“MY LITTLE ARITHMETICIANS!”
Pedagogic Ideals in Dutch Mathematics Textbooks
1790 – 1850

D.J. Beckers

Report No. 0001 (January 2000)
“My little Arithmeticians!”
Pedagogic Ideals in Dutch Mathematics Textbooks
1790 – 1850

D.J. Beckers

Report No. 0001 (January 2000)
“My little arithmeticians!”

Pedagogic ideals in Dutch mathematics textbooks, 1790-1850.

Danny Beckers

Abstract

During the first half of the 19th century mathematics textbooks for lower and middle class education were often used to confront the pupil with middle class values. Attitude towards study, general knowledge and morals made their appearance in mathematics textbooks. This form of mathematics education gradually disappeared by the middle of the century. Three hypothesis will be formulated which could explain the disappearance of the phenomenon.

The quotation above is taken from a Dutch arithmetic textbook, published in 1804. It is followed by the question how long both cities had been buried under the ashes of the volcano. During the first half of the 19th century many textbooks destined for lower (primary) and middle class (primary and secondary) mathematics education contained elements which clearly not only served a didactical purpose. In the 1840s and '50s they gradually disappeared.

In this article the appearance and disappearance of this phenomenon will be discussed against its pedagogical and didactical background. To begin, the general trends in mathematics education and the Dutch educational background will be briefly sketched.

*D.J. Beckers, Department of Mathematics, Katholieke Universiteit Nijmegen, Postbus 9010, 6500 GL Nijmegen, The Netherlands.

1 L. Oling, Rekenkundige Voorstellingen I (1804), p. 28. Free translation: “On August 24 of the year 79 A.D., mount Vesuvius erupted, and the unfortunate cities Herculanum and Pompeji were covered by over 30 feet of ashes, stones, and lava. —and an accidental discovery of the location of the first in 1711 resulted in the excavation of a large part of Herculanum in 1738, and Pompeji in 1743.” Italics as in the original.
1 The Dutch educational system

Unlike most European countries, the educational system in the 18th-century Dutch republic was locally organized and mostly in the hands of private schoolteachers. A serious revision took place during the French occupation of the Netherlands (1795-1813). The 1806 Education Act decreed a centrally controlled system of lower class education which was to be effective until 1857. Teachers had to obtain a degree, which only state examinations could provide, and the curriculum for primary education was laid down. A revision of the educational system had been called for since 1784, when the middle class Society for the common good (usually called: 't Nut) was founded. This Society drew its support from a huge part of Dutch bourgeois circles, was taken seriously by the Government, and thereby became the founding father of modern Dutch primary education.

The ideals of 't Nut towards primary education were largely inspired by the will to enlighten the masses. The lower class people — according to the members of 't Nut — living in terrible, almost bestial circumstances, had to be educated to become good citizens. Good citizenship and enlightenment were more or less identified, knowledge would generate virtue, didactics and pedagogy were seen more or less as the same thing. Good citizenship was promoted by knowledge of the national history, with an emphasis on the glorious 17th century.

The most noteworthy changes that were stimulated by 't Nut — apart from the many good things that were established on other fields, such as public libraries and insurances — were the new didactics and the broader education. The new didactics presented itself in a classroom to all children (thus emphasising their equality), in improved methods for learning how to read and write. The broader education, taking place in a society convinced that knowledge would enhance virtue, broke with the tradition of mainly presenting Bible reading to the lower classes. The new curriculum also contained national history, geography, writing, and arithmetic.

The changes in Dutch education — although the legislation mainly restricted itself to public primary education and left the bourgeois private institutes unhampered — were not restricted to the education of the lower classes. The new educational ideals originated from middle class mentality, and in fact constituted a body of bourgeois values, paternalistically adapted to the lower classes: the call for broad and thorough education, which would yield good and intellectually well-equipped citizens, helped introducing arithmetic and ‘geometrical reasoning’ in lower class education. Also education in the schools for bourgeois children changed tremendously. These so-called changes in Dutch education — although the legislation mainly restricted itself to public primary education and left the bourgeois private institutes unhampered — were not restricted to the education of the lower classes. The new educational ideals originated from middle class mentality, and in fact constituted a body of bourgeois values, paternalistically adapted to the lower classes: the call for broad and thorough education, which would yield good and intellectually well-equipped citizens, helped introducing arithmetic and ‘geometrical reasoning’ in lower class education. Also education in the schools for bourgeois children changed tremendously. These so-called changes in Dutch education — although the legislation mainly restricted itself to public primary education and left the bourgeois private institutes unhampered — were not restricted to the education of the lower classes. The new educational ideals originated from middle class mentality, and in fact constituted a body of bourgeois values, paternalistically adapted to the lower classes: the call for broad and thorough education, which would yield good and intellectually well-equipped citizens, helped introducing arithmetic and ‘geometrical reasoning’ in lower class education. Also education in the schools for bourgeois children changed tremendously. These so-called
French schools were a more practical alternative to the Latin schools of the élite. The curricula of the French schools focussed on modern languages (often French) and other knowledge that was deemed useful in the children’s future careers, such as arithmetic. During the 18th century these institutions had become extremely popular in middle class circles \(^8\). Around 1800 broad education became an important goal in many French schools. Mathematics education changed accordingly, and all this we will find expressed in the mathematics textbooks of those days.

2 Mathematics, Didactics and pedagogics

During the 18th century mathematics was not a compulsory subject in middle class education. Surveyors, merchants and navigators of course all needed some mathematical knowledge in order to do their jobs and they paid an extra fee to learn the ‘art of mathematics’. This mathematics was presented in rote-learning courses, offering the student a load of examples with emphasis on those he would meet in his future career. The merchant’s son would do exercises about the price of a certain amount of goods, some unit price given. The surveyor in training was presented exercises about the calculation of the sidelengths of a triangle, given one of its sides and (two of) its angles \(^9\). Exercises of a different nature did pop up in the recreational vein \(^10\), but were not present in textbooks. The various applications of the rule of three were treated in different chapters in the most common arithmetic textbooks by W. Bartjens and A. van Lintz \(^11\), so that pupils who were not going into the jewelry business would not have to do exercises about the costs of various alloys of gold and silver.

