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Soft-Collinear Effects in Threshold and Joint Resummation

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Abstract Large perturbative corrections, which appear in perturbative expressions for many QCD observables, either at small Q_T or at partonic threshold, can be organized by way of all order resummation. Joint resummation allows simultaneous resummation of threshold and recoil effects and its impact has been assessed upto leading log, next-to-leading log (NLL) and in some cases up to NNLL accuracy. We discuss another class of terms, called soft-collinear effects, which give rise to corrections of the form $\frac{\ln^j N}{N}$ and their impact on the joint resummed calculations of prompt photon production cross section.

1 Introduction

Particle Physics research at high energy colliders depends on our ability to calculate cross sections with ever increasing theoretical accuracy, which is achieved by incorporating higher and higher order corrections in perturbative expansion of the observable of interest. Factorization theorems of perturbative QCD play a key role in calculating these corrections and allow expressing cross section for the process $AB \to FX$ to be factorized into convolutions

$$\sigma = \Sigma_{a,b} \int dx_a f_{a/A}(x_a, \mu_F) \int dx_b f_{b/B}(x_b, \mu_F) \hat{\sigma}_{ab \to FX} \tag{1}$$

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where $f_{a/A}(N, \mu_F)$ and $f_{b/B}(N, \mu_F)$ are the standard parton distribution functions. To calculate hadronic cross sections, we need to calculate the partonic cross section $\hat{\sigma}$ as accurately as possible, which can be achieved order by order in perturbation theory using Feynman diagrams. The perturbation series in powers of α_s can be calculated up to Lowest order (LO), Next-to-leading order (NLO) and so on. Ideally, asymptotic series converges rapidly and a LO or NLO calculation is sufficient. However, sometimes the series contains powers of some numerically large logarithm L and can then take a form containing single logs or double logs. Resummation methods organize these logs in perturbative expansions [1,2]

$$\hat{\sigma} = \hat{\sigma}_0 \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots \right] C(\alpha_s) \tag{2}$$

in terms of an effective expansion parameter $\alpha_s L$ or $\alpha_s^2 L$, where, $g_1, g_2, \ldots$ are computable functions giving results up to leading log (LL), next-to-leading-log(NLL)…accuracy.

Threshold logs of the kind $L^2 = \ln^2(1 - \frac{Q^2}{s})$ appear near the threshold and are resummed in Mellin Space or N-space using techniques of threshold resummation [3–5], while logs arising due to recoil effects are resummed in impact parameter or b-space using recoil resummation formalism [6]. Joint resummation [7–10] combines the threshold and recoil resummations and reproduces the threshold resummed and recoil resummed cross sections in the limits $N >> p^+ b$ and $p^+ b >> N$ respectively, where p^+ is the longitudinal momentum of the hadron [6].

Here, we discuss another class of sub leading contributions to threshold and joint resummed cross section of prompt photon production,which are termed soft-collinear effects and lead to terms of the form $\frac{\ln^j N}{N}$ [11–14]. In the following sections, we illustrate, in case of prompt photon production, how the soft-collinear effects are included in the resummation formalism. We review earlier work where the impact of these terms was assessed for E706 and Tevatron kinematics and also present some preliminary results on the impact of these soft-collinear terms for LHC kinematics.

2 Threshold Resummation

The p_T distribution of prompt photons in hadronic process

$$h_A(p_A) + h_B(p_B) \to \gamma(p_c) + X\,, \tag{3}$$

can be written as

$$\frac{p_T^3 d\sigma_{AB\to\gamma+X}}{dp_T} = \frac{p_T^3 d\sigma^{(\text{direct})}_{AB\to\gamma+X}}{dp_T} + \frac{p_T^3 d\sigma^{(\text{frag})}_{AB\to\gamma+X}}{dp_T} \tag{4}$$

where $\frac{p_T^3 d\sigma^{(\text{direct})}_{AB\to\gamma+X}}{dp_T}$ is the direct part and $\frac{p_T^3 d\sigma^{(\text{frag})}_{AB\to\gamma+X}}{dp_T}$ is the fragmentation part in which the photon in the final state results due to fragmentation of an outgoing quark in QCD hard scattering. The lowest order QCD processes producing the prompt photon at partonic cm energy $\sqrt{s}$

$$q(p_a) + \bar{q}(p_b) \to \gamma(p_c) + g(p_d)$$
$$g(p_a) + q(p_b) \to \gamma(p_c) + q(p_d)$$

contribute to the direct part, while the fragmentation part gets contributions from $2 \to 2$ hard scattering processes in which one of the final state partons fragments into photon.

