly from the simulations within the numerical error (see Paper II). The total energy per reduced mass, \( \epsilon \), of two bodies orbiting each other under the influence of their gravity is given by the sum of the kinetic energy and the potential energy of the system,

\[
\epsilon = \frac{v}{2} - \frac{G(M_d + M_a)}{r}.
\] (5.6)

In addition, the angular momentum per reduced mass of the system can be written as:

\[
\ell = |r \times v|.
\] (5.7)

On the other hand, the orbital energy per reduced mass and specific angular momentum of a system in a Keplerian orbit are given by,

\[
\epsilon = -\frac{G(M_d + M_a)}{2a},
\] (5.8)

and

\[
\ell = G(M_d + M_a) \sqrt{\frac{e^2 - 1}{2\epsilon}}.
\] (5.9)

By combining Eqs. 5.6 and 5.8, we can determine \( a \) at any given time in our simulations. A similar calculation can be done for \( e \) by combining Eqs. 5.9, 5.8 and 5.7.

Figs. 5.5 and 5.6 show the evolution of \( a \) and \( e \), as computed above, for the last 5 orbits of our simulations. We assume that similar to Paper I, where we simulated circular orbits, after the fourth orbit a quasi-steady state is reached. For models Q2e02 to Q2e08, in which angular-momentum loss is relatively small, a similar long-term trend for \( a \) and \( e \) is observed where the semi-major axis increases during the evolution of the system and the eccentricity decreases. We note that the short-timescale variations seen in Figs. 5.5 and 5.6 have little physical meaning, because \( a \) and \( e \) are only well-defined for a complete orbit. Models Q2e06v1 and MMe05 show the opposite trend in \( a \) to models Q2e02 to Q2e08. Since more angular momentum is lost in these systems their orbits are seen to be shrinking. However, model Q2e06v1 shows a decrease in eccentricity, whereas in model MMe05 the eccentricity increases. This likely results from where the exchange of angular momentum takes place during the orbit. In model Q2e06v1 (and the other Q2e0i models) it appears from the high density in the accretion wake at periastron that most of the exchange of angular momentum takes place while the stars are at their closest distance, which will result in a decrease in eccentricity since the companion star will be slowed down at periastron. However, if the strongest torque occurs at apastron (as appears to be the case for model MMe05 from Figure 5.2), the eccentricity will be pumped since the companion star will be slowed down before it moves through periastron again.
The change in orbital separation and eccentricity over a time interval \( \Delta t \) can be derived from Eqs. 5.8 and 5.9, yielding:

\[
\frac{\Delta a}{a} = -\frac{\Delta \epsilon}{\epsilon} + \frac{\Delta M}{M},
\tag{5.10}
\]

where \( M = M_d + M_a \), and

\[
\frac{\Delta \epsilon}{\epsilon} = \frac{1 - e^2}{e^2} \left( \frac{1}{2} \frac{\Delta a}{a} + \frac{1}{2} \frac{\Delta M}{M} - \frac{\Delta \ell}{\ell} \right).
\tag{5.11}
\]

The change in \( \epsilon \) and \( \ell \) is derived from the relative velocity and separation of the stars, which we know at any time in our simulations (Eqs. 5.6 and 5.7). We estimate \( \dot{a} \) and \( \dot{\epsilon} \) as the average rate of change given by Eqs. 5.10 and 5.11 over the last 5 orbits. The resulting values of \( \dot{a}/a \) and \( \dot{\epsilon}/\epsilon \) are shown in Table 5.4. For models Q2e02 to Q2e08, we observe that as the change in angular momentum approaches the isotropic regime (for systems with high eccentricities), the rate at which the system widens becomes larger. For instance, over the ten orbits we run our simulations, in model Q2e08 the orbit expands by about \( \pi 0.08 \) AU. On the other hand, systems with small eccentricities (Q2e02 and Q2e04) widen slowly. This contrasts with the results obtained for their circular counterpart (model Q2e0), where we find that the system shrinks due to the relatively large amount of angular momentum lost.

Model MMe05 is shrinking very fast compared to the rest of the models, and its eccentricity is increasing at a high rate. We should observe however, that even though the stellar parameters for this system are different from the rest of the models we haven chosen a mass-loss rate similar to the other models. As the donor star in this model has a smaller radius it should actually be losing mass at a slower rate. We have verified that by setting a lower mass-loss rate \( 10^{-6} \, \text{M}_\odot \, \text{yr}^{-1} \), as in Paper I) the angular-momentum loss rate and the mass-accretion rate scale down proportionally, i.e. the rate at which the semi-major axis and eccentricity change per unit mass lost is similar.

Our results for models Q2e02 to Q2e08 show an agreement in the sign of \( \dot{a} \) and \( \dot{\epsilon} \) with the analytical derivations for these quantities by Dosopoulou & Kalogera (2016, eqs. 81 and 82) for fast winds. Notice that eq. 82 from Dosopoulou & Kalogera (2016) for \( \dot{\epsilon} \) was first derived by Eggleton (2006). To make the comparison between our results and the orbital secular evolution from Dosopoulou & Kalogera (2016), we replace the assumed Bondi-Hoyle accretion rate with the average mass-accretion rate obtained from our hydrodynamical models. For models Q2e02 to Q2e08, the analytical derivation by Dosopoulou & Kalogera (2016) predicts a faster widening of the orbit, which is a factor \( \approx 1.1 - 2.24 \) larger for models Q2e04 to Q2e08, with the smallest disagreement occurring for Q2e08. Only for model Q2e02, the disagreement in \( \dot{a} \) between the analytical model and the numerical simulation is very large by a factor of \( \approx 7 \). Regarding the eccentricity, the model by Dosopoulou & Kalogera (2016) predicts a faster decrease in the eccentricity of the orbit for models Q2e02 to Q2e08, which is a factor \( \approx 1.44 - 2 \) larger compared to our models. The smallest discrepancy in \( \dot{\epsilon} \) also occurs for model Q2e08. For models Q2e06v1 and MMe05 our results differ from the theoretical
derivation of secular evolution by Dosopoulou & Kalogera (2016). Whereas the sign for the eccentricity change in model Q2e06v1 agrees with their analytical model and is only a factor 1.18 smaller than approximated by their study, their model predicts an increase in the semi-major axis. On the other hand, according to Dosopoulou & Kalogera (2016) the semi-major axis of model MMMe05 should be shrinking a factor \( \approx 2.17 \) slower and the eccentricity should be decreasing, because in their model an increase in eccentricity only occurs for binaries with \( 0 < q < 0.78 \).

5.4. Discussion

In this section, we discuss how the geometry of the outflow of our hydrodynamical models compares with observations of AGB binary systems which are thought to be in eccentric orbits, and how our exploratory results for the rate of change of the eccentricity compare to the tidal circularisation timescales. In addition, we briefly mention some of the numerical and physical effects that may affect our results, and some of the future work that could be performed in order to have a better understanding of how wind mass transfer can impact the eccentricity of binary stars with a red-giant component.

5.4.1. Comparison to observations

Our work shows that wind mass transfer in eccentric binaries results in geometries of the outflow which differ considerably from the circular binary case when \( e \gtrsim 0.5 \). The present study thus confirms the potential of numerical models for constraining the eccentricity in observed interacting binary stars. For instance, a morphology of the outflow which shows a disrupted ring, has been observed in the inner region of the spiral pattern of the binary system AFGL 3068. Hydrodynamical models have shown that the geometry of this system can be reproduced if the eccentricity of the system is about 0.8 (Kim et al., 2017). However, the bifurcation in the spiral described by these models and also observed in AFGL 3068 is not found in our work. We note that a direct comparison between our models and the observations of AFGL 3068 cannot be made. On the one hand, the mass ratio of AFGL 3068 is different to the one studied in this work. On the other hand the spiral pattern of this object extends up to \( \approx 60 \) times the mean orbital separation of the system, and we remove particles at short distances from the binary. Another system for which incomplete ring patterns have been observed is the carbon star CIT 6, which is believed to contain a star evolving from the AGB to the post-AGB phase (Kim et al., 2013). To explain the observed geometry of the outflow of CIT 6, a binary companion with a very high eccentricity has been suggested (Kim et al., 2015). However, for similar reasons to the ones mentioned above (different mass ratios and the fact that we remove particles close to the binary) no direct comparison can be made between our models and those observations.
CHAPTER 5: THE ECCENTRIC BEHAVIOUR OF WINDY BINARY STARS

<table>
<thead>
<tr>
<th>Model</th>
<th>(\dot{a}/a)</th>
<th>(\dot{e}/e)</th>
<th>((\dot{e}/e)_{\text{tides}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q2e0</td>
<td>(-8.43 \times 10^{-7})</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Q2e02</td>
<td>(8.28 \times 10^{-7})</td>
<td>(-1.18 \times 10^{-6})</td>
<td>(-1.10 \times 10^{-4})</td>
</tr>
<tr>
<td>Q2e04</td>
<td>(2.77 \times 10^{-6})</td>
<td>(-9.58 \times 10^{-7})</td>
<td>(-1.91 \times 10^{-5})</td>
</tr>
<tr>
<td>Q2e06</td>
<td>(4.56 \times 10^{-6})</td>
<td>(-5.37 \times 10^{-7})</td>
<td>(-4.41 \times 10^{-6})</td>
</tr>
<tr>
<td>Q2e08</td>
<td>(6.15 \times 10^{-6})</td>
<td>(-2.16 \times 10^{-7})</td>
<td>(-9.22 \times 10^{-7})</td>
</tr>
<tr>
<td>Q2e06v1</td>
<td>(-1.58 \times 10^{-6})</td>
<td>(-1.66 \times 10^{-6})</td>
<td>(-4.41 \times 10^{-6})</td>
</tr>
<tr>
<td>MMe05</td>
<td>(-6.88 \times 10^{-6})</td>
<td>(6.41 \times 10^{-6})</td>
<td>(-5.10 \times 10^{-6})</td>
</tr>
</tbody>
</table>

**NOTES:** \(\dot{a}/a\) corresponds to the change in semi-major axis as measured dynamically from the simulations. \(\dot{e}/e\) is the change in the eccentricity as measured dynamically from the simulations. \((\dot{e}/e)_{\text{tides}}\) corresponds to the expected change in the eccentricity due to tidal interaction, is derived from Eq. 5.12.

### 5.4.2. Orbital evolution timescales

Our results shed some light on the evolution of the orbital parameters when wind mass transfer occurs. However, we note that some of the physical processes occurring in these systems have been neglected in our models. For instance, given the large sizes of AGB stars, for close binary systems circularisation of the orbit is likely to occur due to tidal effects. Since the stars in our models are approximated by point particles, this effect is not taken into account.