It was not only a didactical choice that made mathematics education look like this. Well into the 18th century, mathematics (with the possible exception of Euclidean geometry) was viewed, much more than today, as an applied subject. The mathematical structures were a tool in, rather than an object of study. In particular in the Netherlands, where since 1600 mathematicians at the universities presented engineering courses \(^12\), this view of mathematics was very common. During the 18th century this view was gradually changing. Enlightenment philosophers, seeking a perfect example of sound reasoning, found a pretty good start in 18th century mathematics. By trying to purify it from all uncertainties (knowledge based on pictures or experience)
the enlightenment mathematicians actually created a new subject: around 1800 the
notion of pure mathematics arose. In pure mathematics the mathematical structures
themselves became interesting and useful, because they were absolutely certain, and
could be applied to a variety of cases. During the 18th century several of these cases
had been presented as separé rules. But most of all, pure mathematics was useful
because it was assumed that it taught a man how to reason13. In France the new,
‘pure’ mathematics found its way into engineering education very quickly, and was
most clearly visible in the curriculum of the École Polytechnique11.

Also in Dutch middle class circles pure mathematics was soon held in high esteem.
Since the last decades of the 18th century the Dutch bourgeoisie stimulated the new
form of mathematics by founding so-called ‘mathematical institutes’ for the educa-
tion of their children15. But these people were more practical-minded than the French
géomètres. On the one hand, they gladly accepted the new mathematics as a perfect
example of sound reasoning, and were convinced that serious mathematics educa-
tion would generate better human beings. On the other hand, they also wanted the
practical results visible in the studies16: they held pure mathematics in high esteem
also because it had so many useful applications in their all-day practice. This aspect
of the appreciation the bourgeois had for mathematics shows in Dutch mathematics
textbooks. To stress the importance of the subject, the pupil in his mathematics
textbook was presented a wide variety of applications—for merchant, surveyor and
jeweller; no longer just one of them.

Opposition arose against the introduction of (pure) mathematics at the Latin
schools, either because the new subject was regarded as unsuitable for the children
of the elite or because it was believed to corrupt the good taste —mathematics,
after all, was a working class tool, that possibly made people unsuitable to enjoy
the beauty of the Latin and Greek languages17. People from the lower classes did
not really care for their children learning arithmet18. At the Military Academy
the new form of mathematics education led to a row between the director and the
professor of mathematics. The first held a more old-fashioned view of mathematics
than the latter, who stressed the importance of pure mathematics which had to be
learned before any application could be discussed. But although the mathematician
lost his job over it, by 1830 the curriculum was organized more or less according to his
whishes19. Mathematics became a prerequisite to the important engineering positions

14Bruno Belhoste, Amy Dahan-Dalmedico et Antoine Picon (dir.), La formation polytechnicienne,
1794-1994 (Paris 1994), 9 43
15D.J. Beckers, ‘Het is al Mathesis dat de klok slaat’ in: De Negentiende Eeuw 22, pp. 220 234
16H.J. Smid, Een onbekoekte nieuwheid, Delft (1997). pp. 64 70 noticed that little discussion
existed about what the role of mathematics should be (formative vs. practical prerequisite) at the
French schools.
17Cf. the quarrel between De Gelder and rector Bosse at the Leiden Latin School. D.J. Beckers,
‘Mathematics as a way of life—a biography of the mathematician J. de Gelder’ in: Nieuw Archief
voor Wiskunde (IV) 15 (1996), pp. 275 297
18It was one of the problems teachers were prepared for during their teacher training and which
was discussed regularly in teacher magazines, which suggests that teachers were confronted with this
problem quite frequently. See for example: Bijdragen 1815, p. 141
financed by the Dutch Government. This stimulated the acceptance of mathematics as a school discipline. In all cases, lower, middle class, and elite education the irrefutable character of mathematics, its alleged mind forming capacities, banishing all prejudice and supporting sound reasoning, was brought up to support the need for pure mathematics in the curricula.

Although the formative aspect of (pure) mathematics was brought up time and time again in discussion, so was its wide— if not universal— range of useful applications. Before this had not been an issue: mathematics was by definition an applied subject to the 18th-century mathematicians. In 19th-century mathematics teaching this was no longer the case. But application remained an important goal in mathematics instruction, as the teacher P.J. Baudet (1778-1858) made his pupils realise by presenting them the following dialogue in his 1824 geometry textbook:

DE LEERLINGEN. Wij zijn vol verlangen, Meester, om die schoone toepassingen te zien, welke Meester ons beloofd heeft.

MEESTER. Ik verblijd mij, waarde Jongelingen! dat gij in diergelijke werkzaamheden belangfelt: gij verdient daardoor de achting van alle weldenkende menfchen.21

What changed in the 1790s was not only that pupils started doing exercises on alloys, whether they were destined to become jewellers or not. First of all they also were confronted with the pure mathematical background of the problems they were supposed to solve. This, of course, had its effect on the didactical approach. Second, new kind of exercises appeared. Doing mathematics exercises one can do plain exercises and word exercises. Plain exercises (for example: solve $x$ from $2x - 1 = 5$) hardly occurred in mathematics textbooks before 1790: once children knew how to calculate they were taught the recipes which solved relevant practical problems—by definition these were word problems. The 19th century witnessed the emergence of more plain exercises in mathematics textbooks—these belonged to the pure parts of mathematics. But at the same time word problems appeared in a greater variety!


21 P.J. Baudet, Meetkundig Schoolboek, Deventer (1824), p. 63; free translation: “The pupils. We are very eager, teacher, to learn all these beautiful applications you promised us. Teacher, I am glad, my dear pupils, that you show so much interest in these matters; for this you deserve the esteem of all intelligent people.” —italics and smallcaps as in the original. In the arithmetic textbook by Aeneae the pupil is—in between theoretical passages—even confronted with abbreviations that were customary in merchant’s practice. According to the author this was good, since the pupil also had to know the way he was expected to do his reckoning later.

22See the textbooks by Bartjens and Van Lintz, but also: Abraham de Graaf, De geheele Mathesis ofte Wiskonst, Amsterdam (1717); J.H. Knoop, Werkdudge Meetkond, Den Haag (1757); and many others. Notable exceptions are some of the books in the recreational vein, and books destined for university education. But these served a completely different purpose.

In fact, completely new characteristics appeared in the mathematics textbooks, that were not in any way at all connected to the pupil's future discipline. I have divided these new characteristics into three types, which will be treated separately, although, of course, some passages might belong to more than one category.

