Measurement of the Effective Weak Mixing Angle in $p\bar{p} \to Z/\gamma^* \to e^+ e^-$ Events

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We present a measurement of the effective weak mixing angle parameter \( \sin^2 \theta_{\text{eff}} \) in \( p \bar{p} \rightarrow Z/\gamma^* \rightarrow \mu^+\mu^- \) events at a center-of-mass energy of 1.96 TeV, collected by the D0 detector at the Fermilab Tevatron Collider and corresponding to 8.6 fb\(^{-1}\) of integrated luminosity. The measured value of \( \sin^2 \theta_{\text{eff}}^{\mu\mu} = 0.23016 \pm 0.00064 \) is further combined with the result from the D0 measurement in \( p \bar{p} \rightarrow Z/\gamma^* \rightarrow e^+e^- \) events, resulting in \( \sin^2 \theta_{\text{eff}}^{\mu\mu}[\text{comb}] = 0.23095 \pm 0.00040 \). This combined result is the most precise measurement from a single experiment at a hadron collider and is the most precise determination using the coupling of the \( Z/\gamma^* \) to light quarks.

The weak mixing angle \( \theta_W \) is a fundamental parameter of the standard model (SM). It governs the mechanism of spontaneous symmetry breaking of \( SU(2) \times U(1) \) in which the original vector boson fields \( W \) and \( B_0 \) are transformed to the physical \( W^\pm, Z, \) and \( \gamma \) states. At tree level and in all orders of the on-shell renormalization scheme, the weak mixing angle also relates the \( W \) and \( Z \) boson masses by \( \sin^2 \theta_W = 1 - M_Z^2/M_W^2 \). To include higher-order electroweak radiative corrections and allow comparison with experimental measurements, the effective weak mixing angle can be defined \cite{1} in terms of the relative strengths of the axial vector and vector couplings, \( g_A^f \) and \( g_V^f \), of the \( Z \) boson to fermions, \( f \):

\[
\sin^2 \theta_{\text{eff}}^{f} = \frac{1}{4 |Q_f|} \left( 1 - \frac{g_A^f}{g_V^f} \right),
\]

where \( Q_f \) is the electric charge of the fermions.

It is customary to quote the charged-lepton effective weak mixing angle parameter \( \sin^2 \theta_{\text{eff}}^{e}, \) determined by measurements of observables around the \( Z \)-boson mass pole \( (M_Z) \). The effective mixing angle was precisely measured by the LEP Collaborations and the SLD Collaboration in different physics processes. The combined LEP and SLD result \cite{1} gives a value of \( \sin^2 \theta_{\text{eff}}^{e} = 0.23153 \pm 0.00016 \) at the energy scale \( \mu = M_Z \). The two most precise individual measurements are from the measurement of \( b \)-quark forward-backward asymmetry at LEP \( (\sin^2 \theta_{\text{eff}}^{e} = 0.23221 \pm 0.00029) \) and the measurement of the left-right polarization asymmetry at SLD \( (\sin^2 \theta_{\text{eff}}^{e} = 0.23098 \pm 0.00026) \). An independent determination of the effective weak mixing angle at hadron colliders that is based on different combinations of fermions in the initial and final state from those in the \( e^+e^- \)
The weak mixing angle can be measured in the Drell-Yan process \( p\bar{p} \rightarrow Z/\gamma^* \rightarrow e^+e^- \) through a forward-backward charge asymmetry, \( A_{FB} \), defined by \( A_{FB} = (N_F - N_B)/(N_F + N_B) \), where \( N_F \) and \( N_B \) are the numbers of forward and backward events. Forward (F) or backward (B) events are defined as those for which \( \cos \theta^* > 0 \) or \( \cos \theta^* < 0 \), where \( \theta^* \) is the angle between the negatively charged lepton direction and the incoming proton direction in the Collins-Soper frame [2].

