The version of the following full text has not yet been defined or was untraceable and may differ from the publisher's version.

For additional information about this publication click this link.
http://hdl.handle.net/2066/18707

Please be advised that this information was generated on 2019-12-25 and may be subject to change.
“COME CHILDREN!”
Some changes in Dutch arithmetic textbooks
1750–1850

D.J. Beckers

Report No. 9902 (January 1999)
“Come Children!”
Some changes in Dutch arithmetic textbooks, 1750–1850.

Danny Beckers

Abstract

In this paper two Dutch arithmetic textbooks will be discussed. The first being the successor of the second, both textbooks may be viewed as good representatives of their times. The discussion of these textbooks will be focused on the way they illustrate the totally different educational systems, educational thought, and view of mathematics their respective writers represented.

Kom Jeugd! befeed Uw tyt met vlyt aan nuttig Werk,
De Weetenfchap is eel, zy fchraagt en Land en Kerk.
De Rekenkonft verfiert de ftoet van Weysheits Maagden.
Het geen Euclides al van overlang behaagden.1

Both in Eighteenth and Nineteenth Century arithmetic textbooks we may find words such as in the quotation above. It reflects a deeply felt necessity to learn arithmetic, and to do so because it would bring more than “just” the ability of being able to solve certain exercises. The ideas about what this extra thing was, however, changed considerably around 1800. This paper aims at analyzing this striking difference by looking at two quite different textbooks, which may both be called exponents of their time. They were both arithmetic textbooks for middle class education, meant to be read by pupils aged 10 to 15. Both textbooks knew a lot of “imitations”: textbooks that showed great similarity to the “original.” The existence of these imitations permits speaking about a trend in teaching. In this article I will focus on an Eighteenth and an early Nineteenth Century example to illustrate the changes that took place around 1800 in middle class secondary mathematics education. First the Dutch school system will be briefly discussed.

1Hermans Willemsz., *Arithmetica ofte Reken-Konst*, Enchuyzen (ca. 1760); free translation: “Come children! Spend your time willingly on useful Works. Science is a worthy cause, it supports both Nation and Church. Arithmetic is the finest of all Wisdom’s virgins. As Euclid already agreed.”

2I don’t want to get involved in the discussion which textbook actually was the first one to opt for a certain approach: which author imitated whom etcetera. The mere existence of a set of more or less equivalent books in my opinion allows to speak of a trend, and thereby allows the choosing of a representative of this trend. For this reason I put “imitations” between quotation marks.
1 Mathematics in the Dutch school system

Around 1800, the Dutch school system went through a fundamental revision. In the Eighteenth Century Republic all the school laws had been local issues. During and after the occupation by the French, the Dutch nation was remodeled into a centrally controlled modern state. The school became an issue of great importance to the freshly installed king, William I. Not only did he obtain a firm grip on the school system, well aware of the importance of education in the forming of the citizens in his modern state, he also maintained a system of quality control that had been installed by the French during their rule. Since 1806, all teachers were subjected to state exams, and all schools were regularly visited by government officials, checking the quality of the teaching.3

Those schools that we would now call secondary, for children of ages roughly 10–18, were in fact divided into two categories: higher and lower education, by the social standing of the pupils that the school was for. In elementary school pupils already learned to write and count. For this purpose many alphabet-books also devoted a few pages to arithmetic. Pupils of all social standings thus already knew a bit of arithmetic when they came to secondary school.4 This allowed the teacher to focus on the using of the number system. His pupils even knew how to score in several (number) systems of measures and weights, since non-metrical measures remained in use until the 1850's.

Before 1815, mathematics was not a compulsory subject for any form of education. It had, however, by popular demand, become quite common for middle class pupils to learn some arithmetic, and in engineering education it was customary to study some elementary geometry and algebra.5 Mathematics was basically identified with middle class activities: when in 1815 the Dutch Government decreed the teaching of mathematics to the pupils at the Latin schools (i.e. children of the élite) it took quite some struggle with unwilling rectors before this law was effectuated.6

The two textbooks to be discussed, by Willem Bartjens (± 1590–1673) and Jacob de Gelder (1765–1848), were very common from 1750 until 1850. The first one was one of the most popular textbooks of the Eighteenth Century; the latter became popular after 1815, although Bartjens' textbook did not totally disappear until the 1840's. They opt for totally different approaches in teaching; because they aimed at quite different goals, and because they thought very differently about how children learned. In doing so, they illustrate nicely the changes in Dutch mathematics education around 1800, and thereby some of the aspects of the tremendous changes that the Dutch educational system went through.

