Search for exclusive Higgs and $Z$ boson decays to $\phi\gamma$ and $\rho\gamma$ with the ATLAS detector

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ABSTRACT: A search for the exclusive decays of the Higgs and $Z$ bosons to a $\phi$ or $\rho$ meson and a photon is performed with a $pp$ collision data sample corresponding to an integrated luminosity of up to 35.6 fb$^{-1}$ collected at $\sqrt{s} = 13$ TeV with the ATLAS detector at the CERN Large Hadron Collider. These decays have been suggested as a probe of the Higgs boson couplings to light quarks. No significant excess of events is observed above the background, as expected from the Standard Model. Upper limits at 95% confidence level were obtained on the branching fractions of the Higgs boson decays to $\phi\gamma$ and $\rho\gamma$ of $4.8 \times 10^{-4}$ and $8.8 \times 10^{-4}$, respectively. The corresponding 95% confidence level upper limits for the $Z$ boson decays are $0.9 \times 10^{-6}$ and $25 \times 10^{-6}$ for $\phi\gamma$ and $\rho\gamma$, respectively.

KEYWORDS: Hadron-Hadron scattering (experiments), Higgs physics

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1 Introduction

Following the observation [1, 2] of a Higgs boson, $H$, with a mass of approximately 125 GeV [3] by the ATLAS and CMS collaborations at the Large Hadron Collider (LHC), the properties of its interactions with the electroweak gauge bosons have been measured extensively [4–6]. The coupling of the Higgs boson to leptons has been established through the observation of the $H \rightarrow \tau^+\tau^-$ channel [4, 7, 8], while in the quark sector indirect evidence is available for the coupling of the Higgs boson to the top-quark [4] and evidence for the Higgs boson decays into $b\bar{b}$ has been recently presented [9, 10]. Despite this progress, the Higgs boson interaction with the fermions of the first and second generations is still to be confirmed experimentally. In the Standard Model (SM), Higgs boson interactions to fermions are implemented through Yukawa couplings, while a wealth of beyond-the-SM theories predict substantial modifications. Such scenarios include the Minimal Flavour Violation framework [11], the Froggatt-Nielsen mechanism [12], the Higgs-dependent Yukawa couplings model [13], the Randall-Sundrum family of models [14], and the possibility of the Higgs boson being a composite pseudo-Goldstone boson [15]. An overview of relevant models of new physics is provided in ref. [16].
The rare decays of the Higgs boson into a heavy quarkonium state, $J/\psi$ or $\Upsilon(nS)$ with $n = 1, 2, 3$, and a photon have been suggested for probing the charm- and bottom-quark couplings to the Higgs boson [17–20] and have already been searched for by the ATLAS Collaboration [21], resulting in 95% confidence level (CL) upper limits of $1.5 \times 10^{-3}$ and $(1.3, 1.9, 1.3) \times 10^{-3}$ on the branching fractions, respectively. The $H \to J/\psi \gamma$ decay mode has also been searched for by the CMS Collaboration [22], yielding the same upper limit. The corresponding SM predictions for these branching fractions [23] are $B(H \to J/\psi \gamma) = (2.95 \pm 0.17) \times 10^{-6}$ and $B(H \to \Upsilon(nS)\gamma) = (4.6^{+1.7}_{-1.2}; 2.3^{+0.8}_{-1.0}; 2.1^{+0.8}_{-1.1}) \times 10^{-9}$. The prospects for observing and studying exclusive Higgs boson decays into a meson and a photon with an upgraded High Luminosity LHC [16] or a future hadron collider [24] have also been studied.

Currently, the light ($u, d, s$) quark couplings to the Higgs boson are loosely constrained by existing data on the total Higgs boson width, while the large multijet background at the LHC inhibits the study of such couplings with inclusive $H \to q\bar{q}$ decays. Rare exclusive decays of the Higgs boson into a light meson, $M$, and a photon, $\gamma$, have been suggested as a probe of the couplings of the Higgs boson to light quarks and would allow a search for potential deviations from the SM prediction [23, 25, 26]. Specifically, the observation of the Higgs boson decay to a $\phi$ or $\rho(770)$ (denoted as $\rho$ in the following) meson and a photon would provide sensitivity to its couplings to the strange-quark, and the up- and down-quarks, respectively. The expected SM branching fractions are $B(H \to \phi \gamma) = (2.31 \pm 0.11) \times 10^{-6}$ and $B(H \to \rho\gamma) = (1.68 \pm 0.08) \times 10^{-5}$ [23]. The decay amplitude receives two main contributions that interfere destructively. The first is referred to as “direct” and proceeds through the $H \to q\bar{q}$ coupling, where subsequently a photon is emitted before the $q\bar{q}$ hadronises exclusively to $M$. The second is referred to as “indirect” and proceeds via the $H \to \gamma\gamma$ coupling followed by the fragmentation $\gamma^* \to M$. In the SM, owing to the smallness of the light-quark Yukawa couplings, the latter amplitude dominates, despite being loop induced. As a result, the expected branching fraction predominantly arises from the “indirect” process, while the Higgs boson couplings to the light quarks are probed by searching for modifications of this branching fraction due to changes in the “direct” amplitude.

