GW170817: Implications for the Stochastic Gravitational-Wave Background from Compact Binary Coalescences

The LIGO Scientific Collaboration and The Virgo Collaboration
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The LIGO Scientific and Virgo Collaborations have announced the first detection of gravitational waves from the coalescence of two neutron stars. The merger rate of binary neutron stars estimated from this event suggests that distant, unresolved binary neutron stars create a significant astrophysical stochastic gravitational-wave background. The binary neutron star background will add to the background from binary black holes, increasing the amplitude of the total astrophysical background relative to previous expectations. In the Advanced LIGO-Virgo frequency band most sensitive to stochastic backgrounds (near 25 Hz), we predict a total astrophysical background with amplitude $\Omega_{GW}(f = 25 \text{ Hz}) = 1.8^{+2.7}_{-1.3} \times 10^{-9}$ with 90% confidence, compared with $\Omega_{GW}(f = 25 \text{ Hz}) = 1.1^{+1.2}_{-0.7} \times 10^{-9}$ from binary black holes alone. Assuming the most probable rate for compact binary mergers, we find that the total background may be detectable with a signal-to-noise-ratio of 3 after 40 months of total observation time, based on the expected timeline for Advanced LIGO and Virgo to reach their design sensitivity.

Introduction — On 17 August 2017, the Laser Interferometer Gravitational-Wave Observatory (LIGO) [1] and Virgo [2] Collaborations detected a new gravitational-wave source: the coalescence of two neutron stars [3]. This event, GW170817, comes almost two years after GW150914, the first direct detection of gravitational waves from the merger of two black holes [4]. In total, four high confidence detections and one sub-threshold candidate from binary black hole merger events have been reported: GW150914 [4], GW151226 [5], LVT151012 [6], GW170104 [7] and GW170814 [8]. The last of these events was reported as a three-detector observation by the two Advanced LIGO detectors located in Hanford, WA and Livingston, LA in the United States and the Advanced Virgo detector, located in Cascina, Italy.

In addition to loud and nearby events that are detectable as individual sources, there is also a population of unresolved events at greater distances. The superposition of these sources will contribute an astrophysical stochastic background, which is discernible from detector noise by cross-correlating the data streams from two or more detectors [9, 10]. For a survey of the expected signature of astrophysical backgrounds, see [11–17]. A recent review of data analysis for stochastic backgrounds is given in [18].

Following the first detection of gravitational waves from GW150914, the LIGO and Virgo Collaborations calculated the expected stochastic background from a binary black hole (BBH) population with similar masses to the components of GW150914 [19]. These calculations were updated at the end of the first Advanced LIGO observing run (O1) [20] to take into account the two other black hole merger events observed during the run: GW151226, a high-confidence detection; and LVT151012, an event of lower significance. The results indicated that the background from black hole mergers would likely be detectable by Advanced LIGO and Advanced Virgo after a few years of their operation at design sensitivity. In the most optimistic case, a detection is possible even before instrumental design sensitivity is reached.

In this paper, we calculate the contribution to the background from a binary neutron star (BNS) population using the measured BNS merger rate derived from GW170817 [3]. Also, we update the previous predictions of the stochastic background taking into account the most recent published statements about the rate of BBH mergers [7]. We find the contributions to the background from BBH and BNS mergers to be of similar magnitude. As a consequence, an astrophysical background (including contributions from BNS and BBH mergers) may be detected earlier than previously anticipated.

Background from compact binary mergers — The energy-density spectrum of a gravitational wave background can be described by the dimensionless quantity

$$\Omega_{GW}(f) = \frac{f}{\rho_c} \frac{d\rho_{GW}}{df},$$

which represents the fractional contribution of gravitational waves to the critical energy density of the Universe [9]. Here $d\rho_{GW}$ is the energy density in the frequency interval $f$ to $f + df$, $\rho_c = 3H_0^2c^2/(8\pi G)$ is the critical energy density of the Universe, and the Hubble parameter $H_0 = 67.9 \pm 0.55 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is taken from Planck [21].