For the Z-to-fermion couplings, both \( g_A = I_A^f \) and \( g_V = I_V^f = \sqrt{3}/2 \) exist, whereas for the photon-to-fermion couplings there is only a vector coupling. \( I_A^f \) is the third component of the weak isospin of the fermion. The parity violation implicit in the forward-backward asymmetry arises from the interference between the vector and axial vector couplings. As the main subprocess for Drell-Yan production is the quark-antiquark annihilation \( q\bar{q} \rightarrow e^+e^- \), \( A_{FB} \) depends upon both the couplings to light quarks and the couplings to leptons. The asymmetry can be measured as a function of the invariant mass of the dilepton pair. Since only the vector coupling of the Z boson depends on \( \sin^2 \theta_W \), the information on \( \sin^2 \theta_W \) comes from the asymmetry in the vicinity of the Z-boson pole. Away from the Z-boson mass pole, the asymmetry results from the interference of the axial vector Z coupling and vector photon coupling and depends upon the parton distribution functions (PDFs).

Measurements of \( \sin^2 \theta^*_W \) corresponding to the full data set at the Fermilab Tevatron Collider were performed by the CDF Collaboration using the Z/\( \gamma^* \rightarrow \mu^+\mu^- \) channel [3] and the Z/\( \gamma^* \rightarrow e^+e^- \) channel [4], and by the D0 Collaboration in the Z/\( \gamma^* \rightarrow e^+e^- \) channel [5]. The weak mixing angle was also measured at the Large Hadron Collider (LHC) by the ATLAS, CMS, and LHCb Collaborations [6–8]. Because the directions of the initial quarks and antiquarks in the dominant subprocess \( q\bar{q} \rightarrow Z/\gamma^* \rightarrow e^+e^- \) are unknown and have to be estimated in \( pp \) collisions, the precision of the LHC results is not as good as that of the Tevatron even with higher statistics.

This Letter reports a measurement of the effective weak mixing angle from the \( A_{FB} \) distribution as a function of the dimuon invariant mass using 8.6 fb\(^{-1}\) of data collected by the D0 detector at the Fermilab Tevatron Collider using the Z/\( \gamma^* \rightarrow \mu^+\mu^- \) channel. The Z/\( \gamma^* \rightarrow e^+e^- \) measurement is then combined with the D0 Z/\( \gamma^* \rightarrow e^+e^- \) measurement [5].

The D0 detector comprises a central tracking system, a calorimeter, and a muon system [9–11]. The central tracking system consists of a silicon microstrip tracker and a scintillating fiber tracker, both located within a 1.9 T superconducting solenoidal magnet and optimized for tracking and vertexing capabilities for detector pseudorapidities of \( |\eta| < 3 \) [12]. Outside the solenoid, three liquid-argon and uranium calorimeters provide coverage for \( |\eta| < 3.5 \) for electrons. The muon system is located outside of the calorimeters, providing coverage for \( |\eta| < 2.0 \). It consists of drift chambers and scintillators and 1.8 T iron toroidal magnets. The solenoid and toroid polarities are reversed every two weeks on average to reduce detector-induced asymmetries. Muons are identified using information from both the tracking system and the muon system. Muon momenta are measured using the tracking system information.

To maximize the event sample, data collected with all triggers are used in this analysis. Events are required to have at least two muon candidates reconstructed in the tracking system and the muon system. Both muon candidates [13] are required to have transverse momentum \( p_T > 15 \) GeV/c and \( |\eta| < 1.8 \) with at least one muon within \( |\eta| < 1.6 \). The two muon candidates must be isolated from jets in the event by requiring the sum of transverse momenta of tracks in the tracking system or transverse energy in the calorimeter within cones surrounding the muon candidate to be small. Muons must have a track in the tracking system matched with one in the muon system. To suppress backgrounds, the two matched tracks are required to point to the same \( pp \) interaction vertex and to have opposite charges. Events with muons nearly back to back are removed to reduce the cosmic ray background. Events are further required to have a reconstructed dimuon invariant mass \( 74 < M_{\mu\mu} < 110 \) GeV/c\(^2\). The number of events satisfying these requirements is 481 239.

