An LSL to PVS Compiler

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ABSTRACT
A compiler which translates axiomatic specifications written in the Larch Shared Language (LSL) to theories of the Prototype Verification System (PVS) is described by means of examples. Besides giving an equivalent axiomatic specification in PVS, the compiler enables one to check that an LSL specification has a model in the higher-order logic of PVS.


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1. Introduction

The Larch Shared Language (LSL)[GH93] is an axiomatic specification language based on a multi-sorted first-order logic with equality. Theories axiomatically specified in LSL, by means of traits, are similar to conventional algebraic specifications; in these traits, sorts and operators are introduced together with assertions (properties which are known to hold) and consequences (properties which are thought to be derivable). Normally, traits are translated to input for the Larch Prover (LP), a first order theorem prover, where one can prove the intended consequences from the asserted properties given in the trait. The Prototype Verification System (PVS)[ORSH95] is a theorem prover based on classic higher-order logic. In this paper, we describe a compiler which translates LSL traits to theories of the PVS language.

There is a well-known problem with axiomatic specifications: inconsistent specifications are easily given in an axiomatic specification language, this is also discussed by Jim Horning in [Hor96]. The compiler helps to address this problem by translating LSL traits to two PVS theories: besides an axiomatization of the trait in which stated consequences are to be proven, a second theory is generated in which it is to be shown that the axiomatization can be modeled by a conservative extension.

There are a number of ways in which the compiler can be used. The most interesting is that users of LP may use the compiler to show that an axiomatic specification has a model within the logic of PVS. Also, users of PVS can use the compiler to work with LSL theories; or, it is possible to use the compiler to have properties proven by either LP or PVS.

In this paper, no formal claims are made about semantical equivalence of LSL traits and PVS theories generated; an important question to be addressed when combining two proof tools. However, since LSL has straightforward set-theoretic semantics, a simple translation to PVS was easily found which
we believe preserves semantical equivalence.

The outline of the paper is as follows. In section two, a number of examples show how LSL traits are translated to PVS. In the third section, we explain work done and future work on the LSL compiler. We end with the conclusions in the last section, section four.

2. Translating LSL to PVS

In this section we informally describe LSL and the translation to PVS.

2.1 Traits

First, let us consider a simple example, the specification of a group.

A group is an algebra \(< S, o, 1 >\) in which

1. \(o\) is a binary, associative operator,
2. \(o\) has the identity 1 in \(S\),
3. every element \(b\) of \(S\) has an inverse, denoted as \(b^{-}\).

As stated before, LSL supports a multi-sorted first-order logic with equality. Sorts are non-empty sets of objects. Below we show the LSL specification of a group. LSL's basic unit of specification is a trait; the name of the trait in the example is Group. In the introduces clause, the operators \(o\), \(unit\), and \(inv\) are declared; the sort \(S\) is implicitly declared by its occurrence. The asserts clause defines their properties conform the definition given above; in the implies clause, purported consequences of these properties are given.

```
Group : trait
  introduces
    o : S, S -> S,
    unit : -> S,
    inv : S -> S
  asserts with a, b, c : S .
    o(o(a, b), c) == o(a, o(b, c));
    o(unit, a) == a;
    o(a, unit) == a;
    o(inv(a), a) == unit;
    o(a, inv(a)) == unit
  implies with a, b, c, d, x : S .
    b == inv(inv(b));
    o(b, d) = o(c, d) == b = c;
    o(d, b) = o(d, c) == b = c;
    o(b, x) = c == x = o(inv(b), c);
    o(x, b) = c == x = o(c, inv(b));
    b = c == o(d, b) = o(d, c);
    b = c == o(b, d) = o(c, d);
  \E x : S . o(b, x) = c;
  \E x : S . o(x, b) = c
```

First, we show the axiomatic theory generated in PVS below. As stated in the trait, \(S\) is a nonempty set (as specified by the PVS keyword TYPE+, and the operators \(o\), \(unit\), \(inv\) are introduced as uninterpreted constants. Meaning is given to the constants by a number of AXIOMs, the translation of the assertions of the trait; and intended consequences are translated to LEMMAS.
2. Translating LSL to PVS

