

# Infinite order quantum-gravitational correlations

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**ABSTRACT:** A new approximation scheme for nonperturbative renormalisation group equations for quantum gravity is introduced. Correlation functions of arbitrarily high order can be studied by resolving the full dependence of the renormalisation group equations on the fluctuation field (graviton). This is reminiscent of a local potential approximation in  $O(N)$ -symmetric field theories. As a first proof of principle, we derive the flow equation for the “graviton potential” induced by a conformal fluctuation and corrections induced by a gravitational wave fluctuation. Indications are found that quantum gravity might be in a non-metric phase in the deep ultraviolet. The present setup significantly improves the quality of previous fluctuation vertex studies by including infinitely many couplings, thereby testing the reliability of schemes to identify different couplings to close the equations, and represents an important step towards the resolution of the Nielsen identity. The setup further allows in principle to address the question of putative gravitational condensates.

**KEYWORDS:** Models of Quantum Gravity, Renormalisation Group, Nonperturbative Effects, Nonperturbative Renormalisation

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## 1 Introduction

Einstein's metric theory of gravity is known not to be perturbatively renormalisable. Many different approaches to overcome this difficulty were proposed in the past decades. One remarkably economic proposal is due to Weinberg [1], he conjectured that gravity might be nonperturbatively renormalisable by an interacting fixed point of its renormalisation group flow. If the critical hypersurface of this fixed point has a finite dimension, the resulting theory is as predictive as an asymptotically free theory.

Only in the 90s, with the advent of functional renormalisation group (RG) equations [2-4], this proposal could be tested in  $d = 4$ . Since then, a plethora of approximations and

aspects were studied, including different approximations on the Einstein-Hilbert sector [5–25], higher derivatives and  $f(R)$  [26–53], the two-loop counterterm [54], aspects of unitarity [55, 56], different variables [16, 21, 57–59] and the coupling to matter [58, 60–92]. All pure gravity studies so far are compatible with a fixed point that can control the ultraviolet (UV) behaviour of quantum gravity, and most works that also studied the inclusion of matter arrive at a similar conclusion for matter content compatible with the Standard Model.

A central technical tool to investigate gravitational (or gauge) RG equations is the background field method. In the context of functional RG equations, severe problems arise from that, since the regularisation breaks the split Ward identity, as it depends on the background and the fluctuation separately. It is known that if not treated with enough care, this can even change the universal 1-loop beta function in Yang-Mills theory [69, 93], or destroy the well-known Wilson-Fisher fixed point of the Ising model in  $d = 3$  [94]. A modified version of the split Ward identity exists, which accommodates this deficit [4, 49, 65, 94–108], but is inherently difficult to solve. A more hands-on approach is to resolve the dependence of the effective action on both the background and the fluctuation field, which was systematically developed and applied to quantum gravity in [50, 78, 80, 82, 84, 92, 109–114], for related approaches see also [40, 65, 97, 98, 105, 115–118].

So far in this bimetric setting, it was only possible to derive beta functions for a finite number of couplings. To close these equations, some couplings had to be identified. A priori, it is not clear how this should be done, and whether such a procedure is as stable as in *e.g.* scalar field theories, where one can find the Wilson-Fisher fixed point by a (low order) Taylor expansion of the potential around the vacuum expectation value with accurate estimates for the first critical exponent. The aim of this work is to develop the techniques to lift this restriction, and thus to treat an arbitrary dependence on the fluctuation field. With this it is then possible to study whether the graviton acquires a nonvanishing vacuum expectation value, *i.e.* if there is gravitational condensation.

This work is structured as follows: in Section 2 we introduce the basic ideas of our approach and some technical prerequisites necessary for the subsequent discussion. Section 3 contains the setup and approximations that we use to study the UV structure of quantum gravity. In section 4 we present the numerical results for the fixed point structure, followed by a discussion of the physical significance of these results and potential shortcomings of the approximations in section 5. We conclude with a short summary in section 6. The appendices collect some technical details.

## 2 Functions of the fluctuation field

Lagrangian formulations of the quantisation of gravity often rely on the background field method. For this, the full metric  $g$ , which contains all information, is split into a nondynamical background metric  $\bar{g}$ , and quantum fluctuations around this background, parameterised by a symmetric tensor field  $h$ . Many calculations rely on a so-called linear split or parameterisation,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \quad (2.1)$$

More recently, other types of parameterisation have been studied [22–24, 44–46, 48, 76, 78, 79, 106, 119, 120], *e.g.* the exponential split

$$g_{\mu\nu} = \bar{g}_{\mu\rho} (\exp(\bar{g}^{-1}h))^\rho{}_\nu \equiv \bar{g}_{\mu\rho} (\exp(\mathfrak{h}))^\rho{}_\nu, \quad (2.2)$$

which plays a distinguished role in two-dimensional quantum gravity [121–125], and in general has the virtue of being a one-to-one mapping between metrics  $g$  and symmetric fluctuation tensors  $h$  [119]. Here, we introduced the  $(1, 1)$ -tensor  $\mathfrak{h} = \bar{g}^{-1}h$  for convenience. The special role of  $\mathfrak{h}$  in the exponential parameterisation was already pointed out in [106], where it was called  $X$ . For us,  $\mathfrak{h}$  is useful since powers of it are automatically background covariant. It is also useful to introduce a traceless decomposition,

$$\mathfrak{h} = \mathfrak{h}^{\text{TL}} + \frac{1}{4} \mathbf{h} \mathbb{1}_4, \quad (2.3)$$

where  $\mathbf{h}$  is the trace of  $\mathfrak{h}$ . Here, we already specified four spacetime dimensions, although the following discussion easily extends to other dimensions.

We will limit our discussion in this work to scalar invariants of  $\mathfrak{h}$  under  $GL(4)$ , *i.e.* we don't consider invariants built with derivatives. Since  $\mathfrak{h}$  can be interpreted as a usual matrix, the scalar invariants are exactly the four eigenvalues. Clearly, this is of minor use in a functional language, and we have to find a useful way to form invariants. For this task, we use the Cayley-Hamilton theorem (CHT), which is reviewed briefly in appendix A. Essentially, it states that, if we replace the eigenvalue by the matrix itself in the characteristic equation of the matrix, we get the zero matrix. For  $\mathfrak{h}^{\text{TL}}$ , it reads

$$[\mathfrak{h}^{\text{TL}}]^4 - \frac{1}{2} \text{tr}([\mathfrak{h}^{\text{TL}}]^2) [\mathfrak{h}^{\text{TL}}]^2 - \frac{1}{3} \text{tr}([\mathfrak{h}^{\text{TL}}]^3) \mathfrak{h}^{\text{TL}} + \det(\mathfrak{h}^{\text{TL}}) \mathbb{1}_4 = 0. \quad (2.4)$$

This immediately gives a basis of monomials - the four lowest nonnegative powers of the matrix - and an algorithm to expand any higher power of the matrix to a linear combination of these low powers. Scalar invariants of the matrix are then the determinant and the traces of the first three powers of the matrix. Employing the traceless decomposition, the invariants are

$$\begin{aligned} \mathfrak{h}_1 &= \mathbf{h}, \\ \mathfrak{h}_2 &= \text{tr}([\mathfrak{h}^{\text{TL}}]^2), \\ \mathfrak{h}_3 &= \text{tr}([\mathfrak{h}^{\text{TL}}]^3), \\ \mathfrak{h}_4 &= \det(\mathfrak{h}^{\text{TL}}). \end{aligned} \quad (2.5)$$

Thus, the most general parameterisation of the full metric which doesn't introduce a scale or uses derivatives can be written as

$$g = \bar{g} \left( \mathcal{A}_0 \mathbb{1}_4 + \mathcal{A}_1 \mathfrak{h}^{\text{TL}} + \mathcal{A}_2 [\mathfrak{h}^{\text{TL}}]^2 + \mathcal{A}_3 [\mathfrak{h}^{\text{TL}}]^3 \right). \quad (2.6)$$

No higher powers of  $\mathfrak{h}^{\text{TL}}$  appear since by the CHT they can be reduced to lower powers, and are thus included in this ansatz. The functions  $\mathcal{A}_i$  are free functions of the four invariants (2.5), and determine the parameterisation. The only constraints on them are that  $g$  should

be invertible, *i.e.*  $\det g \neq 0$ , and  $g = \bar{g}$  if  $h = 0$ , which fixes  $\mathcal{A}_0 = 1 + \mathcal{O}(h)$ . The inverse metric  $g^{-1}$  has a similar exact representation,

$$g^{-1} = \left( \mathcal{B}_0 \mathbb{1}_4 + \mathcal{B}_1 \mathfrak{h}^{\text{TL}} + \mathcal{B}_2 [\mathfrak{h}^{\text{TL}}]^2 + \mathcal{B}_3 [\mathfrak{h}^{\text{TL}}]^3 \right) \bar{g}^{-1}. \quad (2.7)$$

The functions  $\mathcal{B}_i$  can be expressed explicitly in terms of the  $\mathcal{A}_i$ , the full expressions are collected in appendix B. As an example, for the linear split,

$$\mathcal{A}_0 = 1 + \frac{1}{4}\mathfrak{h}_1, \quad \mathcal{A}_1 = 1, \quad \mathcal{A}_2 = \mathcal{A}_3 = 0, \quad (2.8)$$

and we find

$$\begin{aligned} \mathcal{B}_0 &= \frac{\mathcal{A}_0^3 - \frac{1}{2}\mathcal{A}_0\mathfrak{h}_2 + \frac{1}{3}\mathfrak{h}_3}{\mathcal{A}_0^4 - \frac{1}{2}\mathcal{A}_0^2\mathfrak{h}_2 + \frac{1}{3}\mathcal{A}_0\mathfrak{h}_3 + \mathfrak{h}_4}, \\ \mathcal{B}_1 &= -\frac{\mathcal{A}_0^2 - \frac{1}{2}\mathfrak{h}_2}{\mathcal{A}_0^4 - \frac{1}{2}\mathcal{A}_0^2\mathfrak{h}_2 + \frac{1}{3}\mathcal{A}_0\mathfrak{h}_3 + \mathfrak{h}_4}, \\ \mathcal{B}_2 &= \frac{\mathcal{A}_0}{\mathcal{A}_0^4 - \frac{1}{2}\mathcal{A}_0^2\mathfrak{h}_2 + \frac{1}{3}\mathcal{A}_0\mathfrak{h}_3 + \mathfrak{h}_4}, \\ \mathcal{B}_3 &= -\frac{1}{\mathcal{A}_0^4 - \frac{1}{2}\mathcal{A}_0^2\mathfrak{h}_2 + \frac{1}{3}\mathcal{A}_0\mathfrak{h}_3 + \mathfrak{h}_4}. \end{aligned} \quad (2.9)$$

It is further straightforward to calculate the determinant of the general metric (2.6). Again, the full expression is deferred to the appendix. For the linear split (2.8),

$$\det g = \left[ \mathcal{A}_0^4 - \frac{1}{2}\mathcal{A}_0^2\mathfrak{h}_2 + \frac{1}{3}\mathcal{A}_0\mathfrak{h}_3 + \mathfrak{h}_4 \right] \det \bar{g}. \quad (2.10)$$

Let us stress again that the expressions just presented are *exact*, and follow directly from the CHT. In appendix B we collect some formulas for the exponential split.

It is in fact advantageous to use the traceless decomposition to define the scalars  $\mathfrak{h}_i$ , because they are in some sense orthogonal, which makes it possible to choose fluctuations  $h$  such that only some of the invariants have a nonvanishing value. This comes in useful if one wants to employ approximations, where one only considers the dependence on a subset of these scalars. As an example, a gravitational wave fluctuation,

$$\mathfrak{h}^{\text{TL}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (2.11)$$

gives

$$\begin{aligned} \mathfrak{h}_2 &= 2(h_+^2 + h_\times^2), \\ \mathfrak{h}_3 &= 0, \\ \mathfrak{h}_4 &= 0. \end{aligned} \quad (2.12)$$

Here,  $h_+$  and  $h_\times$  are the two polarisation states of the gravitational wave in transverse traceless gauge. As claimed, it is thus easily possible to truncate the invariants (2.5) by restricting the considered fluctuations to special choices. One should also notice that  $\mathfrak{h}_2 \geq 0$ , and moreover  $\mathfrak{h}_2 = 0$  implies  $\mathfrak{h}_3 = \mathfrak{h}_4 = 0$ , which can easily be seen if the scalars are expressed in terms of the eigenvalues of the matrix  $\mathfrak{h}^{\text{TL}}$ .

### 3 Functional RG and local potential approximation in quantum gravity

In this work we use the functional renormalisation group to calculate nonperturbative beta functions. The central object of study is the effective average action which interpolates between the classical action in the ultraviolet (UV) and the standard effective action in the infrared (IR), modulo some subtleties [3, 126–128]. It is regularised by a momentum-dependent effective mass term, which renders any infinitesimal RG step finite. This follows the Wilsonian idea of integrating out momentum modes shell by shell. The effective average action (or effective action for short in the following) in our setup is a functional of both the background metric and the fluctuation field,  $\Gamma = \Gamma[\bar{g}; h]$ . The individual dependence on  $\bar{g}$  and  $h$  is necessary to define the regulator and the gauge fixing, and thus gives rise to the split-Ward identity. Still, background diffeomorphism invariance can be explicitly maintained. The RG flow of the effective action is governed by the exact flow equation [2–4]

$$k\partial_k\Gamma \equiv \dot{\Gamma} = \frac{1}{2}\text{STr} \left[ \left( \Gamma^{(2)} + \mathfrak{R} \right)^{-1} \mathfrak{R} \dot{\mathfrak{R}} \right]. \quad (3.1)$$

Here,  $k$  is the IR cutoff scale, an overdot indicates  $k$  times the derivative w.r.t.  $k$ ,  $\mathfrak{R}$  is the regulator kernel,  $\text{STr}$  stands for the supertrace, summing over all dynamical fields and discrete indices, integrating over continuous indices and multiplying a minus sign for Grassmann-valued fields, and  $\Gamma^{(2)}$  is the Hessian of the effective action w.r.t. the fluctuation field(s). For reviews of the flow equation in quantum gravity see *e.g.* [13, 17, 96, 129–131].

#### 3.1 Nielsen identity

The standard effective action, given by the effective average action in the limit  $k \rightarrow 0$ , can only depend on one field, the metric, *i.e.* it is diffeomorphism invariant. For finite  $k$ , this is broken by the regulator and the gauge-fixing. The amount of breaking can be expressed by the so-called Nielsen or split-Ward identity. It relates the derivatives of the effective action w.r.t. the fluctuation field to the ones w.r.t. the background metric. Schematically, it reads

$$\frac{\delta\Gamma}{\delta\bar{g}} - \frac{\delta\Gamma}{\delta h} = \mathcal{R} + \mathcal{G}, \quad (3.2)$$

where  $\mathcal{R}$  and  $\mathcal{G}$  are the breaking terms arising from the regulator and the gauge fixing, respectively, which depend on both the background metric and the fluctuation field individually, more specifically *not* in the combination of the full metric. An introduction to the most important points can be found in [113], for more general discussions, see *e.g.* [4, 49, 65, 94–100, 106–108]. For us, the central observation is that the background independence of observables and the nontrivial Ward identity necessitate the direct computation of fluctuation correlation functions.