This paper describes a search for Higgs boson decays into the exclusive final states $\phi\gamma$ and $\rho\gamma$. The decay $\phi \to K^+K^-$ is used to reconstruct the $\phi$ meson, and the decay $\rho \to \pi^+\pi^-$ is used to reconstruct the $\rho$ meson. The branching fractions of the respective meson decays are well known and are accounted for when calculating the expected signal yields. The presented search uses approximately 13 times more integrated luminosity than the first search for $H \to \phi\gamma$ decays [27], which led to a 95% CL upper limit of $B(H \to \phi\gamma) < 1.4 \times 10^{-3}$, assuming SM production rates of the Higgs boson. Currently, no other experimental information about the $H \to \rho\gamma$ decay mode exists.

The searches for the analogous decays of the $Z$ boson into a meson and a photon are also presented in this paper. These have been theoretically studied [28, 29] as a unique precision test of the SM and the factorisation approach in quantum chromodynamics (QCD), in an environment where the power corrections in terms of the QCD energy scale over the vector boson’s mass are small [29]. The large $Z$ boson production cross section at the LHC
means that rare $Z$ boson decays can be probed at branching fractions much smaller than for Higgs boson decays into the same final states. The SM branching fraction predictions for the decays considered in this paper are $\mathcal{B}(Z \to \phi \gamma) = (1.04 \pm 0.12) \times 10^{-8}$ \cite{28,29} and $\mathcal{B}(Z \to \rho \gamma) = (4.19 \pm 0.47) \times 10^{-8}$ \cite{29}. The first search for $Z \to \phi \gamma$ decays by the ATLAS Collaboration was presented in ref. \cite{27} and a 95% CL upper limit of $\mathcal{B}(Z \to \phi \gamma) < 8.3 \times 10^{-6}$ was obtained. So far no direct experimental information about the decay $Z \to \rho \gamma$ exists.

2 ATLAS detector

ATLAS \cite{30} is a multi-purpose particle physics detector with a forward-backward symmetric cylindrical geometry and near $4\pi$ coverage in solid angle.\footnote{ATLAS uses a right-handed coordinate system with its origin at the nominal interaction point (IP) in the centre of the detector and the $z$-axis along the beam pipe. The $x$-axis points from the IP to the centre of the LHC ring, and the $y$-axis points upward. Cylindrical coordinates ($r,\phi$) are used in the transverse plane, $\phi$ being the azimuthal angle around the $z$-axis. The pseudorapidity is defined in terms of the polar angle $\theta$ as $\eta = -\ln \tan(\theta/2)$.} It consists of an inner tracking detector surrounded by a thin superconducting solenoid, electromagnetic and hadronic calorimeters, and a muon spectrometer.

The inner tracking detector (ID) covers the pseudorapidity range $|\eta| < 2.5$, and is surrounded by a thin superconducting solenoid providing a 2 T magnetic field. At small radii, a high-granularity silicon pixel detector covers the vertex region and typically provides three measurements per track. A new innermost pixel-detector layer, the insertable B-layer, was added before 13 TeV data-taking began in 2015 and provides an additional measurement at a radius of about 33 mm around a new and thinner beam pipe \cite{31}. The pixel detectors are followed by a silicon microstrip tracker, which typically provides four space-point measurements per track. The silicon detectors are complemented by a gas-filled straw-tube transition radiation tracker, which enables radially extended track reconstruction up to $|\eta| = 2.0$, with typically 35 measurements per track.

The calorimeter system covers the pseudorapidity range $|\eta| < 4.9$. A high-granularity lead/liquid-argon (LAr) sampling electromagnetic calorimeter covers the region $|\eta| < 3.2$, with an additional thin LAr presampler covering $|\eta| < 1.8$ to correct for energy losses upstream. The electromagnetic calorimeter is divided into a barrel section covering $|\eta| < 1.475$ and two endcap sections covering $1.375 < |\eta| < 3.2$. For $|\eta| < 2.5$ it is divided into three layers in depth, which are finely segmented in $\eta$ and $\phi$. A steel/scintillator-tile calorimeter provides hadronic calorimetry in the range $|\eta| < 1.7$. LAr technology, with copper as absorber, is used for the hadronic calorimeters in the endcap region, $1.5 < |\eta| < 3.2$. The solid-angle coverage is completed with forward copper/LAr and tungsten/LAr calorimeter modules in $3.1 < |\eta| < 4.9$, optimised for electromagnetic and hadronic measurements, respectively.