In order to estimate the circularisation timescale predicted by tidal evolution, we use eq. 10 from Hut (1981):

\[
\frac{\dot{e}}{e} = -27 \left( \frac{k}{T} \right) q^{-1} (1 + q^{-1}) \left( \frac{R_d}{a} \right)^8 \frac{1}{(1 - e^2)^{13/2}} \times f_3(e^2) - \frac{11}{18} (1 - e^2)^{3/2} f_4(e^2) \frac{\Omega_{\text{spin}}}{\Omega_{\text{bin}}},
\]

(5.12)

where \(k\) is the apsidal motion constant of the donor star, \(T\) is the time-scale on which significant changes in the orbit take place through tidal evolution, \(f_3\) and \(f_4\) are polynomial functions of \(e^2\) given by Hut (1981), \(\Omega_{\text{spin}}\) is the angular velocity of the donor star and \(\Omega_{\text{bin}} = 2\pi/P\) is the mean angular velocity of the binary. We take \((k/T)\) as in eq. 30 from Hurley et al. (2002), with the mass of the envelope equal to \(M_{\text{env}} = 0.55 M_\odot\) in models where \(M_d = 1.2 M_\odot\), and \(M_{\text{env}} = 2.4 M_\odot\) for \(M_d = 3 M_\odot\). For an AGB star the size of the core is negligible compared to the convective envelope, thus we approximate the radius of the envelope as \(R_{\text{env}} = R_d\). Note that in Eq. 5.12 the sign of \(\dot{e}\) is determined by the last factor containing \(\Omega_{\text{spin}}/\Omega_{\text{bin}}\), i.e. \(\dot{e} > 0\) only occurs for high rotation rates (see Hut, 1981, for a discussion).

Table 5.4 shows the estimates for the circularisation timescales for systems with binary parameters as in our simulations. For models Q2e02 and Q2e04, tidal circularisation is much...
more effective than the circularisation induced by wind interaction, whereas for models with $e \gtrsim 0.6$ the tidal circularisation timescale is similar to the hydrodynamical circularisation timescale. We can roughly estimate by how much the eccentricity of these models will decrease by the time the star leaves the AGB phase. A star with the characteristics of our donor star will spend another $\approx 3 \times 10^4$ yr in the superwind phase (see Paper II for the method used to evolve this star). By assuming that the tidal circularisation timescale is constant during the remaining time the star is in the AGB phase, we find that by the time the star leaves the AGB phase the binary will have an eccentricity of $\approx 0.007$ for model Q2e02 and $e(t = 3 \times 10^4$yr) = 0.23 for model Q2e04. However, given the long circularisation timescales from both tidal interaction and wind interaction, for cases Q2e06 and Q2e08 the change in the eccentricity will be almost negligible before the donor star leaves the AGB phase.

Similar to Q2e06, in model Q2e06v1 the induced change in the eccentricity due to tidal forces will be modest while the star is still in the AGB phase. However, since the orbit in this model is shrinking, tidal effects will become more effective during the evolution of the system. For model MMMe05, our hydrodynamical models predict an increase in eccentricity. In this case, the tidal circularising timescale is of the same order of magnitude as the hydrodynamical eccentricity pumping timescale. Therefore it is possible that these effects counteract each other leading only to a small change in the eccentricity. However, as mentioned in Sect. 5.3.3 the mass-loss rate of this system is assumed for the superwind phase of an AGB star with the stellar parameters as in model Q2e06 and not for a donor star with the parameters of model MMMe05. By assuming a lower mass-loss rate for this model, $\dot{M}_d = 10^{-6}$ $M_\odot$ yr$^{-1}$ (as in Paper I), we find that $\dot{e}/e = 4.55 \times 10^{-7}$ yr$^{-1}$. In that case, the eccentricity pumping timescale from the hydrodynamical models will not be able to compete with the tidal circularisation timescale. Furthermore, similar to model Q2e06v1 since the semi-major axis is decreasing in this system, tidal forces will become stronger as the system evolves. From Eq. 5.12, we see that tidal evolution would only be able to pump the eccentricity in a system like MMMe05 if $\Omega_{\text{spin}} = \frac{1}{1.1}\Omega_{\text{orb,pe}}$, where $\Omega_{\text{orb,pe}}$ is the angular velocity at periastron.

### 5.4.3. Model MMMe05

Model MMMe05 shows that it is possible to find a regime where the eccentricity increases due to wind mass transfer, and this trend is quite robust against various tests we have performed. We have tested that regardless of the assumed mass-loss rate of the donor star, the sink radius$^4$, and the temperature profile, the result always lead to an increase in eccentricity. However, there are many other characteristics of this model which make it difficult to compare to the more realistic models Q2e0i. On the one hand, unlike models Q2e0i where the

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$^4$We performed a test where we considered a sink of radius equal to 10% the Roche lobe of the companion star for a circular orbit with $a = 5$ AU. We find that the average mass-accretion efficiency increased compared to the value given in Table 5.2 for model MMMe05, but the angular-momentum loss remained constant. This resulted in an increase of the eccentricity and a decrease of the semi-major axis.
stellar parameters of the donor were taken from a stellar evolution code, the stellar parameters of the AGB star for model MMe05 were chosen arbitrarily to match the parameters of the systems studied in Paper I. Furthermore, in order to compare our results for this model to its circular counterpart from Paper I, we neglected the possibility of pseudo-synchronisation of the donor star. As seen in Paper II, rotation of the donor star for low $v_{\infty}/v_{\text{orb}}$ can modify the morphology of the outflow resulting in a different angular-momentum loss than when the star is non-rotating. This could potentially affect the evolution of the orbital parameters of the binary. On the other hand, whereas in Paper I we assumed a constant velocity profile of the wind, in this work the AGB wind feels an acceleration due to gas pressure, which results in a different wind velocity profile (see Paper II). However, we verified in a test that by taking a predefined terminal wind velocity, as in Paper I, this also results in an increase of the eccentricity. Another difference compared to models Q2e0i is that the radius of the donor is much smaller which could also impact the results.

We note that model MMe05 could potentially counteract tidal circularisation in the region of interest for the progeny of AGB binary systems, since the orbital period in this model is of $\approx 1900$ day. However, we should keep in mind that in this model the stellar parameters were chosen arbitrarily and that since the system is shrinking, tidal interaction will become stronger. In order to verify if the results from this model are physically possible a larger grid of simulations where the stellar parameters of the donor star are computed with a stellar evolution code are necessary.

5.4.4. Other numerical and physical aspects

Besides tidal interaction, there are other physical mechanisms and numerical aspects that could influence our results of the change in the orbital elements of the system. For instance, some physical processes which have not been taken into account in this work are pulsations of the AGB star, dust formation and radiative transfer. By considering these processes, the wind velocity profile will be different to that assumed in this work, which could result in a stronger interaction between the companion star and the wind, especially at periastron. This may affect the amount of angular momentum lost from the binary, and in consequence impact the evolution of the orbital parameters of the system.

From the numerical point of view, we find that the large size of the sink particle results in an enhancement in the mass-accretion rates at different phases during the orbit of some systems. One way to overcome this problem is by setting a smaller sink radius. However, in order to prevent numerical noise with this setup the SPH resolution will have to be much larger than used in this work. As seen in Paper I, a larger sink radius will also result in a larger mass-accretion rate, but will not have a strong effect on the average angular-momentum loss. In the numerical simulations of wind mass transfer by Mohamed (2010), the accretion process is modelled in a smooth fashion, which according to Mohamed (2010) provides a better numerical performance when the mass accretion rates are not constant as in the case
of eccentric orbits. For instance, her numerical models of eccentric binaries show that unlike most of our models in which the accretion disk is engulfed by the large sink, the accretion process occurs mainly when the stars are at their closest distance and that in most cases the mass-accretion rates are much larger than the BHL prediction.

Another numerical aspect that may be of importance is the conservation of angular momentum. Table 5.5 we show the error in the angular momentum budget for our calculations. Similar to Paper II, we find that angular momentum is not exactly conserved and that the error is larger for models in which strong interaction between the gas and the stars occurs. However, we stress that the errors shown in Table 5.5 correspond to the total angular momentum of the particles in the system (stars and gas), and that the error in the orbital angular momentum is only a fraction of this quantity, which unfortunately cannot be disentangled.

We finish by noting that it is possible that not a single mechanism is responsible for the observed puzzling orbital periods and eccentricities of the descendants of AGB stars, such as Ba stars, CEMP-s and post-AGB stars. In order to verify if an increase in eccentricity can occur in a regime with physically realistic parameters, a larger grid of numerical simulations where different binary masses, wind velocities, mass-loss rates, semi-major axes, and eccentricities are studied is needed to explain under which circumstances wind mass transfer will effectively counteract tidal forces and to understand under which circumstances other proposed eccentricity pumping mechanisms, such as the interaction with a circumbinary disk, may become important.

### 5.5 Summary

In this work we present the first exploratory study of the impact of AGB wind mass transfer on the orbital parameters of eccentric low- and intermediate-mass binary systems. In order to do so we perform numerical simulations using the AMUSE framework to couple a hydrodynamics code, which we use to model the wind dynamics, and an N-body code, which is used to model the dynamics of the stars.

We find that for large eccentricities \( e \gtrsim 0.5 \) the morphology of the outflow can be quite different from the circular case. The spiral patterns found in the circular models or systems with small eccentricities become disrupted rings which move outward as the companion star makes its way through apastron. Furthermore, for large \( e \) the outflow resembles the spherically symmetric wind case when the stars are near apastron.

For models Q2e02 to Q2e08, where \( q = 2 \) and the eccentricity is varied between 0.2 and 0.8, we observe a similar trend in their orbital evolution in which \( \dot{a} > 0 \) and \( \dot{e} < 0 \). On the other hand, in system Q2e06v1, where we keep the same parameters as in model Q2e06, but the initial wind velocity is \( v_{\text{init}} = 1 \text{ km s}^{-1} \), we observe that as the star approaches periastron a structure similar to wind Roche lobe overflow is formed. In this case the interaction between the wind and the companion star, as observed from the outflow morphology, is stronger than
in case Q2e06. In addition, we find that the average angular-momentum loss as well as the mass-accretion efficiency are higher than in model Q2e06 and $\dot{a} < 0$, while $\dot{e} < 0$.

Model MMe05, where $q = 2$ but the stars are more massive and the stellar radius of the donor smaller, shows the most complex morphology among our models. In this case the orbit is shrinking and the strongest interaction between the gas and the star appears to occur when the companion star is at apastron, which results in an increase in eccentricity.

Our results show a good agreement with the secular evolution derived for isotropic-winds by Dosopoulou & Kalogera (2016) as long as the outflow approximates the spherically symmetric wind case. However, as the wind morphology deviates from that case, the changes in the orbital parameters of the system are not properly described by the fast-wind approximation.

We also find that the relation derived in Paper II for the angular-momentum loss as a function of the mass ratio and $v_\infty/v_{\text{orb}}$ for the circular models agrees within $\approx 2-20\%$ when applied to the eccentric models by taking the average orbital velocity. The best agreement occurs when the wind velocity and the eccentricity of the system are large.