So far, this direct computation was done in a vertex expansion, both for pure gravity [50, 109–114] as well as gravity-matter systems [78, 80–82, 91, 92]. Related approaches are bimetric calculations [65, 97, 115–117] and efforts to solve the Nielsen identity directly or indirectly [49, 95, 98–108]. The most advanced vertex calculation on a flat background resolved parts of the four-point function [113], whereas in [114], fluctuation-curvature-correlations were resolved for the first time. All of these calculations rely on coupling

identifications of higher vertices to close the flow equation (3.1). With the scheme that we present below, we can overcome this deficit. For the first time, this gives direct access to correlation functions of arbitrary order, and a means to check in how much the coupling identifications are a stable approximation. By this we also automatically provide the explicit mapping between the “bimetric” and the “fluctuation” language, *i.e.* the different possibilities between spanning the effective action with the background and the full metric, or with the background metric and the fluctuation field. The remainder of this section is devoted to introduce our approximation in which we solve the flow equation (3.1).

### 3.2 Einstein-Hilbert part

Our ansatz for the kinetic and potential part of the effective action reads

$$\Gamma_{\text{fluc}} = \frac{1}{16\pi G_N} \int d^4x \left( -\sqrt{\det g} R + \sqrt{\det \bar{g}} 2\mathcal{V}(\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3, \mathfrak{h}_4) \right). \quad (3.3)$$

In this ansatz,  $G_N$  is the (running) Newton’s constant,  $R$  is the Ricci scalar of the full metric  $g$  and  $\mathcal{V}$  is the fluctuation potential, which is also  $k$  dependent. In previous studies, the latter part was approximated by the classical Einstein-Hilbert structure, partly resolving different vertex couplings (conventionally called  $\lambda_i$  for the coupling of the  $i$ -th vertex). For the first time, we go beyond this, resolving the full fluctuation field dependence of the constant part of all vertices. This also extends earlier similar work in conformally reduced gravity [99, 101, 132–136]. The goal of this work is to derive and solve the beta function for  $\mathcal{V}$  in some approximation.

### 3.3 Gauge-fixing and ghosts

The action (3.3) has to be complemented by a gauge fixing term,

$$\Gamma_{\text{gf}} = \frac{1}{32\pi G_N \alpha} \int d^4x \sqrt{\det \bar{g}} \bar{g}^{\mu\nu} F_\mu F_\nu, \quad (3.4)$$

with the gauge fixing condition

$$F_\mu = \mathcal{F}_\mu^{\rho\sigma}[\bar{g}] h_{\rho\sigma} = \left( \delta_\mu^\beta \bar{D}^\alpha - \frac{1+\beta}{4} \bar{g}^{\rho\sigma} \bar{D}_\mu \right) h_{\rho\sigma}. \quad (3.5)$$

By  $\bar{D}$  we understand the covariant derivative constructed from the background metric  $\bar{g}$ , and  $\alpha$  and  $\beta$  are gauge fixing parameters. Eventually, we are interested in the Landau limit  $\alpha \rightarrow 0$  which implements the gauge fixing strictly. In Landau gauge, neither of the gauge fixing parameters flows, no matter the value of  $\beta$  [114, 137].

With the introduction of a gauge fixing, we have to account for the corresponding change in the measure by introducing ghost fields. The resulting ghost action reads

$$\Gamma_{\text{gh}} = \int d^4x \sqrt{\det \bar{g}} \bar{c}_\mu \bar{g}^{\mu\nu} \mathcal{F}_\nu^{\rho\sigma}[\bar{g}] \delta_Q h_{\rho\sigma}. \quad (3.6)$$

Since we gauge-fix  $h$  directly, independently of the parameterisation, the quantum gauge transformation  $\delta_Q h$  is in general nontrivial, and not simply given by the Lie derivative

of the full metric along the ghost vector field  $c$ . Only for the special case of a linear parameterisation, we arrive at the familiar form

$$\delta_Q h_{\mu\nu} = \delta_Q g_{\mu\nu} = \mathcal{L}_c g_{\mu\nu} = D_\mu c_\nu + D_\nu c_\mu. \quad (3.7)$$

We show how to derive the relevant general expression in appendix D. In particular, the quantum gauge transformation for the exponential parameterisation is given in (D.9). In our investigations it turned out that if we gauge-fix the full metric instead, the results rely heavily on the choice of the regulator.

### 3.4 Regulators

We finally have to fix the regularisation of both the gravitons and the ghost fields. In this, we closely follow the strategy of [50, 109, 111–114], choosing the regulator proportional to the two-point function, with the potential  $\mathcal{V}$  and the background curvature  $\bar{R}$  set to zero:

$$\Delta S_{\text{grav}} = \frac{1}{2} \int d^4x \sqrt{\det \bar{g}} h_{\mu\nu} \frac{\mathfrak{R}(\bar{\Delta})}{\bar{\Delta}} \left[ (\Gamma_{\text{fluc}} + \Gamma_{\text{gf}})^{(2)\mu\nu\rho\sigma} \Big|_{\mathcal{V}=h=\bar{R}=0} \right] h_{\rho\sigma}, \quad (3.8)$$

$$\Delta S_{\text{gh}} = \int d^4x \sqrt{\det \bar{g}} \bar{c}_\mu \frac{\mathfrak{R}(\bar{\Delta})}{\bar{\Delta}} \bar{g}^{\mu\nu} \mathcal{F}_\nu^{\rho\sigma}[\bar{g}] \delta_Q h_{\rho\sigma} \Big|_{h=\bar{R}=0}. \quad (3.9)$$

For convenience, we introduced the Laplacean of the background covariant derivative by  $\bar{\Delta} = -\bar{D}^2$ . Since we gauge-fix  $h$  instead of  $g$ , we can also employ the minimal regulator recently introduced in [114], which only changes the background part of the flow, *i.e.* the part at vanishing fluctuation field, resulting in an overall shift of the potential  $\mathcal{V}$ . We checked explicitly that this is indeed the case to all orders in the fluctuation field in the truncation that we discuss subsequently.

### 3.5 Flow equations

Now, we are in the situation to calculate the flow equation for the fluctuation potential  $\mathcal{V}$ . It is enough to use a flat background metric  $\bar{g} = \eta$  after the hessian has been calculated. To simplify matters, we don't derive a flow equation for the Newton's constant, rather treating it as a parameter. The huge amount of tensor algebra is dealt with by the Mathematica package *xAct* [138–143].

Before we carry on, let us discuss our choice of parameterisation. First consider the linear split. It turns out that the constraints on the positivity of the determinant of the metric severely impacts the accessible fluctuation space, and always gives rise to a singular line where  $\det g = 0$ , see (2.10), which cuts the space spanned by (2.5). Put differently, there are fluctuations  $h$  such that the full metric is degenerate or even has the opposite sign, and these fluctuations should tentatively be excluded from the path integral. Since it is a huge technical and numerical hurdle to treat the restricted variable space and the singular line, we will instead consider the exponential split in the following, since it doesn't give rise to any singularities for finite fluctuations due to being a one-to-one map between metrics and fluctuations. We will further assume that the path integral measure is trivial for the exponential split, for a discussion of this see [23].



It is clearly a formidable task to derive the beta function of the full fluctuation potential. As a proof of concept, we shall make a further approximation where we can derive the flow equation with manageable effort, and give comments on the quality and the potential impact of improvements of the approximation later. We thus restrict ourselves in the following to a trace fluctuation together with the leading order of a gravitational wave fluctuation. Our ansatz reads

$$\Gamma_{\text{trunc}} = \frac{1}{16\pi G_N} \int d^4x \left[ -\sqrt{\det g} R + \sqrt{\det \bar{g}} 2 (\mathcal{V}(\mathfrak{h}_1) + \mathfrak{h}_2 \mathcal{W}(\mathfrak{h}_1)) \right] + \Gamma_{\text{gf}} + \Gamma_{\text{gh}}. \quad (3.10)$$

To obtain the flow equations for  $\mathcal{V}$  and  $\mathcal{W}$ , we take the full second functional derivative of this action, and only afterwards project onto fluctuations which include the full  $\mathfrak{h}_1$ -dependence and all terms up to linear order in  $\mathfrak{h}_2$ .

A further technical problem arises when calculating the propagator and eventually the trace, which we mention before we finally present the explicit flow equations. The inversion of the regularised two-point function gives rise to terms where  $\mathfrak{h}^{\text{TL}}$  is contracted with the momentum  $p$ , e.g.  $p^\mu h_{\mu\nu}^{\text{TL}} p^\nu$ . These then appear in denominators, and we have to clarify how to integrate over the momenta, eventually expressing them in terms that only involve the scalar invariants (2.5). In the special case of a gravitational wave fluctuation, we can simply insert the explicit matrix representation (2.11), which with our truncation can be rewritten as

$$\mathfrak{h}^{\text{TL}} = \text{diag} \left( 0, \sqrt{\mathfrak{h}_2/2}, -\sqrt{\mathfrak{h}_2/2}, 0 \right), \quad (3.11)$$

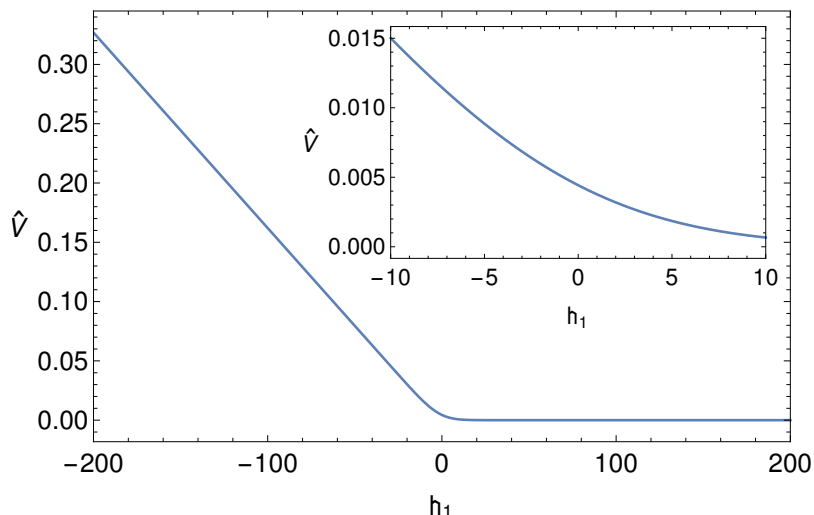
where we can without loss of generality set  $h_\times = 0$ . With this, coordinates for the loop momentum can be chosen in order to calculate the loop integral. Still, in a more general setting, it is useful to have general formulas to treat these kinds of expressions. We will collect some aspects of this in appendix E.

To present the explicit flow equations, we first switch to dimensionless quantities by appropriate rescalings with powers of  $k$ ,

$$V = k^{-2}\mathcal{V}, \quad W = k^{-2}\mathcal{W}, \quad g = k^2 G_N. \quad (3.12)$$

The explicit flow equation for  $V$  in the Landau limit  $\alpha \rightarrow 0$  then reads

$$\begin{aligned} \dot{V}(\mathfrak{h}_1) = & -4V(\mathfrak{h}_1) - \frac{g}{6\pi(\beta-3)^4 \left(1 - e^{\frac{\mathfrak{h}_1}{4}}\right)^3} \left( 4(\beta-3)^4 \left(e^{\frac{\mathfrak{h}_1}{4}} - 1\right)^3 + (\beta-3)^2 \left(1 - e^{\frac{\mathfrak{h}_1}{4}}\right) \times \right. \\ & \left( (\beta-3)^2 \left(-35e^{\frac{\mathfrak{h}_1}{4}} + 4e^{\frac{\mathfrak{h}_1}{2}} + 13\right) + 288V''(\mathfrak{h}_1) - 72(\beta^2 - 10\beta + 15)W(\mathfrak{h}_1) \right) \\ & + 3 \left( (\beta-3)^2 - 96V''(\mathfrak{h}_1) - 16\beta^2 W(\mathfrak{h}_1) \right) \left[ (\beta-3)^2 \left(2e^{\frac{\mathfrak{h}_1}{4}} - 1\right) - 96V''(\mathfrak{h}_1) \right. \\ & \left. - 16\beta^2 W(\mathfrak{h}_1) \right] \ln \left( \frac{(\beta-3)^2 - 96V''(\mathfrak{h}_1) - 16\beta^2 W(\mathfrak{h}_1)}{(\beta-3)^2 e^{\frac{\mathfrak{h}_1}{4}} - 96V''(\mathfrak{h}_1) - 16\beta^2 W(\mathfrak{h}_1)} \right) \\ & \left. + 15(\beta-3)^4 (8W(\mathfrak{h}_1) + 1) \left(2e^{\frac{\mathfrak{h}_1}{4}} + 8W(\mathfrak{h}_1) - 1\right) \ln \left( \frac{8W(\mathfrak{h}_1) + 1}{e^{\frac{\mathfrak{h}_1}{4}} + 8W(\mathfrak{h}_1)} \right) \right). \quad (3.13) \end{aligned}$$



**Figure 1.** Fixed point solution to (4.1) for the rescaled coupling  $\hat{g} = 1/36$ , obtained with pseudo-spectral methods. The solution is a monotonically decreasing function, which rises linearly for large negative arguments, and drops exponentially for large positive arguments.

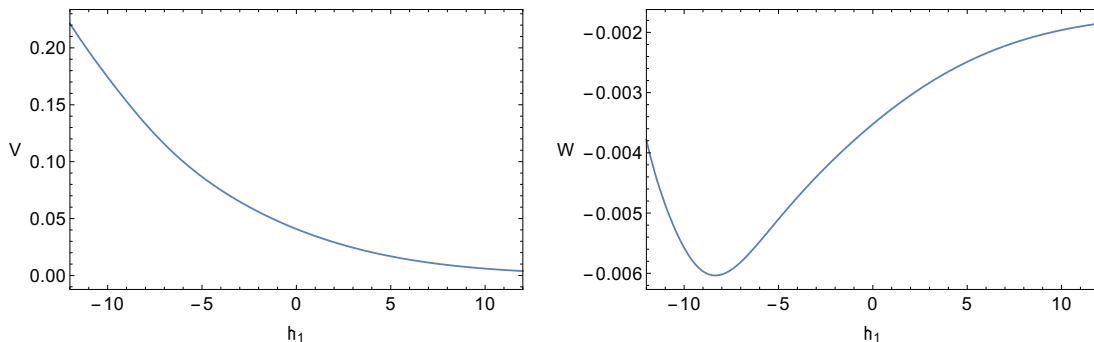
The flow equation for  $W$  is even lengthier and will not be presented here. In this, we assumed that the dimensionless Newton’s constant  $g$  is at an interacting fixed point,  $\dot{g} = 0$  and  $g \neq 0$ . We also chose the Litim regulator [144, 145] to evaluate the integrals. Finally we shifted  $V$  such that the quantum contribution to  $\dot{V}$  vanishes in the limit  $\mathfrak{h}_1 \rightarrow \infty$  if  $V = W = 0$ .

## 4 Fixed point structure

We will now study the fixed point structure of equation (3.13) and the corresponding equation for  $\dot{W}$ . First, we discuss a truncation with  $V$  alone, setting  $W = 0$ , afterwards studying the coupled system. The subsequent numerical results are obtained with pseudo-spectral methods, which have been systematically developed in the context of functional RG flows in [77, 146], and successfully been employed in *e.g.* [147–151]. As numerical parameters, we choose  $g = 1/4$  and  $\beta = 0$ . The precise choice for these parameters is for purely illustrational purpose, but motivated by values obtained in recent studies [114].