The muon spectrometer surrounds the calorimeters and comprises separate trigger and high-precision tracking chambers measuring the deflection of muons in a magnetic field provided by three air-core superconducting toroids.
A two-level trigger and data acquisition system is used to provide an online selection and record events for offline analysis [32]. The level-1 trigger is implemented in hardware and uses a subset of detector information to reduce the event rate to 100 kHz or less from the maximum LHC collision rate of 40 MHz. It is followed by a software-based high-level trigger which filters events using the full detector information and records events for detailed offline analysis at an average rate of 1 kHz.

3 Data and Monte Carlo simulation

The search is performed with a sample of $pp$ collision data recorded at a centre-of-mass energy $\sqrt{s} = 13$ TeV. Events are retained for further analysis only if they were collected under stable LHC beam conditions and the detector was operating normally. This results in an integrated luminosity of 35.6 and 32.3 fb$^{-1}$ for the $\phi\gamma$ and $\rho\gamma$ final states, respectively. The integrated luminosity of the data sample has an uncertainty of 3.4% derived using the method described in ref. [33].

The $\phi\gamma$ and $\rho\gamma$ data samples used in this analysis were each collected with a specifically designed trigger. Both triggers require an isolated photon with a transverse momentum, $p_T$, greater than 35 GeV and an isolated pair of ID tracks, one of which must have a $p_T$ greater than 15 GeV, associated with a topological cluster of calorimeter cells [34] with a transverse energy greater than 25 GeV. The photon part of the trigger follows the same process as the inclusive photon trigger requiring an electromagnetic cluster in the calorimeter consistent with a photon and is described with more detail in ref. [32], while requirements on the ID tracks are applied in the high-level trigger through an appropriately modified version of the $\tau$-lepton trigger algorithms which are described in more detail in ref. [35]. The trigger for the $\phi\gamma$ final state was introduced in September 2015. This trigger requires that the invariant mass of the pair of tracks, under the charged-kaon hypothesis, is in the range 987–1060 MeV, consistent with the $\phi$ meson mass. The trigger efficiency for both the Higgs and $Z$ boson signals is approximately 75% with respect to the offline selection, as described in section 4. The corresponding trigger for the $\rho\gamma$ final state was introduced in May 2016. This trigger requires the invariant mass of the pair of tracks, under the charged-pion hypothesis, to be in the range 475–1075 MeV to include the bulk of the broad $\rho$ meson mass distribution. The trigger efficiency for both the Higgs and $Z$ boson signals is approximately 78% with respect to the offline selection.

Higgs boson production through the gluon-gluon fusion ($ggH$) and vector-boson fusion (VBF) processes was modelled up to next-to-leading order (NLO) in $\alpha_S$ using the POWHEG-Box v2 Monte Carlo (MC) event generator [36–40] with CT10 parton distribution functions [41]. POWHEG-Box was interfaced with the PYTHIA 8.186 MC event generator [42, 43] to model the parton shower, hadronisation and underlying event. The corresponding parameter values were set according to the AZNLO tune [44]. Additional contributions from the associated production of a Higgs boson and a $W$ or $Z$ boson (denoted by $WH$ and $ZH$, respectively) are modelled by the PYTHIA 8.186 MC event generator with NNPDF23LO parton distribution functions [45] and the A14 tune for hadronisation and the underlying event [46]. The production rates and kinematic distributions for the
SM Higgs boson with $m_H = 125$ GeV are assumed throughout. These were obtained from
ref. [16] and are summarised below. The $ggH$ production rate is normalised such that it re-
produces the total cross section predicted by a next-to-next-to-leading-order QCD
calculation with NLO electroweak corrections applied [47–50]. The VBF production rate
is normalised to an approximate NNLO QCD cross section with NLO electroweak correc-
tions applied [51–53]. The $WH$ and $ZH$ production rates are normalised to cross sections
calculated at next-to-next-to-leading order (NNLO) in QCD with NLO electroweak corrections
[54, 55] including the NLO QCD corrections [56] for $gg \rightarrow ZH$. The expected signal
yield is corrected to include the 2% contribution from the production of a Higgs boson in
association with a $t\bar{t}$ or a $b\bar{b}$ pair.

The Powheg-Box v2 MC event generator with CT10 parton distribution functions
was also used to model inclusive $Z$ boson production. Pythia 8.186 with CTEQ6L1 parton
distribution functions [57] and the AZNLO parameter tune was used to simulate parton
showering and hadronisation. The prediction is normalised to the total cross section ob-
tained from the measurement in ref. [58], which has an uncertainty of 2.9%. The Higgs
and $Z$ boson decays were simulated as a cascade of two-body decays, respecting angular
momentum conservation. The meson line shapes were simulated by Pythia. The branch-
ing fraction for the decay $\phi \rightarrow K^+K^-$ is $(48.9 \pm 0.5)\%$ whereas the decay $\rho \rightarrow \pi^+\pi^-$ has
a branching fraction close to 100% [59]. The simulated events were passed through the
detailed Geant 4 simulation of the ATLAS detector [60, 61] and processed with the same
software used to reconstruct the data. Simulated pile-up events (additional $pp$ collisions
in the same or nearby bunch crossings) are also included and the distribution of these is
matched to the conditions observed in the data.