The Monte Carlo (MC) Drell-Yan Z/\( \gamma^* \rightarrow \mu^+\mu^- \) sample is generated using leading-order PYTHIA [14] with the NNPDF3.0 [15] PDFs, followed by a GEANT-based simulation [16] of the D0 detector. Events from randomly selected beam crossings with the same instantaneous luminosity profile as data are overlaid on the simulated events to model detector noise and contributions from the presence of additional \( pp \) interactions. The PYTHIA MC samples are used to study the detector’s geometric acceptance and the momentum scale and resolution of muons. Separate MC samples are generated for the four different polarity combinations of the solenoids and toroidal magnetic fields.

The effective weak mixing angle, which is extracted from \( A_{FB} \) as a function of \( M_{\mu\mu} \), depends strongly on the dimuon mass calibration. Therefore, it is critical to have a precise muon momentum measurement and a consistent measured mean value of \( M_{\mu\mu} \) for all \( \eta \), and each muon charge sign \( q \) and solenoid polarity \( S \). The D0 muon momentum calibration and resolution smearing procedure [13] is applied to the MC simulation, so as to give agreement of the overall width and peak value of the \( M_{\mu\mu} \) distribution with data. However, the muon momentum measurement, especially the scale of the reconstructed muon momentum, still depends on the charge and \( \eta \) of the muons due to imperfect alignment of the detector [17]. Such dependence would translate into a large systematic...
Additional corrections and reweightings are applied to the MC simulation to improve the agreement with data. The ratio between the MC and data efficiencies for the muon identification is measured using the tag-and-probe method [13] and applied to the MC distributions as a function of muon $\eta$. The simulation is further corrected for higher-order effects not included in PYTHIA by reweighting the MC events at the generator level in two dimensions ($p_T$ and rapidity $y$ of the $Z$ boson) to match RESBOS [18] predictions. In addition, next-to-next-to-leading-order QCD corrections are applied as a function of $Z$-boson mass [18,19].

The sign of the track matched to the muon is used to determine the charge of the muon and to classify the event as forward or backward. The charge misidentification rate measured in the data is smaller than 0.4%. Since the opposite charge sign requirement is applied in the event selection, the probability of both muons charges being misidentified, thus transforming a forward event into a backward event or vice versa, is negligibly small.

Background is suppressed by the strict requirements on the muon tracks. The main remaining contribution is from multijet events, in which jets are misidentified as muons, which is estimated from data by selecting events with reversed muon isolation cuts in order to study the shape of the mass distribution of multijet events. The normalization of the multijet background is assumed to be the same as that of the selected same-sign events after correcting for the presence of the misidentified signal events and the additional background contributions described below. The $W +$ jets background is generated using ALPGEN [20] interfaced to PYTHIA for showering and hadronization. The $Z/\gamma^* \rightarrow \tau\tau$, diboson ($WW$ and $WZ$), and $t\bar{t}$ backgrounds are estimated using PYTHIA. In the dimuon mass range used for the effective weak mixing angle measurement, the multijet background is $0.68\% \pm 0.68\%$. A $100\%$ uncertainty is used to safely cover the bias due to corrections for the misidentified signal events. The sum of the $W +$ jets,

uncertainty on the $A_{FB}$ measurement. To reduce this dependence, an additional correction to the muon momentum, $\alpha(q, \eta, S)$, is applied to the data and MC separately. This factor is determined by requiring the mean of the $M_{\mu\mu}$ distribution over the full mass range in each $(q, \eta, S)$ region to be consistent with the corresponding nominal value obtained from a generator-level MC sample after applying the same kinematic and acceptance cuts as those applied to the data. After the calibration, the mean values of $M_{\mu\mu}$ in data and MC samples are consistent to within statistical fluctuations. The additional calibration, together with the D0 muon calibration and resolution smearing procedure [13], reduces not only the $q, \eta, S$ dependence, but also the potential effect from an imperfect modeling of the final-state radiation in the PYTHIA generator. The residual difference between data and MC $M_{\mu\mu}$ mean values is propagated to the uncertainty of the weak mixing angle measurement.