### 4.1 Conformal fluctuation potential

If we set  $W = 0$  in (3.13), and further employ the rescalings  $g \rightarrow (\beta - 3)^2 \hat{g}$ ,  $V \rightarrow (\beta - 3)^2 \hat{V}$ , all occurrences of the gauge parameter  $\beta$  drop out. The flow equation for  $\hat{V}$  is thus gauge



**Figure 2.** Fixed point solution to the coupled system of flow equations for  $V$  and  $W$  for the coupling  $g = 1/4$  and the gauge fixing parameter  $\beta = 0$ , obtained with pseudo-spectral methods. The qualitative picture for  $V$  is the same as with  $W = 0$ , which is reflected by the fact that  $W$  itself is numerically small and only slowly varying.

independent,

$$\begin{aligned} \dot{\hat{V}}(\mathfrak{h}_1) = & -4\hat{V}(\mathfrak{h}_1) + \frac{\hat{g}}{2\pi(e^{\mathfrak{h}_1/4} - 1)^3} \left[ 3 + 96\hat{V}''(\mathfrak{h}_1) + 9e^{\mathfrak{h}_1/2} + \frac{5}{4}\mathfrak{h}_1 - \frac{1}{2}e^{\mathfrak{h}_1/4}(24 + 5\mathfrak{h}_1) \right. \\ & \left. - 96e^{\mathfrak{h}_1/4}\hat{V}''(\mathfrak{h}_1) - \left(-1 + 2e^{\mathfrak{h}_1/4} - 96\hat{V}''(\mathfrak{h}_1)\right) \left(-1 + 96\hat{V}''(\mathfrak{h}_1)\right) \ln \frac{1 - 96\hat{V}''(\mathfrak{h}_1)}{e^{\mathfrak{h}_1/4} - 96\hat{V}''(\mathfrak{h}_1)} \right]. \end{aligned} \quad (4.1)$$

We find a single global solution for the fixed point condition  $\dot{\hat{V}} = 0$ , which is shown in Figure 1. This solution is a monotonically decreasing function. An asymptotic expansion around  $\mathfrak{h}_1 = -\infty$  is possible, where subleading terms are suppressed by powers of  $(e^{\mathfrak{h}_1/4})$ . The leading order is linear in  $\mathfrak{h}_1$ , in contrast to the naive expectation  $\hat{V} \propto \sqrt{\det g / \det \bar{g}} = e^{\mathfrak{h}_1/2}$ , thus we have strong fluctuation effects. Clearly, due to the  $\beta$ -dependent rescaling, all gauge dependence is hidden in the effective coupling  $\hat{g}$ . From recent studies [114] we infer that this dependence is rather weak, signalling the stability of this result upon variations of  $\beta$ . The qualitative picture of the solution is already manifest if one expands  $\hat{V}$  in powers of  $\hat{g}$ , keeping only few terms, and thus the solution varies essentially linearly with  $\hat{g}$ , for small  $\hat{g}$ .

## 4.2 Corrections by gravitational wave fluctuations

We now discuss the coupled system of  $V$  and  $W$ . Again, we find a single solution for the fixed point condition  $\dot{V} = \dot{W} = 0$ , which is shown in Figure 2. The qualitative picture stays the same as discussed above:  $V$  is monotonically decreasing, with similar asymptotics as with  $W = 0$ . The difference in the absolute value is due to the fact that we didn't rescale  $V$  as above, since the gauge dependence doesn't drop out in this extended approximation. On the other hand, the function  $W$  is numerically small, and decreases exponentially in both limits  $\mathfrak{h}_1 \rightarrow \pm\infty$ . This property is not visible in Figure 2 for large negative  $\mathfrak{h}_1$ . The reason for this is that the coupled set of equations is numerically very difficult to solve for these values of  $\mathfrak{h}_1$ .

## 5 Discussion

We now interpret the results presented above and try to assess the reliability of the approximation. The first observation is that, since  $V$  is monotonically decreasing with  $V'(\infty) = 0$ , the natural expansion point around the minimum of the potential would be  $\mathfrak{h}_1 = \infty$ . This immediately raises doubts whether the standard approach, namely an expansion around vanishing fluctuation field, is physically justified. A potential explanation for this is that quantum gravity in the deep UV is in a non-metric phase, and potentially more fundamental building blocks as used in causal dynamical triangulations (CDT) [152–170] or causal sets [171–180] are the true degrees of freedom. On the other hand, we find that the linear correction due to gravitational wave fluctuations is strictly negative. This could potentially “cure” the first observation, in the sense that the true minimum of the potential might be at a point with finite coordinates  $\mathfrak{h}_1, \mathfrak{h}_2 \neq 0$ . Second, the solution deviates strongly from the naive expectation involving the exponential parameterisation, *i.e.*  $V \propto \sqrt{\det g / \det \bar{g}} = e^{\mathfrak{h}_1/2}$ . In particular, our solution rises linearly for large negative arguments, whereas it decreases exponentially fast for large positive arguments. This indicates very strong quantum fluctuations and emphasises the need for the present approach to resolve the full potential.

Let us now assess the quality of the present approximation. Since the graviton fluctuation is dimensionless, anomalous dimensions can play a very important role in the discussion of the fixed point structure. We will now try to analyse some scenarios that are possible if anomalous dimensions are included. For the discussion we will assume that  $\mathfrak{h}_3$  and  $\mathfrak{h}_4$  only play a subdominant role, thus we assume that  $V(\mathfrak{h}_1, \mathfrak{h}_2)$  is a decent approximation to the true potential. The main impact of the anomalous dimensions  $\eta_{\text{Tr}}$  and  $\eta_{\text{TL}}$  of the trace and traceless mode, respectively, is the additional contribution to the canonical scaling,

$$\dot{V}(\mathfrak{h}_1, \mathfrak{h}_2) = -4V(\mathfrak{h}_1, \mathfrak{h}_2) + \frac{\eta_{\text{Tr}}}{2}\mathfrak{h}_1\partial_{\mathfrak{h}_1}V(\mathfrak{h}_1, \mathfrak{h}_2) + \eta_{\text{TL}}\mathfrak{h}_2\partial_{\mathfrak{h}_2}V(\mathfrak{h}_1, \mathfrak{h}_2) + \mathcal{O}(g). \quad (5.1)$$

Note that  $\eta_{\text{TL}}$  is essentially the anomalous dimension of the physical transverse traceless mode, and its momentum dependence was calculated in [111], with strictly positive sign and a value of  $\mathcal{O}(1)$ , although in a linear parameterisation. For large  $\mathfrak{h}_2$ , the canonical scaling term indicates that  $V \propto \mathfrak{h}_2^{4/\eta_{\text{TL}}}$  for  $\mathfrak{h}_2 \rightarrow \infty$  at the fixed point where  $\dot{V} = 0$ . On the other hand, for a well-defined propagator, we need that  $\partial_{\mathfrak{h}_2}V > c_1$  with a finite constant  $c_1$ . This is the analogue of the singularity at  $\Lambda = 1/2$  in standard calculations. We thus conclude that for large  $\mathfrak{h}_2$ , the potential should rise like a power law. Together with the indications of the above results that  $W < 0$ , this indeed strengthens the hint towards a minimum of the potential at a finite value of  $\mathfrak{h}_2$ . Let us however stress that in this analysis we assumed that the quantum contribution is subleading in the limit of large  $\mathfrak{h}_2$ , which might not be the case.

Now we discuss the impact of  $\eta_{\text{Tr}}$ . Again assuming that the quantum term is subleading, we are lead to the conclusion that  $V \propto \mathfrak{h}_1^{8/\eta_{\text{Tr}}}$  for large  $|\mathfrak{h}_1|$  at the fixed point. On the other hand, this time we have an upper bound on the second derivative for a well-defined propagator,  $\partial_{\mathfrak{h}_1}^2V < c_2$ . There are hence several distinct possibilities. If  $\eta_{\text{Tr}} < 0$ , we conclude from the above results that for large negative  $\mathfrak{h}_1$  actually the quantum term

dominates, whereas for large positive arguments, the solution decreases by a power law instead of exponentially. Thus, the situation is qualitatively similar to the case analysed above. By contrast, if  $0 < \eta_{\text{Tr}} < 2$ , the asymptotic scaling is a power law with exponent larger than two, thus the prefactor is necessarily negative, and any putative fixed point potential is unbounded from below. It is not clear to the author how to interpret this case. One might argue that the trace mode is anyway not a propagating physical degree of freedom, and thus the physical part of the graviton potential isn't influenced by this unboundedness. Finally, if  $\eta_{\text{Tr}} > 2$ , there is the possibility for a fixed point potential which is bounded from below and raising like a power law asymptotically. Necessarily, this gives rise to a minimum at a finite value of  $\mathfrak{h}_1$ . Together with the above observations that  $W < 0$  and  $\eta_{\text{TL}} > 0$ , this gives a good chance that the potential admits a minimum at finite values of both invariants. Note however that such a large anomalous dimension could invalidate the standard way of regularisation, for a discussion of this aspect in quantum gravity coupled to matter, see [80]. The authors of [91] reported small, but positive values for  $\eta_{\text{Tr}}$ , however the calculation was done in a linear parameterisation, further neglecting the gaps of the graviton fluctuation. We thus cannot give a definite conclusion on which of the possibilities discussed above is the one realised in a full computation.

Eventually we shall discuss the relation of the present approach to the Nielsen identity. As  $k$  goes to zero, the effective average action only depends on one metric, and full diffeomorphism invariance is restored. In our fluctuation language, this means that when we flow towards the IR, the potential  $\mathcal{V}$  needs to cancel the determinant of the background metric and replace it by the determinant of the full metric in our ansatz for the effective action (3.3), so that

$$\lim_{k \rightarrow 0} \mathcal{V} = \Lambda_{\text{IR}} \sqrt{\frac{\det g}{\det \bar{g}}}, \quad (5.2)$$

with  $\Lambda_{\text{IR}}$  the observed macroscopic value of the cosmological constant. This amounts to a fine-tuning problem in the UV [113], and can give rise to severe constraints on the allowed trajectories in theory space, so that the number of free parameters can be less than the number of relevant directions of the fixed point. It is even feasible that potentially viable fixed points have to be discarded if they don't admit a proper diffeomorphism invariant IR regime. For the Asymptotic Safety conjecture to work, it is necessary that diffeomorphism invariance can be combined with staying in the critical hypersurface of the fixed point. In this way, even a fixed point with a priori infinitely many relevant directions could be physically viable, if the constraint imposed by the Nielsen identity reduces the number of actually free parameters to a finite number.

## 6 Summary

The present work laid the foundation for the study of gravitational correlation functions of arbitrary order. The central ingredient is the CHT, which allows to rewrite many tensorial expressions in terms of scalar invariants and a small number of basis tensors. As a proof of principle, we derived and solved the flow equation for the graviton potential in

an approximation where we retained the full dependence on conformal fluctuations and first order perturbations by gravitational wave fluctuations. The results indicate strong quantum effects, emphasising the need of the present approach to reliably study the UV limit. Some hints are found that there might be a finite graviton vacuum expectation value, or even a non-metric UV phase, depending on the sign of the trace anomalous dimension. Further studies are however necessary to give a definite result. On the technical side, our approach gives a significant contribution towards the resolution of the split-Ward identity, and clarifies the relation between the bimetric and the fluctuation language.

Future studies should try to resolve the full  $\mathfrak{h}_2$ -dependence of the potential and the anomalous dimensions, together with a self-consistent flow equation for the Newton coupling. This allows for the self-consistent determination of critical exponents. This enhancement is technically very involved, as it needs the resolution of lots of tensor structures to calculate the flow equation. Nevertheless, we put forward the necessary ingredients to implement such a calculation.

Another interesting open point is the integration of a flow towards the IR, which is necessary for the resolution of the Nielsen identity, and might shed light on the question of graviton condensation. The latter might however need momentum dependent invariants, *e.g.*  $\text{tr}(\mathfrak{h}^{\text{TL}}\bar{\Delta}\mathfrak{h}^{\text{TL}})$ , which are related to the curvature of the full metric. For a related discussion of gravitational condensates in the Regge-Wheeler lattice formulation of quantum gravity, see [181].

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## A Cayley-Hamilton theorem

Most technical results of this and the following appendices are part of the author’s Ph.D. thesis [182]. Due to the central importance of the Cayley-Hamilton theorem in this work, we shall state it here and collect some explicit formulas for the relevant case of  $4 \times 4$ -matrices. The general theorem can be stated as follows. Consider the characteristic polynomial  $p$  of a matrix  $\mathbb{A}$ ,

$$p(\lambda) = \det(\lambda\mathbb{1} - \mathbb{A}) . \tag{A.1}$$

The Cayley-Hamilton theorem now states that if one replaces  $\lambda$  by the matrix  $\mathbb{A}$  itself in this polynomial, one gets the zero matrix,

$$p(\mathbb{A}) = 0 . \tag{A.2}$$

Stated differently, a matrix is completely characterised by its eigenvalues, up to similarity transformations. Moreover, the theorem provides an explicit algorithm to convert powers of the matrix  $\mathbb{A}$  which are higher than its dimension to a linear combination of lower powers of  $\mathbb{A}$  and the unit matrix.

Let us now specify to  $4 \times 4$ -matrices. In that case,

$$\mathbb{A}^4 - (\text{tr } \mathbb{A})\mathbb{A}^3 + \frac{1}{2} [(\text{tr } \mathbb{A})^2 - \text{tr}(\mathbb{A}^2)] \mathbb{A}^2 - \frac{1}{6} [(\text{tr } \mathbb{A})^3 - 3(\text{tr } \mathbb{A}) \text{tr}(\mathbb{A}^2) + 2 \text{tr}(\mathbb{A}^3)] \mathbb{A} + \det \mathbb{A} \mathbb{1}_4 = 0. \quad (\text{A.3})$$

As shown in the main text, it is beneficial to divide matrices into traceless and trace parts. Thus, let  $\mathbb{A} = \mathbb{B} + \frac{1}{4} \text{tr}(\mathbb{A}) \mathbb{1}_4$ , where  $\mathbb{B}$  is the traceless part of  $\mathbb{A}$ . Then,

$$\mathbb{B}^4 - \frac{1}{2} \text{tr}(\mathbb{B}^2)\mathbb{B}^2 - \frac{1}{3} \text{tr}(\mathbb{B}^3)\mathbb{B} + \det \mathbb{B} \mathbb{1}_4 = 0. \quad (\text{A.4})$$

Any scalar function  $f$  of a constant matrix  $\mathbb{A}$  can thus be parameterised as  $f(\text{tr } \mathbb{A}, \text{tr}(\mathbb{B}^2), \text{tr}(\mathbb{B}^3), \det \mathbb{B})$ . This parameterisation is useful as it allows for controlled approximations, *e.g.*  $\det \mathbb{B} = \text{tr}(\mathbb{B}^3) = \text{tr}(\mathbb{B}^2) = 0$ . An analogous approximation with  $\text{tr}(\mathbb{A}^2) = 0$  would already entail  $\mathbb{A} \equiv 0$  for a real and symmetric  $\mathbb{A}$ .