4 Event selection for $\phi\gamma \rightarrow K^+K^-\gamma$ and $\rho\gamma \rightarrow \pi^+\pi^-\gamma$ final states

The $\phi\gamma$ and $\rho\gamma$ exclusive final states are very similar. Both final states consist of a pair
of oppositely charged reconstructed ID tracks. The difference is that for the former the
mass of the pair, under the charged-kaon hypothesis for the two tracks, is consistent with
the $\phi$ meson mass, while for the later, under the charged-pion hypothesis for the tracks,
it is consistent with the $\rho$ meson mass. Events with a $pp$ interaction vertex reconstructed
from at least two ID tracks with $p_T > 400$ MeV are considered in the analysis. Within an
event, the primary vertex is defined as the reconstructed vertex with the largest $\sum p_T^2$ of
associated ID tracks.

Photons are reconstructed from clusters of energy in the electromagnetic calorimeter.
Clusters without matching ID tracks are classified as unconverted photon candidates while
clusters matched to ID tracks consistent with the hypothesis of a photon conversion into
$e^+e^-$ are classified as converted photon candidates [62]. Reconstructed photon candidates
are required to have $p_T > 35$ GeV, $|\eta^\gamma| < 2.37$, excluding the barrel/endcap calorimeter
transition region $1.37 < |\eta^\gamma| < 1.52$, and to satisfy “tight” photon identification crite-
rin [62]. An isolation requirement is imposed to further suppress contamination from jets.
The sum of the transverse momenta of all tracks within $\Delta R = \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2} = 0.2$
of the photon direction, excluding those associated with the reconstructed photon, is re-
quired to be less than 5% of $p_T^\gamma$. Moreover, the sum of the transverse momenta of all calorimeter energy deposits within $\Delta R = 0.4$ of the photon direction, excluding those associated with the reconstructed photon, is required to be less than $2.45 \text{ GeV} + 0.022 \times p_T^\gamma$.

To mitigate the effects of multiple $pp$ interactions in the same or neighbouring bunch crossings, only ID tracks which originate from the primary vertex are considered in the photon track-based isolation. For the calorimeter-based isolation the effects of the underlying event and multiple $pp$ interactions are also accounted for on an event by event basis using an average underlying event energy density determined from data, as described in ref. [62].

Charged particles satisfying the requirements detailed below are assumed to be a $K^\pm$ meson in the $\phi \gamma$ analysis and a $\pi^\pm$ meson in the $\rho \gamma$ analysis. No further particle identification requirements are applied. In the following, when referring to charged particles collectively the term “charged-hadron candidates” is used, while when referring to the charged particles relevant to the $\phi \gamma$ and the $\rho \gamma$ analyses the terms “kaon candidates” and “pion candidates” are used, respectively, along with the corresponding masses. A pair of oppositely-charged charged-hadron candidates is referred to collectively as $M$.

Charged-hadron candidates are reconstructed from ID tracks which are required to have $|y| < 2.5$, $p_T > 15$ GeV and to satisfy basic quality criteria, including a requirement on the number of hits in the silicon detectors [63]. The $\phi \rightarrow K^+ K^-$ and $\rho \rightarrow \pi^+ \pi^-$ decays are reconstructed from pairs of oppositely charged-hadron candidates; the candidate with the higher $p_T$, referred to as the leading charged-hadron candidate, is required to have $p_T > 20$ GeV.

Pairs of charged-hadron candidates are selected based on their invariant masses. Those with an invariant mass, under the charged-kaon hypothesis, $m_{K^+ K^-}$ between 1012 MeV and 1028 MeV are selected as $\phi \rightarrow K^+ K^-$ candidates. Pairs with an invariant mass, under the charged-pion hypothesis, $m_{\pi^+ \pi^-}$ between 635 MeV and 915 MeV are selected as $\rho \rightarrow \pi^+ \pi^-$ candidates. The candidates where $m_{K^+ K^-}$ is consistent with the $\phi$ meson mass are rejected from the $\phi \gamma$ analysis. This requirement rejects a negligible fraction of the signal in the $\phi \gamma$ analysis. Selected $M$ candidates are required to satisfy an isolation requirement: the sum of the $p_T$ of the reconstructed ID tracks from the primary vertex within $\Delta R = 0.2$ of the leading charged hadron candidate (excluding the charged-hadron candidates defining the pair) is required to be less than 10% of the $p_T$ of the $M$ candidate.