The fragmentation component, which is also of $O(\alpha\alpha_s)$ is expected to contribute to the cross section substantially [15]. Near partonic threshold, soft gluon radiation leads to corrections to $\frac{d\hat{\sigma}}{dp_T}$ as large as $\alpha_s^k \ln^{2k}(1-\hat{x}_T^2)\hat{\sigma}^{Born}$ at order α_s^k in perturbation theory, where $x_T = \frac{2p_T}{\sqrt{S}}$. Threshold resummation organizes these logs to all orders in perturbation theory and is performed by going over to Mellin-transform space or N-space

$$\sigma_{\gamma,N}(E_T) = \int_0^1 dx_T^2 (x_T^2)^{N-1} \frac{p_T^3 d\sigma^{(\text{resum})}_{AB\to\gamma+X}}{dp_T} \tag{5}$$

Convolutions in factorized cross section become ordinary products in Mellin space leading to simple factorised form [4]

$$\sigma_{\gamma,\,N}(E_T) = \sum_{a,b} f_{a/A,\,N+1}(\mu_F^2)\; f_{b/B,\,N+1}(\mu_F^2) \times \Bigg\{ \hat{\sigma}_{ab\to\gamma,\,N}(\alpha_s(\mu^2);\, E_T^2,\, \mu^2,\, \mu_F^2,\, \mu_f^2) + \sum_c \hat{\sigma}_{ab\to c,\,N}(\alpha_s(\mu^2);\, E_T^2,\, \mu^2,\, \mu_F^2,\, \mu_f^2)\; d_{c/\gamma,\,2N+3}(\mu_f^2) \Bigg\} \tag{6}$$

in terms of the moments of each of the functions, where μ_F, μ_f and μ are the the factorziation scale , the fragmentation scale and the renormalization scale respectively. Mellin moments of plus distributions in the cross section give rise to powers of $\ln N$ in the Mellin space expressions. Soft gluon correction terms of the form $\sim \alpha_s^k \ln^{2k} N$ are termed leading log (LL), terms of the form $\alpha_s^k \ln^{2k-1} N$ are called next-to-leading log (NLL) and so on. Threshold resummation exponentiates these logarithmic corrections leading to all order resummed expressions [4,5]. For example,

$$\hat{\sigma}^{(\mathrm{res})}_{q\bar{q}\to\gamma,\,N}(\alpha_s(\mu^2);\, E_T^2,\, \mu^2,\, \mu_F^2,\, \mu_f^2) = \alpha\; \alpha_s(\mu^2)\; \hat{\sigma}^{(0)}_{q\bar{q}\to g\gamma,\,N}\; C_{q\bar{q}\to\gamma}(\alpha_s(\mu^2),\, Q^2/\mu^2;\, Q^2/\mu_F^2)\; \Delta^{q\bar{q}\to g\gamma}_{N+1}(\alpha_s(\mu^2),\, Q^2/\mu^2;\, Q^2/\mu_F^2) \tag{8}$$

where $C_{ab\to\gamma}$ are N independent hard coefficients and $\Delta^{q\bar{q}\to g\gamma}_{N+1}$ are radiative factors containing the $\ln N$ dependence. The expressions for these up to NLL can be found in Ref. [5].

3 Joint Resummation

In joint resummation, the direct component of the cross section is given by [11]

$$\frac{p_T^3 d\sigma^{(\mathrm{direct})}_{AB\to\gamma+X}}{dp_T} = \sum_{ab} \frac{p_T^4}{8\pi S^2} \int_C \frac{dN}{2\pi i}\; f_{a/A}(N,\mu_F) f_{b/B}(N,\mu_F) \times \int_0^1 d\tilde{x}_T^2\, \left(\tilde{x}_T^2\right)^N \frac{|M_{ab}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}}\; C^{(ab\to\gamma d)}((\mu,\tilde{x}_T^2) \times \int \frac{d^2\mathbf{Q}_T}{(2\pi)^2}\, \Theta\,(\bar{\mu}-Q_T) \left(\frac{S}{4{\mathbf{p}'_T}^2}\right)^{N+1} \times \int d^2\mathbf{b}\, \mathrm{e}^{i\mathbf{b}\cdot\mathbf{Q}_T}\, \Sigma^{(\mathrm{resum})}_{ab\to\gamma d}(N,b,\mu,\mu_F,Q) \tag{9}$$