Finally, we find that the hydrodynamical circularisation timescales are either longer ($e \lesssim 0.4$) or similar ($e \gtrsim 0.6$) than tidal circularisation timescales. Given the time remaining that a donor star such as assumed in our models spends in the AGB phase, a strong decrease in eccentricity will only occur for models with $e \lesssim 0.4$, whereas for models with larger eccentricity the circularisation due to tidal interaction will be modest or nearly negligible.

Only for model MMe05 the hydrodynamical timescale could potentially counteract tidal circularisation. However, since the orbit in this model is shrinking, it is likely that tidal effects become stronger during the evolution of the system. A larger grid of numerical simulations where different binary masses, wind velocities, semi-major axes and eccentricities are studied is required to explain under which circumstances wind mass transfer will effectively counteract tidal forces.

**Acknowledgements**

MIS wants to thank Frank Verbunt, Chris Tout, Elliot Lynch, Avishai Gilkis, and Glenn-Michael Oomen for the science discussions during the development of this project.
### Table 5.5: Numerical error in the angular momentum conservation

<table>
<thead>
<tr>
<th>Model</th>
<th>$\delta J$</th>
<th>$J_{\text{init\ yr}^{-1}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td></td>
<td>J</td>
</tr>
<tr>
<td>Q2e0</td>
<td>8.62 x 10^{-6}</td>
<td>1</td>
</tr>
<tr>
<td>Q2e02</td>
<td>8.79 x 10^{-6}</td>
<td>2</td>
</tr>
<tr>
<td>Q2e04</td>
<td>8.94 x 10^{-6}</td>
<td>4</td>
</tr>
<tr>
<td>Q2e06</td>
<td>9.74 x 10^{-6}</td>
<td>6</td>
</tr>
<tr>
<td>Q2e08</td>
<td>4.96 x 10^{-6}</td>
<td>8</td>
</tr>
<tr>
<td>Q2e06v1</td>
<td>1.68 x 10^{-5}</td>
<td>10</td>
</tr>
<tr>
<td>MMe05</td>
<td>8.52 x 10^{-6}</td>
<td></td>
</tr>
</tbody>
</table>
In this thesis we have presented the results of hydrodynamical modelling of wind mass transfer during the asymptotic giant branch (AGB) phase of low- and intermediate-mass binary stars, with the aim of understanding the impact on the orbital evolution of the system. In this final chapter we summarise our main findings, and we briefly discuss where our results stand in the context of binary evolution.

6.1. AGB winds

AGB winds are slow and dense outflows of material which are driven away from the star by a combination of stellar pulsations and radiation pressure on dust particles. Modelling in detail these complex winds is computationally challenging because many physical processes need to be invoked. In Chapter 2 we present the STELLAR_WIND.PY module developed within the AMUSE framework (Portegies Zwart et al., 2009; Pelupessy et al., 2013; Portegies Zwart et al., 2013) to model different types of winds around stellar objects. STELLAR_WIND.PY creates wind particles around stars, and their attributes such as temperature and velocity depend on the type of star that is undergoing mass loss via winds. Once these wind particles are created, they are given to a smoothed-particle hydrodynamics (SPH) code to evolve their dynamics. Instead of modelling the physical details of the acceleration process of AGB winds, the "accelerating mode" in STELLAR_WIND.PY contains several acceleration functions that reproduce analytically the empirical velocity profiles of different stellar winds. Throughout this thesis, we simply use the accelerating wind mode to balance the gravity of the donor star in the equation of motion of the wind (Chapters 4 and 5), or to balance the pressure exerted by the gas on the wind with the aim of injecting particles with a predefined constant velocity (Chapter 3). It is left for future studies to investigate how the results for wind mass transfer depend on using different prescriptions for the wind acceleration mechanism. Nonetheless, we should note that since stellar pulsations of the donor star are not...
CHAPTER 6: SUMMARY AND PROSPECTS

included in \texttt{stellar\_wind.py}, a steady flow will be found in the vicinity of the stars, which may impact the structure of the outflow near the stars. For a more realistic prescription of the wind acceleration mechanism, stellar pulsations of the donor star need to be included. An advantage of \texttt{stellar\_wind.py} being publicly available within the \texttt{AMUSE} framework is that new prescriptions for the stellar wind can be easily added and incorporated into SPH codes.

We also note that many other types of astrophysical phenomena involving outflows of material, such as wind mass transfer in massive binary stars or colliding winds in different stellar configurations, can be approximated using this module. Chapter 2 shows several examples of how the different wind modes available in \texttt{stellar\_wind.py} can be applied to different astrophysical problems.

### 6.2. Wind mass transfer in circular binaries

In Chapters 3 and 4, we perform hydrodynamical simulations of wind mass transfer in AGB binaries with the aim of understanding the puzzling orbital periods of the descendants of AGB binary interaction. In Chapter 3, we validate our models by reproducing the results of binary interaction in AGB binaries by Theuns & Jorissen (1993) for an adiabatic and an isothermal equation of state (EoS). In addition, we add a prescription for the cooling of the gas and find, in agreement with Theuns & Jorissen (1993), that the mass-accretion efficiency onto the companion star depends on the EoS of the gas and the numerical resolution. An important contribution of this chapter is the computation of the angular momentum which is removed by material lost from the binary potential. By performing a convergence test, we find that unlike the mass-accretion efficiency, the specific angular momentum lost from the system remains constant regardless of the numerical resolution. For a given binary mass ratio, we study the angular-momentum loss and the mass-accretion efficiency by varying the wind terminal velocity, $v_\infty$. We find that as $v_\infty$ becomes lower than the orbital velocity of the binary, $v_{\text{orb}}$, the morphology of the outflow becomes more complex than in the spherically symmetric case resulting in a larger amount of angular-momentum loss and a larger mass-accretion efficiency. In this chapter we provide a relation between these two parameters and the wind-terminal-velocity-to-orbital-velocity ratio $v_\infty/v_{\text{orb}}$. Similar results have been found in the works of Jahanara et al. (2005) and Chen et al. (2018) in which different assumptions are made and a different method is used. However, we note that the results of Chen et al. (2018) show that for low $v_\infty/v_{\text{orb}}$ a circumbinary disk is formed, which is not observed in our models. It should be investigated if the formation of such disk is a result of the radiative transfer, pulsations and/or dust formation that Chen et al. (2018) include in their models and that we have neglected.

For $v_\infty/v_{\text{orb}} > 1$, we find that both the specific angular-momentum loss and the mass-accretion efficiency tend to the values predicted by the isotropic wind mode, whereas for $v_\infty/v_{\text{orb}} < 1$ these values are much larger. With these values we compute the change in the
orbital separation of the system, and we find that contrary to what the isotropic-wind mode predicts, for \( v_\infty / v_{orb} < 1 \) the orbits shrink. Finally, in this chapter we also find that for the same \( v_\infty / v_{orb} \) the specific angular momentum lost in units of the orbital angular momentum per reduced mass is similar. The fitting formula provided in this chapter for the angular-momentum loss in terms of \( v_\infty / v_{orb} \) has been tested in a population synthesis code by Abate et al. (2018), who try to reproduce the observed orbital periods of carbon-enhanced metal-poor (CEMP-s) stars determined by Hansen et al. (2016a). Abate et al. (2018) find that our fit in combination with wind Roche-lobe overflow (WRLOF; Mohamed & Podsiadlowski, 2007) accretion efficiencies lead to an increase in the CEMP-s population at orbital periods <2500 days. However, the increase is not enough to reproduce the observations by Hansen et al. (2016a).

In Chapter 4, we extend our parameter study to different mass ratios, orbital separations and wind velocities, and we include the possibility of corotation of the donor star. The adopted stellar parameters of the binary stars are chosen in such a way that they match the characteristics of the possible progenitors of CEMP-s stars. We find that the angular-momentum loss and the mass-accretion efficiency are not only a function of the ratio \( v_\infty / v_{orb} \), but also of the binary mass ratio. Both the angular-momentum loss and the mass-accretion efficiency increase as the mass ratio of the stars approaches unity. Our results show that the orbits of several systems shrink compared to the isotropic-wind mode, especially when \( v_\infty / v_{orb} \) is small and the stars have comparable masses. However, the mass-accretion efficiencies we find are only up to a factor \( \approx 1.6 \) larger than predicted by the Bondi-Hoyle-Lyttleton (BHL) formalism (Hoyle & Lyttleton, 1939; Bondi & Hoyle, 1944; Bondi, 1952), which as shown by Abate et al. (2015a,b) is not enough to reproduce the observed abundances of the CEMP-s stars.

The WRLOF mass-transfer mechanism predicts large amounts of material accreted onto the companion star (e.g. Mohamed & Podsiadlowski, 2007; Chen et al., 2017). However, in the circular models studied in Chapter 3 we do not find such a mechanism of interaction. In Chapter 4 we find that when the initial wind velocity is very low (\( \lesssim 5 \) km s\(^{-1}\)) and the donor star is corotating, the morphology of the outflow is strongly modified resembling the WRLOF mechanism. However, the mass-accretion efficiencies we find in these models are either of the same order or lower than BHL predicts. The low mass-accretion efficiencies found in this thesis may be related to the steady flow in the vicinity of the stars compared to Mohamed (2010) and Chen et al. (2017), in which pulsations of the donor star are included. This should be investigated in future work.

We find that a decrease in the orbital separation of wind-interacting AGB binaries can occur, which opens a new window in the evolution of binary stars. However, in order to verify if our results can help to explain the puzzling orbital periods of barium stars, extrinsic S stars, CEMP-s, post-AGB stars and other offspring of the AGB binaries, the fitting formulae given in Chapter 4 for the mass-accretion efficiency and angular-momentum loss in terms of the mass ratio and \( v_\infty / v_{orb} \) need to be implemented in binary population synthesis codes. We should
also remark that we find that the wind velocity plays an important role in determining the amount of interaction in the binary (angular-momentum loss and mass-accretion efficiency). Therefore it would be helpful if a realistic wind velocity profile is implemented in binary population synthesis codes.

Although this work has been developed with the aim of being applied to low- and intermediate-mass binary systems interacting via AGB winds, because our results are a function of the mass ratio and $v_\infty/v_{\text{orb}}$, they could in principle also apply to compact massive binary stars interacting via winds. We note that in massive binary systems, the winds can be much faster than for AGB winds (Owocki, 2004), but the orbital periods for some of these systems are much shorter (of the order of a few days) and in some cases they favour equal masses (Pinsonneault & Stanek, 2006; Moe & Di Stefano, 2016). Therefore, if $v_\infty/v_{\text{orb}}$ is small and the masses are comparable, large amounts of angular momentum may be lost which will imply a shrinkage of the orbit. This in consequence will impact the detection rates of gravitational waves from neutron stars and stellar-mass black hole mergers. In order to see if our results also apply to such systems, hydrodynamical simulations of these phenomena need to be done.