The $M$ candidates are combined with the photon candidates, to form $M \gamma$ candidates. When multiple combinations are possible, a situation that arises only in a few percent of the events, the combination of the highest-$p_T$ photon and the $M$ candidate with an invariant mass closest to the respective meson mass is selected. The event is retained for further analysis if the requirement $\Delta \phi(M, \gamma) > \pi/2$ is satisfied. The transverse momentum of the $M$ candidates is required to be greater than a threshold that varies as a function of the invariant mass of the three-body system, $m_{M\gamma}$. Thresholds of 40 GeV and 47.2 GeV are imposed on $p_T^M$ for the regions $m_{M\gamma} < 91$ GeV and $m_{M\gamma} \geq 140$ GeV, respectively. The threshold is varied from 40 GeV to 47.2 GeV as a linear function of $m_{M\gamma}$ in the region $91 \leq m_{M\gamma} < 140$ GeV. This approach ensures good sensitivity for both the Higgs and $Z$ boson searches, while keeping a single kinematic selection.
For the $\phi(\to K^+K^-)\gamma$ final state, the total signal efficiencies (kinematic acceptance, trigger and reconstruction efficiencies) are 17% and 8% for the Higgs and Z boson decays, respectively. The corresponding efficiencies for the $\rho\gamma$ final state are 10% and 0.4%. The difference in efficiency between the Higgs and Z boson decays arises primarily from the softer $p_T$ distributions of the photon and charged-hadron candidates associated with the $Z \to M \gamma$ production, as can be seen for the $\phi\gamma$ case by comparing figures 1(a) and 1(b).

The overall lower efficiency in the $\rho\gamma$ final state is a result of the lower efficiency of the $m_M$ requirement due to the large $\rho$-meson natural width and the different kinematics of the $\rho$ decay products, as presented in figures 1(c) and 1(d). Meson helicity effects have a relatively small impact for the $\phi \to K^+K^-$ decays, where the kaons carry very little momentum in the $\phi$ rest frame. Specifically, the expected Higgs ($Z$) boson signal yield in the signal region is 6% larger (9% smaller) than in the hypothetical scenario where the meson is unpolarised. For the $\rho \to \pi^+\pi^-$ decays the yields are increased by 33% (decreased by 83%).

The average $m_M$ resolution is 1.8% for both the Higgs and Z boson decays. The Higgs boson signal $m_M$ distribution is modelled with a sum of two Gaussian probability density functions (pdf) with a common mean value, while the Z boson signal $m_M$ distribution is modelled with a double Voigtian pdf (a convolution of relativistic Breit-Wigner and Gaussian pdfs) corrected with a mass-dependent efficiency factor.

The $m_{K^+K^-}$ distribution for the selected $\phi\gamma$ candidates, with no $m_{K^+K^-}$ requirement applied, is shown in figure 2(a) exhibiting a visible peak at the $\phi$ meson mass. The $\phi$ peak is fitted with a Voigtian pdf, while the background is modelled with a function typically used to describe kinematic thresholds [64]. The experimental resolution in $m_{K^+K^-}$ is approximately 4 MeV, comparable to the 4.3 MeV [59] width of the $\phi$ meson. In figure 2(b), the corresponding distribution for the selected $\rho\gamma$ candidates is shown, where the $\rho$ meson can also be observed. The $\rho$ peak is fitted with a single Breit-Wigner pdf, modified by a mass-dependent width to match the distribution obtained from PYTHIA [42]. The background is fitted with the sum of a combinatoric background, estimated from events containing a same-sign di-track pair, and other backgrounds determined in the fit using a linear combination of Chebychev polynomials up to the second order. Figure 2 only qualitatively illustrates the meson selection in the studied final state, and is not used any further in this analysis.

## 5 Background

For both the $\phi\gamma$ and $\rho\gamma$ final states, the main sources of background in the searches are events involving inclusive photon + jet or multijet processes where an $M$ candidate is reconstructed from ID tracks originating from a jet.

From the selection criteria discussed earlier, the shape of this background exhibits a turn-on structure in the $m_M$ distribution around 100 GeV, in the region of the Z boson signal, and a smoothly falling background in the region of the Higgs boson signal. Given the complex shape of this background, these processes are modelled in an inclusive fashion with a non-parametric data-driven approach using templates to describe the relevant distributions. The background normalisation and shape are simultaneously extracted from a
Figure 1. Generator-level transverse momentum ($p_T$) distributions of the photon and of the charged-hadrons, ordered in $p_T$, for (a) $H \rightarrow \phi \gamma$, (b) $Z \rightarrow \phi \gamma$, (c) $H \rightarrow \rho \gamma$ and (d) $Z \rightarrow \rho \gamma$ simulated signal events, respectively. The hatched histograms denote the full event selection while the dashed histograms show the events at generator level that fall within the analysis geometric acceptance (both charged-hadrons are required to have $|\eta| < 2.5$ while the photon is required to have $|\eta| < 2.37$, excluding the region $1.37 < |\eta| < 1.52$). The dashed histograms are normalised to unity, and the relative difference between the two sets of distributions corresponds to the effects of reconstruction, trigger, and event selection efficiencies. The leading charged-hadron candidate $h = K, \pi$ is denoted by $p_T^h$ and the sub-leading candidate by $p_T^{h2}$. 
fit to the data. A similar procedure was used in the earlier search for Higgs and Z boson decays into $\phi\gamma$ [27] and the search for Higgs and Z boson decays into $J/\psi\gamma$ and $\Upsilon(nS)\gamma$ described in ref. [21].