while the fragmentation component is [7,16]

$$\frac{p_T^3 d\sigma^{(\mathrm{frag})}_{AB\to\gamma+X}}{dp_T} = \sum_{abc} \frac{p_T^4}{8\pi S^2} \int_C \frac{dN}{2\pi i}\; f_{a/A}(N+1,\mu_F) f_{b/B}(N+1,\mu_F) D_{\gamma/c}(2N+3,\mu_F^2) \times \int \frac{d^2\mathbf{Q}_T}{(2\pi)^2}\, \Theta\,(\bar{\mu}-Q_T) \left(\frac{S}{4{\mathbf{p}'_T}^2}\right)^{N+1} \times \int d^2\mathbf{b}\, \mathrm{e}^{i\mathbf{b}\cdot\mathbf{Q}_T}\, \Sigma^{(\mathrm{resum})}_{ab\to cd}(N,b,\mu,\mu_F,Q)\;. \tag{10}$$

where $D_{\gamma/c}(2N+3,\mu_F^2)$ is the fragmentation function and $\mu_f = \mu_F$.

The resummed exponents for the partonic process $ab \to \gamma d$ and the process $ab \to cd$ in combined Mellin-impact parameter space are given by

$$\Sigma^{(\text{resum})}_{ab\to\gamma d}(N,b) = \exp\left[E^{\text{PT}}_a(N,b,Q,\mu,\mu_F) + E^{\text{PT}}_b(N,b,Q,\mu,\mu_F) + F_d(N,Q,\mu)\right] \quad (11)$$

and

$$\begin{aligned}\Sigma^{(\text{resum})}_{ab\to cd}(N,b) = exp\Big[&E^{\text{PT}}_a(N,b,Q,\mu,\mu_F) + E^{\text{PT}}_b(N,b,Q,\mu,\mu_F) \\ &+E^{\text{PT}}_c(N,b,Q,\mu,\mu_F) + F_d(N,Q,\mu)\Big] \\ &\times\left[\sum G^I_{ab\to cd}\exp\left(\Gamma^{(int)ab\to cd}_{IN}\right)\right]\sigma^{(\text{Born})}_{ab\to cd}(N-1,b)\end{aligned} \quad (12)$$

Here, $E^{\text{PT}}_a(N,b,Q,\mu,\mu_F)$, $E^{\text{PT}}_b(N,b,Q,\mu,\mu_F)$ and $E^{\text{PT}}_c(N,b,Q,\mu,\mu_F)$ represent the effects of soft gluon radiation collinear to initial partons a and b and the observed final state parton c respectively. $F_d(N,Q,\mu)$ represents the collinear, soft or hard, emission by the non observed parton d. The last brackett is associated with wide angle soft radiation. The sum runs over all possible color configurations I with $G^I_{ab\to cd}$ representing a weight for each color configuration such that $\sum G^I_{ab\to cd} = 1$. The expressions for resummed exponents and NLL expansion of anomalous dimension matrix $\Gamma^{(int)ab\to cd}_{IN}$ are given in Refs. [11,16].

4 Soft-Collinear Effects

Another important class of potentially large terms are of the form

$$\alpha_s^i \sum_j^{2i-1} d_{ij}\frac{\ln^j N}{N}. \quad (13)$$

which have soft-collinear origin. We consider such terms arising from two sources

1. The singular plus distributions $[\ln^{2j-1}(1-z)/(1-z)]_+$, which can be included by keeping the subleading terms in Mellin transform of plus distributions. For example

$$\int_0^1 dzz^N\left[\frac{\ln(1-z)}{1-z}\right]_+ = \frac{1}{2}\ln^2 N - \frac{1}{2}(\ln N + 1)\frac{1}{N} + \cdots$$

2. The singular but integrable $\ln^{2j-1}(1-z)$, which have purely collinear origin and which can be incorporated by including the regular part of Altarelli–Parisi splitting function [18]. Their effect can be incorporated in threshold resummation by the replacement

$$\frac{z^{N-1}-1}{1-z}A_i^{(1)} \to \left[\frac{z^{N-1}-1}{1-z} - z^{N-1}\right]A_i^{(1)} + \mathcal{O}(\frac{1}{N^2}), \quad (14)$$

in each of the radiative factors. Here, $A^{(1)}$ is the leading order term in anomalous dimension and is equal to C_F and C_A in case of quarks and gluons respectively. This replacement is equivalent to exchanging at order j one soft collinear gluon (corresponding to one factor $\alpha_s \ln^2 N$) for a hard-collinear one (corresponding to a factor $\alpha_s \ln N/N$