## B Parameterisation and inverse metric

Let us now present the full formulas for the inverse and the determinant of the full metric in an arbitrary parameterisation. The coefficients  $\mathcal{B}_i$  of (2.7) read

$$\begin{aligned} \mathcal{B}_0 = \frac{1}{216\Delta} & [216\mathcal{A}_0^2 (\mathcal{A}_2\mathfrak{h}_2 + \mathcal{A}_3\mathfrak{h}_3) + 9\mathcal{A}_0 (-12\mathcal{A}_1^2\mathfrak{h}_2 - 12\mathcal{A}_1 (2\mathcal{A}_2\mathfrak{h}_3 + \mathcal{A}_3 (\mathfrak{h}_2^2 - 4\mathfrak{h}_4)) \\ & + 4\mathcal{A}_2\mathcal{A}_3\mathfrak{h}_2\mathfrak{h}_3 + 6\mathcal{A}_2^2 (\mathfrak{h}_2^2 + 4\mathfrak{h}_4) + \mathcal{A}_3^2 (-3\mathfrak{h}_2^3 + 24\mathfrak{h}_4\mathfrak{h}_2 + 8\mathfrak{h}_3^2)) \\ & + 216\mathcal{A}_2\mathcal{A}_3^2\mathfrak{h}_4^2 + 2\mathfrak{h}_3 (4\mathcal{A}_3^3\mathfrak{h}_3^2 + 9\mathcal{A}_1 ((\mathcal{A}_3\mathfrak{h}_2 + 2\mathcal{A}_1)^2 - 2\mathcal{A}_2^2\mathfrak{h}_2) \\ & - 6\mathcal{A}_2\mathfrak{h}_3 (\mathcal{A}_3 (\mathcal{A}_3\mathfrak{h}_2 + 6\mathcal{A}_1) - 2\mathcal{A}_2^2)) - 18\mathfrak{h}_4 (3\mathcal{A}_2 ((\mathcal{A}_3\mathfrak{h}_2 + 2\mathcal{A}_1)^2 - 2\mathcal{A}_2^2\mathfrak{h}_2) \\ & + 4\mathcal{A}_3\mathfrak{h}_3 (\mathcal{A}_2^2 - \mathcal{A}_3 (\mathcal{A}_3\mathfrak{h}_2 + 2\mathcal{A}_1))) + 216\mathcal{A}_0^3], \end{aligned} \quad (\text{B.1})$$

$$\begin{aligned} \mathcal{B}_1 = \frac{1}{72\Delta} & [ -72\mathcal{A}_3^3\mathfrak{h}_4^2 + \mathfrak{h}_2 (4\mathcal{A}_3^3\mathfrak{h}_3^2 + 9\mathcal{A}_1 ((\mathcal{A}_3\mathfrak{h}_2 + 2\mathcal{A}_1)^2 - 2\mathcal{A}_2^2\mathfrak{h}_2) \\ & - 6\mathcal{A}_2\mathfrak{h}_3 (\mathcal{A}_3 (\mathcal{A}_3\mathfrak{h}_2 + 6\mathcal{A}_1) - 2\mathcal{A}_2^2)) - 18\mathfrak{h}_4 (2\mathcal{A}_1 - \mathcal{A}_3\mathfrak{h}_2) \times \\ & (\mathcal{A}_3 (\mathcal{A}_3\mathfrak{h}_2 + 2\mathcal{A}_1) - 2\mathcal{A}_2^2) - 12\mathcal{A}_0 (-\mathfrak{h}_3 (\mathcal{A}_3^2\mathfrak{h}_2 + 2\mathcal{A}_2^2) \\ & + 2\mathcal{A}_1 (3\mathcal{A}_2\mathfrak{h}_2 + \mathcal{A}_3\mathfrak{h}_3) + 12\mathcal{A}_2\mathcal{A}_3\mathfrak{h}_4) - 72\mathcal{A}_0^2\mathcal{A}_1], \end{aligned} \quad (\text{B.2})$$

$$\begin{aligned} \mathcal{B}_2 = \frac{1}{216\Delta} & [ -18\mathcal{A}_0 (-3 (\mathcal{A}_3\mathfrak{h}_2 + 2\mathcal{A}_1)^2 + 6\mathcal{A}_2 (\mathcal{A}_2\mathfrak{h}_2 + 2\mathcal{A}_0) + 4\mathcal{A}_2\mathcal{A}_3\mathfrak{h}_3) \\ & - 36\mathfrak{h}_4 (-3\mathcal{A}_3\mathcal{A}_2 (\mathcal{A}_3\mathfrak{h}_2 + 4\mathcal{A}_1) + 2\mathcal{A}_3^2 (\mathcal{A}_3\mathfrak{h}_3 + 3\mathcal{A}_0) + 6\mathcal{A}_2^3)], \end{aligned} \quad (\text{B.3})$$

$$\begin{aligned} \mathcal{B}_3 = \frac{1}{36\Delta} & [ -4\mathcal{A}_3^3\mathfrak{h}_3^2 - 18\mathcal{A}_3\mathfrak{h}_4 (\mathcal{A}_3 (\mathcal{A}_3\mathfrak{h}_2 + 2\mathcal{A}_1) - 2\mathcal{A}_2^2) \\ & - 9 (4\mathcal{A}_3\mathcal{A}_1^2\mathfrak{h}_2 + \mathcal{A}_1 (\mathfrak{h}_2 (\mathcal{A}_3^2\mathfrak{h}_2 - 2\mathcal{A}_2^2) - 8\mathcal{A}_0\mathcal{A}_2) + 4\mathcal{A}_1^3 + 4\mathcal{A}_0^2\mathcal{A}_3) \\ & + 6\mathfrak{h}_3 (\mathcal{A}_3\mathcal{A}_2 (\mathcal{A}_3\mathfrak{h}_2 + 6\mathcal{A}_1) - 2\mathcal{A}_2^3 - 4\mathcal{A}_0\mathcal{A}_3^2)], \end{aligned} \quad (\text{B.4})$$

where we already used the ratio of determinants

$$\begin{aligned}
\Delta \equiv \frac{\det g}{\det \bar{g}} &= \mathcal{A}_0^3 (\mathcal{A}_2 \mathfrak{h}_2 + \mathcal{A}_3 \mathfrak{h}_3) + \frac{1}{24} \mathcal{A}_0^2 (-12 \mathcal{A}_1^2 \mathfrak{h}_2 - 12 \mathcal{A}_1 (2 \mathcal{A}_2 \mathfrak{h}_3 + \mathcal{A}_3 (\mathfrak{h}_2^2 - 8 \mathfrak{h}_4)) \\
&\quad + 4 \mathcal{A}_2 \mathcal{A}_3 \mathfrak{h}_2 \mathfrak{h}_3 + 6 \mathcal{A}_2^2 (\mathfrak{h}_2^2 + 8 \mathfrak{h}_4) + \mathcal{A}_3^2 (-3 \mathfrak{h}_2^3 + 36 \mathfrak{h}_4 \mathfrak{h}_2 + 8 \mathfrak{h}_3^2)) \\
&\quad + \frac{1}{108} \mathcal{A}_0 (432 \mathcal{A}_2 \mathcal{A}_3^2 \mathfrak{h}_4^2 - 18 \mathfrak{h}_4 (3 \mathcal{A}_2 (\mathcal{A}_3^2 \mathfrak{h}_2^2 - 2 (\mathcal{A}_2^2 - 2 \mathcal{A}_1 \mathcal{A}_3) \mathfrak{h}_2 + 8 \mathcal{A}_1^2) \\
&\quad + \mathcal{A}_3 \mathfrak{h}_3 (2 \mathcal{A}_2^2 - \mathcal{A}_3 (3 \mathcal{A}_3 \mathfrak{h}_2 + 10 \mathcal{A}_1))) \\
&\quad + \mathfrak{h}_3 (4 \mathcal{A}_3^3 \mathfrak{h}_3^2 + 9 \mathcal{A}_1 ((\mathcal{A}_3 \mathfrak{h}_2 + 2 \mathcal{A}_1)^2 - 2 \mathcal{A}_2^2 \mathfrak{h}_2) \\
&\quad - 6 \mathcal{A}_2 \mathfrak{h}_3 (\mathcal{A}_3 (\mathcal{A}_3 \mathfrak{h}_2 + 6 \mathcal{A}_1) - 2 \mathcal{A}_2^2))) \\
&\quad + \frac{1}{36} \mathfrak{h}_4 (36 \mathcal{A}_3^4 \mathfrak{h}_4^2 + \mathcal{A}_1 (4 \mathcal{A}_3^3 \mathfrak{h}_3^2 + 9 \mathcal{A}_1 ((\mathcal{A}_3 \mathfrak{h}_2 + 2 \mathcal{A}_1)^2 - 2 \mathcal{A}_2^2 \mathfrak{h}_2) \\
&\quad - 6 \mathcal{A}_2 \mathfrak{h}_3 (\mathcal{A}_3 (\mathcal{A}_3 \mathfrak{h}_2 + 6 \mathcal{A}_1) - 2 \mathcal{A}_2^2)) + 6 \mathfrak{h}_4 (-3 \mathcal{A}_3 \mathcal{A}_2^2 (\mathcal{A}_3 \mathfrak{h}_2 + 8 \mathcal{A}_1) \\
&\quad + 2 \mathcal{A}_3^3 \mathcal{A}_2 \mathfrak{h}_3 + 6 \mathcal{A}_1 \mathcal{A}_3^2 (\mathcal{A}_3 \mathfrak{h}_2 + 2 \mathcal{A}_1) + 6 \mathcal{A}_2^4)) + \mathcal{A}_0^4. \tag{B.5}
\end{aligned}$$

We shall also derive the exact representation of the exponential parameterisation. For this, we first derive a recursion for the  $n$ -th power of  $\mathfrak{h}^{\text{TL}}$  for  $n \geq 4$ . Making the ansatz

$$[\mathfrak{h}^{\text{TL}}]^n = a_n \mathbb{1}_4 + b_n \mathfrak{h}^{\text{TL}} + c_n [\mathfrak{h}^{\text{TL}}]^2 + d_n [\mathfrak{h}^{\text{TL}}]^3, \tag{B.6}$$

and with the initial conditions from the CHT,

$$a_4 = -\mathfrak{h}_4, \quad b_4 = \frac{1}{3} \mathfrak{h}_3, \quad c_4 = \frac{1}{2} \mathfrak{h}_2, \quad d_4 = 0, \tag{B.7}$$

we obtain the recursion

$$\begin{aligned}
a_{n+1} &= -\mathfrak{h}_4 d_n, \\
b_{n+1} &= a_n + \frac{1}{3} \mathfrak{h}_3 d_n, \\
c_{n+1} &= b_n + \frac{1}{2} \mathfrak{h}_2 d_n, \\
d_{n+1} &= c_n.
\end{aligned} \tag{B.8}$$

This recursion can be solved by Mathematica, and we shall not present the result for arbitrary  $n$ . Rather, focusing on the exponential parameterisation, one can transform this set of recursion relations to a set of differential equations in a fiducial variable  $x$ . For this, we introduce the functions

$$A(x) = \sum_{n=4}^{\infty} \frac{a_n}{n!} x^n, \quad B(x) = \sum_{n=4}^{\infty} \frac{b_n}{n!} x^n, \quad C(x) = \sum_{n=4}^{\infty} \frac{c_n}{n!} x^n, \quad D(x) = \sum_{n=4}^{\infty} \frac{d_n}{n!} x^n. \tag{B.9}$$

The metric in exponential parameterisation then reads

$$\begin{aligned}
g^{\text{exp}} &= \bar{g} e^{\mathfrak{h}_1/4} \times \\
&\quad \left[ \mathbb{1}_4 (1 + A(1)) + \mathfrak{h}^{\text{TL}} (1 + B(1)) + [\mathfrak{h}^{\text{TL}}]^2 \left( \frac{1}{2} + C(1) \right) + [\mathfrak{h}^{\text{TL}}]^3 \left( \frac{1}{6} + D(1) \right) \right]. \tag{B.10}
\end{aligned}$$



The functions  $A, B, C, D$  are the solutions to the set of ordinary differential equations obtained by multiplying the recursion relations with  $\frac{x^n}{n!}$  and summing over  $n$  from 4 to  $\infty$ . Doing so, one arrives at

$$\begin{aligned} A'(x) &= \mathfrak{h}_4 \left( \frac{x^3}{6} - D(x) \right), \\ B'(x) &= A(x) + \frac{1}{3} \mathfrak{h}_3 \left( \frac{x^3}{6} + D(x) \right), \\ C'(x) &= B(x) + \frac{1}{2} \mathfrak{h}_2 \left( \frac{x^3}{6} + D(x) \right), \\ D'(x) &= C(x). \end{aligned} \tag{B.11}$$

Initial conditions for the functions at  $x = 0$  follow from their definition. This set of differential equations can be solved by Mathematica. To present the result, we first introduce the polynomial

$$p(y) = 6\mathfrak{h}_4 - 2\mathfrak{h}_3 y - 3\mathfrak{h}_2 y^2 + 6y^4, \tag{B.12}$$

and define the operator  $\mathcal{RS}$  (for *RootSum*) which maps a function to the sum of the values of this function at the roots of  $p$ ,

$$\mathcal{RS}[f(y)] = \sum_{y_i: p(y_i)=0} f(y_i). \tag{B.13}$$

With a final abbreviation,

$$\rho_n = \mathcal{RS} \left[ \frac{e^y y^n}{-\mathfrak{h}_3 - 3\mathfrak{h}_2 y + 12y^3} \right], \tag{B.14}$$

we find for the functions at  $x = 1$ ,

$$\begin{aligned} A(1) = \mathcal{RS} \left[ \frac{1}{12y^4 (-\mathfrak{h}_3 + 12y^3 - 3\mathfrak{h}_2 y)} \right. & \left. (\mathfrak{h}_4 e^{-y} (y(y(y+3) + 6) - 6e^y + 6) \times \right. \\ & (2(2\mathfrak{h}_2^2 \rho_0 - 9\rho_3 y^3 + 9\mathfrak{h}_4 (\rho_2 + y(\rho_1 + \rho_0 y)) + 3\mathfrak{h}_3 (2\rho_3 + y(\rho_2 + \rho_1 y))) \\ & \left. \left. - 9\mathfrak{h}_2^2 \rho_1 y + 3\mathfrak{h}_2 (-6\mathfrak{h}_4 \rho_0 + 2\mathfrak{h}_3 (\rho_0 y - \rho_1) + 3y(2\rho_3 + y(\rho_2 + 2\rho_1 y)))) \right) \right], \end{aligned} \tag{B.15}$$

$$\begin{aligned} B(1) = \mathcal{RS} \left[ \frac{1}{12y^4 (-\mathfrak{h}_3 + 12y^3 - 3\mathfrak{h}_2 y)} \right. & \left. (e^{-y} (y(y(y+3) + 6) - 6e^y + 6) \times \right. \\ & (3\mathfrak{h}_2 (\mathfrak{h}_4 (\rho_0 (6y^3 - 2\mathfrak{h}_3) + 6(\rho_3 + y(\rho_2 + \rho_1 y))) - \mathfrak{h}_3 \rho_2 y^2) \\ & + 2(9\mathfrak{h}_4 (\mathfrak{h}_4 (\rho_1 + \rho_0 y) - y^2 (\rho_3 + \rho_2 y)) + 3\mathfrak{h}_3 (\mathfrak{h}_4 (\rho_2 + \rho_0 y^2) - \rho_3 y^3) \\ & \left. \left. - \mathfrak{h}_3^2 y (\rho_2 + \rho_1 y) - 9\mathfrak{h}_4 \mathfrak{h}_2^2 (\rho_1 + \rho_0 y))) \right) \right], \end{aligned} \tag{B.16}$$

$$\begin{aligned} C(1) = \mathcal{RS} \left[ -\frac{1}{24y^4 (-\mathfrak{h}_3 + 12y^3 - 3\mathfrak{h}_2 y)} \right. & \left. (e^{-y} (y(y(y+3) + 6) - 6e^y + 6) \times \right. \\ & (9\mathfrak{h}_2^2 \rho_2 y^2 + 4(3\mathfrak{h}_3 (y^2 (\rho_3 + \rho_2 y) - \mathfrak{h}_4 (\rho_1 + \rho_0 y)) + \mathfrak{h}_3^2 \rho_1 y \\ & + 9\mathfrak{h}_4 (y(\rho_3 + y(\rho_2 + \rho_1 y)) - \mathfrak{h}_4 \rho_0)) \\ & \left. \left. + 6\mathfrak{h}_2 (3\rho_3 y^3 - 3\mathfrak{h}_4 (\rho_2 + \rho_0 y^2 + 2\rho_1 y) + \mathfrak{h}_3 y (\rho_2 + \rho_1 y)))) \right) \right], \end{aligned} \tag{B.17}$$

$$D(1) = \mathcal{RS} \left[ -\frac{1}{4y^4 (-\mathfrak{h}_3 + 12y^3 - 3\mathfrak{h}_2y)} (e^{-y} (y(y(y+3) + 6) - 6e^y + 6) \times \right. \\ \left. (\mathfrak{h}_4 (\rho_0 (-4\mathfrak{h}_3 + 6y^3 - 6\mathfrak{h}_2y) + 6 (\rho_3 + \rho_1 (y^2 - \mathfrak{h}_2) + \rho_2y)) \right. \\ \left. + y (3\mathfrak{h}_2y (\rho_3 + \rho_2y) + 2\mathfrak{h}_3 (\rho_3 + y (\rho_2 + \rho_1y)))) \right]. \quad (\text{B.18})$$

For the special case where we neglect the invariants  $\mathfrak{h}_3$  and  $\mathfrak{h}_4$ , we find  $A = B = \mathcal{O}(\mathfrak{h}_3, \mathfrak{h}_4)$  and

$$C(1) = -\frac{4 + \mathfrak{h}_2 - 4 \cosh \sqrt{\frac{\mathfrak{h}_2}{2}}}{2\mathfrak{h}_2} + \mathcal{O}(\mathfrak{h}_3, \mathfrak{h}_4), \quad (\text{B.19})$$

$$D(1) = -\frac{1}{6} - \frac{2}{\mathfrak{h}_2} + \frac{2\sqrt{2} \sinh \sqrt{\frac{\mathfrak{h}_2}{2}}}{\mathfrak{h}_2^{3/2}} + \mathcal{O}(\mathfrak{h}_3, \mathfrak{h}_4). \quad (\text{B.20})$$

These expressions admit a Taylor expansion in  $\mathfrak{h}_2$  around zero and are thus regular also for  $\mathfrak{h}_2 \rightarrow 0$ .