2.1 Attitude to study

One of the most eye-catching changes in the mathematical textbooks is that since the 1790s attention is paid to the attitude of the pupil to his studies. Eighteenth-century textbooks restricted themselves purely to exercises and a short description of the recipes needed to solve them; if any attention was paid to the way the pupil should study, then it was presented in a few rhymes that encouraged the pupil to do his best. In nineteenth-century textbooks, however, the pupil was told in some detail how he should feel about the subjects he was taught — the quotation from Baudet's textbook above is a nice example — and even how he should study.

The new ideas on education and mathematics resulted in a rejection of the rote-learning didactics of the past centuries, and wanted to endow the pupils with real and clear notions of the sciences, before applying them in real life. For mathematics the idea was quite obvious: the pupil had to learn pure mathematics, understand his reckoning, and be able to give, or at least understand, proofs. That was the only way a child could become a truly civilized, reasonable human being.

In the rather popular arithmetic textbook by 't Nut, written by H. Aeneae (1743-1810) and published 1791-1794, the student was confronted with the dialogues between a student and his teacher. The use of dialogue in teaching is as old as Plato — the novelty here was the style of the dialogue which pressed the reader to study in a certain way. The student in Aeneae's dialogues was an ideal student, who asked the right kind of questions, was always very attentive, and asked for more examples, so that he could practice on his own. A reviewer in a middle class magazine was not impressed by the new didactical approach: he was afraid the student would learn too much of the mathematical structures that underlay his actions, and would not become a quick calculator. When in 1810, however, a list was made of schoolbooks that were considered to be major assets to any schoolteacher, Aeneae's book was among them; indeed, it would become one of the most highly valued teacher's guides during the early 19th century.

Many other textbooks explicitly striving for understanding — as opposed to rote learning — appeared, most notably the arithmetic textbook by Jan Brunt (1777-1803), which knew many editions, and the textbooks by the famous

---

24 For example: Hermans Willemsz., *Arithmetica ofte Reken-Kunst*, Enchuyzen (1751)
26 Vaderslandsche Bibliotheek 1795-I, pp. 82–87
27 It appeared in the *Algemeene lijst* in the government magazine *Bijdragen* (1810) where it was recommended for the use of teachers, and it was still valued for that reason in the preface of P.J. Baudet, *Rekenboek voor de scholen* dl. 1, Deventer (1826). Baudet's opinion was subscribed by a reviewer of his textbook in *Bijdragen* 1827, pp. 367–372, and also by the publishers of the *Tijdschrift voor aankomend onderwijsers*, who in the first issue of their magazine presented a catalogue of a suitable teacher's library, where the textbook by Aeneae may be found accompanied by the textbooks by De Gelder, Baudet, Strabbe and Lacroix; see this magazine 1 nr. 1 (1836).
Haarlem teacher trainer P.J. Prinsen (1777-1854); both were reprinted and used until the 1830s.

All these textbooks abandoned the idea of confronting the student with only those kind of exercises he would need in his future career: the child had to have an ample supply of exercises which should encourage him to think. That was achieved by a gradual increase of the level of difficulty, but also by presenting many different phrasings, in several settings. The pupil who really understood, would be able to see the mathematical structure behind the problem. Furthermore, the arithmetic textbooks often addressed the children in person: “now do these exercises, my dear child” and similar phrases were not uncommon. Pupils were encouraged to do the exercises very thoroughly, and they also were urged to enjoy studying:

Zijt gij niet verheugd, mijne jonge rekemaars! dat wij nu met het leeren van den Regel van Driëen een' aanvang zullen maken? Natuurlijk, zegt gij, wie zoude niet verheugd zijn, wanneer hij van dag tot dag meer en meer in verschillende wetenschappen vordert?

It was not unusual in textbooks to simply ask a pupil to do the next question, or check if a given solution was correct. This way of addressing the pupil directly, and giving him the right example of eagerness for knowledge, were quite common until the 1840s, but reprints of some of these textbooks still appeared in the 1850s and 60s.

Practically all mathematics textbooks destined for lower and middle class education in some form paid attention to the pupil’s attitude towards his study. The school in one of the textbooks by R.G. Rijkens (1795-1855), filled with merry little arithmeticians, is situated in Children’s Joy. The popular arithmetic textbooks by P.J. Baudet encouraged pupils to study thoroughly. The algebra textbook by J. de Gelder (1765-1848) contained a lengthy introduction which the pupil was urged to read over and over again telling the student how he should study: never go on to the next theorem before you have grasped fully the previous one; if an exercise seems too hard, try substituting numbers for the variables and see if you can solve it:

---

28This kind of remarks were made by J. de Gelder, P.J. Baudet and other mathematics teachers.
29R.G. Rijksens, Practisch Rekenboek, ten dienste van jongens en meisjes II (1830), quoted from the second edition (1839), p. 1; free translation: “Are you not eager, my little arithmeticians! to start learning the Rule of Threes? Of course I am, you say, who would not be eager to learn more and more, from day to day?” —italics as in the original.
30For example in: H.G. Backer, Rekenboekje voor Eerstbeginnenden, Zalt-Bommel (1842); and in the reprint of De liefhebber van het rekenen (1842) by H. Hemkes, where children at the end of the book were practically begging for a new arithmetic textbook.
31X. van Callegoed, Rekenboek ten dienste der Scholen, Amsterdam (1854); H. Kromer, Meetkundig Rekenboek voor de Jeugd, Groningen (1831, 1868); another striking example was P.J. Andriessen, De Historie van Twee Appeljongens, Amsterdam (1863, 1868), but this was not a textbook in the strict sense. It was sold as a children’s book.
et cetera.

Generally algebra and geometry textbooks no longer address the pupils as “dear children” —this, as might be expected, seems age-depending.

Whether poured in the form of dialogues giving the good example, or direct instruction in an introduction to the book, advice on the best way to study was always present. Eagerness and perseverance were the prerequisites of a good attitude. The new didactic approach (the new goal) in mathematics, however, also needed a new kind of attention from the pupil: he had to face the difficulty of understanding why the reckoning was done in this way and not in another. This he was told in many textbooks, either directly, or in the ‘perfect student’ dialogue, where the imaginary student was always asking (the right) questions.