In order to model the background from all the binary mergers in the Universe, we follow a similar approach to [19]. A population of binary merger events may be characterized by a set of average source parameters $\theta$ (such as component masses or spins). If such a population merges at a rate $R_m(z; \theta)$ per unit comoving volume per unit source time at a given redshift $z$, then the total gravitational-wave energy density spectrum from all the
sources is given by (see, e.g. [11][19])

\[
\Omega_{GW}(f, \theta) = \frac{f}{\rho_c H_0} \int_0^{z_{\text{max}}} dz \frac{dE_{GW}(f, \theta)/df_s}{(1+z)E(\Omega_M, \Omega_A, z)},
\]

Here \(dE_{GW}(f, \theta)/df_s\) is the energy spectrum emitted by a single source evaluated in terms of the source frequency \(f_s = (1+z)f\). The function \(E(\Omega_M, \Omega_A, z) = \sqrt{\Omega_M(1+z)^3 + \Omega_A}\) accounts for the dependence of co-moving volume on redshift assuming the best-fit cosmology from Planck [21], where \(\Omega_M = 1 - \Omega_A = 0.3065\). We choose to cut off the redshift integral at \(z_{\text{max}} = 10\). Redshifts larger than \(z = 5\) contribute little to the integral because of the \([(1+z)E(z)]^{-1}\) factor in Eq. 2 as well as the small number of stars formed at such high redshift, see for example [11][17][22].

The energy spectrum \(dE_{GW}/df_s\) is determined from the strain waveform of the binary system. The dominant contribution to the background comes from the inspiral phase of the binary merger, for which \(dE/df_s \propto M_c^{5/3} f^{-1/3}\), where \(M_c = (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}\) is the chirp mass for a binary system with component masses \(m_1\) and \(m_2\). In the BNS case we only consider the inspiral phase, since neutron stars merge at \(\sim 2\) kHz, well above the sensitive band of stochastic searches. We introduce a frequency cutoff at the innermost stable circular orbit. For BBH events, we include the merger and ringdown phases using the waveforms from [12][23] with the modifications from [24].

The merger rate \(R_m(z; \theta)\) is given by

\[
R_m(z; \theta) = \int_{t_{\min}}^{t_{\max}} R_f(z_f; \theta) P(t_d; \theta) dt_d,
\]

where \(t_d\) is the time delay between formation and merger of a binary, \(P(t_d; \theta)\) is the time delay distribution given parameters \(\theta\), \(z_f\) is the redshift at the formation time \(t_f = t(z) - t_d\), and \(t(z)\) is the age of the Universe at merger. We assume that the binary formation rate \(R_f(z; \theta)\) scales with the star formation rate. For the BNS background, we make similar assumptions to those used in [19], which are outlined in what follows below. We adopt the star formation model of [25], which produces very similar results as compared to the model described by [26]. We assume a time delay distribution \(P(t_d) \propto 1/t_d\), for \(t_{\min} < t_d < t_{\max}\). Here \(t_{\min}\) is the minimum delay time between the binary formation and merger. We assume \(t_{\min} = 20\) Myr [27]. The maximum time delay \(t_{\max}\) is set to the Hubble time [28][30]. We also need to consider the distribution of the component masses to calculate \(\Omega_{GW}\). We assume that each mass of the binary is drawn from uniform distribution ranging from 1 to 2 \(M_\odot\). The value of \(R_m\) at \(z = 0\) is normalized to the median BNS merger rate implied by GW170817, which is \(1540^{+3200}_{-1200}\) Gpc\(^{-3}\) yr\(^{-1}\) [8].

The calculation of the BBH background is similar, with the following differences. We assume \(t_{\min} = 50\) Myr for the minimum time delay [19][38]. Massive black holes are formed preferentially in low-metallicity environments. For binary systems where at least one black hole has a mass larger than \(30 M_\odot\), we therefore re-weight the star formation rate \(R_f(z)\) by the fraction of stars with metallicities \(Z \leq Z/2\). Following [19], we adopt the mean metallicity-redshift relation of [20], with appropriate scalings to account for local observations [25][37]. It is also important to specify the mass distribution. We use a power-law distribution of the primary (i.e., larger mass) component \(p(m_1) \propto m_1^{-2.35}\) and a uniform distribution of the secondary [6][7]. In addition, we require that the component masses take values in the range 5 – 95 \(M_\odot\) with \(m_1 + m_2 < 100 M_\odot\) and \(m_2 < m_1\), in agreement with the observations of BBHs to date [4]. For the rate of BBH mergers, we use the most recent published result associated with the power-law mass distribution \(103^{+110}_{-83}\) Gpc\(^{-3}\) yr\(^{-1}\) [7][8]. As shown in [20], using a flat-log mass distribution instead of the power-law only affects \(\Omega_{GW}(f)\) at frequencies above 100 Hz, which has very little impact on the detectability of the stochastic background with LIGO and Virgo. Frequencies below 100 Hz contribute to more than 99% of the sensitivity of the stochastic search [20].