## C Curvature identities

In this appendix, we present some useful formulas related to the curvature tensors. First, we rewrite the kinetic part of the Einstein-Hilbert action into a form of which the second derivative with respect to the fluctuation simplifies tremendously. The Ricci scalar can be expressed as

$$R = g^{\alpha\beta} \left[ \Gamma^\gamma_{\gamma\delta} \Gamma^\delta_{\alpha\beta} - \Gamma^\gamma_{\alpha\delta} \Gamma^\delta_{\gamma\beta} - \partial_\alpha \Gamma^\gamma_{\gamma\beta} + \partial_\gamma \Gamma^\gamma_{\alpha\beta} \right], \quad (\text{C.1})$$

where  $\Gamma$  is the Christoffel symbol of the metric  $g$ , and  $\partial$  denotes the standard partial derivative. Using basic identities from differential geometry, partial integration and dropping boundary terms, we can rewrite

$$\begin{aligned} & \int d^d x \sqrt{\det g} g^{\alpha\beta} [-\partial_\alpha \Gamma^\gamma_{\alpha\beta} + \partial_\gamma \Gamma^\gamma_{\alpha\beta}] \\ &= \int d^d x \left[ \Gamma^\gamma_{\gamma\beta} \partial_\alpha (\sqrt{\det g} g^{\alpha\beta}) - \Gamma^\gamma_{\alpha\beta} \partial_\gamma (\sqrt{\det g} g^{\alpha\beta}) \right] \\ &= \int d^d x \left[ \Gamma^\gamma_{\gamma\beta} \left( -\sqrt{\det g} g^{\mu\nu} \Gamma^\beta_{\mu\nu} \right) - \Gamma^\gamma_{\alpha\beta} \sqrt{\det g} \left( \Gamma^\delta_{\gamma\delta} g^{\alpha\beta} - \left( \Gamma^\alpha_{\gamma\delta} g^{\delta\beta} + \Gamma^\beta_{\gamma\delta} g^{\alpha\delta} \right) \right) \right] \\ &= \int d^d x \sqrt{\det g} g^{\alpha\beta} \left[ -2\Gamma^\gamma_{\gamma\delta} \Gamma^\delta_{\alpha\beta} + 2\Gamma^\gamma_{\alpha\delta} \Gamma^\delta_{\gamma\beta} \right], \end{aligned} \quad (\text{C.2})$$

and thus

$$\int d^d x \sqrt{\det g} R = \int d^d x \sqrt{\det g} g^{\alpha\beta} \left[ -\Gamma^\gamma_{\gamma\delta} \Gamma^\delta_{\alpha\beta} + \Gamma^\gamma_{\alpha\delta} \Gamma^\delta_{\gamma\beta} \right]. \quad (\text{C.3})$$

The virtue of this rewriting is the following: for the flow equation, we need the second variation of the action with respect to the fluctuation. In the present work, we project the flow equation onto constant  $h$  and flat background  $\bar{g} = \delta$ . Since the Christoffel symbols are linear in derivatives, the only contribution to the second variation comes from the combination where all Christoffel symbols are varied. As a side remark, the same result

can be obtained if one treats the Christoffel symbols as (1, 2)-tensors and rewrites the partial derivatives as covariant derivatives plus the corresponding Christoffel symbols, finally dropping the terms with covariant derivatives.

For future reference, we also write formulas for the Ricci scalar, Ricci tensor and Riemann tensor in terms of the background curvatures and derivatives in a background-covariant way. A simple calculation shows that

$$\begin{aligned}
R_{\mu\nu} &= \bar{R}_{\mu\nu} + \frac{1}{2}g^{\alpha\beta} (2\bar{D}_\beta\bar{D}_{(\mu}g_{\nu)\alpha} - \bar{D}_\alpha\bar{D}_\beta g_{\mu\nu} - \bar{D}_\mu\bar{D}_\nu g_{\alpha\beta}) \\
&+ \frac{1}{4}g^{\alpha\beta}g^{\gamma\delta} \left[ (2\bar{D}_\beta g_{\gamma\delta}\bar{D}_{(\mu}g_{\nu)\alpha} + \bar{D}_\mu g_{\alpha\gamma}\bar{D}_\nu g_{\beta\delta} - \bar{D}_\alpha g_{\mu\nu}\bar{D}_\beta g_{\gamma\delta}) \right. \\
&\left. + 2(\bar{D}_\alpha g_{\mu\nu}\bar{D}_\delta g_{\beta\gamma} + 2\bar{D}_\gamma g_{\mu\alpha}\bar{D}_{[\delta}g_{\beta]\nu} - 2\bar{D}_\delta g_{\beta\gamma}\bar{D}_{(\mu}g_{\nu)\alpha}) \right], \tag{C.4}
\end{aligned}$$

from which we immediately get the Ricci scalar by a contraction with  $g^{\mu\nu}$ . For the Riemann tensor, we have

$$\begin{aligned}
R_{\mu\nu\rho\sigma} &= g_{\alpha[\mu}\bar{R}_{\nu]\rho\sigma} - \bar{D}_\rho\bar{D}_{[\mu}g_{\nu]\sigma} + \bar{D}_\sigma\bar{D}_{[\mu}g_{\nu]\rho} \\
&+ \frac{1}{2}g^{\alpha\beta} \left[ \bar{D}_\alpha g_{\mu[\sigma}\bar{D}_{|\beta|}g_{\rho]\nu} + \bar{D}_\alpha g_{\nu[\sigma}\bar{D}_{|\mu|}g_{\rho]\beta} + \bar{D}_{[\mu}g_{|\sigma\beta|}\bar{D}_{\nu]}g_{\rho\alpha} + \bar{D}_\alpha g_{\mu[\rho}\bar{D}_{|\nu|}g_{\sigma]\beta} \right. \\
&\left. + \bar{D}_\beta g_{\nu[\sigma}\bar{D}_{\rho]}g_{\mu\alpha} + \bar{D}_\mu g_{\beta[\sigma}\bar{D}_{\rho]}g_{\nu\alpha} + \bar{D}_\nu g_{\beta[\rho}\bar{D}_{\sigma]}g_{\mu\alpha} + \bar{D}_{[\rho}g_{|\nu\beta|}\bar{D}_{\sigma]}g_{\mu\alpha} + \bar{D}_\alpha g_{\mu[\rho}\bar{D}_{\sigma]}g_{\nu\beta} \right]. \tag{C.5}
\end{aligned}$$

## D Quantum gauge transformation

In this appendix we discuss some aspects of our choice of gauge fixing. In particular, we choose to gauge-fix the fluctuation field  $h$  instead of the full metric  $g$ , since then the vertices are not affected by the gauge fixing, which might yield results that are less sensitive to the specific gauge choice. For the corresponding ghost action, we have to derive how the “quantum gauge transformation” of  $h$  looks like in terms of the variation of  $g$  [22]. The latter is nothing else than a BRST transformation along the ghost field  $c$ ,

$$\delta_Q g_{\mu\nu} = \mathcal{L}_c g_{\mu\nu} = D_\mu c_\nu + D_\nu c_\mu, \tag{D.1}$$

$\mathcal{L}$  being the Lie derivative. We will show that this can be done practically in all generality, again with the help of the CHT, and also derive the explicit expression that we need in the main text.

To illustrate the general calculation, we start with (2.6), where we however do not use a traceless decomposition,

$$g = \bar{g} \left( \tilde{\mathcal{A}}_0 \mathbb{1}_4 + \tilde{\mathcal{A}}_1 \mathfrak{h} + \tilde{\mathcal{A}}_2 \mathfrak{h}^2 + \tilde{\mathcal{A}}_3 \mathfrak{h}^3 \right). \tag{D.2}$$

The  $\tilde{\mathcal{A}}_i$  are understood depend on the traces and the determinant of  $\mathfrak{h}$ , *i.e.*  $\tilde{\mathcal{A}}_i = \tilde{\mathcal{A}}_i(\text{tr } \mathfrak{h}, \text{tr } \mathfrak{h}^2, \text{tr } \mathfrak{h}^3, \det \mathfrak{h})$ . A quantum gauge transformation of this equation gives (remember that this means  $\delta_Q \bar{g} = 0$ )

$$\begin{aligned}
\delta_Q g &= \bar{g} \left( (\delta_Q \tilde{\mathcal{A}}_0) \mathbb{1}_4 + (\delta_Q \tilde{\mathcal{A}}_1) \mathfrak{h} + \tilde{\mathcal{A}}_1 \delta_Q \mathfrak{h} + (\delta_Q \tilde{\mathcal{A}}_2) \mathfrak{h}^2 + \tilde{\mathcal{A}}_2 [(\delta_Q \mathfrak{h}) \mathfrak{h} + \mathfrak{h} \delta_Q \mathfrak{h}] \right. \\
&\left. + (\delta_Q \tilde{\mathcal{A}}_3) \mathfrak{h}^3 + \tilde{\mathcal{A}}_3 [(\delta_Q \mathfrak{h}) \mathfrak{h}^2 + \mathfrak{h} (\delta_Q \mathfrak{h}) \mathfrak{h} + \mathfrak{h}^2 \delta_Q \mathfrak{h}] \right), \tag{D.3}
\end{aligned}$$

with

$$\begin{aligned}
\delta_Q \tilde{\mathcal{A}}_i &= \tilde{\mathcal{A}}_i^{(1,0,0,0)} \text{tr}(\delta_Q \mathfrak{h}) + 2\tilde{\mathcal{A}}_i^{(0,1,0,0)} \text{tr}((\delta_Q \mathfrak{h}) \mathfrak{h}) + 3\tilde{\mathcal{A}}_i^{(0,0,1,0)} \text{tr}((\delta_Q \mathfrak{h}) \mathfrak{h}^2) \\
&+ \frac{1}{24} \tilde{\mathcal{A}}_i^{(0,0,0,1)} \left[ 4(\text{tr } \mathfrak{h})^3 \text{tr}(\delta_Q \mathfrak{h}) - 12(\text{tr } \mathfrak{h}) \text{tr}(\mathfrak{h}^2) \text{tr}(\delta_Q \mathfrak{h}) + (\text{tr } \mathfrak{h})^2 \text{tr}(\mathfrak{h} \delta_Q \mathfrak{h}) \right. \\
&\quad \left. + 12 \text{tr}(\mathfrak{h}^2) \text{tr}(\mathfrak{h} \delta_Q \mathfrak{h}) + 8 \text{tr}(\mathfrak{h}^3) \text{tr}(\delta_Q \mathfrak{h}) + 24 \text{tr}(\mathfrak{h}) \text{tr}(\mathfrak{h}^2 \delta_Q \mathfrak{h}) - 24 \text{tr}(\mathfrak{h}^3 \delta_Q \mathfrak{h}) \right].
\end{aligned} \tag{D.4}$$

The superscripts indicate the number of derivatives w.r.t. the respective arguments, *i.e.*  $\tilde{\mathcal{A}}_i^{(1,0,0,0)} = \partial_{\text{tr } \mathfrak{h}} \tilde{\mathcal{A}}_i$  and so on. The task is to solve (D.3) for  $\delta_Q \mathfrak{h}$ . Clearly,  $\delta_Q \mathfrak{h}$  will be linear in  $\delta_Q g$ , and in general contains all possible products of  $\delta_Q g$  with  $\mathfrak{h}$ . Due to the CHT, there is only a finite number of independent products, and in four dimensions, this number is 32. By making an ansatz for  $\delta_Q \mathfrak{h}$  as linear combination of the elements of these products, inserting this ansatz into (D.3), and using the CHT, we can solve for the coefficients. We calculated the solution explicitly with the command *SolveConstants* of xTras [143], but the result is too bulky to be presented here. Recently, an order-by-order calculation has been discussed in [91].

It is obvious that in a similar fashion we can derive the relation between the variation of  $h$  and the variation of the background metric,  $\delta_B \bar{g}$ , with fixed variation of the full metric,  $\delta_B g = 0$ . This corresponds to a background gauge transformation, and plays a role in the construction of explicit solutions to the split-Ward identity, see *e.g.* [106].