The eagerness for learning and thorough knowledge was also one of the most profitable results the teachers believed (mathematics) education to lead to. It would be narrow-minded to view the phenomenon only as a didactical ideal. There is also an educational aspect to it. The good citizen had to possess all the middle class virtues: he had to be an enlightened person. Sound reasoning was essential, and from his sound reasoning originated his strong belief in God and his country, his rejection of superstition and his actions that could bring the nation so much good. Mathematics instruction in this way contributed to middle class ideals.

2.2 The fields of knowledge for the gentleman

Whether solving ‘life-like’ word problems was seen as the final goal in education (during the 18th century), or as a means of showing the pupil how useful mathematics was, this type of question confronted the pupil with the additional problem of reducing the exercise to its bare (mathematical) essentials. The question about Pompeii at the beginning of this article is far less realistic than a question on the profit of a merchant who has bought an amount of silk for 79 and sold it for 1738 shillings. The word ‘potatoes’ might be substituted for ‘silk’, or we may choose other numbers without really changing the exercise; these ‘realistic’ kind of exercises I will call ‘standard word problems’. The ‘Pompeii’ exercise is not a standard word problem: one cannot easily change the numbers (or the names of the cities) in the exercise and have a similar word problem — at least not without giving false information. The story offered extra historical information, knowledge that was considered to be suitable for a good (early 19th century) citizen.

The question involving Pompeii is an example of, what I will call, a ‘formative question’, offering information from another field of knowledge, deemed relevant for the pupil but clearly not intended as being of practical use in his future career. And this is certainly not the only example: P.J. Prinsen tells in his arithmetic textbook...
how large mirrors are made, that near Sala in the kingdom of Sweden there is a 12th century silver mine; Koster\textsuperscript{36} is introduced as the inventor of the printing press (in a question which treats the dimensions of his statue in Haarlem), and we learn that the Senegalese monkey-bread tree grows the largest trunks\textsuperscript{37}. Lucas Oling (1750-1806) tells us about Huygens and about the discovery of New Holland by the Dutch navigator Abel Tasman. Oling also gives several astronomical facts, apart from exercises on fountains, and the expansion coefficients of several metals\textsuperscript{38}. Jan Brunt narrates some biographical facts about Michiel de Ruyter, and tells about the discoveries made by Spanish and Portuguese sailors\textsuperscript{39}, while Jacob de Gelder in his algebra textbook tells about Alexander the Great, Newton, Huygens and the rise and fall of the Roman empire\textsuperscript{40} and in his arithmetic about the discoveries by Columbus, Vasco da Gama and Magelhaes, the Julian and Gregorian calendars, the \textit{System of Nature} by Linnaeus, and about the surveying work by Méchain and Delambre\textsuperscript{41}.

Many of the exercises relate to physical topics, mostly to the planetary system. In the geometry textbook by H. Kremer, for example, the pupil was presented a lot of exercises concerning the distances and speeds of the planets. Most noteworthy, the pupil was presented an exercise in which he had to assume that the Ptolemaic system was correct, and given the necessary data, was asked to calculate the average speed of the sun going around the earth. The absurd number resulting, was probably meant to convince the child of the truth of the Copernican system. This textbook also contained exercises about the monkey-bread tree (this tree was popular indeed), the Egyptian pyramids, a short history of approximations for \( \pi \), and balloons —including the story of the first balloon by the Montgolfier brothers\textsuperscript{42}.

These problems are so far-fetched that they can hardly have been meant to show the usefulness of mathematics. The occurrence of such kind of questions illustrates that the Dutch middle classes in those days did not only strive for a (truly 18th century) encyclopedic ideal of knowledge, but also that they were convinced that the striving for this ideal could be continued during the mathematics lessons, or —at least for some— any lessons. P.J. Baudet, for example, published both mathematics and French textbooks. In the preface to one of his French textbooks Baudet claimed that it would be very useful to offer the children general knowledge in brief passages meant as exercises in translation. His reviewer agreed\textsuperscript{43}. He might have made similar remarks about arithmetic, algebra and geometry, for in his mathematics textbooks Baudet offered his pupils the kind of exercises discussed above\textsuperscript{44}. W.F. Andriessen

\textsuperscript{36}Laurens Janszoon Coster or Koster was in Dutch historiography the inventor of the printing press. Probably patriotism made that Gutenberg’s claim was not taken seriously.
\textsuperscript{38}L. Oling, \textit{Rekenkundige Voorstellen I}, p. 29 and III, pp. 45, 46, 79 respectively.
\textsuperscript{39}Jan Brunt, \textit{Eerste Beginnissen der Rekenkunde II} (1807\textsuperscript{4}), pp. 33, 39.
\textsuperscript{40}J. de Gelder, \textit{Beginnissen der Stelkunst}, Amsterdam / Den Haag (1836\textsuperscript{5}), pp. 259-260.
\textsuperscript{41}J. de Gelder, \textit{Beginnissen der Cijferkunst} dl. I, pp. 15, 35 and 58.
\textsuperscript{42}II. Kremer, \textit{Meetkundig Rekenboek voor de Jongel}, Groningen (1821).
\textsuperscript{43}\textit{Bijdragen} 1834-41, p. 679. Literally: "Wij zeggen met den Heer Baudet: ‘het veld van algemene kundigheden is zoo ruim, dat men de(n) jonge(1) lieden, te gelijk met de taal, zeer wel eenige nuttige denkbeelden kan mededeelen,’ zonder de opstellen met laffe en zouteloze samenvoegingen, of afgetrokkenne redeneringen, te vullen”.
\textsuperscript{44}P.J. Baudet, \textit{Rekenboek voor de scholen I}, Deventer (1826), p. 11; II, Deventer (1826), p. 104.
in the late 1840s, early 1850s even made his pupils recall things from the history or religion lessons in his arithmetic textbook, by asking them questions like: “find the factorization of the number which indicates the year of the beginning of the 80 years war”\(^\text{45}\) and “How many hours old was Methuselah when he died”\(^\text{46}\); most textbooks supplied the necessary data for such a problem.