4E.P. de Booy, De Weldaet der Scholen, Utrecht (1977), pp. 58–60
5Most noteworthy in this respect were the schools of the charitable institutions of Renswoude: a middle class creation where orphans were educated to become engineers. In these institutions mathematics, and the mathematician teaching it, played an important role. These schools illustrate the growth of a group within middle class circles, that attributed more importance to mathematics. Cf. E.P. de Booy and J. Engel, Van erfenis tot studiebeurs, Delft (1985)
2 The “Cyfferinge” by W. Bartjens

The *Cyfferinge* (Calculations) by Willem Bartjens was actually an early Seventeenth Century textbook. The original knew a huge number of reprints, revised by several authors — where the main revision made was practically always the name of the revisor appearing on the title page. It keeps popping up in book lists of secondary schools until the beginning of the Nineteenth Century\(^7\). During the Eighteenth Century Bartjens’ book and its imitations became very common. In fact, many towns had their own “Bartjens”\(^8\). For example: a publisher from Enkhuysen was proud to be able to sell the one and only “Enkhuyzer” arithmetic, which appeared for the first time in 1616, but knew quite some Eighteenth Century reprints too\(^8\). None of these textbooks, however, had such an impressive printing history as did the textbook by Bartjens\(^9\).

Bartjens’ textbook was published in two volumes. The first contained the most common arithmetical operations: the pupils learned adding, subtracting, multiplying and dividing by means of a few examples, from which they had to learn the correct recipe. From a present-day point of view the first volume of Bartjens’ *Cyfferinge* contained these four arithmetical operations and some applications in whole numbers and fractions. The second volume was for those who longed for more and contained a lot of applications, sometimes with small extensions, like how to do reckoning with percentages, and the rule of false. The second volume was printed less often than the first, which suggests that most pupils restricted themselves to the first volume.

The recipe-presentation by Bartjens is best illustrated with an example. For this purpose the most crucial rule in the first part of his textbook is discussed: the so-called rule of three. This rule taught the pupil how to translate numerical information from one set of units into another, for example: Rhenish to Amsterdam miles. Learning this so crucial rule of three, the pupil was taught that there were always four quantities involved, one of which was to be calculated. Three were known, and fairly easily recognizable as being numbers with a certain meaning. Pairwise these quantities had the same name (e.g. “miles” or “feet”). The pupil was then taught to write down the unknown, the quantity with the same name as the unknown, the quantity belonging to the unknown, and the quantity belonging to the second quantity in a certain fixed order, thus obtaining \(a : b = c : x\), with \(a\), \(b\) and \(c\) known and \(x\) the unknown. To illustrate this, the teacher showed a few examples: two horses cost three pounds, how much do three horses cost? This would yield the equation:

\[
2 \text{ horses} : 3 \text{ horses} = £3 : £x
\]

The pupil was told that the equation was solved by multiplying the second and third numbers, and dividing the result by the first.

---


\(^8\) H. Willemsz., *Arithmetica ofte Reken-Konst*, Enchuyzen: R. Callebach Klenck (1751) and later editions.

\(^9\) For the Eighteenth Century this printing history has been reasonably well described in: P.J. Buijsters en L. Buijsters-Smets, *Bibliografie van Nederlandsche school- en kinderboeken 1700–1800*, Zwolle: Waanders (1997)
After a few pages with exercises the pupil was confronted with another example: two men work for one hour to finish a job; how long would one man need to finish the same job? Applying the same process here the pupil would obtain the equation:

\[ 2 \text{ men} : 1 \text{ man} = 1 \text{ hour} : x \text{ hours}. \]

This — of course — is false. But in Eighteenth Century didactics this was considered another face of the same rule: there was the (right) rule of three, and the false rule of three. The pupil was taught to check if the problem that was posed belonged to the right or to the wrong rule of three: this depended on an expected value of \( x \). Since three horses would cost more than two, the pupil would recognize this rule to be right. Since, however, one man would have to work longer than two men, this rule was wrong. He already knew what to do when the rule was right, now he was also taught what to do when the rule was wrong. If the rule was wrong, he had to multiply the first and third, and divide by the second to obtain the desired answer.

The Eighteenth Century teacher of mathematics did not want his pupil to get confused over the equation. Equations were difficult enough; since the teacher did not want to overload his pupil’s brain with the meanings of all these symbols (in fact, the notation was quite rudimentary), the aforementioned way of teaching was the most sensible approach. This is exactly the idea that can be read between the lines of the arithmetic textbook by Bartjens: a question was posed; it was solved in a number of successive operations, and afterwards the pupil could apply the solution of the example to a number of exercises. These exercises could be solved a few times to allow the pupil to practice. At the end of his lessons the pupil mastered a lot of recipes to apply in several situations.