### 6.3. Eccentric binaries

In Chapter 5, we perform an exploratory study of the effect of wind mass transfer on the orbital parameters of eccentric binaries interacting via winds. We find that the morphology of the outflow differs considerably from the non-eccentric case. This may be of importance to help constrain the eccentricities of observed interacting binary systems. We find that for most of our models in which the binary parameters match those of CEMP-s star progenitors, wind mass transfer results in an increase of the orbital separation and a decrease in eccentricity. For larger initial eccentricities, we find that the angular-momentum loss and the mass-accretion efficiency approach the spherically symmetric wind case because the star spends most of its time near apastron where $v_{\text{wind}}/v_{\text{orb}}$ is large. Only for one case we find that the effect of the wind on the binary increases the eccentricity, at a similar rate as tidal interaction circularises the orbit. However, the stellar parameters of this model are chosen arbitrarily. A complete study where more realistic parameters are chosen will be needed to verify these results and see if the effect of wind mass transfer can help to explain the large eccentricities observed in the descendants of AGB binaries.

We should note that it is possible that more than one mechanism is responsible for the large eccentricities observed in the offspring of AGB binary stars. For instance, Artymowicz et al. (1991) showed that the eccentricity of the binary can be pumped by the interaction of the stars with a circumbinary disk. This idea is observationally supported by the fact that most post-AGB binary systems show evidence for a circumbinary disk (van Winckel, 2003b; de Ruyter et al., 2006). However, according to Rafikov (2016), in order for the circumbinary disk to effectively counteract tidal circularisation, it needs to be more massive than the
observed ones, and longer lived. Furthermore, numerical simulations by Chen et al. (2017) suggest that only for low $\nu_\infty/\nu_{\text{orb}}$ a circumbinary disk can be formed. Therefore, it is possible that different mechanisms or a combination of mechanisms pump the eccentricity or counteract tidal interaction in AGB binary systems.

This thesis has opened a new window in our understanding of wind mass transfer in binary stars, as it has shown that the orbits of low- and intermediate-mass binary stars interacting via AGB winds can evolve towards shorter periods, and that wind mass transfer could in principle counteract tidal circularisation. The next step is to implement these results in a binary population synthesis code to compare our numerical models to the observed orbital properties of the descendants of AGB binary interaction. Additional improvements that can be made to obtain a more self-consistent model for the stellar wind include pulsations of the donor star, dust formation, and radiative transfer. More broadly, our results are not restricted to the study of the puzzling orbits of the offspring of AGB binary stars. For instance, if more binary stars evolve towards shorter periods, it is likely that a larger number of binaries will go into common envelope (Iben & Livio, 1993) and these systems will shrink even more, which could in principle increase the number of cataclysmic binaries, double white dwarfs, Type I supernovae formed via the single or double degenerate scenarios, and the mergers of white dwarfs due to gravitational-wave emission.
the Pacific Conference Series, page 333.


Sterren zijn heldere objecten in de ruimte die altijd evolueren gedurende hun leven. Op een bepaald ondergaan bijna alle sterren een periode van massa verlies vanop hun oppervlak, in wat een sterrenwind genoemd wordt. Als de ster geïsoleerd evolueert, vliegt dit materiaal ongestoord de ruimte in. In tegenstelling tot onze zon zijn veel sterren in het universum deel van een sterrenpaar, waarbij twee sterren rondom elkaar draaien onder de invloed van hun gemeenschappelijk zwaartekrachtsveld. Zulke systemen noemen we binaire sterren of dubbelsterren. Als een sterrenwind ontstaat bij een van de dubbelsterren kan een deel van deze wind op de kompaan terechtkomen en diens atmosfeer vervuilen. Meer zelfs, aangezien massa verloren gaat aan het dubbelster systeem zullen de positie en de baan van de sterren veranderen (dit wordt later verder uitgelegd). Het doel van deze thesis is om te begrijpen hoe de positie en de vorm van de baan evolueren wanneer een van de sterren in een dubbelster systeem massa verliest in de vorm van een sterrenwind.

Een sterrenwind

Bijna alle sterren ondergaan een periode in hun leven waarin materie constant van hun oppervlak weg beweegt. De hoeveelheid materiaal dat wordt weggeblazen, de samenstelling en de snelheid hangen af van de massa en de leeftijd van de ster. De zon is een voorbeeld van een ster die momenteel massa verliest in een sterrenwind. We noemen die wind ook wel de zonnewind, en de gevolgen kunnen we vanop Aarde waarnemen als auroras, wanneer de deeltjes in de zonnewind zoals electronen en protonen botsen met de bovenste lagen van het magneetveld van de Aarde. De zon stoot meer dan $2 \times 10^{16}$ kg materie per jaar uit, wat neerkomt op 60 miljoen keer de massa van het Empire State building. De zonnewind heeft een snelheid tussen 250 km/s en 750 km/s (900 tot 27000 keer sneller dan de gemiddelde snelheidslimiet op de autostrade).
SAMENVATTING

Sterren en hun grootte

Sterren worden ingedeeld op basis van hun massa. Sterren met een massa tussen 0.8 en \( \sim 2.2 \) keer de massa van de zon worden lage massa sterren genoemd, terwijl sterren met een massa tussen \( \sim 2.2 \) en 8 keer de zonsmassa gemiddelde massa sterren genoemd worden. Sterren met een massa groter dan 8 zonsmassa's worden zware sterren genoemd. Er zijn zelfs sterren in het heelal met een massa meer dan 100 zonsmassa's! Het leven van een ster, onafhankelijk van haar massa, is een constant gevecht tegen de zwaartekracht die probeert om de ster te doen inkrimpen. Het grootste deel van hun leven balanceren sterren deze inwaardse kracht met een naar buiten gerichte druk gecreëerd door kernfusie van waterstof naar helium in zijn midden. Deze levensfase word de hoofdreeks genoemd. 

De energie die geproduceerd wordt door kernfusie reacties binnenin de ster zorgt voor de nodige druk om te voorkomen dat de ster in elkaar klap. Nadat het zich door de lagen van de verplaatst wordt de energie uitgestraald vanuit de oppervlak van de ster en wordt het zichtbaar als een schijnende ster in het heelal. De lengte van de hoofdreeks in de tijd wordt bepaald door de massa van de ster. Hoe zwaarder de ster, hoe sneller alle waterstof opgebrand zal zijn. Het leven van een zware ster duurt enkele miljoenen jaren, terwijl een lage massa ster miljarden jaren kan blijven schijnen. Onze zon bevindt zich momenteel op de hoofdreeks en zal nog 4.5 miljard jaar in deze fase blijven. Nu zullen we bondig beschrijven hoe sterren met verschillende massa's evolueren.

De evolutie van lage en gemiddelde massa sterren

Net zoals mensen verschillende levensfases doormaken, zoals de kindertijd, puberteit, volwassenheid en pensionering, zo gaan sterren ook door verschillende levensfases. Lage en gemiddelde massa sterren evolueren gelijkwaardig: als alle waterstof in de kern is opgebruikt kan de ster de zwaartekracht niet langer tegenhouden en begint ze te inkrimpen. Wanneer de kern krimpt wordt er verse waterstof toegevoegd aan de buitenlagen en deze kunnen opnieuw ontbranden in een schil rond de kern wanneer de temperatuur en druk van de buitenlagen van de kern hoog genoeg worden voor kernfusie. De stralingsdruk die daarmee gepaard gaat zorgt ervoor dat de buitenlagen van de ster opzwellen tot enkele honderden keren hun originele grootte\(^1\). Hoe meer helium geproduceerd word in deze schil, die dan wordt toegevoegd aan de kern, hoe groter de druk en temperatuur zullen worden en uiteindelijk zullen er nieuwe kernreacties starten in de kern. De "assen" van de helium verbranding zijn koolstof en zuurstof, en de stralingsdruk voorkomt de inkringening van de kern. Wanneer alle helium opgebrand is in de kern bereikt de ster de zogenaamde asymptotische reuzentak (AGB, voor asymptotic giant branch). Omdat de temperatuur in de kern niet voldoende hoog

\(^{1}\)Wanneer alle waterstof in de kern van onze zon opgebrand is, zullen haar buitenlagen Mercurius en Venus (en mogelijk ook de Aarde) opslokken.
Samenvatting

De AGB (Asymptotic Giant Branch) is een levensfase van een ster met een massa tussen de 0,3 en 8 zonnemassa's waarbij de ster meer gas verliest dan een gewone bolvormige ster. De ster krimpt dan af, samenvoelt zich en verloopt door de kernfusie van koolstof en zuurstof in zwaardere elementen. De kern krimpt opnieuw en koolstof en zuurstof worden ontbrand. De stofdeeltjes vliegen weg in een sterrenwind, terwijl de ster verder verkleint zich. In de laatste levensfase van een AGB ster wordt de kern ioniseerd en straalt het weggeblazen materiaal, inclusief de koolstof en zuurstof kern, aanzienlijk. De straling van de kern verlicht de materiaal nevel, alsof het een planetaire nevel is. De ster wordt langzaam maar zeker een witte dwerg.
De evolutie van zware sterren

Zware sterren volgen een ander levenspad dan lage en gemiddelde massa sterren. We gaan hier echter niet diep op in omdat de focus van deze thesis bij de lage en gemiddelde massa sterren ligt. In een notendop zijn zware sterren wel in staat om kernfusie reacties met elementen zwaarder dan koolstof en zuurstof te ontbranden in hun kern. Een zware ster bereikt zijn levensinde wanneer de hele kern bestaat uit ijzer. Omdat de fusie van elementen zwaarder dan ijzer meer energie kost dan er vrijkomt en de ster dus niet langer in staat is om de zwaartekracht tegen te werken, valt ze in elkaar. Wanneer de kern in elkaar valt noemt men dit een supernova explosie. Zulke ontploffingen zijn erg helder, en sommigen zijn zelfs waargenomen vanop Aarde tijdens het daglicht². Afhankelijk van hoe zwaar de ster was tijdens de hoofdreeks zal haar leven eindigen als een neutronenster of als zwart gat.

Dubbelsterren

Zoals al eerder aangegeven leeft onze zon in relatieve isolatie van andere sterren, maar veel andere gelijkarmige sterren bevinden zich in dubbelster systemen. De banen van de dubbelster kunnen zowel cirkelvormig alsook excentrisch³ zijn. Als een van de componenten in een dubbelster massa verliest in de vorm van een sterrenwind, zoals het geval is voor AGB sterren, zal de wind in de richting van de kompaan geblazen worden en gedeeltelijk op diens oppervlak terechtkomen. Dit zal de kompaan vervuilen en kan ervoor zorgen dat deze sneller gaat draaien rond haar as. Het massa verlies van de AGB donor ster kan ook de banen van het hele systeem veranderen. In deze sectie beschrijven we de mogelijke baanveranderingen als functie van hoe de donor ster (in ons geval een AGB ster) massa verliest.