Group: THEORY
BEGIN

S : TYPE+;
o : [\[S, S\]->S];
unit : S;
inv : [\[S\]->S]

assertion0 : AXIOM FORALL (a : S, b : S, c : S) : (o(o(a,b),c) = o(a,o(b,c)));
assertion1 : AXIOM FORALL (a : S) : (o(unit,a) = a);
assertion2 : AXIOM FORALL (a : S) : (o(a,unit) = a);
assertion3 : AXIOM FORALL (a : S) : (o(inv(a),a) = unit);
assertion4 : AXIOM FORALL (a : S) : (o(a,inv(a)) = unit);

consequence0 : LEMMA FORALL (b : S) : (b = inv(inv(b)));
consequence1 : LEMMA FORALL (b : S, d : S, c : S) : ((o(b,d) = o(c,d)) = (b = c));
consequence2 : LEMMA FORALL (d : S, b : S) : ((o(d,b) = o(d,c)) = (b = c));
consequence3 : LEMMA FORALL (b : S, x : S, c : S) : ((o(b,x) = c) = (x = o(inv(b),c)));
consequence4 : LEMMA FORALL (x : S, b : S, c : S) : ((o(x,b) = c) = (x = o(c,inv(b))));
consequence5 : LEMMA FORALL (b : S, c : S, d : S) : ((b /= c) = (o(d,b) /= o(d,c)));
consequence6 : LEMMA FORALL (b : S, c : S, d : S) : ((b /= c) = (o(b,d) /= o(c,d)));
consequence7 : LEMMA FORALL (b : S, c : S) : EXISTS (x : S) : (o(b,x) = c);
consequence8 : LEMMA FORALL (b : S, c : S) : EXISTS (x : S) : (o(x,b) = c);

END Group

We show the proof of “for all b in S we have b = (b^-)” (consequence0) mathematically together with the corresponding PVS proof script.

Take an arbitrary b in S (SKOLEM 1 "b"). Then

(b^-) = < 1 is the left identity >
LEMA "assertion1" ("a" "inv(inv(b))") (REPLACE -1 1 RL :HIDE? T)
1o(b^-) = < definition of right inverse >
(LEMA "assertion4" ("a" "b") (REPLACE -1 1 RL :HIDE? T)
(bob^-)o(b^-) = < associativity o >
(USE "assertion0") (REPLACE -1 1 :HIDE? T)
bo(b^-)o(b^-) = < definition of right inverse >
(USE "assertion4") (REPLACE -1 1 :HIDE? T)
bo1 = < 1 is the right identity >
(USE "assertion2") (ASSERT)
b

Below, we show a generated “model” theory; the user is expected to provide (type in) instantiations for the uninterpreted constants such that the lemmas generated hold. Of course, we expect that instantiations given are taken (or constructed) out of the logic of PVS in a conservative manner: i.e., we expect them to be well-defined. In this case, we show that <Z, +, 0> is a group by providing the necessary instantiations. In the example, all proofs of the lemmas are trivially done by PVS with the proof strategy (GRIND).

GroupModel: THEORY
BEGIN

S : TYPE+ = int;
o : [[S, S]->S] = +;
2. Translating LSL to PVS

unit : S = 0;
inv : [[S]—>S] = -;

assertion0 : LEMMA FORALL (a : S, b : S, c : S) : (o(o(a,b),c) = o(a,o(b,c)));
assertion1 : LEMMA FORALL (a : S) : (o(unit,a) = a);
assertion2 : LEMMA FORALL (a : S) : (o(a,unit) = a);
assertion3 : LEMMA FORALL (a : S) : (o(inv(a),a) = unit);
assertion4 : LEMMA FORALL (a : S) : (o(a,inv(a)) = unit);

END GroupModel

2.2 Inductive Sorts

The previous example was very basic. LSL allows one to specify more complex sorts, such as sorts which are inductively generated by a set of operators. Below we give an example, a trait specifying a sort of unary numbers U by means of two generators: an initial constructor 0 and a successor operator Succ.

UnaryNumbers : trait
introduces
  0: -> U,
  Succ: U —> U
asserts
  sort U generated by 0: -> U, Succ: U —> U

The generated by assertion states that every object in U can be generated by a finite number of (legal) applications of the two generators 0 and Succ.

Below the PVS axiomatization of the above trait is given. Two axioms are generated: U_cover states that every object in U can be constructed from one of the two generators, the next axiom U_induction states the induction proof principle over the generators. Note that the first axiom follows from the latter; however, we prefer to give both for convenience.