In a 1784 edition of Bartjens’ textbook the rule of three is literally introduced as follows:

Wont alcool genemt / om drie deel getallen begrept. Het achtere tegen te rechterhand t’ gene gij begreent te betten; ende dat hem een t’ nem gelijk fit / set boor tegen te linkerhand: het treet set is t’ nemen. Multipliceert t’ getal dat tegen te rechterhand het met t’ middelste getal: t’ product dihecreet voor t’ getal dat tegen te linkerhand het / ende te wythoone alse produkt sal van lache vorder zijn. 11

And after a few pages of examples, a few other rules and a lot of exercises, the false rule of three is introduced:

Deze regel werd also genemt / om dat deze werd gemaakt contraär de rechte Regcl van Beye; want men Multipliceert achere set hoore met het middelste getal / en’t product dihecreet men boar

10Since exercises were made on a slate, pupils could not, like nowadays, read back their former solutions. Doing exercises more than once thus made perfect sense. It remained in practice until well into the Nineteenth Century, as can be read from the pedagogical journal Nieuwe Bijdragen, for example in the review of an arithmetic textbook by P.J. Baudet in the 1827 issue, pp. 367–372.

11Klaas Bosch, De vernieuwde Cijfferinge van Mr. Willem Bartjens, Amsterdam (1784), p. 36; free translation: “It is called like this / because three numbers are involved. Put to the far right the number you desire to know [meant is: the number in name equal to the unknown], and the number that belongs to it / put to the far left: put the third number in the middle. Multiply the number on the right with the number in the middle and divide it by the number on the left / and the result will be the desired value.”
The rule of three with fractions is dealt with in the same way. A lot of exercises follow.

Striking to us, is that Bartjens at first uses the rule of three only in examples of the costs of certain quantities of goods. Using the rule of three to calculate the value of Rotterdam to Amsterdam coins and vice versa is treated as if it were a different rule in another paragraph, and also the reckoning with the rule of threes in alloys of gold and silver is treated in a separate section of the book. Thus the pupil who was going in trade would not have to bother about reckoning with alloys, and the pupil who would enter the jewelry business, would not have to bother about changing the weights and currency of one city into those of another.

All the exercises in Bartjens’ textbook are applications: actually, most of the exercises in both volumes are phrased in such a way that they are likely to pop up in practice. This, of course, illustrates the main goal that Bartjens was striving for: preparing pupils for practice. For Eighteenth Century teachers, being completely dependent on the parents of their pupils for their income, did exactly what these parents demanded: make their sons reliable merchants, jewellers or surveyors by making them acquainted with the age-old rules of the profession they had learned. This situation changed completely around 1800.

3 The “Cijferkunst” by J. de Gelder

The Leyden professor Jacob de Gelder published an arithmetic textbook in two volumes as well, 1812–1814, called Allereerste gronden der Cijferkunst (First principles of arithmetic). It was to become popular, and —like Bartjens— was imitated quite often: the textbooks by J.C.J. Kempees and H. Strootman from the 1830’s bore practically the same title. Also there was the anonymous Nieuw Schoolboek der Rekenkunde (New Arithmetic Textbook), published in the late 1810’s and a textbook published in the southern part of the country (nowadays Belgium), which was even by the reviewers at the time recognized as an imitation of De Gelder’s textbook.

These textbooks were used during the first half of the Nineteenth Century. Since De Gelder explicitly rejected the use of the textbooks by Bartjens and his imitators it looks very promising to research their difference.

12Ibidem, p. 152; free translation: “This rule is named like this / because the way it is done is exactly opposite to the one of the right Rule of Three; because here you multiply the first and the middle number, and divide by the last / as is shown in the next example. You could also use the right rule of three if you write it down backwards.”

13J.C.J. Kempees, Beginselen der Cijferkunst, Breda: Broese & Comp. (1862–1863); H. Strootman, Beginselen der Cijferkunst, Breda: Gulick & Hermans (1847–1848)

14Nieuw Schoolboek der Rekenkunde, Arnhem: Paulus Nijhoff (1818–1819)

15Nieuwe Bijdragen (1827), pp. 28–29; the textbook was Eerste Gronden der Rekenkunst, Brugge: Bogaert du Mortier (1826)

16J. de Gelder, Allereerste Gronden der Cijferkunst I, Den Haag / Amsterdam (1824), pp. VIII–IX
One striking difference between Bartjens' textbook and De Gelder's was their printing history: like Bartjens', De Gelder's textbook was published in two volumes, but these two volumes were no longer really separately published. If the first volume went through a new edition, then the second volume soon followed. This illustrates that both volumes of De Gelder's textbook were seen as an indivisible part, used in an educational system that aimed at the same knowledge for all the pupils. For Bartjens' textbook this was not the case.