Let us now discuss the exponential split in particular, and derive the quantum transformation of  $h$  to quadratic order in  $\mathfrak{h}^{\text{TL}}$ , including all information on the trace. For this, we start with the definition of the exponential split,

$$g_{\mu\nu} = \bar{g}_{\mu\rho} e^{\mathfrak{h}_1/4} \left[ e^{\mathfrak{h}^{\text{TL}}} \right]^\rho{}_\nu = \bar{g}_{\mu\rho} e^{\mathfrak{h}_1/4} \sum_{n=0}^{\infty} \frac{1}{n!} \left[ (\mathfrak{h}^{\text{TL}})^n \right]^\rho{}_\nu, \tag{D.5}$$

and take a quantum variation with fixed background metric,

$$\begin{aligned}
\delta_Q g_{\mu\nu} &= \bar{g}_{\mu\rho} e^{\mathfrak{h}_1/4} \left[ \frac{1}{4} \delta_Q \mathfrak{h}_1 \sum_{n=0}^{\infty} \frac{1}{n!} \left[ (\mathfrak{h}^{\text{TL}})^n \right]^\rho{}_\nu + \sum_{n=0}^{\infty} \frac{1}{n!} \delta_Q \left[ (\mathfrak{h}^{\text{TL}})^n \right]^\rho{}_\nu \right] \\
&= \bar{g}_{\mu\rho} e^{\mathfrak{h}_1/4} \left[ \frac{1}{4} \delta_Q \mathfrak{h}_1 \sum_{n=0}^{\infty} \frac{1}{n!} \left[ (\mathfrak{h}^{\text{TL}})^n \right]^\rho{}_\nu + \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{l=0}^{n-1} \left[ (\mathfrak{h}^{\text{TL}})^l (\delta_Q \mathfrak{h}^{\text{TL}}) (\mathfrak{h}^{\text{TL}})^{n-l-1} \right]^\rho{}_\nu \right].
\end{aligned} \tag{D.6}$$

Now we use that, by definition,  $\delta_Q \mathfrak{h}^{\text{TL}} = \delta_Q \mathfrak{h} - \frac{1}{4} \mathbb{1}_4 \delta_Q \mathfrak{h}_1$ , and we see that the part including  $\delta_Q \mathfrak{h}_1$  exactly cancels, leaving us with

$$\delta_Q g_{\mu\nu} = \bar{g}_{\mu\rho} e^{\mathfrak{h}_1/4} \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{l=0}^{n-1} \left[ (\mathfrak{h}^{\text{TL}})^l (\delta_Q \mathfrak{h}) (\mathfrak{h}^{\text{TL}})^{n-l-1} \right]^\rho{}_\nu. \tag{D.7}$$

We can further reorganise the sums to yield the final expression

$$\delta_Q g_{\mu\nu} = \bar{g}_{\mu\rho} e^{\mathfrak{h}_1/4} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \frac{1}{(l+m+1)!} \left[ (\mathfrak{h}^{\text{TL}})^l (\delta_Q \mathfrak{h}) (\mathfrak{h}^{\text{TL}})^m \right]^\rho{}_\nu. \tag{D.8}$$

For the truncation in the main text, we need  $\delta_Q h$  including up to second order in  $h^{\text{TL}}$ . It is easy to show that the solution is

$$\begin{aligned}
e^{h_1/4} \delta_Q h_{\mu\nu} &= \delta_Q g_{\mu\nu} - \frac{1}{2} \left( h_{\mu\alpha}^{\text{TL}} \bar{g}^{\alpha\beta} \delta_Q g_{\beta\nu} + (\delta_Q g_{\mu\alpha}) \bar{g}^{\alpha\beta} h_{\beta\nu}^{\text{TL}} \right) \\
&+ \frac{1}{12} \left( h_{\mu\alpha}^{\text{TL}} \bar{g}^{\alpha\beta} h_{\beta\gamma}^{\text{TL}} \bar{g}^{\gamma\sigma} \delta_Q g_{\sigma\nu} + 4 h_{\mu\alpha}^{\text{TL}} \bar{g}^{\alpha\beta} (\delta_Q g_{\beta\gamma}) \bar{g}^{\gamma\sigma} h_{\sigma\nu}^{\text{TL}} + (\delta_Q g_{\mu\alpha}) \bar{g}^{\alpha\beta} h_{\beta\gamma}^{\text{TL}} \bar{g}^{\gamma\sigma} h_{\sigma\nu}^{\text{TL}} \right) \\
&+ \mathcal{O}(h^{\text{TL}3}).
\end{aligned} \tag{D.9}$$

## E Loop momentum integration

In this appendix, we discuss some technical aspects in how to treat the integrals over loop momenta. Most of the discussion will be done in arbitrary dimension  $d$  and in flat space. For the discussion, let us introduce  $\mathcal{P} = p_\mu T^\mu{}_\nu p^\nu$ , where  $T$  is an arbitrary tensor and  $p$  is the loop momentum. The first step is to calculate expressions of the type

$$\int d^d p g(p) \mathcal{P}^n, \tag{E.1}$$

for general functions  $g(p)$  which shall only depend on the absolute value of  $p$ . Inserting the definition of  $\mathcal{P}$ , we need to calculate

$$\int d^d p g(p) p_{\mu_1} \cdots p_{\mu_{2n}}. \tag{E.2}$$

By Lorentz invariance, the tensor structure (in a flat space) is given by the symmetrised product of  $n$  metrics,

$$\int d^d p g(p) p_{\mu_1} \cdots p_{\mu_{2n}} = \alpha_n \eta_{(\mu_1 \mu_2} \cdots \eta_{\mu_{2n-1} \mu_{2n})} \int d^d p g(p) p^{2n}. \tag{E.3}$$

To calculate the constants  $\alpha_n$ , we multiply this equation by another product of  $n$  metrics; the result is

$$\alpha_n = \frac{(2n-1)!!(d-2)!!}{(d+2(n-1))!!}. \tag{E.4}$$

Now, we have to carry out the contraction of the metrics with the product of  $T$ s. It is straightforward to show that

$$T^{\mu_1 \mu_2} \cdots T^{\mu_{2n-1} \mu_{2n}} \eta_{(\mu_1 \mu_2} \cdots \eta_{\mu_{2n-1} \mu_{2n})} = \frac{2^n}{(2n-1)!!} B_n \left( \frac{0!}{2} \tau_1, \frac{1!}{2} \tau_2, \dots, \frac{(n-1)!}{2} \tau_n \right). \tag{E.5}$$

Here, we introduced the traces  $\tau_i = \text{tr}(T^i)$ , and  $B_n$  stands for the  $n$ -th complete Bell polynomial. The complete Bell polynomials are defined by

$$B_n(x_1, \dots, x_n) = \sum_{k=1}^n \sum_{j_1 \dots j_{n-k+1}} \frac{n!}{j_1! \cdots j_{n-k+1}!} \left( \frac{x_1}{1!} \right)^{j_1} \cdots \left( \frac{x_{n-k+1}}{(n-k+1)!} \right)^{j_{n-k+1}}, \tag{E.6}$$

where the inner sum extends over all non-negative integers  $j_l$  subject to the two conditions

$$\sum_{l=1}^{n-k+1} j_l = k, \quad \sum_{l=1}^{n-k+1} l j_l = n. \quad (\text{E.7})$$

Combining this, one gets

$$\int d^d p g(p) (p_\mu p_\nu T^{\mu\nu})^n = \frac{2^n (d-2)!!}{(d+2(n-1))!!} B_n \int d^d p g(p) p^{2n} = \frac{1}{\left(\frac{d}{2}\right)_n} B_n \int d^d p g(p) p^{2n}. \quad (\text{E.8})$$

We suppressed the arguments of the Bell polynomial, and used the Pochhammer symbol  $(x)_n = \Gamma(x+n)/\Gamma(x)$ . In fact, the Bell polynomials with the arguments as above can be evaluated explicitly by use of the exponential generating function,

$$\exp\left(\sum_{n=1}^{\infty} \frac{a_n}{n!} x^n\right) = \sum_{n=0}^{\infty} \frac{B_n(a_1, \dots, a_n)}{n!} x^n. \quad (\text{E.9})$$

Inserting the arguments, we get on the left-hand side

$$\begin{aligned} \exp\left(\sum_{n=1}^{\infty} \frac{a_n}{n!} x^n\right) &= \exp\left(\frac{1}{2} \text{tr} \sum_{n=1}^{\infty} \frac{1}{n} (xT)^n\right) \\ &= \exp\left(-\frac{1}{2} \text{tr} \ln(\mathbb{1} - xT)\right) \\ &= [\det(\mathbb{1} - xT)]^{-1/2}. \end{aligned} \quad (\text{E.10})$$

We conclude that the complete Bell polynomials can be obtained by the Taylor expansion coefficients of this expression in  $x$  around zero, and thus finally

$$T^{\mu_1 \mu_2} \dots T^{\mu_{2n-1} \mu_{2n}} \eta_{(\mu_1 \mu_2} \dots \eta_{\mu_{2n-1} \mu_{2n})} = \frac{2^n (d-2)!!}{(d+2(n-1))!!} \partial_x^n \frac{1}{\sqrt{\det(\mathbb{1} - xT)}} \Big|_{x=0}. \quad (\text{E.11})$$

Clearly, we can assume that  $T$  is traceless - if it is not, we introduce a traceless decomposition in the beginning, then only the traceless part will give rise to a nontrivial angular dependence. In four dimensions, we get thus by use of the CHT,

$$\det(\mathbb{1} - xT) = 1 - \frac{1}{2} \tau_2 x^2 - \frac{1}{3} \tau_3 x^3 + (\det T) x^4. \quad (\text{E.12})$$

If the determinant of  $T$  is neglected, then the coefficients of the Taylor expansion can be calculated explicitly. A lengthy calculation yields

$$\begin{aligned} \frac{1}{\sqrt{1 - \frac{\tau_2}{2} x^2 - \frac{\tau_3}{3} x^3}} &= \sum_{n=0}^{\infty} \frac{\left(\frac{\tau_2}{2}\right)^{n/2} \mathbf{i}^n}{\left(\frac{n-3\mu}{2}\right)!} \pi \left(\frac{\sqrt{2}\tau_3}{3\mathbf{i}\tau_2^{3/2}}\right)^\mu x^n \times \\ &\quad {}_3F_2^{\text{reg}}\left(\frac{2+\mu-n}{6}, \frac{4+\mu-n}{6}, \frac{7\mu-n}{6}; \mu + \frac{1}{2}, \frac{1+\mu-n}{2} \middle| \frac{6\tau_3^2}{\tau_2^3}\right), \end{aligned} \quad (\text{E.13})$$

with  $\mu = n \bmod 2$ ,  ${}_3F_2^{\text{reg}}$  is the regularised generalised hypergeometric function and  $\mathbf{i}^2 = -1$ .

In fact, one can generalise (E.11) to the symmetric contraction of  $n$  different tensors:

$$T_1^{\mu_1\mu_2} \dots T_n^{\mu_{2n-1}\mu_{2n}} \eta_{(\mu_1\mu_2} \dots \eta_{\mu_{2n-1}\mu_{2n})} = \frac{2^n(d-2)!!}{(d+2(n-1))!!} \partial_{x_1} \dots \partial_{x_n} \frac{1}{\sqrt{\det\left(\mathbb{1} - \sum_{i=1}^n x_i T_i\right)}} \Big|_{x_i=0}. \quad (\text{E.14})$$

This can be proven as follows. Define

$$\mathcal{T} = \sum_{i=1}^n x_i T_i, \quad (\text{E.15})$$

then by definition

$$T_1^{\mu_1\mu_2} \dots T_n^{\mu_{2n-1}\mu_{2n}} \eta_{(\mu_1\mu_2} \dots \eta_{\mu_{2n-1}\mu_{2n})} = \frac{1}{n!} \partial_{x_1} \dots \partial_{x_n} \mathcal{T}^{\mu_1\mu_2} \dots \mathcal{T}^{\mu_{2n-1}\mu_{2n}} \eta_{(\mu_1\mu_2} \dots \eta_{\mu_{2n-1}\mu_{2n})} \Big|_{x_i=0}. \quad (\text{E.16})$$

For the right-hand side, we can insert (E.11) and obtain

$$T_1^{\mu_1\mu_2} \dots T_n^{\mu_{2n-1}\mu_{2n}} \eta_{(\mu_1\mu_2} \dots \eta_{\mu_{2n-1}\mu_{2n})} = \frac{1}{n!} \frac{2^n(d-2)!!}{(d+2(n-1))!!} \partial_{x_1} \dots \partial_{x_n} \partial_y^n \frac{1}{\sqrt{\det(\mathbb{1} - y\mathcal{T})}} \Big|_{x_i=y=0}. \quad (\text{E.17})$$

We can now commute the derivatives freely. Realising that

$$\frac{1}{\sqrt{\det(\mathbb{1} - y\mathcal{T})}} = f(yx_1, \dots, yx_n), \quad (\text{E.18})$$

we find

$$\partial_{x_1} \dots \partial_{x_n} \partial_y^n f(yx_1, \dots, yx_n) \Big|_{x_i=y=0} = \partial_y^n y^n f^{(1, \dots, 1)}(0, \dots, 0) \Big|_{y=0} = n! f^{(1, \dots, 1)}(0, \dots, 0), \quad (\text{E.19})$$

and thus follows (E.14), as claimed.

## References

- [1] S. Weinberg, *Ultraviolet divergences in quantum theories of gravitation*, *General Relativity: An Einstein centenary survey*, Eds. Hawking, S.W., Israel, W; Cambridge University Press (1979) 790–831.
- [2] C. Wetterich, *Exact evolution equation for the effective potential*, *Phys.Lett.* **B301** (1993) 90–94.

- [3] T. R. Morris, *The Exact renormalization group and approximate solutions*, *Int. J. Mod. Phys. A* **9** (1994) 2411–2450, [[hep-ph/9308265](#)].
- [4] M. Reuter, *Nonperturbative evolution equation for quantum gravity*, *Phys.Rev.* **D57** (1998) 971–985, [[hep-th/9605030](#)].
- [5] S. Falkenberg and S. D. Odintsov, *Gauge dependence of the effective average action in Einstein gravity*, *Int. J. Mod. Phys. A* **13** (1998) 607–623, [[hep-th/9612019](#)].
- [6] W. Souma, *Nontrivial ultraviolet fixed point in quantum gravity*, *Prog. Theor. Phys.* **102** (1999) 181–195, [[hep-th/9907027](#)].
- [7] O. Lauscher and M. Reuter, *Ultraviolet fixed point and generalized flow equation of quantum gravity*, *Phys.Rev.* **D65** (2002) 025013, [[hep-th/0108040](#)].
- [8] O. Lauscher and M. Reuter, *Is quantum Einstein gravity nonperturbatively renormalizable?*, *Class. Quant. Grav.* **19** (2002) 483–492, [[hep-th/0110021](#)].
- [9] M. Reuter and F. Saueressig, *Renormalization group flow of quantum gravity in the Einstein-Hilbert truncation*, *Phys. Rev.* **D65** (2002) 065016, [[hep-th/0110054](#)].
- [10] D. F. Litim, *Fixed points of quantum gravity*, *Phys. Rev. Lett.* **92** (2004) 201301, [[hep-th/0312114](#)].
- [11] O. Lauscher and M. Reuter, *Fractal spacetime structure in asymptotically safe gravity*, *JHEP* **10** (2005) 050, [[hep-th/0508202](#)].
- [12] M. Reuter and J.-M. Schwindt, *A Minimal length from the cutoff modes in asymptotically safe quantum gravity*, *JHEP* **01** (2006) 070, [[hep-th/0511021](#)].
- [13] M. Niedermaier and M. Reuter, *The Asymptotic Safety Scenario in Quantum Gravity*, *Living Rev.Rel.* **9** (2006) 5–173.
- [14] K. Groh and F. Saueressig, *Ghost wave-function renormalization in Asymptotically Safe Quantum Gravity*, *J. Phys.* **A43** (2010) 365403, [[1001.5032](#)].
- [15] D. Benedetti, K. Groh, P. F. Machado and F. Saueressig, *The Universal RG Machine*, *JHEP* **1106** (2011) 079, [[1012.3081](#)].
- [16] E. Manrique, S. Rechenberger and F. Saueressig, *Asymptotically Safe Lorentzian Gravity*, *Phys. Rev. Lett.* **106** (2011) 251302, [[1102.5012](#)].
- [17] M. Reuter and F. Saueressig, *Quantum Einstein Gravity*, *New J.Phys.* **14** (2012) 055022, [[1202.2274](#)].
- [18] U. Harst and M. Reuter, *The 'Tetrad only' theory space: Nonperturbative renormalization flow and Asymptotic Safety*, *JHEP* **05** (2012) 005, [[1203.2158](#)].
- [19] D. Litim and A. Satz, *Limit cycles and quantum gravity*, [1205.4218](#).
- [20] A. Nink and M. Reuter, *On quantum gravity, Asymptotic Safety, and paramagnetic dominance*, *Int.J.Mod.Phys.* **D22** (2013) 138–157, [[1212.4325](#)].
- [21] S. Rechenberger and F. Saueressig, *A functional renormalization group equation for foliated spacetimes*, *JHEP* **03** (2013) 010, [[1212.5114](#)].
- [22] A. Nink, *Field Parametrization Dependence in Asymptotically Safe Quantum Gravity*, *Phys.Rev.* **D91** (2015) 044030, [[1410.7816](#)].
- [23] H. Gies, B. Knorr and S. Lippoldt, *Generalized Parametrization Dependence in Quantum Gravity*, *Phys. Rev.* **D92** (2015) 084020, [[1507.08859](#)].



