Search for Tensor, Vector, and Scalar Polarizations in the Stochastic Gravitational-Wave Background

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The detection of gravitational waves with Advanced LIGO and Advanced Virgo has enabled novel tests of general relativity, including direct study of the polarization of gravitational waves. While general relativity allows for only two tensor gravitational-wave polarizations, general metric theories can additionally predict two vector and two scalar polarizations. The polarization of gravitational waves is encoded in the spectral shape of the stochastic gravitational-wave background, formed by the superposition of cosmological and individually unresolved astrophysical sources. Using data recorded by Advanced LIGO during its first observing run, we search for a stochastic background of generically polarized gravitational waves. We find no evidence for a background of any polarization, and place the first direct bounds on the contributions of vector and scalar polarizations to the stochastic background. Under log-uniform priors for the energy in each polarization, we limit the energy densities of tensor, vector, and scalar modes at 95% credibility to $\Omega_T < 5.58 \times 10^{-8}$, $\Omega_V < 6.35 \times 10^{-8}$, and $\Omega_S < 1.08 \times 10^{-7}$ at a reference frequency $f_0 = 25$ Hz.

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Introduction.—The direct detection of gravitational waves offers novel opportunities to test general relativity in previously unexplored regimes. Already, the compact binary mergers [1–5] observed by Advanced LIGO (the Laser Interferometer Gravitational Wave Observatory) [6,7] and Advanced Virgo [8] have enabled improved limits on the graviton mass, experimental measurements of post-Newtonian parameters, and inference of the speed of gravitational waves, among other tests [3,9–11].

Another central prediction of general relativity is the existence of only two gravitational-wave polarizations: the tensor plus and cross modes, with spatial strain tensors

$$\hat{e}_+ = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \hat{e}_x = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \hat{e}_y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \hat{e}_l = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (1)$$

(assuming waves propagating in the $+\hat{z}$ direction). Generic metric theories of gravity, however, can allow for up to four additional polarizations: the $x$ and $y$ vector modes and the breathing and longitudinal scalar modes, with basis strain tensors [12–14]

$$\hat{e}_b = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \hat{e}_l = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2)$$

The observation of vector or scalar modes would be in direct conflict with general relativity, and so the direct measurement of gravitational-wave polarizations offers a promising avenue by which to test theories of gravity [14].

Recently, the Advanced LIGO-Virgo network has succeeded in making the first direct statement about the polarization of gravitational waves. The gravitational-wave signal GW170814, observed by both the Advanced LIGO and Virgo detectors, significantly favored a model assuming pure tensor polarization over models with pure vector or scalar polarizations [4,15]. In general, however, the ability of the Advanced LIGO-Virgo network to study the polarization of gravitational-wave transients is limited by several factors. First, the LIGO-Hanford and LIGO-Livingston detectors are nearly co-oriented, preventing Advanced LIGO from sensitively measuring more than a single polarization mode [4,9,10,15]. Second, at least five detectors are needed to fully characterize the five polarization degrees of freedom accessible to quadrupole detectors. Quadrupole detectors (those measuring differential arm motion) have degenerate responses to breathing and...
longitudinal modes, and can therefore measure only a single linear combination of scalar breathing and longitudinal polarizations [14–17].

Beyond compact binary mergers, another target for Advanced LIGO and Virgo is the stochastic gravitational-wave background. An astrophysical stochastic background is expected to arise from the population of distant compact binary mergers [18–23], core-collapse supernovae [24–26], and rapidly rotating neutron stars [27–29]. In particular, the astrophysical background from compact binary mergers is likely to be detected by LIGO and Virgo at their design sensitivities [23]. A background of cosmological origin may also be present, due to cosmic strings [30,31], inflation [32–35], and phase transitions in the early Universe [32,33,36–38].

Long duration gravitational-wave sources, like the stochastic background [39–42] or persistent signals from rotating neutron stars [43–45], offer a viable means of searching for nonstandard gravitational-wave polarizations. Unlike gravitational-wave transients, which sample only a single point on the LIGO/Virgo antenna response patterns, long-duration signals contain information about many points on the antenna patterns. Long-duration signals, therefore, enable the direct measurement of gravitational-wave polarizations using the current generation of gravitational-wave detectors, without the need for additional detectors or an independent electromagnetic counterpart. The stochastic background is thus a valuable laboratory for polarization-based tests of general relativity [42].