5.1 Background modelling

The background modelling procedure for each final state exploits a sample of approximately 54,000 $K^+K^-$ and 220,000 $\pi^+\pi^-\gamma$ candidate events in data. These events pass all the kinematic selection requirements described previously, except that the photon and $M$ candidates are not required to satisfy the nominal isolation requirements, and a looser $p_T^M > 35$ GeV requirement is imposed. This selection defines the background-dominated "generation region" (GR). From these events, pdfs are constructed to describe the distributions of the relevant kinematic and isolation variables and their most important correlations. In this way, in the absence of appropriate simulations, pseudocandidate events are generated, from which the background shape in the discriminating variable is derived.

This ensemble of pseudocandidate events is produced by randomly sampling the distributions of the relevant kinematic and isolation variables, which are estimated from the data in the GR. Each pseudocandidate event is described by $M$ and $\gamma$ four-momentum vectors and the associated $M$ and photon isolation variables. The $M$ four-momentum vector is constructed from sampled $\eta_M$, $\phi_M$, $m_M$ and $p_T^M$ values. For the $\gamma$ four-momentum vector, the $\eta_\gamma$ and $\phi_\gamma$ are determined from the sampled $\Delta\phi(M,\gamma)$ and $\Delta\eta(M,\gamma)$ values whereas $p_T^\gamma$ is sampled directly.

The most important correlations among these kinematic and isolation variables in background events are retained in the generation of the pseudocandidates through the following sampling scheme, where the steps are performed sequentially:

i) Values for $\eta_M$, $\phi_M$, $m_M$ and $p_T^M$ are drawn randomly and independently according to the corresponding pdfs.
ii) The distribution of $p_T$ values is parameterised in bins of $p_T^M$, and values are drawn from the corresponding bins given the previously generated value of $p_T^M$. The $M$ isolation variable is parameterised in bins of $p_T^M(p_T^\gamma)$ for the $\phi\gamma$ ($\rho\gamma$) model and sampled accordingly. The difference between the two approaches for the $\phi\gamma$ and $\rho\gamma$ accounts for the difference in the observed correlations arising in the different datasets.

iii) The distributions of the values for $\Delta\eta(M,\gamma)$, photon calorimeter isolation, normalised to $p_T^\gamma$, and their correlations are parameterised in a two-dimensional distribution. For the $\phi\gamma$ analysis, several distributions are produced corresponding to the $p_T^M$ bins used earlier to describe the $p_T^\gamma$ and $M$ isolation variables, whereas for the $\rho\gamma$ final state the two-dimensional distribution is produced inclusively for all $p_T^M$ values.

iv) The photon track isolation, normalised to $p_T^\gamma$, and the $\delta\phi(M,\gamma)$ variables are sampled from pdfs generated in bins of relative photon calorimeter isolation and $\Delta\eta(M,\gamma)$, respectively, using the values drawn in step iii).

The nominal selection requirements are imposed on the ensemble, and the surviving pseudocandidates are used to construct templates for the $m_{M\gamma}$ distribution, which are then smoothed using Gaussian kernel density estimation [65]. It was verified through signal injection tests that the shape of the background model is not affected by potential signal contamination.

5.2 Background validation

To validate the background model, the $m_{M\gamma}$ distributions in several validation regions, defined by kinematic and isolation requirements looser than the nominal signal requirements, are used to compare the prediction of the background model with the data. Three validation regions are defined, each based on the GR selection and adding one of the following: the $p_T^M$ requirement (VR1), the photon isolation requirements (VR2), or the meson isolation requirement (VR3). The $m_{M\gamma}$ distributions in these validation regions are shown in figure 3. The background model is found to describe the data in all regions within uncertainties (see section 6).

Potential background contributions from $Z \rightarrow \ell\ell\gamma$ decays and inclusive Higgs decays were studied and found to be negligible for the selection requirements and dataset used in this analysis.