- [24] K. Falls, *On the renormalisation of Newton's constant*, *Phys. Rev.* **D92** (2015) 124057, [[1501.05331](#)].
- [25] K. Falls, *Critical scaling in quantum gravity from the renormalisation group*, [1503.06233](#).
- [26] O. Lauscher and M. Reuter, *Flow equation of quantum Einstein gravity in a higher derivative truncation*, *Phys. Rev.* **D66** (2002) 025026, [[hep-th/0205062](#)].
- [27] A. Codello and R. Percacci, *Fixed points of higher derivative gravity*, *Phys. Rev. Lett.* **97** (2006) 221301, [[hep-th/0607128](#)].
- [28] D. Benedetti, P. F. Machado and F. Saueressig, *Asymptotic safety in higher-derivative gravity*, *Mod. Phys. Lett.* **A24** (2009) 2233–2241, [[0901.2984](#)].
- [29] K. Groh, S. Rechenberger, F. Saueressig and O. Zanusso, *Higher Derivative Gravity from the Universal Renormalization Group Machine*, *PoS EPS-HEP2011* (2011) 124, [[1111.1743](#)].
- [30] S. Rechenberger and F. Saueressig, *The  $R^2$  phase-diagram of QEG and its spectral dimension*, *Phys. Rev.* **D86** (2012) 024018, [[1206.0657](#)].
- [31] N. Ohta and R. Percacci, *Higher Derivative Gravity and Asymptotic Safety in Diverse Dimensions*, *Class. Quant. Grav.* **31** (2014) 015024, [[1308.3398](#)].
- [32] P. F. Machado and F. Saueressig, *On the renormalization group flow of  $f(R)$ -gravity*, *Phys.Rev.* **D77** (2008) 124045, [[0712.0445](#)].
- [33] A. Codello, R. Percacci and C. Rahmede, *Ultraviolet properties of  $f(R)$ -gravity*, *Int.J.Mod.Phys.* **A23** (2008) 143–150, [[0705.1769](#)].
- [34] A. Bonanno, A. Contillo and R. Percacci, *Inflationary solutions in asymptotically safe  $f(R)$  theories*, *Class. Quant. Grav.* **28** (2011) 145026, [[1006.0192](#)].
- [35] M. Demmel, F. Saueressig and O. Zanusso, *Fixed-Functionals of three-dimensional Quantum Einstein Gravity*, *JHEP* **11** (2012) 131, [[1208.2038](#)].
- [36] J. A. Dietz and T. R. Morris, *Asymptotic safety in the  $f(R)$  approximation*, *JHEP* **1301** (2013) 108, [[1211.0955](#)].
- [37] K. Falls, D. Litim, K. Nikolakopoulos and C. Rahmede, *A bootstrap towards asymptotic safety*, [1301.4191](#).
- [38] J. A. Dietz and T. R. Morris, *Redundant operators in the exact renormalisation group and in the  $f(R)$  approximation to asymptotic safety*, *JHEP* **07** (2013) 064, [[1306.1223](#)].
- [39] K. Falls, D. F. Litim, K. Nikolakopoulos and C. Rahmede, *Further evidence for asymptotic safety of quantum gravity*, *Phys. Rev.* **D93** (2016) 104022, [[1410.4815](#)].
- [40] M. Demmel, F. Saueressig and O. Zanusso, *RG flows of Quantum Einstein Gravity in the linear-geometric approximation*, *Annals Phys.* **359** (2015) 141–165, [[1412.7207](#)].
- [41] M. Demmel, F. Saueressig and O. Zanusso, *RG flows of Quantum Einstein Gravity on maximally symmetric spaces*, *JHEP* **06** (2014) 026, [[1401.5495](#)].
- [42] A. Eichhorn, *The Renormalization Group flow of unimodular  $f(R)$  gravity*, *JHEP* **1504** (2015) 096, [[1501.05848](#)].
- [43] M. Demmel, F. Saueressig and O. Zanusso, *A proper fixed functional for four-dimensional Quantum Einstein Gravity*, *JHEP* **08** (2015) 113, [[1504.07656](#)].

- [44] N. Ohta, R. Percacci and G. P. Vacca, *Flow equation for  $f(R)$  gravity and some of its exact solutions*, *Phys. Rev.* **D92** (2015) 061501, [[1507.00968](#)].
- [45] N. Ohta and R. Percacci, *Ultraviolet Fixed Points in Conformal Gravity and General Quadratic Theories*, *Class. Quant. Grav.* **33** (2016) 035001, [[1506.05526](#)].
- [46] N. Ohta, R. Percacci and G. P. Vacca, *Renormalization Group Equation and scaling solutions for  $f(R)$  gravity in exponential parametrization*, *Eur. Phys. J.* **C76** (2016) 46, [[1511.09393](#)].
- [47] K. Falls, D. F. Litim, K. Nikolakopoulos and C. Rahmede, *On de Sitter solutions in asymptotically safe  $f(R)$  theories*, [1607.04962](#).
- [48] K. Falls and N. Ohta, *Renormalization Group Equation for  $f(R)$  gravity on hyperbolic spaces*, *Phys. Rev.* **D94** (2016) 084005, [[1607.08460](#)].
- [49] T. R. Morris, *Large curvature and background scale independence in single-metric approximations to asymptotic safety*, *JHEP* **11** (2016) 160, [[1610.03081](#)].
- [50] N. Christiansen, *Four-Derivative Quantum Gravity Beyond Perturbation Theory*, [1612.06223](#).
- [51] S. Gonzalez-Martin, T. R. Morris and Z. H. Slade, *Asymptotic solutions in asymptotic safety*, *Phys. Rev.* **D95** (2017) 106010, [[1704.08873](#)].
- [52] Y. Hamada and M. Yamada, *Asymptotic safety of higher derivative quantum gravity non-minimally coupled with a matter system*, *JHEP* **08** (2017) 070, [[1703.09033](#)].
- [53] S. Nagy, B. Fazekas, Z. Peli, K. Sailer and I. Steib, *Regulator dependence of fixed points in quantum Einstein gravity with  $R^2$  truncation*, [1707.04934](#).
- [54] H. Gies, B. Knorr, S. Lippoldt and F. Saueressig, *Gravitational Two-Loop Counterterm Is Asymptotically Safe*, *Phys. Rev. Lett.* **116** (2016) 211302, [[1601.01800](#)].
- [55] A. Nink and M. Reuter, *The unitary conformal field theory behind 2D Asymptotic Safety*, *JHEP* **02** (2016) 167, [[1512.06805](#)].
- [56] D. Becker, C. Ripken and F. Saueressig, *On avoiding Ostrogradski instabilities within Asymptotic Safety*, [1709.09098](#).
- [57] J. Biemans, A. Platania and F. Saueressig, *Quantum gravity on foliated spacetimes: Asymptotically safe and sound*, *Phys. Rev.* **D95** (2017) 086013, [[1609.04813](#)].
- [58] J. Biemans, A. Platania and F. Saueressig, *Renormalization group fixed points of foliated gravity-matter systems*, *JHEP* **05** (2017) 093, [[1702.06539](#)].
- [59] W. B. Houthoff, A. Kurov and F. Saueressig, *Impact of topology in foliated Quantum Einstein Gravity*, *Eur. Phys. J.* **C77** (2017) 491, [[1705.01848](#)].
- [60] R. Percacci and D. Perini, *Constraints on matter from asymptotic safety*, *Phys. Rev.* **D67** (2003) 081503, [[hep-th/0207033](#)].
- [61] R. Percacci and D. Perini, *Asymptotic safety of gravity coupled to matter*, *Phys. Rev.* **D68** (2003) 044018, [[hep-th/0304222](#)].
- [62] O. Zanusso, L. Zambelli, G. P. Vacca and R. Percacci, *Gravitational corrections to Yukawa systems*, *Phys. Lett.* **B689** (2010) 90–94, [[0904.0938](#)].
- [63] J.-E. Daum, U. Harst and M. Reuter, *Running Gauge Coupling in Asymptotically Safe Quantum Gravity*, *JHEP* **01** (2010) 084, [[0910.4938](#)].

- [64] G. Narain and R. Percacci, *Renormalization Group Flow in Scalar-Tensor Theories. I*, *Class.Quant.Grav.* **27** (2010) 075001, [[0911.0386](#)].
- [65] E. Manrique, M. Reuter and F. Saueressig, *Matter Induced Bimetric Actions for Gravity*, *Annals Phys.* **326** (2011) 440–462, [[1003.5129](#)].
- [66] G. P. Vacca and O. Zanusso, *Asymptotic Safety in Einstein Gravity and Scalar-Fermion Matter*, *Phys. Rev. Lett.* **105** (2010) 231601, [[1009.1735](#)].
- [67] U. Harst and M. Reuter, *QED coupled to QEG*, *JHEP* **1105** (2011) 119, [[1101.6007](#)].
- [68] A. Eichhorn and H. Gies, *Light fermions in quantum gravity*, *New J.Phys.* **13** (2011) 125012, [[1104.5366](#)].
- [69] S. Folkerts, D. F. Litim and J. M. Pawłowski, *Asymptotic freedom of Yang-Mills theory with gravity*, *Phys.Lett.* **B709** (2012) 234–241, [[1101.5552](#)].
- [70] P. Dona and R. Percacci, *Functional renormalization with fermions and tetrads*, *Phys. Rev.* **D87** (2013) 045002, [[1209.3649](#)].
- [71] B. Dobrich and A. Eichhorn, *Can we see quantum gravity? Photons in the asymptotic-safety scenario*, *JHEP* **06** (2012) 156, [[1203.6366](#)].
- [72] A. Eichhorn, *Quantum-gravity-induced matter self-interactions in the asymptotic-safety scenario*, *Phys. Rev.* **D86** (2012) 105021, [[1204.0965](#)].
- [73] P. Donà, A. Eichhorn and R. Percacci, *Matter matters in asymptotically safe quantum gravity*, *Phys.Rev.* **D89** (2014) 084035, [[1311.2898](#)].
- [74] T. Henz, J. M. Pawłowski, A. Rodigast and C. Wetterich, *Dilaton Quantum Gravity*, *Phys.Lett.* **B727** (2013) 298–302, [[1304.7743](#)].
- [75] A. Eichhorn and M. M. Scherer, *Planck scale, Higgs mass, and scalar dark matter*, *Phys. Rev.* **D90** (2014) 025023, [[1404.5962](#)].
- [76] R. Percacci and G. P. Vacca, *Search of scaling solutions in scalar-tensor gravity*, *Eur. Phys. J.* **C75** (2015) 188, [[1501.00888](#)].
- [77] J. Borchardt and B. Knorr, *Global solutions of functional fixed point equations via pseudospectral methods*, *Phys. Rev.* **D91** (2015) 105011, [[1502.07511](#)].
- [78] P. Donà, A. Eichhorn, P. Labus and R. Percacci, *Asymptotic safety in an interacting system of gravity and scalar matter*, *Phys. Rev.* **D93** (2016) 044049, [[1512.01589](#)].
- [79] P. Labus, R. Percacci and G. P. Vacca, *Asymptotic safety in  $O(N)$  scalar models coupled to gravity*, *Phys. Lett.* **B753** (2016) 274–281, [[1505.05393](#)].
- [80] J. Meibohm, J. M. Pawłowski and M. Reichert, *Asymptotic safety of gravity-matter systems*, *Phys. Rev.* **D93** (2016) 084035, [[1510.07018](#)].
- [81] A. Eichhorn, A. Held and J. M. Pawłowski, *Quantum-gravity effects on a Higgs-Yukawa model*, *Phys. Rev.* **D94** (2016) 104027, [[1604.02041](#)].
- [82] J. Meibohm and J. M. Pawłowski, *Chiral fermions in asymptotically safe quantum gravity*, *Eur. Phys. J.* **C76** (2016) 285, [[1601.04597](#)].
- [83] A. Eichhorn and S. Lippoldt, *Quantum gravity and Standard-Model-like fermions*, *Phys. Lett.* **B767** (2017) 142–146, [[1611.05878](#)].
- [84] T. Henz, J. M. Pawłowski and C. Wetterich, *Scaling solutions for Dilaton Quantum Gravity*, *Phys. Lett.* **B769** (2017) 105–110, [[1605.01858](#)].

- [85] N. Christiansen and A. Eichhorn, *An asymptotically safe solution to the  $U(1)$  triviality problem*, *Phys. Lett.* **B770** (2017) 154–160, [[1702.07724](#)].
- [86] N. Christiansen, A. Eichhorn and A. Held, *Is scale-invariance in gauge-Yukawa systems compatible with the graviton?*, *Phys. Rev.* **D96** (2017) 084021, [[1705.01858](#)].
- [87] A. Eichhorn and A. Held, *Viability of quantum-gravity induced ultraviolet completions for matter*, [1705.02342](#).
- [88] A. Eichhorn and A. Held, *Top mass from asymptotic safety*, [1707.01107](#).
- [89] C. Wetterich, *Graviton fluctuations erase the cosmological constant*, *Phys. Lett.* **B773** (2017) 6–19, [[1704.08040](#)].
- [90] A. Eichhorn and F. Versteegen, *Upper bound on the Abelian gauge coupling from asymptotic safety*, [1709.07252](#).
- [91] A. Eichhorn, S. Lippoldt and V. Skrinjar, *Nonminimal hints for asymptotic safety*, [1710.03005](#).
- [92] N. Christiansen, D. F. Litim, J. M. Pawłowski and M. Reichert, *One force to rule them all: asymptotic safety of gravity with matter*, [1710.04669](#).
- [93] D. F. Litim and J. M. Pawłowski, *Renormalization group flows for gauge theories in axial gauges*, *JHEP* **09** (2002) 049, [[hep-th/0203005](#)].
- [94] I. H. Bridle, J. A. Dietz and T. R. Morris, *The local potential approximation in the background field formalism*, *JHEP* **1403** (2014) 093, [[1312.2846](#)].
- [95] J. M. Pawłowski, *Geometrical effective action and Wilsonian flows*, [hep-th/0310018](#).
- [96] J. M. Pawłowski, *Aspects of the functional renormalisation group*, *Annals Phys.* **322** (2007) 2831–2915, [[hep-th/0512261](#)].
- [97] E. Manrique and M. Reuter, *Bimetric Truncations for Quantum Einstein Gravity and Asymptotic Safety*, *Annals Phys.* **325** (2010) 785–815, [[0907.2617](#)].
- [98] I. Donkin and J. M. Pawłowski, *The phase diagram of quantum gravity from diffeomorphism-invariant RG-flows*, [1203.4207](#).
- [99] J. A. Dietz and T. R. Morris, *Background independent exact renormalization group for conformally reduced gravity*, *JHEP* **04** (2015) 118, [[1502.07396](#)].
- [100] M. Safari, *Splitting Ward identity*, *Eur. Phys. J.* **C76** (2016) 201, [[1508.06244](#)].
- [101] P. Labus, T. R. Morris and Z. H. Slade, *Background independence in a background dependent renormalization group*, *Phys. Rev.* **D94** (2016) 024007, [[1603.04772](#)].
- [102] T. R. Morris and A. W. H. Preston, *Manifestly diffeomorphism invariant classical Exact Renormalization Group*, *JHEP* **06** (2016) 012, [[1602.08993](#)].
- [103] M. Safari and G. P. Vacca, *Covariant and single-field effective action with the background-field formalism*, *Phys. Rev.* **D96** (2017) 085001, [[1607.03053](#)].
- [104] M. Safari and G. P. Vacca, *Covariant and background independent functional RG flow for the effective average action*, *JHEP* **11** (2016) 139, [[1607.07074](#)].
- [105] C. Wetterich, *Gauge invariant flow equation*, [1607.02989](#).
- [106] R. Percacci and G. P. Vacca, *The background scale Ward identity in quantum gravity*, *Eur. Phys. J.* **C77** (2017) 52, [[1611.07005](#)].