UnaryNumbers : THEORY
BEGIN
  U : TYPE+;
  0 : U;
  Succ : [[U]—>U]

  U_cover : AXIOM FORALL (u0 : U) : ((0 = u0) OR EXISTS (u1 : U) : (u0 = Succ(u1)));

  U_induction : AXIOM FORALL (P : [[U]—>bool]) :
    ((P(0) AND FORALL (u0 : U) : (P(u0) => P(Succ(u0))))) => FORALL (u0 : U) : P(u0));

END UnaryNumbers

However, the above specification is too weak as it does not state that all unary numbers generated are different. This can be specified by adding the keyword freely to the specification. This states that every object in the sort is uniquely described by a finite number of applications of the generators.

sort U generated freely by 0: -> U, Succ: U —> U
2. Translating LSL to PVS

If the freely keyword is added to the generated by assertion, two extra axioms are generated.

\[
\begin{align*}
U\text{-disjoint} & : \text{AXIOM} \\
& \quad \text{FORALL } (u_0 : U) : \neg((0 = \text{Succ}(u_0)))
\end{align*}
\]

\[
\begin{align*}
U\text{-injective} & : \text{AXIOM} \\
& \quad \text{FORALL } (u_0 : U, u_1 : U) : ((\text{Succ}(u_0) = \text{Succ}(u_1)) \Rightarrow (u_0 = u_1))
\end{align*}
\]

2.3 Observers on sorts

In LSL, one can state that a set of operators is a set of observers for a sort; that is, each two objects in a sort which give similar results under the observers are equal. Below, an example is given. The example states that two states of a lamp are the same if you cannot observe a difference by looking at it.

In the trait below, the partitioned by assertion states that \textit{look} is an observer over the sort \textit{States}.

\[
\begin{align*}
\text{Lamp} & : \text{trait} \\
& \quad \text{introduces} \\
& \quad \quad \text{on,off} : \text{State} \rightarrow \text{State}, \\
& \quad \quad \text{look} : \text{State} \rightarrow \text{bool} \\
& \quad \text{asserts with } s : \text{State}. \\
& \quad \quad \text{sort } \text{State partitioned by } \text{look} : \text{State} \rightarrow \text{bool} \\
& \quad \quad \text{look}(\text{on}(s)); \\
& \quad \quad \neg\text{look}(\text{off}(s))
\end{align*}
\]

The assertion above is translated to the following axiom.

\[
\begin{align*}
\text{State partition} & : \text{AXIOM FORALL } (s\_varO : \text{State}, s\_var1 : \text{State}): \\
& \qquad ((\text{look}(s\_var0) = \text{look}(s\_varl)) \Rightarrow (s\_var0 = s\_varl));
\end{align*}
\]

2.4 Reusing specifications

LSL allows traits to be reused in other traits in two manners: by including them, in this case the sorts and operators defined in the included traits are added to the including trait, or by assuming traits, then the axioms of the assumed trait are added as assumptions to the assuming trait.

Whenever a trait is included or assumed, sorts and operators in that trait may be renamed. An included or assumed sort coincides with another sort if, possibly after renaming, they have the same name; an included or assumed operator coincides with another operator if, possibly after renaming, they have the same name and the same type. Coinciding sorts, or operators, are understood to denote the same sort, or operator.

Including traits In the example below, the inclusion (and renaming) mechanism of LSL is demonstrated. A sort, or operator, may always be renamed by means of the for construct. However, sometimes it is more convenient to provide a trait with a number of parameters of often renamed sorts, or operators. This is shown in the second trait named transitive.

\[
\begin{align*}
\text{irreflexive} & : \text{trait} \\
& \quad \text{introduces} \\
& \quad \quad \text{less}:T, T\rightarrow\text{bool} \\
& \quad \text{asserts with } x:T. \\
& \quad \quad \neg\text{less}(x, x).
\end{align*}
\]
2. Translating LSL to PVS

transitive (S, less:S,S->bool) : trait
    introduces
    less:S, S->bool
    asserts with x, y, z:S .
    less(x, y)/\less(y, z)=>less(x, z).

partialorder(R, less:R,R->bool) : trait
    includes
    irreflexive(R for T, le:R,R -> bool for less:T,T ->bool),
    transitive(R, le:R,R->bool).

In the resulting theory for the the partialorder trait, one sort R and one operator le are axiomatically described.

partialorder: THEORY
BEGIN
    R : TYPE*;
    le : [[R, R]->bool]

    assertion0 : AXIOM FORALL (x : R) : NOT(le(x,x));
    assertion1 : AXIOM FORALL (x : R, y : R, z : R) : ((le(x,y) AND le(y,z)) => le(x,z));

END partialorder

We would like to note that if a trait is included or assumed, the consequences of that trait become axioms in the including or assuming trait since they are supposed to be derivable.

Assuming traits    Since one can include traits in other traits, it is often convenient to describe that a trait may only be included in a certain context, i.e. when a number of assumptions hold. For instance, in the example below, trait reflexiveclosure states that a reflexive closure operator lesseq only exists whenever < T, less > is a partial order.

reflexiveclosure (T, less:T,T->bool) : trait
    assumes partialorder(T, less:T,T->bool)
    introduces
    lesseq:T, T->bool
    asserts with x,y:T .
    lesseq(x, y) == IF x=y THEN true ELSE less(x,y) FI
    implies
    trait transitive (T, lesseq:T,T->bool).