In this Letter, we present the first direct search for vector and scalar polarizations in the stochastic gravitational-wave background. We analyze data recorded during Advanced LIGO’s first observing run (O1), which has previously been searched for both isotropic and anisotropic backgrounds of standard tensor polarizations [46,47]. First, we describe the O1 data set and its initial processing. We then discuss the stochastic analysis, including the construction of Bayesian odds that indicate the nondetection of a generically polarized stochastic background in our data. Finally, we present upper limits on the joint contributions of tensor, vector, and scalar polarizations to the stochastic gravitational-wave background. Additional details and results are presented in the Supplemental Material [48], available online.

Data.—We search Advanced LIGO data for evidence of a stochastic background, analyzing data recorded between September 18, 2015 15:00 UTC and January 12, 2016 16:00 UTC during LIGO’s O1 observing run. We do not include several days of O1 data recorded prior to September 18, but this has negligible impact on our results. We exclude times containing the binary black hole signals GW150914 and GW151226, as well as the signal candidate LVT151012.

The initial data processing proceeds as in previous analyses [46,49]. Time-domain strain measurements from the LIGO-Hanford and LIGO-Livingston detectors are down-sampled from 16 384 Hz to 4096 Hz and divided into half-overlapping 192-s segments. Each time segment is then Hann-windowed, Fourier transformed, and high-pass filtered using a 16th order Butterworth filter with a knee frequency of 11 Hz. Finally, the strain data are coarse-grained to a frequency resolution of 0.03 125 Hz and restricted to a frequency band from 20–1726 Hz. Within each segment, we compute the LIGO-Hanford and LIGO-Livingston strain auto-power spectral densities using Welch’s method [50].

Standard data quality cuts are performed in both the time and frequency domains to mitigate the effects of non-Gaussian instrumental and environmental noise [46,47,51]. In the time domain, 35% of data is discarded due to nonstationary detector noise, leaving 29.85 days of coincident observing time. In the frequency domain, an additional 21% of data is discarded to remove correlated narrow-band features between LIGO-Hanford and LIGO-Livingston [46,47,51]. These narrow-band correlations are due to a variety of sources, including injected calibration signals, power mains, and GPS timing systems. To estimate possible contamination due to terrestrial Schumann resonances [52–54], we additionally monitored coherences between magnetometers installed at both detectors. Schumann resonances were found to contribute negligibly to the stochastic measurement [46,51].

We assume conservative 4.8% and 5.4% calibration uncertainties on the strain amplitude measured by LIGO-Hanford and LIGO-Livingston, respectively [55]. Phase calibration is a much smaller source of uncertainty and is therefore neglected [46,56]. All results below are obtained after marginalization over amplitude uncertainties; see the Supplemental Material [48] for details.

Method.—To search for a generically polarized stochastic background, we will apply the methodology presented in Ref. [42]. This method is summarized below, and additional details are discussed in the Supplemental Material [48].

The stochastic background may be detected in the form of a correlated signal between pairs of gravitational-wave detectors. We will assume that the stochastic background is stationary, isotropic, and Gaussian. For simplicity, we also assume that the background is uncorrelated between polarization modes. Finally, we assume that the tensor and vector contributions to the background are individually unpolarized (with equal contributions, for instance, from the tensor plus and cross modes). Certain theories may violate one or more of these assumptions. For example, the stochastic background is unlikely to remain strictly unpolarized in the presence of gravitational-wave birefringence, as in Chern-Simons gravity [57–59], while theories violating Lorentz invariance may yield a departure from isotropy [60,61]. The violation of one or more of our assumptions would likely reduce our search’s sensitivity to the stochastic background.
Given the above assumptions, the expected cross-correlation between two detectors in the presence of a stochastic background is of the form [39–41,62]

\[
\langle \tilde{s}_1(f) \tilde{s}_2^*(f') \rangle = \frac{1}{2} \delta(f - f') \sum_A \Gamma_A(f) S_h^A(f). \tag{3}
\]

Here, \( S_h^A(f) \) is the one-sided gravitational-wave strain power spectral density of the net tensor \((A = T)\), vector \((V)\), and scalar \((S)\) contributions to the stochastic background. The detectors’ geometry is encoded in the overlap reduction functions \( \Gamma_A(f) \), defined [39,42,62,63]

\[
\Gamma_A(f) = \frac{1}{8\pi} \sum_{\mathbf{n} \in A} \int d\hat{n} F^a_1(\hat{n}) F^a_2(\hat{n}) e^{2\pi if\cdot\hat{n}\cdot\Delta x/c}. \tag{4}
\]

\( F^a_1(\hat{n}) \) is the antenna response function of detector 1 to signals of polarization \( a \). \( \Delta x \) is the separation vector between detectors, and \( c \) is the speed of light. The integral is taken over all sky directions \( \hat{n} \).