The inclusion of such exercises was seldom explained. In the fourth part of his arithmetic textbook, J. Brunt introduced the metric system. Some of its history is recalled, and Brunt made his readers acquainted with the notions of meridian, poles, earth axis, equator, etcetera. Brunt used these notions in order to narrate briefly how the length of the meter had been determined. He explained his action in a footnote where he said that children would be eager to learn a bit of geography and that he was glad to offer it\(^\text{47}\). Whatever might have been the reason, the formative questions were added consciously: Oling and De Gelder mentioned the fact that they were presenting exercises on several arts and sciences explicitly on the title page of their books.

It might be noted, that also quite practical (non mathematical) information was sometimes presented. In an 1830 arithmetic for farmers, for example, the pupils were taught about the Dutch tax system and about the rights resulting from their paying taxes. These rights depended on the amount of tax paid. The amount of tax paid was a subject of some of the exercises\(^\text{48}\). This information seems to have been meant more as a stimulus to see arithmetic as a useful science, than as a way of quenching the thirst for general knowledge.

One important facet of these questions may not be overlooked: perhaps the authors included these exercises merely to stimulate the interest of the child. Although applications of mathematics are never as straightforward as textbook authors would like us to believe, it was stimulating for the pupil to experience (the illusion of) being able to solve different kind of exercises, and not just a hundred copies of the same. It is unknown how these questions were used in class room practice —if they were used at all. It is, of course, possible to skip such questions, or —while discussing the answer —merely devoting attention to the numerical solution. In these ways the class room situation might have influenced the effects that the authors had intended. But that does not affect the ideals of these authors.

2.3 The virtuous citizen

Stimulating the interest, by introducing some —far-fetched— ‘applications’ of the abstract mathematics that was being taught, may also have been a didactical stim-

\(^{45}\)W.F. Andriessen, *Toegepaste Cijferkunst III*, Amsterdam (1851), p. 9
\(^{47}\)Jan Brunt, *Eerste Beginselen der Rekenkunde IV* (1865)\(^*\), p. 42; literally: “Offchon dit niet tot het onderwerp van dit rekenboekje behoort, heb ik echter niet ondienstig geoordeeld, dit er in te voegen, vertrouwend, dat kinderen, of zulken die van de Aardrijkskunde niets mochten weten, geene gelegenheid verzuimen zullen, om ook dit weinige zich eigen te maken.”
ulus for questions involving morality in mathematics textbooks. The pedagogical intentions here, however, cannot be overlooked. An example from an 1808 algebra textbook:

Drie Kinderen werden in een Tuin met Appelen befoond, die zij zonder schikking na zich nemen. Maar het oudste Kind ziende dat hij de meeste Appelen heeft, geeft aan elk der beide andere Kinderen zoo veel als ieder heeft; even zoo doet ook het tweede en daarna ook het derde Kind, en zij bevinden na de deeling ieder $\frac{8}{9}$ Appelen te hebben.\(^{19}\)

The question was, of course, how many apples each of them had received in the beginning of the story. In this way, solving a set of linear equations was used to promote generosity. It is remarkable that the exercise was omitted from the 1841 revised edition of this textbook.\(^{20}\)

One — judging from the number of reprints — rather popular arithmetic textbook by the Groningen teacher R.G. Rijkens also gave exercises where misbehaving boys were in some way punished. A deceased miser is not mourned; a man, already stealing as a youth, is hanged at later age for another crime; and the joy of throwing a snowball makes a boy guilty of robbing a girl of her eyesight.\(^{21}\) Rijkens also tells his readers the story of two boys who start fighting at the market. One falls into a basket filled with eggs, and instead of paying for the damage, the two molest the owner. They end up in jail, are fined for the damages, and miss three days of wages. The pupil is asked to calculate the financial damage and at the same time is made clear that this kind of behaviour would not do.\(^{22}\)

That these questions really constituted a pedagogical dimension, may be illustrated by the different phrasing of comparable questions in arithmetic textbooks destined for use at schools for girls. During the 19th century middle class girls were destined to become good housewives. The amount of general knowledge and mathematics girls had to possess was a matter under discussion, but there was general agreement on some points: they should not become savants and they should be able to run a household properly.\(^{23}\) Indeed we may find these ideas in the moralistic exercises of the arithmetic textbooks for girls.\(^{24}\)

It is difficult to say whether these questions were an expression of the Biedermeier culture they were produced in, or that they were consciously added. Certainly the

\(^{19}\)P. van Campen, Grundbeginselen der Algebra of Stelkunst, Leiden (1808), pp. 137–138; free translation: “Three children were randomly given some apples. The oldest child, observing that he had most, decided to double the amount of apples that each of his companions had received. The second child acted the same, and finally the third child did so too. After this, they found that each possessed $8$ apples.”

\(^{20}\)De Grondbeginselen der Algebra . . . van P. van Campen, geheel omgewerkt, Leiden (1841).\(^{21}\)R.G. Rijkens, Praktisch Rekenboek, ten dienste van jongens en meisjes I (1837\(^2\)), p. 12; II (1839\(^2\)), pp. 10–11, 71.\(^{22}\)Idem, dl. I (1837\(^2\)), p. 41.

\(^{23}\)Mineke van Essen, Opvoeden met een dubbel doel, Amsterdam (1990), pp. 25 28.

\(^{24}\)For example: N. Anslijn, Rekenboek voor Meisjes, Leiden (1804–1807) 4 volumes, with many reprints; W.A. Baars & P. Joling, Rekenboek voor Meisjes in het koninkrijk der Nederlanden, Leeuwarden (1830 1831) 2 volumes; H. Sluyters, Practisch Cijferboek voor Meisjes, Zierikzee (1837)
‘saving exercises’ must have been consciously added. Usually these were divided into two, placed directly after each other. The first introduced a child who always saved the larger part of his money and only used small amounts for good purposes, like charity or buying arithmetic textbooks. Some reckoning taught that by the end of the year this child had enough money to buy new clothes. The second exercise would introduce another child, who did not save that much, but spent his money every week on candy or unworthy child magazines. Some reckoning showed that the latter did not have enough money to spend on new clothes or other necessary equipment. Saving was one of the issues that was raised by ‘t Nut, and also served a very practical purpose: the middle classes were convinced that many abuses of money led to poverty, and that could be prevented. It’s inconceivable that such a combination of questions, on such an issue, unconsciously ended up in a textbook.