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calibration and resolution smearing, the estimation of the backgrounds, and the efficiency scale factors are themselves also dominated by the limited data samples. The PDF uncertainty is obtained as the standard deviation of the distribution of \( \sin^2 \theta_W \) values given by each of the equal-weighted PDF sets from NNPDF3.0 [15]. The best fit is

\[
\sin^2 \theta_W = 0.22994 \pm 0.00059 \text{(stat)} \pm 0.00005 \text{(syst)} \\
\pm 0.00024 \text{(PDF)}.
\]

The PYTHIA generator assumes that the effective couplings of leptons, \( u \) quarks, and \( d \) quarks are the same [5], and it also ignores the mass-scale dependence and complex-valued calculations of the weak corrections and fermion-loop correction to the photon propagator [21]. To correct for these assumptions and reach the common fermion-loop correction to the photon propagator [21],

we shift the value of \( \sin^2 \theta_W \) by +0.00022 and introduce an additional systematic uncertainty of 0.00004 [21] to get \( \sin^2 \theta_W[\mu] = 0.23016 \pm 0.00064 \).

The D0 \( e^+e^- \) measurement [5] and the \( \mu^+\mu^- \) measurement presented here are used as inputs to a D0 combination result for \( \sin^2 \theta_W \). The \( e^+e^- \) measurement in Ref. [5] has been modified for consistency to incorporate the use of additional higher-order corrections and the NNPDF3.0 PDFs employed in this Letter and in the CDF measurement [4]. The corrected value is \( \sin^2 \theta_W[ee] = 0.23137 \pm 0.00047 \) [21]. The D0 \( e^+e^- \) and \( \mu^+\mu^- \) measurements agree to within 1.4 standard deviations.

The central values and systematic uncertainties of the \( e^+e^- \) and \( \mu^+\mu^- \) channels are combined using the inverse of the squares of the statistical uncertainties as weights. The systematic uncertainties are treated as uncorrelated, except the higher-order correction uncertainty which is treated as 100\% correlated. However, the total combined uncertainty in practice does not depend on whether the systematic uncertainties of the input measurements are taken to be correlated or uncorrelated, because both measurements are dominated by statistical uncertainties. The correlation of the acceptances between the \( e^+e^- \) and \( \mu^+\mu^- \) channels cannot be ignored in treating the PDF uncertainty. Instead of estimating a correlation matrix between \( \sin^2 \theta_W \) results for these two channels, a combined PDF uncertainty is estimated by first estimating the PDF uncertainty on the average of values for the \( e^+e^- \) and \( \mu^+\mu^- \) channels, and then scaling that uncertainty using the linear relation between \( A_{FB} \) and \( \sin^2 \theta_W \) calculated using MC simulations.

The combination is

\[
\sin^2 \theta_W[\text{comb}] = 0.23095 \pm 0.00035 \text{(stat)} \\
\pm 0.00007 \text{(syst)} \pm 0.00019 \text{(PDF)}.
\]

Table II summarizes the inputs and the results of the combination of the \( e^+e^- \) and \( \mu^+\mu^- \) measurements. The measured \( \sin^2 \theta_W \) values from D0 and other experiments are compared to the LEP and SLD average in Fig. 2. The D0 combination has an uncertainty close to the precision of the world’s best measurements performed by the LEP and SLD Collaborations.