$$\alpha_s^k \ln^{2k} N \to \alpha_s^k \frac{\ln^{2k-1} N}{N}.$$

The extra term can be cast in a convenient form [17] adding terms of the form

$$f_q' = \frac{A_q^{(1)}}{2\pi b_0} exp\left(- \frac{\lambda}{\alpha_s b_0}\right)[\ln(1-2\lambda) - \ln(1-\lambda)] \tag{15}$$

$$f_g' = \frac{3A_g^{(1)}}{2\pi b_0} exp\left(- \frac{\lambda}{\alpha_s b_0}\right)[\ln(1-2\lambda) - \ln(1-\lambda)] \tag{16}$$

to the resummed exponents. The initial state $\alpha_s^k \ln^{2k-1} N/N$ terms can also be generated in the context of joint resummation by extending evolution of parton densities to a soft scale [11].

To include the off diagonal $\ln N/N$ effects, we can replace the combination

$$f_{i/A}(\mu_F, N) f_{j/B}(\mu_F, N) \exp\left[E_i^{\rm PT}(N, b, Q, \mu, \mu_F) + E_j^{\rm PT}(N, b, Q, \mu, \mu_F)\right]$$

by

$$\mathcal{C}_{i/A}(Q, b, N)\, \mathcal{C}_{j/B}(Q, b, N)\; \exp\left[E_i^{\rm PT}(N, b, \mu, Q) + E_j^{\rm PT}(N, b, \mu, Q)\right]$$

where

$$\mathcal{C}_{i/H}(Q, b, N) = \sum_k \mathcal{E}_{ik}\,(N, Q/\chi, \mu_F)\; f_{k/H}(N, \mu_F)\;.$$

with the matrix $\mathcal{E}$ being the evolution matrix which implements evolution from scale μ_F to scale Q/χ. We will call this method of including $\ln N/N$ effects the evolution method in the next section, while the method of Ref. [17] will be referred to as exponential method.

5 Numerical Results

We study numerically the impact of including the $\ln N/N$ terms in the resummed cross section of prompt photon production for three kinematic conditions: $p\bar{p}$ collisions at the Tevatron at $\sqrt{S} = 1.96$ TeV [19,20], pN collisions in the E706 fixed target experiment with $E_{\rm beam} = 530$ GeV [21], corresponding to $\sqrt{S} = 31.5$ GeV and at pp collisions at LHC at $\sqrt{S} = 14$ TeV. In Fig. 1, we show the effect of $\ln N/N$ terms on joint as well as threshold resummed cross section, including both the direct as well as fragmentation contributions, for E706 and Tevatron kinematics. We find the contribution to be small for joint resummation (JR) but substantial for threshold resummation as was also observed in case of only direct contribution [11]. A detailed analysis can be found in Ref. [16]. We present peliminary results using both the exponentiation method as well as the evolution method of incorporating the leading soft-collinear effects for LHC kinematics at four scales

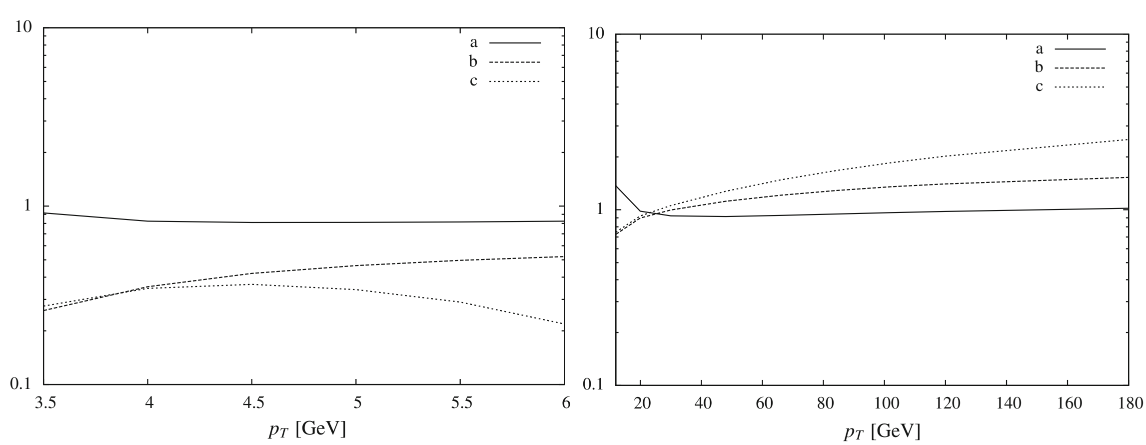

Fig. 1 Comparison of JR without lnN/N with JR with lnN/N (**a**), Threshold without lnN/N (**b**) and Threshold with lnN/N (**c**): left panel—E706 kinematics, right panel—Tevatron kinematics [16]