- [107] N. Ohta, *Background Scale Independence in Quantum Gravity*, *PTEP* **2017** (2017) 033E02, [[1701.01506](#)].
- [108] C. M. Nieto, R. Percacci and V. Skrinjar, *Split Weyl transformations in quantum gravity*, [1708.09760](#).
- [109] N. Christiansen, D. F. Litim, J. M. Pawłowski and A. Rodigast, *Fixed points and infrared completion of quantum gravity*, *Phys.Lett.* **B728** (2014) 114–117, [[1209.4038](#)].
- [110] A. Codello, G. D’Odorico and C. Pagani, *Consistent closure of renormalization group flow equations in quantum gravity*, *Phys.Rev.* **D89** (2014) 081701, [[1304.4777](#)].
- [111] N. Christiansen, B. Knorr, J. M. Pawłowski and A. Rodigast, *Global Flows in Quantum Gravity*, *Phys. Rev.* **D93** (2016) 044036, [[1403.1232](#)].
- [112] N. Christiansen, B. Knorr, J. Meibohm, J. M. Pawłowski and M. Reichert, *Local Quantum Gravity*, *Phys. Rev.* **D92** (2015) 121501, [[1506.07016](#)].
- [113] T. Denz, J. M. Pawłowski and M. Reichert, *Towards apparent convergence in asymptotically safe quantum gravity*, [1612.07315](#).
- [114] B. Knorr and S. Lippoldt, *Correlation functions on a curved background*, *Phys. Rev.* **D96** (2017) 065020, [[1707.01397](#)].
- [115] E. Manrique, M. Reuter and F. Saueressig, *Bimetric Renormalization Group Flows in Quantum Einstein Gravity*, *Annals Phys.* **326** (2011) 463–485, [[1006.0099](#)].
- [116] D. Becker and M. Reuter, *En route to Background Independence: Broken split-symmetry, and how to restore it with bi-metric average actions*, *Annals Phys.* **350** (2014) 225–301, [[1404.4537](#)].
- [117] D. Becker and M. Reuter, *Propagating gravitons vs. ‘dark matter’ in asymptotically safe quantum gravity*, *JHEP* **12** (2014) 025, [[1407.5848](#)].
- [118] C. Wetterich, *Gauge symmetry from decoupling*, *Nucl. Phys.* **B915** (2017) 135–167, [[1608.01515](#)].
- [119] M. Demmel and A. Nink, *Connections and geodesics in the space of metrics*, *Phys. Rev.* **D92** (2015) 104013, [[1506.03809](#)].
- [120] N. Ohta, R. Percacci and A. D. Pereira, *Gauges and functional measures in quantum gravity I: Einstein theory*, *JHEP* **06** (2016) 115, [[1605.00454](#)].
- [121] H. Kawai, Y. Kitazawa and M. Ninomiya, *Quantum gravity in (2+epsilon)-dimensions*, *Prog. Theor. Phys. Suppl.* **114** (1993) 149–174.
- [122] H. Kawai, Y. Kitazawa and M. Ninomiya, *Scaling exponents in quantum gravity near two-dimensions*, *Nucl. Phys.* **B393** (1993) 280–300, [[hep-th/9206081](#)].
- [123] H. Kawai, Y. Kitazawa and M. Ninomiya, *Ultraviolet stable fixed point and scaling relations in (2+epsilon)-dimensional quantum gravity*, *Nucl. Phys.* **B404** (1993) 684–716, [[hep-th/9303123](#)].
- [124] H. Kawai, Y. Kitazawa and M. Ninomiya, *Renormalizability of quantum gravity near two-dimensions*, *Nucl. Phys.* **B467** (1996) 313–331, [[hep-th/9511217](#)].
- [125] T. Aida, Y. Kitazawa, H. Kawai and M. Ninomiya, *Conformal invariance and renormalization group in quantum gravity near two-dimensions*, *Nucl. Phys.* **B427** (1994) 158–180, [[hep-th/9404171](#)].

- [126] E. Manrique and M. Reuter, *Bare Action and Regularized Functional Integral of Asymptotically Safe Quantum Gravity*, *Phys. Rev.* **D79** (2009) 025008, [[0811.3888](#)].
- [127] E. Manrique and M. Reuter, *Bare versus Effective Fixed Point Action in Asymptotic Safety: The Reconstruction Problem*, *PoS CLAQG08* (2011) 001, [[0905.4220](#)].
- [128] T. R. Morris and Z. H. Slade, *Solutions to the reconstruction problem in asymptotic safety*, *JHEP* **11** (2015) 094, [[1507.08657](#)].
- [129] M. Reuter, *Effective average actions and nonperturbative evolution equations*, [hep-th/9602012](#).
- [130] R. Percacci, *Asymptotic Safety*, [0709.3851](#).
- [131] S. Nagy, *Lectures on renormalization and asymptotic safety*, *Annals Phys.* **350** (2014) 310–346, [[1211.4151](#)].
- [132] M. Reuter and H. Weyer, *Background Independence and Asymptotic Safety in Conformally Reduced Gravity*, *Phys. Rev.* **D79** (2009) 105005, [[0801.3287](#)].
- [133] M. Reuter and H. Weyer, *Conformal sector of Quantum Einstein Gravity in the local potential approximation: Non-Gaussian fixed point and a phase of unbroken diffeomorphism invariance*, *Phys. Rev.* **D80** (2009) 025001, [[0804.1475](#)].
- [134] P. F. Machado and R. Percacci, *Conformally reduced quantum gravity revisited*, *Phys. Rev.* **D80** (2009) 024020, [[0904.2510](#)].
- [135] A. Bonanno and F. Guarneri, *Universality and Symmetry Breaking in Conformally Reduced Quantum Gravity*, *Phys. Rev.* **D86** (2012) 105027, [[1206.6531](#)].
- [136] J. A. Dietz, T. R. Morris and Z. H. Slade, *Fixed point structure of the conformal factor field in quantum gravity*, *Phys. Rev.* **D94** (2016) 124014, [[1605.07636](#)].
- [137] D. F. Litim and J. M. Pawłowski, *Flow equations for Yang-Mills theories in general axial gauges*, *Phys.Lett.* **B435** (1998) 181–188, [[hep-th/9802064](#)].
- [138] “xAct: Efficient tensor computer algebra for Mathematica.” <http://xact.es/index.html>.
- [139] D. Brizuela, J. M. Martín-García and G. A. Mena Marugan, *xPert: Computer algebra for metric perturbation theory*, *Gen. Rel. Grav.* **41** (2009) 2415–2431, [[0807.0824](#)].
- [140] J. M. Martín-García, *xPerm: fast index canonicalization for tensor computer algebra*, *Computer Physics Communications* **179** (Oct., 2008) 597–603, [[0803.0862](#)].
- [141] J. M. Martín-García, R. Portugal and L. R. U. Manssur, *The Invar tensor package*, *Computer Physics Communications* **177** (Oct., 2007) 640–648, [[0704.1756](#)].
- [142] J. M. Martín-García, D. Yllanes and R. Portugal, *The Invar tensor package: Differential invariants of Riemann*, *Computer Physics Communications* **179** (Oct., 2008) 586–590, [[0802.1274](#)].
- [143] T. Nutma, *xTras: A field-theory inspired xAct package for mathematica*, *Computer Physics Communications* **185** (June, 2014) 1719–1738, [[1308.3493](#)].
- [144] D. F. Litim, *Optimized renormalization group flows*, *Phys.Rev.* **D64** (2001) 105007, [[hep-th/0103195](#)].
- [145] D. F. Litim, *Critical exponents from optimized renormalization group flows*, *Nucl.Phys.* **B631** (2002) 128–158, [[hep-th/0203006](#)].

- [146] J. Borchardt and B. Knorr, *Solving functional flow equations with pseudo-spectral methods*, *Phys. Rev.* **D94** (2016) 025027, [[1603.06726](#)].
- [147] M. Heilmann, T. Hellwig, B. Knorr, M. Ansorg and A. Wipf, *Convergence of Derivative Expansion in Supersymmetric Functional RG Flows*, *JHEP* **1502** (2015) 109, [[1409.5650](#)].
- [148] J. Borchardt, H. Gies and R. Sondenheimer, *Global flow of the Higgs potential in a Yukawa model*, *Eur. Phys. J.* **C76** (2016) 472, [[1603.05861](#)].
- [149] J. Borchardt and A. Eichhorn, *Universal behavior of coupled order parameters below three dimensions*, *Phys. Rev.* **E94** (2016) 042105, [[1606.07449](#)].
- [150] B. Knorr, *Ising and Gross-Neveu model in next-to-leading order*, *Phys. Rev.* **B94** (2016) 245102, [[1609.03824](#)].
- [151] B. Knorr, *Critical (Chiral) Heisenberg Model with the Functional Renormalisation Group*, [1708.06200](#).
- [152] J. Ambjorn and R. Loll, *Nonperturbative Lorentzian quantum gravity, causality and topology change*, *Nucl. Phys.* **B536** (1998) 407–434, [[hep-th/9805108](#)].
- [153] J. Ambjorn, J. Jurkiewicz and R. Loll, *Spectral dimension of the universe*, *Phys. Rev. Lett.* **95** (2005) 171301, [[hep-th/0505113](#)].
- [154] J. Ambjorn, J. Jurkiewicz and R. Loll, *Quantum Gravity, or The Art of Building Spacetime*, [hep-th/0604212](#).
- [155] D. Benedetti and J. Henson, *Spectral geometry as a probe of quantum spacetime*, *Phys. Rev.* **D80** (2009) 124036, [[0911.0401](#)].
- [156] C. Anderson, S. J. Carlip, J. H. Cooperman, P. Horava, R. K. Kommu and P. R. Zulkowski, *Quantizing Horava-Lifshitz Gravity via Causal Dynamical Triangulations*, *Phys. Rev.* **D85** (2012) 044027, [[1111.6634](#)].
- [157] J. Ambjorn, S. Jordan, J. Jurkiewicz and R. Loll, *A Second-order phase transition in CDT*, *Phys. Rev. Lett.* **107** (2011) 211303, [[1108.3932](#)].
- [158] J. Laiho and D. Coumbe, *Evidence for Asymptotic Safety from Lattice Quantum Gravity*, *Phys. Rev. Lett.* **107** (2011) 161301, [[1104.5505](#)].
- [159] J. Ambjorn, A. Goerlich, J. Jurkiewicz and R. Loll, *Nonperturbative Quantum Gravity*, *Phys. Rept.* **519** (2012) 127–210, [[1203.3591](#)].
- [160] S. Jordan and R. Loll, *Causal Dynamical Triangulations without Preferred Foliation*, *Phys. Lett.* **B724** (2013) 155–159, [[1305.4582](#)].
- [161] J. Ambjørn, A. Görlich, J. Jurkiewicz and R. Loll, *Quantum Gravity via Causal Dynamical Triangulations*, in *Springer Handbook of Spacetime* (A. Ashtekar and V. Petkov, eds.), pp. 723–741. 2014. [1302.2173](#). DOI.
- [162] J. Ambjorn, A. Görlich, J. Jurkiewicz and R. Loll, *Causal dynamical triangulations and the search for a theory of quantum gravity*, *Int. J. Mod. Phys.* **D22** (2013) 1330019.
- [163] J. Ambjorn, A. Görlich, J. Jurkiewicz, A. Kreienbuehl and R. Loll, *Renormalization Group Flow in CDT*, *Class. Quant. Grav.* **31** (2014) 165003, [[1405.4585](#)].
- [164] J. H. Cooperman, *Making the case for causal dynamical triangulations*, [1410.0670](#).
- [165] J. Ambjørn, J. Gizbert-Studnicki, A. Görlich, J. Jurkiewicz, N. Klitgaard and R. Loll, *Characteristics of the new phase in CDT*, *Eur. Phys. J.* **C77** (2017) 152, [[1610.05245](#)].

- [166] L. Glaser, T. P. Sotiriou and S. Weinfurtner, *Extrinsic curvature in two-dimensional causal dynamical triangulation*, *Phys. Rev.* **D94** (2016) 064014, [[1605.09618](#)].
- [167] J. Laiho, S. Bassler, D. Coumbe, D. Du and J. T. Neelakanta, *Lattice Quantum Gravity and Asymptotic Safety*, *Phys. Rev.* **D96** (2017) 064015, [[1604.02745](#)].
- [168] J. Ambjorn, D. Coumbe, J. Gizbert-Studnicki, A. Gorlich and J. Jurkiewicz, *New higher-order transition in causal dynamical triangulations*, *Phys. Rev.* **D95** (2017) 124029, [[1704.04373](#)].
- [169] J. Ambjørn, J. Gizbert-Studnicki, A. Görlich, K. Grosvenor and J. Jurkiewicz, *Four-dimensional CDT with toroidal topology*, *Nucl. Phys.* **B922** (2017) 226–246, [[1705.07653](#)].
- [170] L. Glaser and R. Loll, *CDT and Cosmology*, *Comptes Rendus Physique* **18** (2017) 265–274, [[1703.08160](#)].
- [171] L. Bombelli, J. Lee, D. Meyer and R. Sorkin, *Space-Time as a Causal Set*, *Phys. Rev. Lett.* **59** (1987) 521–524.
- [172] R. D. Sorkin, *Space-time and causal sets*, in *Proceedings, 7th Latin American International Symposium on General Relativity and Gravitation: Cocoyoc, Mexico, December 2-7, 1990*, 1990.
- [173] R. D. Sorkin, *Causal sets: Discrete gravity*, in *Lectures on quantum gravity. Proceedings, School of Quantum Gravity, Valdivia, Chile, January 4-14, 2002*, pp. 305–327, 2003, [[gr-qc/0309009](#), DOI].
- [174] S. Surya, *Evidence for a Phase Transition in 2D Causal Set Quantum Gravity*, *Class. Quant. Grav.* **29** (2012) 132001, [[1110.6244](#)].
- [175] S. Surya, *Directions in Causal Set Quantum Gravity*, [1103.6272](#).
- [176] L. Glaser and S. Surya, *Towards a Definition of Locality in a Manifoldlike Causal Set*, *Phys. Rev.* **D88** (2013) 124026, [[1309.3403](#)].
- [177] L. Glaser and S. Surya, *The Hartle–Hawking wave function in 2D causal set quantum gravity*, *Class. Quant. Grav.* **33** (2016) 065003, [[1410.8775](#)].
- [178] A. Eichhorn, S. Mizera and S. Surya, *Echoes of Asymptotic Silence in Causal Set Quantum Gravity*, *Class. Quant. Grav.* **34** (2017) 16LT01, [[1703.08454](#)].
- [179] L. Glaser, D. O’Connor and S. Surya, *Finite Size Scaling in 2d Causal Set Quantum Gravity*, [1706.06432](#).
- [180] A. Eichhorn, *Towards coarse graining of discrete Lorentzian quantum gravity*, [1709.10419](#).
- [181] H. W. Hamber, *Vacuum Condensate Picture of Quantum Gravity*, [1707.08188](#).
- [182] B. Knorr, *Asymptotic safety in QFT: from quantum gravity to graphene*, Ph.D. thesis, Jena, Oct, 2017.