The raising of morally good citizens was an issue also during the teaching of mathematics: not only did they become good citizens just by learning how to reason, pupils were also confronted with good examples, or bad ones punished. Textbook authors and teachers felt it as their obligation to raise good and civilized human beings. This was the main goal in education — certainly for the lower classes — and therefore the good example had to be present in arithmetic textbooks too. In my view, questions involving morality were intended to convince the pupil that becoming a virtuous citizen was in his and society’s best interest: this conviction was strengthened by the mathematical correct results of the exercises — the mathematical truth was simply exported to the real world. Certainly the ‘saving’ questions were a conscious attempt to export early 19th century middle class ideals to children of all classes. Also in most of the other exercises one would be inclined to see ‘t Nut at work, although unfortunately textbook authors do not say anything about it.

3 Reception

During the first half of the 19th century practically all textbook authors opted for ‘thorough learning’. Although they did not agree in detail on what it meant, and were certainly not unanimous on how this was best achieved, rote learning was (at least ideally) abandoned. In some form dialogue, addressing the “sweet children” either directly or via the teacher in a prologue this new didactics was present in all

---

55 A few of the magazines meant here have indeed survived and can be found in the Dutch Royal Library in The Hague. They go by titles as: De Snakenburgsche Courant and Tijdschriftje voor merkwurde en ware geschiedenissen. Not many of these magazines still remain, but that is to be expected. Their existence illustrates that opposition existed against the Biedermeier sweetness.

56 N. Callegood, Rekenboek ten dienste der scholen I, Amsterdam (1854), p. 62; J. Brunt, Eerste beginselen der Rekenkunde II, Leiden (1804), p. 19, p. 103; pp. 50 51 in later editions; L.J. Baudet, Rekenboek voor de scholen I, Deventer (1826), p. 137 (in this book the exercise is presented in the form of two workers, one saving to buy his family a boar for meat supply during the winter, the other buying meat in small portions. The question is why the first is better off than his colleague, and apart from the obvious calculations about costs and quantity, the pupil is also expected to note that he obtains better food in an easier way, and with less worry!); R.G. Rijkens, Practisch Rekenboek, ten dienste van jongens en meisjes, Groningen (1839), pp. 43 44; H. Acneae, Rekenboek I (1791), pp. 201 202 and others.
the new textbooks. Textbooks containing many exercises of the same kind became rare; at least the various applications of the rule of three were mingled in order to show its versatility. Most textbooks also contained formative exercises, and many paid attention to morals. In arithmetics the latter two phenomena occurred more often than in algebra or geometry books.

How were these new features received? First of all it is good to note that although most of the new books did contain the features discussed above, in some places the 18th century textbooks were still in use. These were, however, not valued by the reviewers in the Government magazine Bijdragen, which was issued for Dutch teachers. They preferred the textbooks by De Gelder, Baudet, Brunt and Prinsen, or the little short-lived exercise book by D. Braakenburg, which was appreciated for its wide variety of topics offering good didactical possibilities. As was suggested by the authors, reviewers thought the variety would keep the children alert, thus avoiding rote learning. Many teachers and reviewers thus made it clear that the pure mathematical background was important but it were the numerous applications, in literally all subjects important to a sensible human being, that made pure mathematics indispensable in education.

Formative exercises were valued. The exercises on the planetary system in the geometry book by Krerner, for example, received praise in the Bijdragen. An exercise in an arithmetic textbook for farmers which offered information on the growing of plants, too: according to the reviewer it would attend the pupils to a subject they would not ponder otherwise. For the reviewers in the Bijdragen there was little or no knowledge that knowledge and virtue were two sides of the same coin. This made the appreciation for these formative questions more or less obvious. One thing reviewers

57The most extraordinary example of this, apart from the Bartjens reprints, was the Vernieuwde Licht des Koophandels by A.B. Strabbe, which was reprinted until 1816.

58The author is working on a list of Dutch mathematical (text)books with a short description of the items. This remark must be read as an impression of the work done so far: realistic statistical data is not yet available.

59Most noteworthy the textbooks by Willem Bartjens, Adam van Lintz and Jan van Oln were reprinted until the 1840s.

60See for example the review in Bijdragen 1810, pp. 200-201.


62Bijdragen 1827, pp. 20-21. Just as an illustration: one of the exercises resulting in the number 1672—ended with the question if this was also an important year in Dutch national history.

63Bijdragen 1822, pp. 583-584.

64Bijdragen 1830, pp. 532-536; the textbook reviewed was W.A. Baars and P. Joling, Rekenboek voor den landman en den landscholen, Sneek (1830). I am not capable of judging if the information offered in this particular exercise—i.e. on the rapidity of growth during different months—is realistic or not; but this is quite insignificant, since the reviewer clearly thinks so. His valuation of this type of exercise stands.

65This idea found many adherents in the Bijdragen, which might be illustrated for example, by the
were very serious about was the level of mathematical difficulty: the pupil had to be making progress. If the questions were difficult enough (and not too hard to solve), the story that was used to wrap the exercise in was an extra touch which drew their attention.

But what about the exercises involving morality? Unfortunately reviewers don’t mention these explicitly. Only in the rejection of the moralistic stories R.G. Rijkens had used in his arithmetic some reviewers pay attention to this aspect. Rijkens’ textbook contained a few exercises which covered a whole page, telling the stories of boys or girls misbehaving sometimes with horrible consequences. According to the reviewer in the Bijdragen these stories belonged in a textbook on morality, although he did like the textbook as a whole. One would expect some discussion on the morality exercises, or at least on the saving exercises, but there is none. A unconsciously added Biedermeyer element after all? The presence of morality might have been so obvious in this time of Van Alphen (1746-1803) and Anslijn (1775-1838) that reviewers did not pay special attention to it. If ethical problems were not treated in textbooks, the reviewers might have taken it for granted that teachers brought the subject up in every suitable occasion.