We will work not directly with \( \Gamma_A(f) \), but rather with the normalized overlap reduction functions \( \gamma_A(f) \propto \Gamma_A(f)/\Gamma_0 \), where the constant \( \Gamma_0 \) is chosen such that \( \gamma_T(f) = 1 \) for co-located and co-oriented detectors. For Advanced LIGO, \( \Gamma_0 = 1/5 \), but in general its value will vary for other experiments like LISA and pulsar timing arrays [64]. The normalized overlap reduction functions for LIGO’s Hanford-Livingston baseline are shown in Fig. 1.

Because tensor, vector, and scalar modes each have distinct overlap reduction functions, the shape of a measured cross-correlation spectrum [Eq. (3)] will reflect the polarization content of the stochastic background [39,42]. Of the three curves in Fig. 1, the scalar overlap reduction function is smallest in magnitude. This reflects the fact that the Advanced LIGO detectors have weaker geometrical responses to scalar-polarized gravitational waves than to tensor- and vector-polarized signals.

Conventionally, gravitational-wave backgrounds are parameterized by their energy-density spectra [62,64]

\[
\Omega(f) = \frac{1}{d \ln f} \frac{d \rho_{GW}}{d \ln f}, \tag{5}
\]

where \( d \rho_{GW} \) is the energy density in gravitational waves per logarithmic frequency interval \( d \ln f \). We normalize Eq. (5) by \( \rho_c = 3H_0^2 c^2/8\pi G \), the closure energy density of the Universe. Here, \( G \) is Newton’s constant and \( H_0 \) is the Hubble constant; we take \( H_0 = 68 \text{ km} \text{s}^{-1} \text{Mpc}^{-1} \) [65]. The precise relationship between \( \Omega(f) \) and \( S_h(f) \) is theory dependent. Under any theory obeying Isaacson’s formula for the stress-energy of gravitational waves [66], the energy-density spectrum is related to \( S_h(f) \) by [42,62,67]

\[
\Omega(f) = \frac{2\pi^2}{3H_0^2} f^3 S_h^S(f). \tag{6}
\]

Equation (6) does not hold in general, however [67]. For ease of comparison with previous studies, we will instead take Eq. (6) as the definition of the canonical energy-density spectra \( \Omega^A(f) \). The canonical energy-density spectra can be directly identified with true energy densities under any theory obeying Isaacson’s formula. For other theories, \( \Omega^A(f) \) can instead be understood simply as a function of the detector-frame observable \( S_h^A(f) \).

Within each 192 s time segment (indexed by \( i \)), we form an estimator of the visible cross power between LIGO-Hanford and LIGO-Livingston:

\[
\hat{C}_i(f) = \frac{1}{\Delta T} \frac{20\pi^2}{3H_0^2} \int f^3 \tilde{s}_{1,i}(f) \tilde{s}_{2,i}(f) df, \tag{7}
\]

normalized such that the estimator’s mean and variance are [42]

\[
\langle \hat{C}_i(f) \rangle = \sum_A \gamma_A(f) \Omega^A(f) \tag{8}
\]

and

\[
\sigma^2_i(f) = \frac{1}{2\Delta T df} \left( \frac{10\pi^2}{3H_0^2} \right)^2 f^6 P_{1,i}(f) P_{2,i}(f), \tag{9}
\]

respectively. Within Eqs. (7) and (9), \( \Delta T \) is the segment duration, \( df \) the frequency bin width, and \( P_{1,i}(f) \) is the one-sided auto-power spectral density of detector 1 in time segment \( i \), defined by

\[
\langle \tilde{s}_{1,i}(f) \tilde{s}_{1,i}^*(f') \rangle = \frac{1}{2} \delta(f - f') P_{1,i}(f). \tag{10}
\]
The normalization of $\hat{C}(f)$ is chosen such that the contribution from each polarization appears symmetrically in Eq. (8); this choice differs by a factor of $\gamma_i(f)$ from the point estimate $\hat{Y}(f)$ typically used in stochastic analyses [42,46,49]. Finally, the cross-power estimators from each segment are optimally combined via a weighted sum to form a single cross-power spectrum for the O1 observing run,

$$\hat{C}(f) = \frac{\sum \hat{C}_i(f)\sigma_i^{-2}(f)}{\sum \sigma_i^{-2}(f)},$$

with the corresponding variance

$$\sigma_i^{-2}(f) = \sum \sigma_i^{-2}(f).$$

Note that, unlike transient gravitational-wave searches, searches for the stochastic background are well described by Gaussian statistics due to the large number of time segments contributing to the final cross-power spectrum [68].