Massa verlies in dubbelsterren

De twee belangrijkste kanalen waarlangs massa-overdracht kan plaatsvinden in dubbelsterren zijn via de zogenaamde Roche lus en wind massa-overdracht, die we nu beurtelings uitleggen, beginnend met het eerste kanaal. Wanneer twee sterren in een kleine baan rond elkaar draaien vindt er interactie plaats tussen de twee sterren waardoor er - zonder een sterrenwind - materiaal van de ene ster naar de andere kan bewegen. De zogenaamde donor ster dijt uit, waardoor haar Roche lus, een equipotentiaal oppervlak in het veld van de gravitationele potentiële energie van de dubbelster, gevuld wordt. De massa die zich buiten de Roche lus bevindt is sterker gravitationeel gebonden aan de kompaan dan aan de ster zelf,

²SN 1054 is een supernova die gebeurt is in het jaar 1054 en waargenomen werd door Chinese sterrenkundigen. Men kon deze explosie waarnemen gedurende ongeveer 2 jaar, zelfs in het zonlicht.
³Excentriciteit beschrijft de vorm van een kegelsnede. Excentriciteiten tussen 0 en 1 beschrijven een ellips. Hoe groter de excentriciteit, hoe meer de ellips afwijkt van een cirkel.

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en zal dus naar de kompaan bewegen (de Roche lus is geen vatbaar gegeven - we kunnen
dit oppervlak niet waarnemen). De kompaan slokt dit materiaal op, in een proces dat we ac-
cretie noemen (de kompaan wordt ook wel de accretor genoemd). Het kan echter gebeuren
dat er zoveel materiaal buiten de Roche lus terecht komt en naar de accretor beweegt dat die
laatste niet langer alle materiaal kan opslokken. In dat geval zullen beide sterren door een
grote mantel van materiaal, die ook beide sterren omvat, bewegen. Zo’n systeem wordt een
contact-dubbelster genoemd, omdat beide sterren (via de mantel) rechtstreeks met elkaar in
contact staan. Wrijving tussen de sterren (die nog steeds rond elkaar draaien) en de mantel
zorgen ervoor dat de baan van de dubbelster steeds kleiner wordt, of anders gezegd de baan-
periode van de dubbelster neemt af 4. Voor kleine baanperiodes zijn de getijdewerkingen5
groot, met als gevolg dat de baan meer cirkelvormig zal worden als deze initieel excentrisch
is.

Het tweede kanaal is dat van wind massa-overdracht. In het eenvoudigste geval kunnen
we aannemen dat de wind die de donor verlaat een bolvormige symmetrie heeft, en een hoge
snelheid bereikt. In dat geval kan de kompaan een gedeelte van deze wind opslakken, maar
de rest van de wind verdwijnt in de lege ruimte. Wanneer we de natuurkundige vergelijkingen
die dit proces beschrijven oplossen blijkt dat de baan van de dubbelster groter wordt: de
sterren bewegen weg van elkaar en hun baanperiodes worden groter.

Wanneer de theorie van deze twee massa-overdrachtsprocessen in beschouwing genomen
wordt in de evolutie van dubbelsterren met kleine banen, komen we tot de conclusie dat er
(theoretisch gezien) geen dubbelstersystemen kunnen bestaan met baanperiodes tussen 1 en
10 jaar, en dat deze systemen zich op cirkelvormige banen zouden moeten bewegen.

Zulke theoretische voorspellingen kunnen getoetst worden aan de realiteit door middel
van observaties van dubbelstersystemen. Een recent voorbeeld hiervan is de voorspelling van
het bestaan van zwaartekrachtsgolven uitgezonden door extreem compacte dubbelsterren,
gedaan door Einstein in 1916, en de observationele meting hiervan 100 jaar later. We kunnen
dus onderzoeken of de theoretische voorspellingen voor de evolutie van dubbelsterren correct
zijn door zulke systemen te observeren en hun eigenschappen te meten. Daarvoor moeten we
uiteraard eerst systemen vinden die in het recente verleden een kleine baanperiode hadden.
Een mogelijkheid is om de chemische samenstelling van dubbelsterren te onderzoeken. In-
dien wind massa-overdracht heeft plaatsgevonden kunnen er sporen zijn van vervuiling met
elementen die specifiek door een AGB ster geproduceerd worden. Eenmaal geïdentificeerd
can de banen van deze objecten bestudeerd worden, en hun eigenschappen vergeleken

4Via de derde wet van Kepler weten we dat als de afstand tussen twee objecten klein is de baanperiode ook
laag is. Als voorbeeld beschouwen we Mercurius en de Aarde: aangezien Mercurius zich dichter bij de zon
bevindt, duurt 1 omwenteling rond de zon 88 dagen, terwijl dat voor de Aarde (die zich verder weg bevindt)
365 dagen duurt.
5De getijdekracht is een pseudokracht die haar oorsprong vindt in de zwaartekracht die een object uitoeft op
een tweede object. Vanwege de eindige groote van de objecten is deze kracht niet hetzelfde voor ieder punt
in een (bijvoorbeeld) bolvorming object, aangezien niet alle punten even ver van het eerste lichaam verwijderd
zijn.
De verschillende massa-overdrachts mechanismes in dubbelsterren. Links wordt de overdracht via de Roche lus afgebeeld. De witte lijnen stellen de Roche lus voor rond een AGB donor ster (rood) en de kompaan (geel). Wanneer de donor zijn Roche lus vult kan materiaal naar de kompaan bewegen via het snijpunt van de witte krommes, ook wel het binnenste Lagrange punt (L1) genoemd. De gele gevulde lussen tonen hoe de gezamenlijke mantel rond beide sterren vormt wanneer de kompaan niet al het materiaal dat van de donor komt kan accreteren. Rechts wordt het wind massa-overdrachts mechanisme getoond dat leidt tot een grotere baan.

worden met de voorspellingen. Wat blijkt? De metingen tonen net het tegenovergestelde van de theoretische voorspellingen! Er zijn veel lage massa dubbelsterren met baanperiodes tussen 1 en 10 jaar, en systemen met kleine baanperiodes hebben excentrische in plaats van cirkelvormige banen. De conclusie is duidelijk: de theorie mist enkele cruciale ingrediënten voor de evolutie van dubbelsternsystemen en/of de aannames die worden gemaakt (bijvoorbeeld een hoge windsnelheid) zijn niet toepasbaar voor de sterrenwinden van AGB sterren.

**Deze thesis: ingrediënten**

Om te onderzoeken welke ingrediënten de theorie mist en welke aannames fout zijn, creëren we computersimulaties van dubbelsterren, zodat we te weten komen hoe de banen van deze systemen evolueren wanneer we sommige parameters veranderen. Enkele voorbeelden van zulke parameters zijn de afstand tussen de sterren, de massa’s, de excentriciteit van de baan of de snelheid van de wind. Het voordeel van deze simulaties is dat we de theorie kunnen aanpassen zoals we zelf willen, om te bepalen welke ingrediënten belangrijk zijn en welke geen invloed op de baan hebben. In onze simulaties gaan we als volgt te werk: 1 component is een AGB ster die grote hoeveelheden massa verliest in een sterrenwind, en haar kompaan is een ster zoals de zon. De wind wordt voorgesteld als deeltjes die het oppervlak van de ster verlaten\(^6\). Ieder winddeeltje heeft eigenschappen zoals massa, dichtheid, positie

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\(^6\)Voor een gedetailleerde beschrijving verwijzen we de lezer naar Hoofdstuk 2.
en snelheid. Voor de snelheid kiezen we een typische waarde voor een AGB sterrenwind. Er zijn nu twee manieren om de verandering in de baan te volgen (we willen weten of de baan groter of kleiner wordt).

In de eerste methode bestuderen we 2 dingen: de hoeveelheid massa die de kompaan kan opslökk en het impulsmoment van de winddeeltjes die niet op de kompaan terechtkomen. Om te weten hoeveel massa de kompaan opslökt, tellen we simpelweg het aantal winddeeltjes en sommen we hun massa's op. Het impulsmoment is echter niet zo eenvoudig te bestuderen. Cruciaal hiervoor is de definitie van ‘deeltjes die het dubbelstersysteem verloren heeft’. Om dit te bepalen, definiëren we een grens relatief ver weg van de sterren; eens een winddeeltje deze grens overschrijdt, zeggen we dat het deeltje (en zijn impulsmoment) verloren gegaan is voor de dubbelster. Met deze twee grootheden (de massa opgeslokt door de kompaan en het impulsmoment verloren aan de dubbelster) kunnen we de vergelijkingen voor de evolutie van de baan oplossen (om de lezer niet af te schrikken herhalen we de vergelijkingen hier niet; zie Hoofdstuk 1).

In de tweede methode meten we de verandering van de baan direct: we volgen de baan van de dubbelster gedurende enkele omwentelingen en kijken hoe aannames over de afstand tussen de sterren en de excentriciteit van de baan de baanevolutie beïnvloeden.

Onze resultaten

Wanneer we onze dubbelster simulatie hebben opgezet, kunnen we alle parameters veranderen. In Hoofdstuk 3 onderzoeken we wat er gebeurt wanneer de windsnelheid van een dubbelster met een cirkelvormige baan en een donorster dubbel zo zwaar als de kompaan veranderd wordt van 10, 15, 30 en 150 km/s (36000, 54000, 108000 en 540000 km/u). Wat betreft de baanevolutie vinden we dat een windsnelheid van 150 km/s dezelfde baanevolutie toont als de analytische oplossing van de theoretische vergelijkingen voor een snelle bolvormige symmetrische wind, namelijk dat de baan groter wordt. Voor een windsnelheid van 30 km/s vinden we dat de oplossing het analytische model ook benadert: de baan vergroot nog steeds. Wanneer we de snelheid verder verlagen (10 en 15 km/s) vinden we echter dat het impulsmoment dat verloren gaat groter is dan de theoretische voorspelling. De massa die wordt opgeslokt door de kompaan is ook groter dan de voorspelling, met als gevolg: de baan wordt kleiner! Dit toont aan dat, hoewel de theoretische modellen dit niet voorspelden, de banen in werkelijkheid wel degelijk kleiner kunnen worden. We tonen ook aan er een verband is tussen de windsnelheid, de hoeveelheid verloren impulsmoment, en de verhouding

7Impulsmoment is een behouden grootheid voor het heelal in zijn geheel. Het is echter mogelijk dat deze grootheid niet behouden is voor 1 specifiek dubbelstersysteem, aangezien een deel van de massa in de wind terecht zal komen in het lege heelal buiten de dubbelster. Aangezien het impulsmoment van de dubbelster mee de baan bepaalt kan het bestuderen van deze grootheid ons meer inzicht verschaffen over de evolutie van de baan.
Samenvatting

van de windsnelheid en de relatieve baansnelheid van de sterren. Voor gelijkwaardige verhoudingen is de hoeveelheid impuls moment ook gelijkwaardig. Dit klinkt misschien trivial, maar als dit inderdaad het geval is kunnen de conclusies van ons werk toegepast worden op sterren met willekeurige massa’s, zolang ze maar dezelfde verhouding hebben. Dit bespaart ons dus een hoop berekeningen!