A further validation of the background modelling is performed using events within a sideband of the $M$ mass distribution. For the $\phi\gamma$ analysis the sideband region is defined by $1.035\text{ GeV} < m_{K^+K^-} < 1.051\text{ GeV}$. For the $\rho\gamma$ analysis the sideband region is defined by $950\text{ MeV} < m_{\pi^+\pi^-} < 1050\text{ MeV}$. All other selection requirements and modelling procedures are identical to those used in the signal region. Figures 4(a) and 4(b) show the $m_{M\gamma}$ distributions for the sideband region. The background model is found to describe the data within the systematic uncertainties described in section 6.
Figure 3. The distribution of $m_{K^+ K^-}$, top ($m_{\pi^+ \pi^-}$, bottom) in data compared to the prediction of the background model for the VR1, VR2 and VR3 validation regions. The background model is normalised to the observed number of events within the region shown. The uncertainty band corresponds to the uncertainty envelope derived from variations in the background modelling procedure. The ratio of the data to the background model is shown below the distributions.

Figure 4. The distribution of $m_{M_{\gamma\gamma}}$ for the (a) $\phi\gamma$ and (b) $\rho\gamma$ selections in the sideband control region. The background model is normalised to the observed number of events within the region shown. The uncertainty band corresponds to the uncertainty envelope derived from variations in the background modelling procedure. The ratio of the data to the background model is shown below the distributions.
6 Systematic uncertainties

Trigger and identification efficiencies for photons are determined from samples enriched with $Z \rightarrow e^+e^-$ events in data [32, 62]. The systematic uncertainty in the expected signal yield associated with the trigger efficiency is estimated to be 2.0%. The photon identification and isolation uncertainties, for both the converted and unconverted photons, are estimated to be 2.4% and 2.6% for the Higgs and $Z$ boson signals, respectively. An uncertainty of 6.0% per $M$ candidate is assigned to the track reconstruction efficiency and accounts for effects associated with the modelling of ID material and track reconstruction algorithms if a nearby charged particle is present. This uncertainty is derived conservatively by assuming a 3% uncertainty in the reconstruction efficiency of each track [66], and further assuming the uncertainty to be fully correlated between the two tracks of the $M$ candidate.

The systematic uncertainties in the Higgs production cross section are obtained from ref. [16] as described in section 3. The $Z$ boson production cross-section uncertainty is taken from the measurement in ref. [58].

The photon energy scale uncertainty, determined from $Z \rightarrow e^+e^-$ events and validated using $Z \rightarrow \ell\ell\gamma$ events [67], is applied to the simulated signal samples as a function of $\eta^\gamma$ and $p_T^\gamma$. The impact of the photon energy scale uncertainty on the Higgs and $Z$ boson mass distributions does not exceed 0.2%. The uncertainty associated with the photon energy resolution is found to have a negligible impact. Similarly, the systematic uncertainty associated with the ID track momentum measurement is found to be negligible. The systematic uncertainties in the expected signal yields are summarised in table 1.

The shape of the background model is allowed to vary around the nominal shape, and the parameters controlling these systematic variations are treated as nuisance parameters in the maximum-likelihood fit used to extract the signal and background yields. Three such shape variations are implemented through varying $p_T^\gamma$, linear distortions of the shape of the $\Delta\phi(M, \gamma)$, and a global tilt of the three-body mass. The first two variations alter the kinematics of the pseudocandidates that are propagated to the three-body mass.

7 Results

The data are compared to background and signal predictions using an unbinned maximum-likelihood fit to the $m_{M\gamma}$ distribution. The parameters of interest are the Higgs and $Z$ boson signal normalisations. Systematic uncertainties are modelled using additional nuisance parameters in the fit; in particular the background normalisation is a free parameter in the model. The fit uses the selected events with $m_{M\gamma} < 300$ GeV. The expected and observed numbers of background events within the $m_{M\gamma}$ ranges relevant to the Higgs and $Z$ boson signals are shown in table 2. The observed yields are consistent with the number of events expected from the background-only prediction within the systematic and statistical uncertainties. The results of the background-only fits for the $\phi\gamma$ and $p\gamma$ analyses are shown in figures 5(a) and 5(b), respectively.

Upper limits are set on the branching fractions for the Higgs and $Z$ boson decays into $M\gamma$ using the CL$_s$ modified frequentist formalism [68] with the profile-likelihood-ratio test.
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<table>
<thead>
<tr>
<th>Source of systematic uncertainty</th>
<th>Yield uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total $H$ cross section</td>
<td>6.3%</td>
</tr>
<tr>
<td>Total $Z$ cross section</td>
<td>2.9%</td>
</tr>
<tr>
<td>Integrated luminosity</td>
<td>3.4%</td>
</tr>
<tr>
<td>Photon ID efficiency</td>
<td>2.5%</td>
</tr>
<tr>
<td>Trigger efficiency</td>
<td>2.0%</td>
</tr>
<tr>
<td>Tracking efficiency</td>
<td>6.0%</td>
</tr>
</tbody>
</table>

**Table 1.** Summary of the relative systematic uncertainties in the expected signal yields. The magnitude of the effects are the same for both the $\phi \gamma$ and $\rho \gamma$ selections.