The measured values of the effective weak mixing angle and the mass of the W boson, \( M_W \) [23], are complementary in the SM global fit and have different sensitivities to new physics scenarios. As an indicative measure of relative precision, we convert \( \sin^2 \theta_W \) into the W-boson mass using the relationship, valid in the framework of the SM and the on-shell renormalization scheme,

\[
\sin^2 \theta_W = |\text{Re}[\kappa_e(M_Z^2)]| \times \left(1 - \frac{M_W^2}{M_Z^2}\right),
\]

where \( |\text{Re}[\kappa_e(M_Z^2)]| \) is a radiative correction calculated using ZFITTER [22]. The calculated value of \( |\text{Re}[\kappa_e(M_Z^2)]| \) is 1.0371 [24]. The main uncertainty on this quantity is due

<table>
<thead>
<tr>
<th>( e^+e^- ) channel</th>
<th>( \mu^+\mu^- ) channel</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin^2 \theta_W )</td>
<td>0.231 37</td>
<td>0.230 16</td>
</tr>
<tr>
<td>Statistical</td>
<td>0.000 43</td>
<td>0.000 59</td>
</tr>
<tr>
<td>Systematic</td>
<td>0.000 09</td>
<td>0.000 06</td>
</tr>
<tr>
<td>PDF</td>
<td>0.000 17</td>
<td>0.000 24</td>
</tr>
<tr>
<td>Total</td>
<td>0.000 47</td>
<td>0.000 64</td>
</tr>
</tbody>
</table>

FIG. 2. Comparison of \( \sin^2 \theta_W(M_Z) \) measured by D0 with results from other experiments. The average of measurements from the LEP and SLD Collaborations [1] is also shown.
to the experimental measurement of the top-quark mass $173.2 \pm 0.9$ GeV/$c^2$ [25]. This translates into an uncertainty of 0.0008 on the value of $\sin^2 \theta_{\text{eff}}$. The values of other input parameters, including the electromagnetic fine-structure constant $\alpha_{\text{em}}$ with a “running” correction from light-quark contributions, the strong-interaction coupling at the $Z$-boson mass $\alpha_s(M_Z^2)$, the Fermi constant $G_F$, and the masses of the $Z$ boson $M_Z$ and the Higgs boson $m_H$, give uncertainties that are negligible compared to the uncertainty arising from the top-quark mass, as discussed in Ref. [21]. By this procedure, we obtain $M_W = 80.396 \pm 0.21$ MeV/$c^2$, with an uncertainty similar to the best direct determination of $M_W$.

In conclusion, we have measured the effective weak mixing angle parameter from the forward-backward charge asymmetry $A_{FB}$ distribution in the process $p\bar{p} \to Z/\gamma^* \to \mu^+\mu^-$ at the Fermilab Tevatron Collider. The primary systematic uncertainty arising from muon momentum calibration is reduced by introducing a charge-$\eta$-solenoid-dependent calibration. The final result using 8.6 fb$^{-1}$ of D0 run II data is $\sin^2 \theta_{\text{eff}}[\mu\mu] = 0.23016 \pm 0.00064$, which is at the level of the best single-channel precision from hadron collider experiments. The D0 combination of the $e^+e^-$ and $\mu^+\mu^-$ measurements is $\sin^2 \theta_{\text{eff}}[\text{comb}] = 0.23095 \pm 0.00040$, which is the most precise single-experiment measurement at hadron colliders and is the most precise result based on the coupling of light quarks to the $Z$ boson.

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[3] T. Aaltonen et al. (CDF Collaboration), Indirect measurement of $\sin^2 \theta_W$ or $M_W$ using $\mu^+ \mu^-$ pairs from $\gamma^* / Z$ bosons produced in $p \bar{p}$ collisions at a center-of-momentum energy of 1.96 TeV, Phys. Rev. D 89, 072005 (2014).


[12] D0 uses a cylindrical coordinate system with the $z$ axis along the beam axis in the proton direction. Angles $\theta$ and $\phi$ are the polar and azimuthal angles, respectively. Pseudorapidity is defined as $\eta = -\ln(\tan(\theta/2))$, where $\theta$ is measured with respect to the interaction vertex. In the massless limit, $\eta$ is equivalent to the rapidity $y = (1/2) \ln[(E + p_z)/(E - p_z)]$, and $\eta_{\text{det}}$ is the pseudorapidity measured with respect to the center of the detector.