Terwijl we de evolutie van de baan volgen kunnen we ook bestuderen hoe de kompaan de wind geometrie beïnvloedt. We vinden dat de aanwezigheid van de kompaan zorgt voor spiraalpatronen in de wind (zie Figuur 3); het effect is groter wanneer de windsnelheid laag is. De oorzaak hiervan is dat wanneer de wind een lage snelheid heeft, de kompaan meer tijd heeft voor de interactie met de wind, en dit resulteert in beter gedefinieerde spiraalstructuren. Dit is belangrijk omdat dit soort simulaties sterrenkundigen kan helpen om de observaties beter te begrijpen.

In Hoofdstuk 4 breiden we onze studie uit naar sterren met verschillende massa’s, afstanden tussen de sterren, en windsnelheden. We bestuderen ook het effect van rotatie van de donorster, in het bijzonder nemen we aan dat de rotatiesnelheid vergelijkbaar is met de baansnelheid. Net zoals in Hoofdstuk 3 zijn de banen cirkelvormig. We vinden dat de hoeveelheid verloren impuls moment en de hoeveelheid massa opgeslokt door de kompaan groter is wanneer de massa’s van de 2 sterren vergelijkbaar zijn. Het tegenovergestelde gebeurt wanneer de massa van de kompaan veel kleiner is dan de massa van de donor. De interactie tussen de sterren wordt sterker naarmate de afstand tussen de sterren afneemt: kleinere separaties betekent meer verloren impuls moment en meer massa opgeslokt door de kompaan. We vinden ook dat wanneer de windsnelheid erg laag is en de donor rotatie ondergaat het mechanisme dat de massa-overdracht reguleert lijkt op een combinatie van wind massa-overdracht en Roche lus overdracht. We vinden ook dat in 15 van onze 26 modellen de banen kleiner worden, in overeenstemming met de metingen. We gaan de goede richting uit!

Ten slotte bestuderen we in Hoofdstuk 5 wat er gebeurt wanneer de banen excentrisch zijn in plaats van cirkelvormig. We bestuderen excentriciteiten van 0,2, 0,4, 0,6 en 0,8 en vergelijken onze resultaten met het geval waarin de excentriciteit 0 is (een cirkelvormige baan). We vinden dat de morfologie betreft dat de ellipticiteit van de baan verschillende geometriën in de wind introduceert (Figuur 3). De spiraalstructuren van het cirkelvormige geval worden nu verstoorde ringen rond de sterren. In de meerderheid van onze simulaties vinden we dat de excentriciteit daalt in de tijd, en dat de banen groter worden. Er is echter ook 1 model waarbij de interactie tussen de wind en de dubbelster de excentriciteit vergroot (dit is net tegenovergesteld aan de theoretische voorspelling). Dit resultaat, in combinatie

8De relatieve baansnelheid is de snelheid waarmee de sterren rondom elkaar bewegen.
9Voor de Aarde is de rotatieperiode 24 uur, en de baanperiode 365 dagen. Hier bestuderen we het geval waarbij de rotatieperiode van de Aarde 365 dagen zou zijn (of andersom: de baanperiode 24 uur), of meer algemeen dat iedere omwenteling van de donor ster om haar eigen as overeenkomt met 1 baanperiode van de dubbelster
Samenvatting

Figure 3: Links tonen we de spiraalarm structuren die gevormd worden wanneer de kompaan, in een cirkelvormige baan, interageert met de wind van de AGB donor ster. Rechts het resultaat van simulaties voor een elliptische baan, die leidt tot een verschillende structuur. De patronen in deze computermodellen zijn ook waargenomen met observaties van dubbelsterren met 1 component in de AGB fase. Door onze modellen te vergelijken met die observaties kunnen we de eigenschappen van deze systemen achterhalen. Beneden zien we een spiraalstructuur gezien door de ALMA telescoop rond de AGB ster LL Pegasi en zijn kompaan [Credit Kim et al. (2017)].
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met de kleiner wordende banen in Hoofdstukken 3 en 4, zou mee kunnen verklaren waarom metingen vaak onverwacht excentrische banen tonen.

Hoewel de thesis hier eindigt, is dit niet het einde van het verhaal. De resultaten in deze thesis hebben aangetoond dat de interactie van een AGB wind in een dubbelstersysteem de baan kan verkleinen, en ook dat de mogelijkheid bestaat om de excentriciteit te vergroten. Dit staat in contrast met de theoretische voorspellingen. We hebben deze simulaties echter gedaan voor een klein aantal sterren. Om deze resultaten op een robuust voetstuk te zetten moeten onze resultaten bevestigd worden door simulaties te bestuderen van een groot aantal sterren.
Resumen

Las estrellas son objetos luminosos en el universo que están constantemente evolucionando durante sus vidas. Casi todas las estrellas experimentan un episodio de pérdida de masa en su superficie en algún momento de sus vidas. A diferencia de nuestro Sol, una gran mayoría de estrellas en el universo con masas más grandes o iguales a las del Sol se encuentran en sistemas dobles o sistemas binarios, es decir, dos estrellas están orbitando la una a la otra mediante su propia influencia gravitacional. Esto significa que si una de las estrellas está perdiendo grandes cantidades de masa en forma de vientos estelares (llamamos a esta estrella la estrella "donadora"), el viento "soplará" hacia su estrella compañera, contami

Vientos estelares

Casi todas las estrellas experimentan un flujo de materia que es lanzado desde su superficie. Los astrónomos llaman a esta pérdida de masa "viento estelar". Los vientos estelares son flujos de materia que son lanzados desde la superficie de las estrellas. La cantidad de material que es lanzado, su composición y velocidad dependen de la masa y la edad de la estrella. Por ejemplo, el Sol está perdiendo masa en forma de vientos estelares, los cuales son llamados "vientos solares". Desde la Tierra estos vientos solares pueden ser observados en forma de auroras boreales, cuando las partículas que lo componen (electrones y protones) colisionan con la atmósfera terrestre. El Sol expulsa alrededor de $2 \times 10^{16}$ kilogramos de material al año, lo cual es equivalente a la masa de 60 millones de edificios similares al Empire State. El viento solar viaja a velocidades que van desde los 250 km/s a los 750 km/s (i.e. 900 km/h a 27.000 veces más rápido que la velocidad promedio en una autopista).
Las estrellas y sus tamaños

Las estrellas en el universo están divididas de acuerdo a su masa. Aquellas estrellas cuya masa está en el rango de 0.8 a 2.2 veces la masa del Sol son llamadas "estrellas de baja masa", y aquellas estrellas cuya masa varía entre los 2.2 y 8 veces la masa del Sol son llamadas "estrellas de masa media". En el universo hay estrellas que pueden llegar a ser hasta cientos de veces más masivas que nuestro Sol! Las estrellas cuya masa es mayor que ocho veces la masa del sol son conocidas como "estrellas masivas".

La vida de una estrella, independientemente de su masa, es una pelea constante contra la gravedad que trata de hacerla colapsar. Durante gran parte de sus vidas, las estrellas contrarrestan a la gravedad con presión que es generada mediante fusiones nucleares que transforman hidrógeno en helio en su interior. Esta fase de la vida de una estrella es conocida por los astrónomos como "secuencia principal". La energía producida por las reacciones nucleares en el interior de una estrella provee la presión necesaria para prevenir que la estrella colapse. Al mismo tiempo esta reacción proporciona de energía suficiente para que la estrella brille. El tiempo que una estrella pasa en la secuencia principal es determinado por su masa. Cuanto más masiva sea una estrella más rápido terminará su combustible (durante la secuencia principal el combustible de la estrella es el hidrógeno que se está quemando en el interior de la estrella). El tiempo de vida de una estrella masiva es del orden de millones de años, pero las estrellas de baja masa pueden llegar a vivir miles de millones de años. Nuestro Sol está actualmente en la secuencia principal y seguirá en esta fase por al menos otros 4.5 mil millones de años. A continuación descriptremos brevemente las fases por las que las estrellas de distintas masas pasan.

Evolución de estrellas de baja y mediana masa

Al igual que los humanos pasan por diferentes etapas durante su vida, como la niñez, la adolescencia, la edad adulta y la vejez, las estrellas también pasan por diferentes etapas durante su vida. Las estrellas de baja y mediana masa envejecen de manera similar y su producto final es una "enana blanca". Una vez que el hidrógeno en el núcleo de una estrella se termina, la estrella no es capaz de contrarrestar la gravedad que la está jalando y su interior comienza a colapsar. Conforme el núcleo de la estrella colapsa, hidrógeno "fresco" es suministrado por sus regiones externas y es fusionado en una capa que rodea el núcleo de la estrella, donde la temperatura y la presión son bastante altas para fusionar el hidrógeno. La presión de radiación resultante causa que las capas externas de la estrella se expandan dando lugar a una estrella brillante que es cientos de veces más grande10