Given the measured cross-power spectrum $\hat{C}(f)$, we compute Bayesian evidence for various hypotheses describing the presence and polarization of a possible stochastic signal within our data. Evidence is computed using PyMultiNest [69], a Python interface to the nested sampling code MultiNest [70–74]. We consider several different hypotheses: (i) Gaussian noise (N): No stochastic signal is present in our data, and the measured cross power is due entirely to Gaussian noise. (ii) Signal (SIG): A stochastic background of any polarization(s) is present. (iii) Tensor-polarized (GR): The data contains a purely tensor-polarized stochastic signal, consistent with general relativity. (iv) Nonstandard polarizations (NGR): The data contains a stochastic signal with vector and/or scalar contributions. These evidences are combined to form two Bayesian odds [42]: (1) Odds $O_N^{SIG}$ for the presence of a stochastic signal relative to pure noise, and (2) odds $O_N^{NGR}$ for the presence of nonstandard polarizations versus ordinary tensor modes. $O_N^{SIG}$ quantifies evidence for the detection of a generically polarized stochastic background, and generally depends only on a background’s total power, not its polarization content. $O_N^{NGR}$ indicates if the background’s polarization is inconsistent with general relativity. In particular, the sensitivity of $O_N^{NGR}$ to nonstandard polarizations is not significantly affected by the strength of any tensor polarization which may also be present [42]. See the Supplemental Material [48] for further details about our hypotheses and odds ratio construction, including the priors placed on these hypotheses and their parameters.

Results.—Using the cross power measured between LIGO-Hanford and LIGO-Livingston during Advanced LIGO’s O1 observing run, we obtain odds $\ln O_N^{SIG} = −0.53$ between signal and Gaussian noise hypotheses, indicating a nondetection of the stochastic gravitational-wave background. Additionally, we find $\ln O_{NGR}^{GR} = −0.25$, consistent with values expected in the presence of Gaussian noise [42]. (We will use $\ln$ and $\log$ to denote base-$e$ and base-10 logarithms, respectively.)

Given our nondetection, we place upper limits on the presence of tensor, vector, and scalar contributions to the stochastic background. To simultaneously constrain the properties of each polarization, we will restrict our analysis to a model assuming the presence of tensor, vector, and scalar-polarized signals (this is the TVS hypothesis in the notation of the Supplemental Material [48]). Under this hypothesis, we model the total canonical energy density of the stochastic background as a sum of power laws:

$$\Omega(f) = \Omega_T^f \left(\frac{f}{f_0}\right)^{\alpha_T} + \Omega_V^f \left(\frac{f}{f_0}\right)^{\alpha_V} + \Omega_S^f \left(\frac{f}{f_0}\right)^{\alpha_S}.$$  

Here, $\Omega_A^f$ is the amplitude of polarization $A$ at a reference frequency $f_0$, and $\alpha_A$ is the corresponding spectral index. We take $f_0 = 25$ Hz [46]. Standard tensor-polarized stochastic backgrounds are predicted to be well described by power laws in the Advanced LIGO band. The expected astrophysical background from compact binary mergers, for instance, is well modeled by a power law with $\alpha_T = 2/3$ [18–20,75].

We will consider two different prior distributions for the background amplitudes: a log-uniform prior between $10^{-13} \leq \Omega_A^f \leq 10^{-5}$ and a uniform prior between $0 \leq \Omega_A^f \leq 10^{-5}$. The former (log-uniform) corresponds to the prior adopted in Ref. [42]. The latter (uniform) implicitly reproduces the maximum likelihood analysis used in previous studies, and is included to allow direct comparison to previous stochastic results [46,49]. The upper amplitude bound ($10^{-5}$) is consistent with limits placed by Initial LIGO and Virgo [49]. In order to be normalizable, the log-uniform prior requires a nonzero lower bound; although parameter estimation results will depend on the specific choice of lower bound, in general this dependence is weak [44]. Our lower bound ($10^{-13}$) is chosen to encompass small energy densities well below the reach of LIGO and Virgo at design sensitivity [23,46].

Following Ref. [42], we take our spectral index priors to be $p(\alpha_A) \propto 1 - |\alpha_A|/\alpha_{MAX}$ for $|\alpha_A| \leq \alpha_{MAX}$ and $p(\alpha_A) = 0$ elsewhere. This prior preferentially weights flat energy-density spectra, penalizing spectra which are more steeply positively or negatively sloped in the Advanced LIGO band. We conservatively choose $\alpha_{MAX} = 8$, allowing for energy-density spectra significantly steeper than backgrounds predicted from known astrophysical sources (like compact binary mergers).