In the trait reflexiveclosure, the axioms of the assumed trait are assumed to hold. However, when trait natrefflexive, shown below, includes this trait, the axioms of trait partialorder become proof obligations in the including trait.

natrefflexive : trait
    includes reflexiveclosure(nat for T)
    asserts with x,y : nat .
    less(x,y) == x < y
    implies with x,y : nat .
    lesseq(x,y) == x <= y.
The translation of trait `natrexflexive` now becomes non-trivial to read; it is shown below. First, the assertions of the trait are listed as axioms. In this case, the only assertion is a definition for the `less` operator. After the assertions, the to be proven assumptions of the included trait `reflexiveclosure` are listed as lemmas. Then, the assertions and the consequences of the included trait are listed as axioms, since they are known to hold. Lastly, the only intended consequence stated in the trait `natrexflexive` is listed as a lemma.

```
natreflexive: THEORY
  BEGIN
    less : [[nat, nat]->bool];
    lesseq : [[nat, nat]->bool]
    assertion0 : AXIOM
      FOEALL (x : nat, y : nat) : (less(x,y) = (x < y));
    assumption0 : LEMMA
      FOEALL (x : nat) : NOT(less(x,x));
    assumption1 : LEMMA
      FOEALL (x : nat, y : nat, z : nat) : ((less(x,y) AND less(y,z)) => less(x,z));
    assertion1 : AXIOM
      FOEALL (x : nat, y : nat) : (lesseq(x,y) = IF (x = y) THEN true ELSE less(x,y) ENDIF);
    consequence0 : AXIOM
      FOEALL (x : nat, y : nat, z : nat) : ((lesseq(x,y) AND lesseq(y,z)) => lesseq(x,z));
    consequence1 : LEMMA
      FOEALL (x : nat, y : nat) : (lesseq(x,y) = (x <= y));
  END natreflexive
```

Note that in the previous trait the sort `nat` is not introduced since it is, as is the sort `bool`, a predefined sort of LSL.

**Discharging assumptions** A drawback of including or assuming traits is that assertions or consequences are easily included multiple times through several “paths”. However, when possible, superfluous (syntactically equivalent) properties are discharged by the compiler.

### 2.5 Shorthands

Shorthands abbreviate often used specifications. LSL has shorthands for enumerations, tuples and unions. On the following pages, examples of these shorthands are given together with their intended meaning.

**Enumerations** In the example below, the enumeration shorthand is used to define the sort `YRB` as an enumeration of yellow, red, and blue.

```
Kandinski : trait
  sort YRB enumeration of yellow, red, blue.
```

The above trait is understood to be equivalent to the trait below.

```
Kandinski :trait
```

introduces
  yellow, red, blue: -> YEB,
  succ: YEB -> YEB
asserts
  sort YEB generated freely by yellow, red, blue;
  succ(yellow) = red;
  succ(red) = blue

Tuples In the example below, the tuple shorthand is used to define the sort Edge as an pair of
Nodes named in and out.

Graph : trait
  sort Edge tuple of in: Node, out: Node.

The above trait is understood to be equivalent to the trait below.

Graph: trait
  introduces
    T: Node, Node -> Edge,
    in: Edge -> Node,
    out:Edge -> Node,
    set_in: Edge, Node -> Edge,
    set_out: Edge, Node -> Edge
  asserts with n0,n1,n2 : Node .
    sort Edge generated freely by T: Node, Node->Edge;
    sort Edge partitioned by in, out: Edge -> Node;
    in(T(n0,n1)) = n0;
    out(T(n0,n1)) = n1;
    set_in(T(n0,n1), n2) = T(n2,n1);
    set_out(T(n0,n1), n2) = T(n0,n2).

Unions Below, the union shorthand defines the sort Possession as a distinct product of yours or
mine elements in the sort T.

Possession(T) : trait
  sort Possession[T] union of yours: T, mine : T.

The above trait is understood to be equivalent to the trait below.