10Cuando nuestro Sol agote todo su hidrógeno, sus capas exteriores se expandirán tanto que su tamaño alcanzará las órbitas de Mercurio, Venus y tal vez la Tierra.
FIGURE 1: Vista esquemática de una estrella AGB (no a escala). La estrella está formada por un núcleo de carbón y oxígeno que es rodeado por delgadas capas donde H y He se están fusionando. Una capa intermedia formada principalmente por He las separa. Arriba de estas capas una envoltura altamente convectiva yace. Pulsaciones en la envoltura exterior levantan material a largas distancias. En estas regiones el gas es condensado en granos de polvo debido a las bajas temperaturas. Después la radiación de la estrella ejerce presión sobre los granos de polvo acelerándolos lejos de la estrella. El polvo transfiere momento al gas que se encuentra alrededor mediante colisiones y arrastra al gas lejos de la estrella AGB en forma de viento estelar. Momentum is transferred from the dust to the surrounding gas via collisions dragging the gas away from the AGB star in a stellar wind.
Conforme más helio es producido en el envoltorio del núcleo y añadido al núcleo, la presión y temperatura de este último aumenta, lo cual permite que una empiece una nueva reacción nuclear. Ahora en el núcleo de la estrella las cenizas de helio son quemadas dando lugar a la creación de carbón y oxígeno, lo cual previene que la estrella colapse. Cuando el helio en el núcleo de la estrella se acaba, la estrella se mueve a lo que se conoce como "rama asintótica gigante" (o AGB por sus siglas en inglés). Durante esta fase el núcleo de estrella tiene cenizas de carbón y oxígeno. Dado que la temperatura en el núcleo es muy baja, la estrella no puede producir suficiente energía para iniciar una nueva reacción nuclear que produzca elementos más pesados. Por lo tanto el núcleo de la estrella se contrae y su temperatura incrementa lo cual calienta a las capas que rodean el núcleo. Esta temperatura no es suficiente para iniciar una nueva reacción en el núcleo de la estrella pero sí para iniciar una reacción nuclear en la capa adyacente que ahora quema helio en carbon y oxígeno. Durante este punto de su vida la estrella tiene un núcleo de carbón y oxígeno, una capa que rodea al núcleo hecha de helio, otra capa que envuelve a estos mantos hecha de hidrógeno en la que en algunas ocasiones también hay una reacción nuclear, y una gran envoltura que cubre todo esto hecha de hidrógeno (Figura 1). Durante esta fase, la estrella también presenta "pulsaciones estelares" que son variaciones en el radio y la luminosidad de la estrella. Los astrónomos no entienden muy bien el origen de estas pulsaciones, sin embargo es posible que dado que hay muchos efectos físicos ocurriendo en el interior de la estrella en este punto de su vida (similar a los cambios de humor de un adolescente), la estrella cambia de tamaño porque está tratando de contrarrestar a la gravedad que la intenta colapsar. Las pulsaciones estelares ocasionan que el material en las capas exteriores de la estrella sean elevados de la superficie y empujados lejos de la estrella en forma de vientos estelares. Comparados con los vientos solares, los vientos AGB son muy lentos. Son lanzados con velocidades de alrededor de 5-30 km/s (o 18000-108000 km/h). A pesar de sus velocidades tan bajas, las estrellas AGB pierden mucho más material que el Sol (hasta $\approx 2 \times 10^{25}$ kilogramos al año o la masa de 272 lunas). Eventualmente, toda la envoltura de la estrella es removida dejando al núcleo de carbon y oxígeno desnudo, el cual ioniza la envoltura lanzada produciendo una nebulosa planetaria. Esta es la última fase en la vida de una estrella de masa baja e intermedia antes de que el núcleo de la estrella se enfríe en lo que es llamado una enana blanca.

**Evolución de una estrella masiva**

Las estrellas masivas evolucionan de diferente manera. Dado que el objetivo de esta tesis es estudiar la evolución de aquellos sistemas donde las estrellas son de baja y mediana masa, no discutiremos en detalle la evolución de las estrellas masivas aquí. Las estrellas masivas producen suficiente energía para quemar mediante reacciones nucleares elementos más pesados que el carbón y el oxígeno en su interior. Sin embargo, una vez que hierro es producido en el interior de estas estrellas, una nueva reacción no puede empezar y en
este momento la estrella comienza a morir, ed decir la estrella no es capaz de balancear el colapso gravitacional. Cuando el núcleo de la estrella colapsa produce una explosión que es conocida como "supernova". Este tipo de objetos son muy luminosos e históricamente han sido observados desde la Tierra en plena luz del día durante incluso años. Dependiendo de qué tan masiva fue la estrella cuando era joven (durante la secuencia principal), el producto final de estos objetos será una estrella de neutrones o un agujero negro.

**Estrellas binarias**

Como se mencionó antes, nuestro Sol vive en relativa soledad a comparación de otras estrellas que viven en sistemas dobles o binarios. Las órbitas de dichas estrellas pueden ser tanto circulares como elípticas. Si una de las estrellas en un sistema binario está perdiendo material en forma de vientos estelares, como ocurre con las estrellas AGB, parte del material que la estrella AGB pierde puede terminar en la superficie de su compañera, contaminándola y dándole velocidad angular, i.e. la estrella compañera podrá rotar sobre su propio eje. La contaminación que sufre la compañera también puede afectar su evolución. Por otro lado, la pérdida de material de la estrella donadora, puede perturbar la órbita del sistema, acercando a las estrellas o alejándolas y cambiando la forma de la órbita. En esta sección discutimos los posibles cambios en la órbita que pueden ocurrir conforme la estrella donadora (en este caso la estrella AGB) pierde masa.

**Pérdida de masa en estrellas binarias**

Hay dos mecanismos principales a través de los cuales puede ocurrir la transferencia de masa en estrellas dobles: desbordamiento del lóbulo de Roche y transferencia de masa por vientos estelares. Si las estrellas están muy cercanas, la interacción entre ellas se produce a través de lo que se conoce como desbordamiento del lóbulo de Roche. En este caso, la estrella donadora se expande a medida que evoluciona y llena un volumen llamado lóbulo de Roche, que rodea el sistema binario. Debemos tener en cuenta que el lóbulo de Roche no es una superficie que podamos ver físicamente. El tamaño de dicho volumen depende de cuán lejos estén las estrellas entre sí y de sus masas. Si la estrella crece lo suficiente para llenar su lóbulo de Roche, parte de la envoltura de la estrella donadora podrá dirigirse hacia la estrella compañera (ver Fig. 2). La estrella compañera comenzará a tragar o "acrecentar" el material que viene hacia ella. Sin embargo, puede ocurrir que la estrella compañera no pueda acrecentar toda la masa, entonces el material comenzará a acumularse en la estrella compañera hasta que ambas estrellas queden engullidas en la envoltura de la estrella donadora. Esto es lo que se conoce como una "estrella binaria de contacto". La fricción entre las estrellas (que

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11El objeto astronómico SN 1054 es una suernova que fue observada en 1054 y fue descrita por astrónomos chinos. Permaneció visible por alrededor de dos años en plena luz del día.
se orbitan entre sí) y su envoltura hace que las estrellas se muevan a una distancia más cercana, reduciendo su período orbital.\textsuperscript{12} A la corta distancia a la que se encuentran las estrellas, las fuerzas de marea\textsuperscript{13} son bastante fuertes y su efecto en la órbita es hacerla circular si era originalmente elíptica.

El segundo canal a través del cual puede ocurrir transferencia de masa es a través de vientos estelares. En el caso más simple, suponemos que el viento abandona la estrella donadora de forma esféricamente simétrica y con una velocidad muy grande. La estrella compañera es capaz de tragar parte del material, pero el resto del viento se escapa al espacio. En este caso, cuando resolvemos las ecuaciones que describen este proceso, notamos que la órbita de las estrellas se hace más grande, es decir, las estrellas se separan una de la otra y sus períodos orbitales se hacen más grandes.

Teniendo en cuenta estos dos canales de interacción (el desbordamiento del lóbulo de Roche y la transferencia de masa por vientos estelares), los astrofísicos teóricos predicen que ningún producto del sistema de la interacción de estrellas binarias de baja y media masa debería tener períodos orbitales entre 1 y 10 años. Más aún, aquellos sistemas binarios cercanos, producto de la interacción de estrellas de baja y mediana masa, deberían tener órbitas circulares.

En su mayoría, las predicciones hechas por físicos teóricos pueden ser comprobadas mediante observaciones (un buen ejemplo es la predicción de la existencia de las ondas gravitacionales hecha por Einstein en 1916 y probada cien años más tarde por la colaboración científica LIGO-Virgo). Por lo tanto, una manera de comprobar si los caminos evolutivos para las estrellas binarias propuestos por los astrofísicos son correctos es observando aquellos sistemas que se cree son el producto de la interacción de las estrellas de baja masa e intermedia cuando la estrella más vieja estaba en la fase AGB. Una forma de identificar qué sistemas son descendientes de tal interacción es observando la composición de la estrella más joven en un sistema binario, ya que la estrella compañera más evolucionada es ahora una enana blanca. Si ambas estrellas interaccionaron en el pasado, eso significa que la estrella compañera pudo haber tragado material de la estrella compañera más vieja y ahora está contaminada por elementos producidos durante la fase AGB de su compañera. Al mirar las órbitas de dichos objetos, podemos comprobar si los astrofísicos teóricos tienen razón. Sin embargo, resulta que a los modelos teóricos les faltan algunos ingredientes porque los astrónomos observadores ven todo lo contrario a lo que los teóricos predicen. ¿Hay muchos sistemas binarios de masa baja e intermedia "viejos" cuyos períodos orbitales están en el rango de 1 a 10 años y, lo que es peor, los sistemas binarios cercanos tienen órbitas elípticas? ¡Cómo llegaron ahí?! Antes de

\textsuperscript{12}La tercera ley de Kepler proporciona una relación entre la separación orbital y el período orbital. Si la separación entre dos objetos es pequeña, el período orbital también se vuelve pequeño. Tomemos, por ejemplo, Mercurio y la Tierra en su órbita alrededor del Sol. Dado que Mercurio está más cerca del Sol, solo le toma 88 días completar una órbita, mientras que la Tierra tarda 365 días completar una órbita.

\textsuperscript{13}La fuerza de marea es una fuerza aparente que surge porque la gravedad ejercida sobre un cuerpo extendido por un cuerpo secundario no es uniforme a lo largo del cuerpo primario.
FIGURE 2: Ilustración de los mecanismos de transferencia de masa en estrellas binarias. A la izquierda mostramos el mecanismo de desbordamiento del lóbulo de Roche. El lóbulo de Roche (en blanco) sobresale alrededor de la estrella donadora AGB (en rojo) y la estrella compañera (en amarillo). Una vez que la estrella donadora llena el lóbulo de Roche, se puede transferir material a la estrella compañera a través del punto interior que conecta las superficies llamadas punto Lagrangiano interno ($L_1$). El patrón amarillo que se desborda en ambos lóbulos muestra la envoltura común alrededor de las estrellas. Esto solo ocurre si la estrella compañera no puede tomar todo el material que proviene de su vecina. A la derecha, una ilustración del mecanismo de transferencia de masa de viento que produce un ensanchamiento de la órbita.
que podamos intentar responder a esa pregunta, debemos recalcar que la conclusión sobre la falta de sistemas dentro de ciertos períodos establecidos por la teoría se hace partiendo de varias suposiciones (esto por su simplicidad). Por ejemplo, que las velocidades del viento son muy rápidas, pero como se explicó antes, no es así para los vientos AGB.

