Measurement of longitudinal flow decorrelations in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV with the ATLAS detector

ATLAS Collaboration*

CERN, 1211 Geneva 23, Switzerland

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Abstract Measurements of longitudinal flow correlations are presented for charged particles in the pseudorapidity range $|\eta| < 2.4$ using 7 $\mu$b$^{-1}$ and 470 $\mu$b$^{-1}$ of Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV, respectively, recorded by the ATLAS detector at the LHC. It is found that the correlation between the harmonic flow coefficients $v_n$ measured in two separated $\eta$ intervals does not factorise into the product of single-particle coefficients, and this breaking of factorisation, or flow decorrelation, increases linearly with the $\eta$ separation between the intervals. The flow decorrelation is stronger at 2.76 TeV than at 5.02 TeV. Higher-order moments of the correlations are also measured, and the corresponding linear coefficients for the $k$th-moment of the $v_n$ are found to be proportional to $k$ for $v_3$, but not for $v_2$. The decorrelation effect is separated into contributions from the magnitude of $v_n$ and the event-plane orientation, each as a function of $\eta$. These two contributions are found to be comparable. The longitudinal flow correlations are also measured between $v_n$ of different order in $n$. The decorrelations of $v_2$ and $v_3$ are found to be independent of each other, while the decorrelations of $v_4$ and $v_5$ are found to be driven by the nonlinear contribution from $v_2^3$ and $v_2 v_3$, respectively.

1 Introduction

Heavy-ion collisions at RHIC and the LHC create hot, dense matter whose space-time evolution is well described by relativistic viscous hydrodynamics [1,2]. Owing to strong event-by-event (EbyE) density fluctuations in the initial state, the space-time evolution of the produced matter also fluctuates event by event. These fluctuations lead to correlations of particle multiplicity in momentum space in both the transverse and longitudinal directions with respect to the collision axis. Studies of particle correlations in the transverse plane have revealed strong harmonic modulation of the particle densities in the azimuthal angle: $dN/d\phi \propto 1 + 2 \sum_{n=1}^{\infty} v_n \cos n(\phi - \Phi_n)$, where $v_n$ and $\Phi_n$ represent the magnitude and event-plane angle of the $n^{th}$-order harmonic flow. The measurements of harmonic flow coefficients $v_n$ and their EbyE fluctuations, as well as the correlations between $\Phi_n$ of different order [3–9], have placed important constraints on the properties of the dense matter and on transverse density fluctuations in the initial state [10–15].

Most previous flow studies assumed that the initial condition and space-time evolution of the matter are boost-invariant in the longitudinal direction. Recent model studies of two-particle correlations as a function of pseudorapidity $\eta$ revealed strong EbyE fluctuations of the flow magnitude and phase between two well-separated pseudorapidities, i.e. $v_n(\eta_1) \neq v_n(\eta_2)$ (forward-backward or FB asymmetry) and $\Phi_n(\eta_1) \neq \Phi_n(\eta_2)$ (event-plane twist) [16–18]. The CMS Collaboration proposed an observable based on the ratio of two correlations: the correlation between $\eta$ and $\eta_{ref}$ and the correlation between $-\eta$ and $\eta_{ref}$. This ratio is sensitive to the correlation between $\eta$ and $-\eta$ [19]. The CMS results show that the longitudinal fluctuations lead to a linear decrease of the ratio with $\eta$, and the slope of the decrease shows a strong centrality dependence for elliptic flow $v_2$ but very weak dependences for $v_3$ and $v_4$. This paper extends the CMS result by measuring several new observables based on multi-particle correlations in two or more $\eta$ intervals [20]. These observables are sensitive to the EbyE fluctuations of the initial condition in the longitudinal direction. They are also sensitive to nonlinear mode-mixing effects, e.g. $v_4$ contains nonlinear contributions that are proportional to $v_2^3$ [8,9,21–23]. Furthermore, the measurements are performed at two nucleon–nucleon centre-of-mass collision energies, $\sqrt{s_{NN}} = 2.76$ TeV and 5.02 TeV, to evaluate the $\sqrt{s_{NN}}$ dependence of the longitudinal flow fluctuations. Recent model calculations predict an increase of longitudinal flow fluctuations at lower $\sqrt{s_{NN}}$ [24]. Therefore, measurements of these observables at two collision energies can provide new insights into the initial condition along the longitudinal direction and should help in the development of full three-dimensional viscous hydrodynamic models.

* e-mail: atlas.publications@cern.ch

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Using these new observables, this paper improves the study of the longitudinal dynamics of collective flow in three ways. Firstly, the CMS measurement, which is effectively the first moment of the correlation between \( v_n \) in separate \( \eta \) intervals, is extended to the second and the third moments. Secondly, a correlation between four different \( \eta \) intervals is measured to estimate the contributions from the fluctuations of \( \Phi_n \). Thirdly, correlations between harmonics of different order are also measured, e.g. between \( v_2 \) and \( v_4 \) in different \( \eta \) intervals, to investigate how mode-mixing effects evolve with rapidity. In this way, this paper presents a measurement of flow decorrelation involving \( v_2 \), \( v_3 \), \( v_4 \) and \( v_5 \), using Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) and 5.02 TeV.

2 Observables

This section gives a brief summary of the observables measured in this paper, further details can be found in Refs. [19,20,25]. The azimuthal anisotropy of the particle production in an event is conveniently described by harmonic flow vectors \( V_n = v_n e^{i \Phi_n} \), where \( v_n \) and \( \Phi_n \) are the magnitude and phase (or event plane), respectively. The \( V_n \) are estimated from the observed per-particle normalised flow vector \( q_n \) [5]:

\[
q_n \equiv \frac{\sum w_i e^{i \Phi_i}}{\sum w_i}.
\]

The sums run over all particles in a given \( \eta \) interval of the event, and \( \Phi_i \) and \( w_i \) are the azimuthal angle and the weight assigned to the \( i \)th particle, respectively. The weight accounts for detector non-uniformity and tracking inefficiency.