**Predictions and detectability** — A stochastic background of gravitational-waves introduces a correlated signal in networks of terrestrial detectors. This signal is expected to be much weaker than the detector noise, but can be distinguished from noise by cross-correlating the strain data from two or more detectors. For a network of \(n\) detectors, assuming an isotropic, unpolarized, Gaussian, and stationary background, the optimal signal-to-noise ratio (SNR) of a cross-correlation search is given by

\[
\text{SNR} = \frac{3H_0^2}{10\pi^2} \sqrt{2T} \left[ \int_0^\infty df \sum_{i,j=1}^n \sum_{i,j>1} \frac{\gamma_{ij}^2(f)\Omega_{GW}^2(f)}{f^0P_i(f)P_j(f)} \right]^{1/2},
\]

in which \(i, j\) run over detector pairs, \(P_i(f)\) and \(P_j(f)\) are the one-sided strain noise power spectral densities of the two detectors, and \(\gamma_{ij}(f)\) is the normalized isotropic overlap reduction function between the pair [3][19]. While the cross correlation search is not optimal for non-Gaussian backgrounds, Eq. 4 gives the correct expression for the cross-correlation signal-to-noise ratio irrespective of the Gaussianity of the background [27][39].

On the left hand panel of Fig. 4 we show the estimates on the background energy density \(\Omega_{GW}(f)\) for the BNS and BBH merger populations described in the previous section (red and green curves, respectively). The total (combined) background from BBH and BNS mergers is also plotted (solid blue curve) along with the 90% credible Poisson uncertainties in the local rate (indicated by the grey shaded region). Considering this uncertainty, we predict \(\Omega_{GW}^{\text{tot}}(f = 25\) Hz\) = \(1.8^{+2.7}_{-1.3} \times 10^{-9}\).
The spectrum is well approximated by a power law \( \Omega_{GW}(f) \propto f^{2/3} \) at low frequencies where the contribution from the inspiral phase is dominant. The power law remains a good approximation for \( f \lesssim 100 \) Hz. Indeed, previous work suggests that the deviation from power-law behaviour is probably inaccessible by advanced detectors [10]. The frequency region between 10 Hz and 100 Hz accounts for more than 99% of the accumulated SNR for the Advanced LIGO-Virgo network [20]. The median value of the background from BBH mergers \( [\Omega_{GW}(f = 25 \text{ Hz}) = 1.1^{+1.2}_{-0.7} \times 10^{-9}] \) is comparable with our previous predictions [19] but the statistical uncertainty has decreased by a factor of 32% due to an increase in the number of observed BBH mergers. The median contribution from BNS systems is of the same order \( [\Omega_{GW}^{\text{BNS}}(f = 25 \text{ Hz}) = 7^{+1.5}_{-0.6} \times 10^{-9}] \). The statistical uncertainty of the BNS background is larger than that of the BBH background, since we have only one BNS detection. These results are summarised in Table I.

The estimates on the background energy density shown in Fig. 1 are compared with power-law integrated (PI) curves [11] at various observing sensitivities (O2, O3 and Design). The PI curves represent the expected 1\( \sigma \) sensitivity of the standard cross-correlation search to power-law gravitational-wave backgrounds [11], for example the \( \Omega_{GW}(f) \propto f^{2/3} \) spectrum expected from the inspiral phase of binary mergers. Hence, if a power-law spectrum intersects a PI curve, then it has \( \text{SNR} \geq 1 \) for the corresponding observing run.

Although the stochastic background is dominated by unresolvable sources, the energy-density spectra in Fig. 1 include contributions from the loudest, individually detectable events. Simulations of the astrophysical background given the inferred rate and mass distribution indicate that removing sources that are individually detectable by the LIGO-Virgo network with a combined SNR > 12 has a very small impact on the results. The detectable sources have an even smaller effect than shown by the analysis in [19], which considered only the population of loud, high mass sources. This highlights the fact that the spectrum is dominated by low-mass systems, which are less likely to be detected individually.

Different assumptions are possible on the various distributions that enter the calculation of \( \Omega_{GW} \) (Eq. 2), such as star formation rate, metallicity evolution and delay times. However, we have verified that variations on our assumptions are contained within the Poisson band shown in Fig. 1, consistent with the detailed study in [19]. Additionally, if some of the observed BBH systems were of primordial (rather than stellar) origin, with a different redshift distribution, their contribution to the stochastic background spectrum could be weaker [12,13].