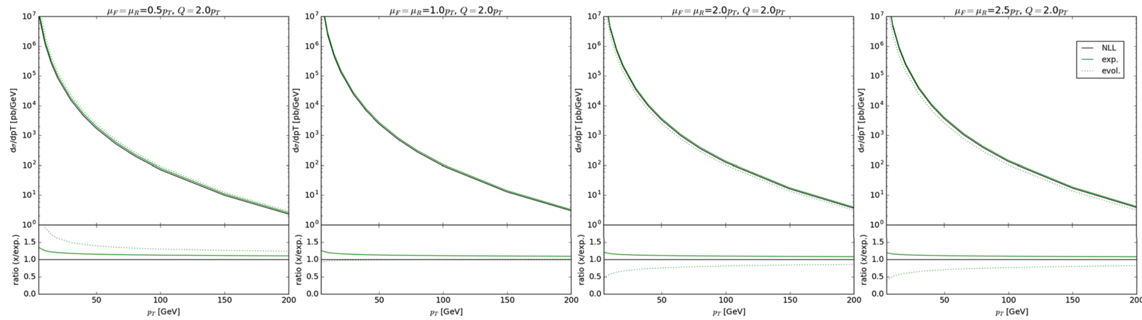

Fig. 2 Comparison of exponentiation and evolution methods for initial state exponents in direct + fragmentation contribution to p_T distribution in JR with lnN/N at NLL when exponentiation method is used for final state: LHC kinematics

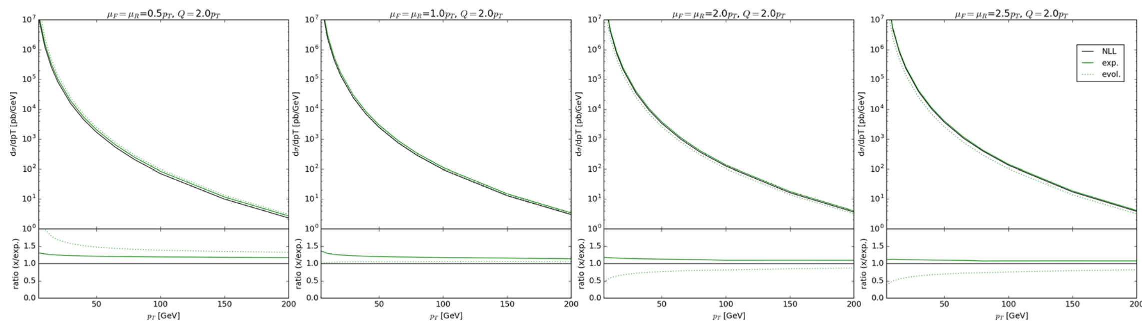

Fig. 3 Comparison of exponentiation and evolution methods for initial state exponents in direct + fragmentation contribution to p_T distribution in JR with lnN/N at NLL when evolution method is used for final state: LHC kinematics

$\mu_R = \mu_F = \frac{p_T}{2}, p_T, 2p_T$ and $2.5p_T$. In all cases, the effect of these terms is found to be appreciable at LL but smaller at NLL. In general, the difference between the LL and NLL result is found to reduce substantially when the $\ln N/N$ effects are included (Fig. 2).

6 Summary

We have discussed improvements possible in joint as well as threshold resummation through summing purely collinear enhancements arising from terms of the form $\ln^i N/N$. The effect of including the leading $\ln N/N$ term in resummed cross section for prompt photon production is found to be non-negligible when both direct and fragmentation processes are taken into account. The effect is found to be appreciable in threshold resummed cross section at E7076 and Tevatron kinematics but smaller in case of JR as the corrections due to recoil effects overshadow soft-collinear effects. For LHC kinematics, we find the difference between results obtained using exponentiation and evolution methods to be small but non-negligible. A detailed study comparing the two methods will be presented in future. Our results indicate that it may be worthwhile to include sub leading terms of the kind $\ln^i N/N$ and assess their impact (Fig. 3).

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