We perform parameter estimation using posterior samples obtained by PyMultiNest. Figure 2 shows posteriors on the tensor, vector, and scalar background amplitudes, under each choice of amplitude prior. The dashed and dot-dashed
curves are proportional to the log-uniform and uniform amplitude priors, respectively; each prior curve has been renormalized by a constant factor to illustrate consistency between our priors and posteriors at small $\Omega_0$. We can now place upper limits on the amplitude of each component at $f_0 = 25$ Hz. The 95\% credible upper limits on the amplitude of each polarization are listed in Table I for each choice of prior (for convenience, we list limits in terms of both $\log \Omega_0$ and $\Omega_0$). As no signal was detected, our posteriors on the spectral indices $\alpha_A$ are dominated by our prior. Full parameter estimation results, including posteriors on $\alpha_A$, are given in the Supplemental Material [48].

Care should be taken when comparing these upper limits to those obtained in previous analyses (e.g., Table I of Ref. [46]). Three important distinctions should be kept in mind. First, the amplitude posteriors in Fig. 2 (and hence the limits in Table I) are obtained after marginalization over spectral index. Previous analyses, on the other hand, typically assume specific fixed spectral indices or present exclusion curves in the $\Omega_0 - \alpha_T$ plane [46]. Second, Bayesian upper limits may be strongly influenced by one’s adopted prior. Uniform amplitude priors, for instance, preferentially weight large signals and therefore yield more conservative upper limits.

signal amplitudes, giving tighter limits. Finally, our results are obtained under a specific signal hypothesis allowing simultaneously for tensor, vector, and scalar polarizations. These limits are not generically identical to those that would be obtained if we allowed for tensor modes alone. In the Supplemental Material [48], we have tabulated upper limits under a variety of signal hypotheses allowing for each unique combination of gravitational-wave polarizations (our results, though, do not vary considerably between hypotheses). We have additionally verified that, under the GR (tensor-only) hypothesis with delta-function priors on the background’s spectral index, we recover upper limits identical to results previously published in Ref. [46].

Conclusion.—The direct measurement of gravitational-wave polarizations may open the door to powerful new tests of gravity. Such measurements largely depend only on the geometry of a gravitational wave’s strain and its direction of propagation, not on the details of any specific theory of gravity. Recently, the Advanced LIGO-Virgo observation of the binary black hole merger GW170814 has enabled the first direct study of gravitational-wave polarizations [4,15]. While LIGO and Virgo are limited in their ability to discern the polarization of gravitational-wave transients, the future construction of additional detectors, like KAGRA [76,77]

TABLE I. 95\% credible upper limits on the log amplitudes of tensor, vector, and scalar modes in the stochastic background at reference frequency $f_0 = 25$ Hz. We assume an energy-density spectrum in which all three modes are present, and present limits following marginalization over the spectral index of each component [see Eq. (13)]. We show results for two different amplitude priors: a log-uniform prior ($dp/d\log \Omega_0 \propto 1$; top row) and a uniform prior ($dp/d\Omega_0 \propto 1$; bottom row). Additional parameter estimation results are shown in the Supplemental Material [48].

<table>
<thead>
<tr>
<th>Prior</th>
<th>$\log \Omega_0^T$</th>
<th>$\log \Omega_0^V$</th>
<th>$\log \Omega_0^S$</th>
<th>$\Omega_0^T$</th>
<th>$\Omega_0^V$</th>
<th>$\Omega_0^S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log uniform</td>
<td>$-7.25$</td>
<td>$-7.20$</td>
<td>$-6.96$</td>
<td>$5.58 \times 10^{-8}$</td>
<td>$6.35 \times 10^{-8}$</td>
<td>$1.08 \times 10^{-7}$</td>
</tr>
<tr>
<td>Uniform</td>
<td>$-6.70$</td>
<td>$-6.59$</td>
<td>$-6.07$</td>
<td>$2.02 \times 10^{-7}$</td>
<td>$2.54 \times 10^{-7}$</td>
<td>$8.44 \times 10^{-7}$</td>
</tr>
</tbody>
</table>
and LIGO-India [78], will help to break existing degeneracies and allow for increasingly precise polarization measurements.

Long-duration signals offer further opportunities to study gravitational-wave polarizations. Detections of continuous sources like rotating neutron stars [44,45] and the stochastic background [42] will offer the ability to directly measure and/or constrain gravitational-wave polarizations, even in the absence of additional detectors. In this Letter, we have conducted a search for a generically polarized stochastic background of gravitational waves using data from Advanced LIGO’s O1 observing run. Although we find no evidence for the presence of a background (of any polarization), we have succeeded in placing the first direct upper limits (listed in Table I) on the contributions of vector and scalar modes to the stochastic background.

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