The right hand panel of Fig. 1 shows the expected accumulated SNR as a function of total observation time, updated from [19]. For O1, which lasted approximately 4 months, we use the actual instrumental sensitivities of the two LIGO detectors (Virgo was not yet operating at that time) [20]. The second observing run (O2) was recently completed and ran for approximately 9 months. For this run we use typical sensitivities of 100 Mpc for Livingston and 60 Mpc for Hanford, assuming a duty cycle of 50%. We do not include Virgo as it does not contribute significantly to the sensitivity of stochastic searches in O2 due to the lower range and one month integration time. For the next planned observing run (O3), and the following stages of sensitivity improvements, we include Virgo and assume a 50% duty cycle for each detector. Following [45], we assume O3 will be 12 months long (2017 – 2018). We define the “Near Design” phase (2019+) to be a 12 month run where Hanford and Livingston operate at design sensitivity and Virgo at late sensitivity. Lastly, we assume that the “Design” phase (2022+) will be 24 months and will have Hanford, Livingston, and Virgo operating at design sensitivity. These assumptions are broadly consistent with [45], though we make specific assumptions about the duration of each observing run for concreteness.

The median total background from a combined BBH and BNS background may be identified with \( \text{SNR} = 3 \), corresponding to false alarm probability \(< 3 \times 10^{-3} \), after approximately 40 months of observing. In the most optimistic scenario allowed by statistical uncertainties, the total background could be identified after 18 months or as early as O3. The most pessimistic case considered here is out of reach of the advanced detector network but is in the scope of third generation detectors [16].

Although the BNS and BBH backgrounds have similar energy-densities, they have extremely different statistical properties. To illustrate this, we plot a simulated strain time-series in Fig. 2 and show an example BNS (red) and BBH (green) background. The BNS events create an approximately continuous background consisting of a superposition of overlapping sources, since the duration of the waveform (190 s on average in the frequency band above 10 Hz) is long compared to the average time interval between two successive events (13\( ^{+49}_{-9} \) s on average). Considering the uncertainty on the rate, the average number of overlapping sources can vary between 3 and 45. In Fig. 2 14 sources overlap on average.
FIG. 1. The left panel shows the predicted median background for the BNS (red) and BBH (green) models described in the text, the total (combined) background (blue), and the Poisson error bars (grey shaded region) for the total background. We also show expected PI curves for observing runs O2, O3, and design sensitivity (see the main text for details about the assumptions made for these observing runs). Virgo is included in O3 and beyond. The PI curves for O3 and beyond cross the Poisson error region, indicating the possibility of detecting this background or placing interesting upper limits on the evolution of the binary merger rates with redshift. In the right panel, we plot the signal-to-noise ratio as a function of observing time for the median total background (blue curve) and associated uncertainty (shaded region). The median of the predicted total background can be detected with SNR = 3 after 40 months of observation time, with LIGO-Virgo operating at design sensitivity (2022 – 2024). The markers indicate the transition between observing runs. We only show 12 months of the Design phase here, although for the calculation of the PI curves it is assumed to be 24 months long (see [45]).

The BBH background is different in nature even though the resulting energy density spectrum is similar. BBH events create a highly non-stationary and non-Gaussian background (sometimes referred to as a popcorn background in the literature), i.e. individual events are well separated in time, on top of the continuous background from contributed BNS inspirals. The duration of the waveform is much smaller for these massive sources (14 s on average in the band above 10 Hz, considering both the power law mass distribution and the distribution in redshift [46]) and much less than the time interval between events (223$^{+352}_{-115}$ s on average) resulting in rare overlaps.

Table 4 shows the estimated energy density at 25 Hz for each of the BNS, BBH and Total backgrounds. We also show the average time between events $\tau$ for each of these backgrounds as well as the average number of overlapping sources at any time $\lambda$, and the associated Poisson error bounds. The inverse of $\tau$ gives the rate of events in Universe in $s^{-1}$.

**Conclusion** — The first gravitational wave detection of a binary neutron star system implies a significant contribution to the stochastic gravitational wave background from BNS mergers. Assuming the median merger rates, the background may be detected with SNR = 3 after 40 months of accumulated observation time, during the Design phase (2022–). In the most optimistic case, an astrophysical background may be observed at a level of $3\sigma$ after only 18 months of observation, during O3, the next observing run.

There are additional factors which may lead to an even earlier detection. First, the presence of additional sources, for example black hole-neutron star systems, will further add to the total background. Even small contributions to the background can decrease the time to detection significantly. Second, the analysis we have presented here assumes the standard cross-correlation search. Specialized non-Gaussian searches may be more sensitive, particularly to the BBH background [47, 48]. Unlike a standard matched filter search, non-Gaussian pipelines do not attempt to find individual events, but rather to measure the rate of sub-threshold events independently of their distribution.

A detection of the astrophysical background allows for a rich set of follow-up studies to fully understand its composition. The difference in the time-domain structure of the BBH and BNS signals may allow the BNS and BBH backgrounds to be measured independently. After detecting the background, stochastic analyses can address whether the background is isotropic [49–51], unpolarized [52], and consistent with general relativity [53]. Finally, understanding the astrophysical background is crucial to subtract it and enable searches for a background of cosmological origin [46].

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