On the other hand, the people of 't Nut saw much more unity between the several sciences: the application of mathematics to geography and history was more or less obvious to them. Morality exercises might also have been seen in the same light as the other applications. One of the goals of historical literature for children was to confront pupils with the perfect example of virtuous citizens and the great things they had accomplished by behaving the way they did. In this case the appreciation for the physics and history exercises may be transferred directly to the problems on morality. In a time when it was customary to identify knowledge and virtue this would not be such a strange idea. A sensible and enlightened human being would see the necessity of behaving in a morally just way. That was what many people within 't Nut believed. This point of view was shared by the people of the Dutch Mathematical Society, who in 1814 published a speech held by one of their members, in which all political horrors of the French revolution were put on the account of people who had abused their wits by following a false philosophy. These events could be prevented in future, according to the Society, by teaching solid mathematics. In a time when it was customary to identify knowledge and virtue this would not be such a strange idea. A sensible and enlightened human being would see the necessity of behaving in a morally just way. That was what many people within 't Nut believed. This point of view was shared by the people of the Dutch Mathematical Society, who in 1814 published a speech held by one of their members, in which all political horrors of the French revolution were put on the account of people who had abused their wits by following a false philosophy. These events could be prevented in future, according to the Society, by teaching solid mathematics. In this view mathematics and morals were not so far apart, and it might have been found obvious to apply mathematical

---

66Bijdragen 1832, pp. 413—419.
67H. van Alphen wrote a series of children’s poems in the 1780s, which were to become extremely popular in the early 19th century; N. Anslijn was well-known for his children books Brave Hendrik and Brave Maria, presenting nouseatingly sweet children as role models to its readers; cf. P.J. Buysters e.a. (ed.), De hele bibelebontse berg, Amsterdam (1989), pp. 237—240.
69J. de Gelder, Redevoering uitgesproken bij de opening van de 35e algemene vergadering van het Wiskundig Genootschap, Amsterdam (1814).
knowledge to morality. As pure mathematics was regarded as a perfect example of sound reasoning, a way of obtaining truth and banishing prejudice, the (obviously correct, certainly from an 18th-century point of view) application of this wonderful science to morality, was regarded as a completely natural extension of its domain.

The opinions expressed in the *Bijdragen* confirm that the phenomena discussed were a part of the ideal way of teaching. A mere glance at the number of reprints of the textbooks by Brunt, De Gelder, Baudet, Hemkes, Kremer and Rijkens, reveals that a considerable number of teachers agreed it was worth trying to put these ideals into practice.

## 4 Disappearance of the phenomenon

By the middle of the 19th century these features gradually vanished. During the 1840s and 1850s arithmetic textbooks for secondary education gradually reduced the amount of text in word problems to an absolute minimum: standard word problems, for example with the goods to be traded simply denoted with letters ‘A’ and ‘B’, began to appear. Standard word problems and pure mathematics exercises took over although the exercises were still mixed to avoid rote learning. The arithmetic textbooks by H. Strootman (1777-1851) and J.C.J. Kempeis (1817-1874) omitted the long prefaces De Gelder had given, but still preserved the formative exercises. In the 1870s the textbooks by Jan Versluys (1845-1920) dominated the market for secondary school textbooks. These only contained plain exercises and standard word problems.

In primary education the textbooks by Brunt and Rijkens were replaced by textbooks that no longer addressed the children in person and omitted formative exercises and questions on moral, like those by A. Boeser, A.W. Alings, L. Bouwman, J. Steynis, J.J.C. Donck and R.H. Pieters. These arithmetic textbooks offered

---

70 P.J. Buijstens e.a. (ed.), *De hele Bibelebontse berg*, pp. 240–241; the same development occurs in children’s books.


74 J. Versluys, *Theorie der Rekenkunde*, Groningen (1870); there has been a discussion between K. Knapper and J. Versluys concerning the percentages Versluys used on several bonds. This, however, was of a practical nature: it concerned the question whether the percentages used were up-to-date. It was completely irrelevant to the education of the pupil, as is the value of the price of cotton in the exercises concerning the buying or selling of cotton —up-to-date prices are relevant only to the merchant.

75 A. Boeser, *Rekenboekje* parts 1-5, Amsterdam (1850-1861) and *Verzameling van rekenkundige voorstellen* parts 1-3, Amsterdam (1852-1866).


77 L. Bouwman, *Algebruik Rekenboek*, Groningen (1857); it has to be noticed that a 'generosity' exercise is printed in this book at p. 63.


only plain and standard word exercises, concentrating almost entirely on the mathematical reasoning. These textbooks gradually replaced the “little arithmeticians” textbooks in the 1850s. Boeser explicitly stated that he thought variety in the type of exercises was good, but he also made it clear that in his arithmetics he did not want to spend time on other subjects than reckoning; in one of his textbooks he even excused himself for not making the tobacco exercises more realistic by adding the tax regulations. He argued that it would take up too much space. The merchant by the name “Honest” (Eerlijk), only appearing in his first arithmetic textbook from 1850, may be considered a relic, where before the reviewers had always expressed the opinion that this kind of exercises could easily be made up by a teacher. In an 1860 review in the Bijdragen there was even a complaint about a textbook which did not contain enough plain exercises, where before the reviewers had always expressed the opinion that this kind of exercises could easily be made up by a teacher. In algebra and geometry textbooks the phenomenon disappeared even earlier than in the arithmetics: by the 1830s De Gelder’s algebra textbook was the only algebra textbook actually in use, containing a long introduction telling the pupil how to study, and exercises containing ‘general knowledge’.

For the disappearance of the phenomenon there are three plausible explanations. First these exercises may have been intended as a way to stimulate the acceptance of mathematics in the primary and secondary schools. This hypothesis could explain the occurrence of the observed phenomena in lower class education. On the other hand: in higher class education mathematics had to compete with Latin and Greek, and no classic languages were merged into the mathematics textbooks. Perhaps this combination was considered too difficult. A discussion about the compatibility of mathematics and Latin did take place, though.

As for middle class education, there is no abundance of source material. We would have to assume that many textbook authors felt the need to convince themselves and their public (and perhaps the children) once more of the validity of their shared ideals—not unusual among people of all times. There was some opposition towards the new form of mathematics that was taught: the long life of the textbooks by Bartjens show that there were people who did not believe in the blessings of pure mathematics. Except for a paper in a middle class journal there is, however, no indication of any strong aversion towards the growing importance of pure mathematics. But if there was, the occurrence of the observed phenomena could be interpreted not only as a pedagogic ideal, but also as having a soothing effect on the opposition. By showing the relevance of pure mathematics and linking it to other fields of knowledge (and

---

81 A. Leen, *De ontwikkeling van het rekenonderwijs op de lagere school*, Groningen (1961)
83 A.L. Boeser, *Eerste Rekenboekje*, Amsterdam (1850). The numerous reprints of this textbook all are identical, so that “Honest” indeed witnessed the end of the 19th century, and in this way outlived his creator Boeser as an anachronistic relic in a primary school textbook the later textbooks by Boeser no longer mention him.
84 *Bijdragen* 1860, pp. 434–435; the textbook reviewed was A. Bierens, *Practisch Cijferboekje voor de lagere scholen*, Breda (1859).
87 Vaderlandsche Letteroefeningen 1827-1, pp. 178–179
among these most notably those that of old had been taught as mathematics) some extra legitimation was created, which by the 1850s was no longer necessary.