**Esta tesis: ingredientes**

Para responder cómo consiguieron esas órbitas los sistemas binarios de baja y mediana masa observados, simulamos un sistema de estrellas binarias (o dobles) en la computadora, para poder estudiar qué sucede con su órbita en diferentes condiciones, como la distancia a la que están una de la otra, sus masas, el tipo de órbita (circular o elíptica) y la velocidad con la que se expulsa el viento. La ventaja de simular estos sistemas "artificiales" es que podemos hacer hasta cierto punto modelos realistas que incluyen más física, ya que es la computadora la que lidia con este problema. La receta que seguimos es la siguiente: una de las componentes es una estrella AGB que está perdiendo una gran cantidad de material en forma de viento estelar, y su estrella compañera es una estrella similar al Sol. Alrededor de la estrella AGB creamos partículas que representan el fluido (viento) que abandona el objeto (en el Capítulo 2 se proporciona una descripción detallada de cómo se crean estas partículas de viento). Cada una de estas partículas de viento tiene ciertas características físicas tales como masa, densidad, posición y velocidad. La velocidad de estas partículas se establece para que sea la típica para los vientos AGB. Hay dos formas a través de las cuales podemos estimar el cambio en la órbita (si se reduce o si se ensancha). En el primer método debemos tener en cuenta dos cantidades: la cantidad de masa (viento) que la estrella compañera toma de su vecina más evolucionada y el momento angular que es extraído del material que la estrella compañera no toma. Para calcular la cantidad de masa que toma la estrella compañera, básicamente contamos cuántas partículas de viento caen sobre la estrella compañera en un intervalo de tiempo determinado y sumamos sus masas. Calcular el momento angular que se pierde es más complicado. La pregunta que surge es ¿cómo o cuándo consideramos que el viento se ha ido del sistema? Para hacer esto, establecemos un límite lejos de las estrellas donde asumimos que el material que lo cruza se ha escapado y se ha llevado consigo cierto momento angular. Usando estos valores, la cantidad de masa que tomó la estrella compañera y la cantidad de momento angular que tomó el viento, podemos resolver una ecuación que nos indica el cambio en la órbita (para propósitos pedagógicos, esta ecuación no se da aquí, pero puede encontrarse en el Capítulo 1). El segundo método para medir el cambio en la órbita es midiendo directamente, es decir, simplemente seguimos el movimiento de las estrellas.

14El momento angular es una cantidad que se conserva en la naturaleza. Sin embargo, dado que los sistemas estudiados aquí están perdiendo masa a través de los vientos estelares, el momento angular no se conserva, es decir, la masa que se pierde quita el momento angular de la órbita de la estrella. Por lo tanto, el cambio en el momento angular nos puede dar una idea de cómo se ve afectada la órbita de un sistema.
en la simulación durante unas pocas órbitas y vemos cómo cambia su separación y el tipo de órbita.

**Nuestros resultados**

Una vez que el sistema binario que pierde masa es simulado, comenzamos a variar ciertos parámetros. En el Capítulo 3 exploramos lo que le sucede a un sistema binario que está en una órbita circular y dónde la estrella donadora es dos veces más masiva que la estrella compañera. Exploramos varios casos cuando la velocidad del viento varía de 10, 15, 30 y 150 km/s (36.000, 54.000, 108.000, and 540.000 km/h). Encontramos que para una velocidad del viento de 150 km/s, la evolución de la órbita en la simulación da un resultado similar al caso de vientos rápidos discutido anteriormente, es decir para vientos rápidos y simétricamente esféricos la órbita de las estrellas se ensancha. Para una velocidad de 30 km/s, encontramos que la órbita se aproxima al modelo del viento esféricamente simétrico y la órbita también se amplía. Sin embargo, a medida que la velocidad del viento disminuye (10 km/s y 15 km/s), detectamos que la pérdida de momento angular es mayor, la masa "comida" por la estrella compañera también es grande y esto se traduce en una reducción del tamaño de la órbita! Por lo tanto, encontramos que incluso cuando la transferencia de masa ocurre a través de vientos estelares, las estrellas pueden acercarse. En este capítulo también encontramos una relación entre la cantidad de momento angular que se pierde en términos de la velocidad del viento y la velocidad orbital relativa de las estrellas (la velocidad a la que los cuerpos giran uno alrededor del otro). Para proporciones de estas velocidades similares se pierde una cantidad parecida de momento angular. Esto puede parecer poco interesante, pero si este es el caso, en lugar de simular miles de sistemas binarios en la computadora para verificar qué sucede en sus órbitas, se puede reducir el trabajo sabiendo que las órbitas de los sistemas binarios que comparten esta característica evolucionarán de manera similar.

Mientras observamos cómo cambia la órbita de las estrellas, también podemos ver cómo la estrella compañera modifica la geometría del viento expulsado por la estrella AGB. Encuentramos que la presencia de la estrella compañera pinta hermosos patrones espirales en el viento (ver Fig. 3), especialmente cuando la velocidad del viento es baja. Esto significa que si el viento se aleja de la estrella donadora a velocidades muy bajas, la estrella compañera tendrá más tiempo para interactuar con ella creando estructuras en espiral más definidas. Lo anterior es importante porque puede ayudarnos a determinar los tipos de sistemas que los astrónomos observan con telescopios.

En el Capítulo 4, extendemos nuestro estudio a estrellas con diferentes masas, diferentes separaciones orbitales, diferentes velocidades del viento y también exploramos la posibilidad de que la estrella donadora esté girando con una velocidad angular similar a la de la estrella binaria, es decir, por cada revolución que da la estrella donadora alrededor de su propio eje, el
FIGURE 3: A la izquierda mostramos los brazos espirales creados por la estrella compañera en el viento aventado por la estrella AGB cuando la órbita es circular. A la derecha vemos que cuando la órbita es elíptica, el patrón creado en el viento es algo diferente. Estos patrones obtenidos por modelos de computadora también se observan con telescopios cuando se dirigen a estrellas dobles en las que una de las componentes se encuentra en la fase AGB. Por lo tanto, al crear estrellas "artificiales" en la computadora podemos inferir las propiedades de los objetos observados por los astrónomos. La imagen de abajo muestra una estructura espiral observada por el telescopio ALMA alrededor de la estrella AGB LL Pegasi y su estrella compañera [Crédito (Kim et al., 2017).]
sistema binario también completa una órbita. Similar al Capítulo 3, las órbitas estudiadas en este capítulo son circulares. Encontramos que la cantidad de momento angular que se pierde y la cantidad de masa que la estrella compañera toma de su vecina es mayor cuando las masas de las dos estrellas son similares. Lo contrario ocurre cuando la estrella compañera es mucho más ligera que la estrella donadora. Como función de la separación orbital, encontramos que cuanto más cerca están las estrellas más fuerte es su interacción, es decir, la estrella compañera agarra más masa y se pierde más momento angular. También descubrimos que cuando la velocidad del viento es muy baja y la estrella donadora está girando, el mecanismo de transferencia de masa se asemeja al "desbordamiento del lóbulo de Roche por vientos estelares". Este es un mecanismo de transferencia de masa intermedio donde el viento lento llena el lóbulo de Roche de la estrella donadora y la masa se transfiere a través de la superficie del lóbulo de Roche. ¡En este capítulo encontramos que la órbita de 15 de los 26 modelos creados en la computadora se reduce! ¡Nos estamos moviendo en la dirección correcta!

Finalmente, en el Capítulo 5 investigamos qué sucede cuando las órbitas de las estrellas son elípticas. Estudiamos excentricidades entre 0.2, 0.4, 0.6 y 0.8, y las comparamos con un caso donde la excentricidad es cero (caso circular). Encontramos que, desde el punto de vista morfológico, el hecho de que la órbita sea elíptica introduce diferentes geometrías en el viento (ver Fig. 3). Por ejemplo, los brazos espirales formados en el caso de órbita circular se convierten en anillos incompletos alrededor de las estrellas. En la mayoría de nuestros modelos, encontramos que la excentricidad del sistema está disminuyendo y que las estrellas se están separando entre sí. Sin embargo, para un modelo encontramos que la interacción entre las estrellas y el viento causa un aumento en la excentricidad! Esto puede ser útil para explicar esas extrañas órbitas observadas por los astrónomos en estrellas binarias de la vida real.

Con estas aportaciones terminamos esta tesis. Sin embargo, este no es el final de la historia. Los resultados de esta tesis han demostrado que el viento soplado por una estrella AGB a su estrella compañera puede producir que las estrellas se acerquen entre sí y que también puede aumentar la excentricidad (elipticidad) de los sistemas. Sin embargo, nuestros resultados aún necesitan ser probados para verificar si pueden explicar las órbitas desconcertantes de las estrellas descendientes de las binarias de masa baja e intermedia. Dicha prueba también deberá invocar modelos de computadora donde nuestros resultados se apliquen a grandes más sistemas estrellas dobles.

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15La Tierra, por ejemplo, tarda aproximadamente 24 h en completar una revolución y 365 días para completar una órbita. En el caso de las estrellas que estudiamos, el tiempo que le toma a la estrella donadora completar una revolución es el mismo tiempo que tardan ambas estrellas en completar una órbita.

16La excentricidad caracteriza la forma de una sección cónica. Las excentricidades mayores que 0 y menores que una corresponden a una elipse. En una elipse, cuanto mayor sea la excentricidad, más se desvíla la elipse de un círculo.
I was born in Toluca, Mexico on April 5th 1990. My academic career started when I was four months old as my father took a sabbatical year at the University of Tamkang in Taiwan. Thanks to my dad I quickly learned what being in academia was and thanks to that I began exploring the world as I was coming with him to many of his conferences or other sabbatical stays in other countries. It was because of the discussions with my father that my eagerness to learn more about how the universe works began.

I did all my basic education in Toluca, and in 2007 I started my first solo adventure as I began my bachelor studies in physics at the Universidad Nacional Autónoma de México in Mexico City. At the beginning of my bachelors I was mainly interested in cosmology and medical physics. However, thanks to the opportunity I was given to attend a summer school (one month) at the San Pedro Mártil Observatory in Ensenada, Mexico and later a summer school at the now Institute for Radioastronomy and Astrophysics in Morelia, Mexico my interest in astronomy or "science fiction" as a dear professor used to call it increased. At the end of my bachelor I started my thesis project in astrophysics under the supervision of Dr. William Lee. For that project I was performing hydrodynamical simulations of the interaction of neutron stars and black holes. I finished my bachelor in 2012, as such studies in Mexico take four and a half years.

After completing my bachelor I had the opportunity to attend a summer program at the Space Telescope Science Institute in Baltimore, USA, where I was working under the supervision of Dr. Alessandra Aloisi, Justin Ely and Azalee Bostroem with the aim of calibrating the Near Ultraviolet channel of the Cosmic Origins Spectrograph of the Hubble Telescope. This experience and the people working at that institute inspired me to pursue a career in astrophysics.

In 2012 I continued my graduate studies in the Erasmus Mundus program for astronomy, Astromundus, where I had the good fortune of taking courses in three universities, the University of Innsbruck, the University of Rome Tor Vergata and I finished my studies at the
University of Goettingen where I performed a really fun project as a masters thesis. At the University of Goettingen I was working under the supervision of Dr. Sandra Jeffers performing numerical simulations whose aim was to see if objects in the Kuiper Belt could smash planets in our Solar system and if a similar thing could occur in other exoplanetary systems that show traces of debris disks.

In 2014 I began my PhD under the supervision of Dr. Onno Pols in Nijmegen, The Netherlands, and I also had the opportunity of being part of the University of Leiden, The Netherlands, thanks to my second supervisor Prof. Dr. Simon Portegies Zwart. During this time I was fortunate to show the results of my research with poster and oral presentations at several national and international conferences in Leiden, Amsterdam and Nunspeet (The Netherlands), Bonn and München (Germany), Nice (France), Cambridge (The UK), and Quy Nhon (Vietnam).
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