De Gelder spent some years of his life teaching at secondary school level and wrote a whole series of textbooks which was highly recommended by the Dutch government. In these textbooks, a lot of space was devoted to the explanation of the theorems, and the pupil was urged to comprehend every step of the proof, convince himself of the truth of every theorem very thoroughly, before going on to the next. According to De Gelder, having a clear idea what was meant was very important, according to De Gelder. Clear notions were half the solution. He emphasised the clear reasoning of the pupil: preferably he had to find a solution himself, and explain why it was correct. De Gelder started his arithmetic textbook by defining a number as the name given to a certain quantity of units. First showing that a number could thus be represented by simple scoring, he remarked that this would not be convenient if the number to score was large. This he took as an excuse to introduce the notation of our number system and the equivalent on the counting board. Next, he introduced the definitions of addition and subtraction in terms of the already established notion of a number. Before the sections on addition and subtraction, De Gelder presented a few examples of what happened if one performed these operations on a counting board. This allowed him to refer to the counting practice when he started treating the quicker way of adding and subtracting with numbers, and helped him in proving, for example, the commutativity of the addition. All this is presented as an abstraction of known things: De Gelder referred to examples from everyday life, and also in the exercises all kind of examples pop up: unlike what was done by Bartjens in his textbook, the examples are not presented in a different section, but are exercises belonging to a part of the theory. Most noteworthy: in the "rule of three" section—which is no longer a rule, but an application to which special attention is devoted—exercises for both merchants and surveyors were presented.

All these didactic novelties did not mean that all rules were abandoned. De Gelder, like Bartjens, presented rules in his arithmetic textbook. These rules, unlike the ones in Bartjens textbook, had the status of a summary: at the end of the explanation the rule came out as being the result. The pupil should be able to check the correctness of the rule, he was not confronted with examples showing how to apply it: that, he should have learned from the preceding sections.

---

17 D.J. Beckers, 'Jacob de Gelder (1765-1848) en de didactiek van de wiskunde' in: Euclides 71 nr. 8 (juni 1996), pp. 372–380
18 In fact, De Gelder was very strict about this (and other) definitions: one of his students in the early 1830's kind of frustrated made a remark about De Gelder in his diary. During an exam De Gelder had asked him what he thought a number was. He had answered: "a quantity of units". De Gelder had reprimanded him for being so slovenly. See: H. van Gelder, Hildebrands voorbereiding, Den Haag (1956), pp. 30–36.
19 J. de Gelder, Allereerste gronden der cijferkunst, Den Haag / Amsterdam (1824), pp. 11–17
De Gelder wanted his pupils to be thorough in their understanding. To test their understanding he made them explain every step they made in search of a solution. Just doing exercises was not enough for him: the pupil had to show that he understood his lessons by explaining why he did what he did, and not something else. De Gelder warned against using the verb “must”. Pupils had to explain their calculations; understanding arithmetic for De Gelder meant

in hunne eigene taal, in hunne eigen woorden, kunnen verklaren en betoogen: hierin bejaat toch het ware kennen van elk ding; zonder deze hoedanigheid te hebben verkregen, kan men niet gehouden worden de Cijferkunst te verstaan. Men zegge nimmer tot een Leerling: met dat getal moet gij multipliceren, hiermede divideren! enz. Men helpe hem, door redenering, zelf zoeken; en vindt men daartoe op het oogenblik geen tijd, late men hem liever niets doen; dit is de alleen ware en veilige weg.20

De Gelder checked the capacities of his pupils individually, and urged the buyers of his books to do the same. Apart from that, he rehearsed regularly by means of a conversation in class. He chose a problem and asked questions of specific pupils until the problem was solved. In an 1826 book on how to teach mathematics he wrote how such a rehearsing lesson ideally should be. He asked the question how to multiply 35 Rhenish rods, 7 feet, 5 inches and 11 lines by 13 and proceeded as follows:

Zeg mij, PIETER! wat zijn 35 R. roeden, 7 voeten, 5 duimen, 11 lijnen?
Hierdoor wordt eene zeker lengte voorgefteld, die eenige geheelen, deelen en onderdeelen bevat.
PauLUS! welke zijn hier de geheelen?
Rijndische roeden.
Welke is de aangenomene verdeeling van zulk eene roede?
Eene Rijndische roede bevat 12 voeten, een voet 12 duimen en een duim 12 lijnen.
Zeg mij nu, Hendrik! wat beteekent het die lengte met 13 5/16 te vermenigvuldigen?
Dat men eene lengte zoeken moet, die dertien maal de gegevene lengte bevat, en daar nog moet bijvoegen een lengte, die gelijk is aan vijf maal één zestiende deel van de gegevene lengte.
Zeer wel geantwoord: maar zeg mij nu eens, wat is vermenigvuldigen?
In geheele getallen die kortere bewerking, om, buiten de gewone optelling, de fom van gelijke getallen te vinden.
Maar, Willem! wij hebben wel geleerd getallen met getallen te vermenigvuldigen; maar hier wordt gevraagd eene lengte met een getal te vermenigvuldigen; hoe moet dit verstaan worden?21

20J. de Gelder, Allereerste gronden der Cijferkunst, Den Haag / Amsterdam (18243), p. XIX; free translation: “in one's own language, in one's own words, being able to explain and give proofs: that is real knowledge; if one doesnot possess that quality, then one doesnot understand arithmetic. Never say to a pupil: you must multiply or divide by this number — instead, help your pupils by means of reasoning to discover a correct approach; and if you cannot find the time to do so, better have them do nothing; that is the only safe way.”

21 Jacob de Gelder, Verhandeling over het verband en den samenhang der natuurlijke en zedelijke
Thus De Gelder was indicating the way how to solve the problem himself, but the pupils were forced to express themselves accurately — knowing precisely what you were talking about, was half the solution in De Gelder's opinion. He wanted to know if the pupils had really understood their reckoning. This he checked by interrogating the pupil. The way how to solve the exercise was more or less indicated by him; he didn't want to be bothered with different ways of tackling the problem. His pupils already knew several ways of tackling the problem: several solutions had been demonstrated, they had done some exercises, and since they were encouraged to think about an exercise for a longer time rather than doing a lot of exercises quickly but mechanically.

4 Concluding remarks

The transition that took place in Dutch mathematics education around 1800 actually created a whole new subject. The Cijferkunst by De Gelder presented to the pupil a more or less formal introduction to the pure mathematical backgrounds of reckoning. Of course attention was paid to the many applications of this theory. Not only in order to show the importance of the mathematics the pupil was studying, but also because mathematics itself was seen as an abstract representation of reality. Contrariwise, Bartjens' textbook, representing the Eighteenth Century view, treated many separate rules. Application was not an issue in Eighteenth Century mathematics education: the subject itself was applied. Only those rules most useful to the pupil were necessary to learn.

The publication history of the two books illustrate the emergence of a new, government controlled, curriculum. The second volume of Bartjens' textbook was not reprinted together with the first, while De Gelder's arithmetics were published in succession. Didactically the difference was also striking: whereas De Gelder was aiming at making the pupils understand what they were doing, and why, Bartjens taught his pupils a recipe which — if properly followed — would yield the correct result, without

---

wetenschappen, Den Haag / Amsterdam (1826), p. 385; free translation:

“Tell me, Peter! what is 35 Rhenish rods, 7 feet, 5 thumbs and 11 lines?
It is a certain length, which consists of some units and parts of this unit.
Paul! which are the units in this case?
Rhenish rods.
And how are the parts of this Rhenish rod defined?
A Rhenish rod consists of 12 feet, a feet of 12 inches and an inch of 12 lines.
Now tell me, Henry! what does it mean to multiply this length by $13\frac{5}{16}$?
It means that we are looking for a length that consists of thirteen times the given length plus five times one sixteenth part of that length.
Very well indeed; but tell me, what do you mean by multiplying?
In whole numbers multiplying means a short algorithm to find the sum of a number of equal terms, without straightforwardly adding them.
But, William! we know how to multiply numbers by numbers; but now we are asked to multiply a length by a number; how must we conceive this?"
This last remark is in sharp contrast with modern educational practice, in which children are even deluded into believing that multiplying 1 meter by 1 meter results in a square meter.

ibidem, p. 365 e.v.
any reflection.

The two textbooks discussed beautifully illustrate three important new aspects that appeared in Dutch Society around 1800: a new centralized educational system, a new didactic approach and the changing view of the Dutch middle class on arithmetic. In my opinion it would also be rewarding to compare a popular textbook with its imitations. It could reveal much of the silent assumptions underneath the teaching methods. Differences and constant factors in these textbooks can tell us a lot about teaching, but this is quite another matter which lies well beyond the scope of this article.

---

A slightly modified version of this paper has been accepted for publication in *Paradigm*, the journal of the Textbook Colloquium. It will appear in the first issue of 1999. Since this journal is not very common in the Netherlands, the author decided to make this version available as well.