<table>
<thead>
<tr>
<th>Observed yields (Mean expected background)</th>
<th>Expected signal yields</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H$</td>
</tr>
<tr>
<td></td>
<td>[$B = 10^{-4}$]</td>
</tr>
<tr>
<td>Mass range [GeV]</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>12051</td>
</tr>
<tr>
<td>81–101</td>
<td>(3500 ± 30)</td>
</tr>
<tr>
<td>120–130</td>
<td>1076 (1038 ± 9)</td>
</tr>
<tr>
<td>$\phi \gamma$</td>
<td>15.6 ± 1.5</td>
</tr>
<tr>
<td>$\rho \gamma$</td>
<td>83 ± 7</td>
</tr>
</tbody>
</table>

**Table 2.** The number of observed events and the mean expected background, estimated from the maximum-likelihood fit and shown with the associated total uncertainty, for the $m_{M\gamma}$ ranges of interest. The expected Higgs and $Z$ boson signal yields, along with the total systematic uncertainty, for $\phi \gamma$ and $\rho \gamma$, estimated using simulations, are also shown in parentheses.

**Figure 5.** The (a) $m_{K^+K^-\gamma}$ and (b) $m_{\pi^+\pi^-\gamma}$ distributions of the selected $\phi \gamma$ and $\rho \gamma$ candidates, respectively, along with the results of the maximum-likelihood fits with a background-only model. The Higgs and $Z$ boson contributions for the branching fraction values corresponding to the observed 95% CL upper limits are also shown. Below the figures the ratio of the data to the background-only fit is shown.
Table 3. Expected and observed branching fraction upper limits at 95% CL for the $\phi\gamma$ and $\rho\gamma$ analyses. The ±1σ intervals of the expected limits are also given.

<table>
<thead>
<tr>
<th>Branching Fraction Limit (95% CL)</th>
<th>Expected</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{B}(H \to \phi\gamma)$ [10^{-4}]</td>
<td>4.2^{+1.8}_{-1.2}</td>
<td>4.8</td>
</tr>
<tr>
<td>$\mathcal{B}(Z \to \phi\gamma)$ [10^{-6}]</td>
<td>1.3^{+0.6}_{-0.4}</td>
<td>0.9</td>
</tr>
<tr>
<td>$\mathcal{B}(H \to \rho\gamma)$ [10^{-4}]</td>
<td>8.4^{+1.1}_{-2.4}</td>
<td>8.8</td>
</tr>
<tr>
<td>$\mathcal{B}(Z \to \rho\gamma)$ [10^{-6}]</td>
<td>33^{+13}_{-9}</td>
<td>25</td>
</tr>
</tbody>
</table>

statistic [69]. For the upper limits on the branching fractions, the SM production cross section is assumed for the Higgs boson [16], while the ATLAS measurement of the inclusive $Z$ boson cross section is used for the $Z$ boson signal [58], as discussed in section 3. The results are summarised in table 3. The observed 95% CL upper limits on the branching fractions for $H \to \phi\gamma$ and $Z \to \phi\gamma$ decays are 208 and 87 times the expected SM branching fractions, respectively. The corresponding values for the $\rho\gamma$ decays are 52 and 597 times the expected SM branching fractions, respectively. Upper limits at 95% CL on the production cross section times branching fraction are also estimated for the Higgs boson decays, yielding 25.3 fb for the $H \to \phi\gamma$ decay, and 45.5 fb for the $H \to \rho\gamma$ decay.

The systematic uncertainties described in section 6 result in a 14% deterioration of the post-fit expected 95% CL upper limit on the branching fraction in the $H \to \phi\gamma$ and $Z \to \phi\gamma$ analyses, compared to the result including only statistical uncertainties. For the $\rho\gamma$ analysis the systematic uncertainties result in a 2.3% increase in the post-fit expected upper limit for the Higgs boson decay, while for the $Z$ boson decay the upper limit deteriorates by 29%.

8 Summary

A search for the decays of Higgs and $Z$ bosons into $\phi\gamma$ and $\rho\gamma$ has been performed with $\sqrt{s} = 13$ TeV $pp$ collision data samples collected with the ATLAS detector at the LHC corresponding to integrated luminosities of up to 35.6 fb$^{-1}$. The $\phi$ and $\rho$ mesons are reconstructed via their dominant decays into the $K^+K^-$ and $\pi^+\pi^-$ final states, respectively. The background model is derived using a fully data driven approach and validated in a number of control regions including sidebands in the $K^+K^-$ and $\pi^+\pi^-$ mass distributions.

No significant excess of events above the background expectations is observed, as expected from the SM. The obtained 95% CL upper limits are $\mathcal{B}(H \to \phi\gamma) < 4.8 \times 10^{-4}$, $\mathcal{B}(Z \to \phi\gamma) < 0.9 \times 10^{-6}$, $\mathcal{B}(H \to \rho\gamma) < 8.8 \times 10^{-4}$ and $\mathcal{B}(Z \to \rho\gamma) < 25 \times 10^{-6}$.

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