Possession: trait
  sort tag[Possession[T]] enumeration of yours, mine
  introduces
    tag: Possession[T] -> tag[Possession[T]],
    yours: T -> Possession[T],
    mine: T -> Possession[T],
    get_yours: Possession[T] -> T,
    get_mine: Possession[T] -> T
  asserts with t0 : T.
    sort Possession[T] generated by
      yours, mine: T -> Possession[T];
    sort Possession[T] partitioned by
      tag: Possession[T] -> tag[Possession[T]],
      get_yours, get_mine: Possession[T] -> T;
3. Related and Future Work

The compiler described is intended to become part of the IOA toolset [GL98]. The toolset is developed around the IOA language, a language for describing distributed systems as input/output automata according to the model of Lynch and Tuttle [LT89] [Lyn96]. Tools under development are a simulator, a compiler to the Larch theorem prover [GH93] and a compiler to the model checker SPIN [Hol91]. My interest is to compile the IOA language to PVS and, if possible, to Isabelle/HOL [Pau94]. Since LSL is part of the IOA language, the LSL to PVS compiler has been developed as a first step in this project. We have chosen to implement a new compiler to be able to support possible future higher-order logic extensions.

Below a picture describing the structure of the compiler is shown. The compiler takes a number of LSL specifications as its input and translates it in a number of steps to a PVS theory file. The boxes describe several components of the compiler, and the arrows the data structures passed between them. Also, it is shown which components can produce errors. The component which translates to an Isabelle/HOL specification file has not been implemented yet.

Below, we discuss different parts of the tool and related work.

- The **scanner** takes an input file and translates it into a sequence of tokens. It was implemented with the javacc compiler generator.

- The **parser** recognizes a grammar and transforms it into a parse tree. The grammar recognized is an LL(2) subset of the original LSL language to ease the translation. This grammar is developed with use of the Grammar Work Bench [NKDvZ92]; a tool which we also use to generate terms in the LSL language to test the parser implemented. Since the language recognized is LL(2), it is implemented in a straightforward manner as a recursive descent parser.

- The **checker** translates the parse tree into a context. The checker checks semantic consistency of the input and performs the actual translation. The context build describes translated input as a collection of terms in an intermediate higher-order logic language. The intermediate language is used to be able to easily translate to different (higher-order logic) proof tools. This intermediate language and the translation is described in a paper available at [http://www.cs.kun.nl/~marcod/](http://www.cs.kun.nl/~marcod/). In this paper, the specification of the checker is given as a

```plaintext
tag(yours(t0)) = yours;
tag(mine(t0)) = mine;
get_yours(yours(t0)) = t0;
get_mine(mine(t0)) = t0.
```
transducer denoted as an attribute grammar where attributes range over terms in this intermediate language.

- The PVS printer or the HOL printer translate the intermediate representation into either PVS or Isabelle/HOL specification files.

The LSL compiler has been developed in Java[AG] in order to be able to easily link this compiler to other tools in the toolset. The following line describes how to start the tool:

```java
java lsl [-/+check][-/+tokens][-/+unparse][-/+context][-/+pvs][-/+hol] fn.lsl
```

The check switch may be used to semantically check an input file. The other switches produce the output of one of the components referred to. For instance, the context switch will produce a printed representation of the context, or the pvs switch will produce PVS output. The compiler is lazy in the sense that if only the unparse switch is set, for instance, it will only print the parse tree but will not perform semantic analysis.

We see two obvious extensions to the LSL tool. One is the addition of labeling of the axioms and consequences denoted in the original LSL specification to facilitate proof development in the theorem provers compiled to. For instance, at the moment, it reads awkward that a theorem named consequence4 holds by application of assertion2 and consequence0. The second extension would be to automatically generate proof strategies. For instance, proof strategies for induction principles generated may automate part of proving efforts.

However, future work will be mainly in the IOA toolset context. The attribute grammar for LSL is to be extended to an attribute grammar which describes a transducer for the whole IOA language, and the LSL compiler is to be extended to an IOA compiler.

### 4. Conclusions

We have presented a compiler which translates axiomatic specifications given in the Larch Shared Language to two theories for PVS: one theory in which it is possible to reason about the axiomatic specification given, and a second theory in which it can be shown that the axiomatic specification has a model. In this manner, a well-known problem with LSL, it is very easy to write down inconsistent axiomatic specifications, can be addressed. No formal claims are made about semantic equivalence of input and output theories; however, it is argued that, since input and output theories have the same set-theoretic underpinnings, we believe that they are semantically equivalent.

The LSL compiler has been developed as part of a toolset for the IOA specification language of which the LSL language forms a subset. There are a number of interesting features which can be implemented in the compiler; either for LSL users (conservative extensions, subsorting) or for PVS users (generation of proof strategies). However, the compiler is first to be extended to recognize and translate the total IOA language.

**Note:** A beta-release of the compiler can be tried at [http://www.cs.kun.nl/~marcod/lsl.html](http://www.cs.kun.nl/~marcod/lsl.html)

### References


References