A second hypothesis which sounds plausible is that, although teachers were still expected to teach all fields of human knowledge, the encyclopaedic ideal of science that the middle class strove for during the first decades of the 19th century, had lost its attractiveness. Accepting this assumption, the disappearance of the pedagogically suitable questions might be seen in terms of the growing gap between the different disciplines, or at least a growing gap between the didactical approaches of the sciences and humanities. Perhaps even as the precursor of the emergence of a didactics separate from pedagogics.

A third hypothesis is that by a change in the pedagogical climate, this pedagogical dimension in mathematics textbooks suddenly became intolerable. Indeed, the pedagogical climate changed. A scientific pedagogy, resembling physics in its methods, seemed within reach. Supported by new psychological theories, this new pedagogy left no place for the sweetness and ideals of the first half of the 19th century. Of course, this was theory, and how fast these theories became accepted by textbook authors has never been the subject of serious research.

Either hypothesis allows an explanation for the fact that the phenomenon disappeared from algebra and geometry earlier than from arithmetic textbooks. Arithmetics were largely written by schoolteachers during the whole century; they were used in primary education. Algebra and geometry textbooks, destined for secondary school, were written by teachers for engineers. In the 1820s it became self-evident for engineers to have some mathematical knowledge. Bourgeois children’s chances for a career were thus determined by the amount of their mathematical knowledge. Indeed, since the 1820s so-called ‘preparation schools’ (voorbereidings instututen, preparing for engineering school) emerged, that focussed on the teaching of mathematics.

The developments fit all three theories. If the phenomenon indeed was an aspect of legitimation, it is simply observed that the engineering schools hastened the disappearance of the opposition against a heavy pure mathematics curriculum. On the other hand, if the phenomenon is an illustration of the disappearance of the encyclopaedic ideal one may observe that indeed the restricted education at the engineering schools stimulated teachers to specialize. Finally, if it was a change in the pedagogical climate that stimulated the disappearance of the phenomenon, we could observe that the teachers at the engineering schools stood much closer to scientist, and often were acquainted with scientists. Thus, they were more likely to be influenced by a change in pedagogical theories than were their counterparts in primary and secondary education. Either way, the phenomenon simply became redundant.

5 Concluding remarks

First of all I would like to remark that the distinction in three aspects as I have done in section 2 is not made to my knowledge by contemporary authors. Probably

89H.J. Smid, Eén onbekoekte nieuwigheid, Delft (1997), pp. 65, 192 201
the contemporaries saw all three phenomena discussed as undivisible part of what we nowadays consider to be pedagogics. Secondly I would (once more) like to stress that the phenomenon may not have played a large role in class room practice. The textbook authors were for the larger part well-educated people. I have shown that they were speaking on behalf of the well-educated middle class: the reviews leave no doubt about the appreciation of the didactic ideals that were described. But it would be a rash decision to interpret these positive reviews as didactic reality. Having made these remarks it is now time for an overview.

From the early 1790s until the 1840s all good Dutch mathematics textbooks for lower and middle class education contained exercises which illustrate the intertwining of didactical and pedagogic ideas of those times. Their value was neither to be sought in the future careers of the pupils (as was the case for 18th century textbooks), nor in a straightforward didactically suitable packaging. They probably were meant to stimulate the interest of the pupil by offering him diverse examples, but that was certainly not the only purpose. These questions were added to allow the pupil to learn history, geography and physics while doing mathematics: these were not conflicting subjects but should be merged in the brain of the well-educated citizen. Even morality was present in the stories that accompanied some of the exercises. Moreover, in the theoretical parts of the textbooks pupils were stimulated to value their studies, and study in a particular way. Thus the children, who according to the ideals of their parents had to receive a broad education, were confronted with a lot more than only mathematics.

The appearance of these phenomena in Dutch mathematical textbooks illustrates the acceptance of pedagogical theories that saw education as a crucial factor in raising a child. The child had to possess knowledge in various fields, which would in turn allow him to behave like a good citizen. What this good citizen was, and how he behaved, was explained by means of famous people from national history, good examples (or bad ones punished) in simple moralistic stories, and by directly addressing the child and pointing him at his obligations (yes, you will love to make these exercises, for you know it will bring you so much good ... ). The mathematical textbooks might thus be seen as an illustration of how close pedagogy and didactics actually were in the early 19th century.

Three plausible explanations for the disappearance of the phenomenon have been offered. The phenomenon can be seen as an attempt to legitimize (the new form of) mathematics as a teaching discipline. Secondly, it can be seen as a final convulsion of the ‘homo universalis’ ideal, as it still lived on within the middle class. Finally, it might have been a change in the pedagogical climate which resulted in new pedagogical idea(s) that no longer valued the Biedermeier elements. It requires more research to decide which hypothesis suits best the developments in Dutch mathematics.

90In this respect it might be good to note that not all teachers were actually good arithmeticians. Most noteworthy: in K.H. ten Berge, *Practisch Rekenboek der Arithmetiek*, Winschoten (1847); and the anonymous *Eerste Beginselen der Rekenkunde* V, Groningen (1858); the authors wrote (mathematically) awful things to explain how ordinary fractions could be transformed in decimal fractions. Class room practice, of course, was virtually completely dependent on teachers —of diverse competence. How each of them interpreted ‘thorough study’ and all other educational ideals discussed, is hard —if not impossible— to find out.
tics) education during the 19th century. Textbooks in other new fields, like geography, could prove useful in this respect. A comparison with developments in other countries might prove useful as well. In any case the Biedermeier elements in mathematical textbooks are a curious and interesting phenomenon, which most of the early 19th century “little arithmeticians” must have been confronted with, and which illustrates how far middle class values in fact permeated society.