The longitudinal flow fluctuations are studied using the correlation between the \( k \)th-moment of the \( n \)th-order flow vectors in two different \( \eta \) intervals, averaged over events in a given centrality interval, \( r_{n|n;k} \), for \( k = 1,2,3 \):

\[
r_{n|n;k} (\eta) = \frac{\langle q_n(\eta) q_n^*(\eta_{ref}) \rangle}{\langle q_n^2(\eta) \rangle},
\]

\[
= \left| \frac{\langle v_n(\eta) v_n(\eta_{ref}) \rangle}{\langle v_n^2(\eta) \rangle} \right|^k \cos k \Phi_n(\eta) - \Phi_n(\eta_{ref}) \right),
\]

where \( \eta_{ref} \) is the reference pseudorapidity common to the numerator and the denominator, the subscript "\( n \)" denotes the \( n \)th-moment of the flow vectors of order \( n \) at \( \eta \), combined with the \( k \)th moment of the conjugate of the flow vector of order \( n \) at \( \eta_{ref} \). The sine terms vanish in the last expression in Eq. (2) because any observable must be an even function of \( \Phi_n(\eta) - \Phi_n(\eta_{ref}) \). A schematic illustration of the choice of the \( \eta \) (|\( \eta | < 2.4 \)) and \( \eta_{ref} \) (|\( \eta_{ref} | < 4.9 \)) to be discussed in Sect. 5, as well as the relations between different flow vectors, are shown in the left panel of Fig. 1. This observable is effectively a 2k-particle correlator between two subevents as defined in Ref. [28], and the particle multiplets containing duplicated particle indices are removed using the cumulant framework, with particle weights taken into account [20].

The observable measured by the CMS Collaboration [19] corresponds to \( k = 1 \), i.e. \( r_{n|n;1} \). It should be noted that \( \langle q_n(\eta) \rangle = 0 \) because the event plane changes randomly from event to event. Hence a direct study of the correlation between \( +\eta \) and \( -\eta \) via a quantity such as \( \langle q_n(\eta) q_n^*(\eta_{ref}) \rangle / \langle q_n^2(\eta) \rangle \) is not possible. One could also consider a quantity like \( \langle q_n(\eta) q_n^*(\eta_{ref}) \rangle / \langle q_n^2(\eta) \rangle \) but the denominator would be affected by short-range correlations. Hence, it is preferable to work with quantities of the type used in Eq. (2), which give a correlator sensitive to the flow decorrelation between \( \eta \) and \( -\eta \) through the reference flow vector \( q_n^k(\eta_{ref}) \).

One important feature of Eq. (2) is that the detector effects at \( \eta_{ref} \) are expected to cancel out to a great extent (see Sect. 5). To ensure a sizeable pseudorapidity gap between

\[\text{(a) } r_{n|n;k}(\eta) = \frac{\langle q_n(\eta_1) q_n^*(\eta_{ref}) \rangle}{\langle q_n^2(\eta_1) \rangle}, \]

\[\text{(b) } r_{n|n;2}(\eta) = \frac{\langle q_n(\eta_2) q_n^*(\eta_{ref}) \rangle}{\langle q_n^2(\eta_2) \rangle}, \]

\[\text{Fig. 1 Schematic illustration of the procedure for constructing the correlators } r_{n|n;k}(\eta) \text{ Eq. (2) (left panel) and } r_{n|n;2}(\eta) \text{ Eq. (5) (right panel). The acceptance coverages for the ATLAS tracker used for } \eta \text{ and reference detector used for } \eta_{ref} \text{ are discussed in Sect. 5.}\]
the flow vectors in both the numerator and denominator of Eq. (2), \( \eta_{\text{ref}} \) is usually chosen to be at large pseudorapidity, e.g. \( \eta_{\text{ref}} > 4 \) or \( \eta_{\text{ref}} < -4 \), while the pseudorapidity of \( q_n(\eta) \) and \( q_n(\eta) \) is usually chosen to be close to mid-rapidity, \( |\eta| < 2.4 \). If flow harmonics from multi-particle correlations factorise into single-particle flow harmonics, e.g. \( \langle V_n^k(\eta) V_n^k(\eta) \rangle = \langle v_n^k(\eta) \rangle \langle v_n^k(\eta) \rangle \), then it is expected that \( r_{n|n;k}(\eta) \approx 1 \). Therefore, a value of \( r_{n|n;k}(\eta) \) different from 1 implies a factorisation-breaking effect due to longitudinal flow fluctuations, and such an effect is generally referred to as “flow decorrelation”.

Based on the CMS measurement [19] and arguments in Ref. [20], the observable \( r_{n|n;k}(\eta) \) is expected to be approximately a linear function of \( \eta \) with a negative slope, and is sensitive to both the asymmetry in the magnitude of \( v_n \) and the twist of the event-plane angles between \( \eta \) and \( \eta \):

\[
r_{n|n;k}(\eta) \approx 1 - 2 F_{n;k}^r \eta, \quad F_{n;k}^r = F_{n;k}^{\text{asy}} + F_{n;k}^{\text{twi}},
\]

where \( F_{n;k}^{\text{asy}} \) and \( F_{n;k}^{\text{twi}} \) represent the contribution from FB \( v_n \) asymmetry and event-plane twist, respectively. The \( r_{n|n;k} \) results obtained in Ref. [19] were for \( k = 1 \) and \( n = 2, 3, 4 \). The measured \( F_{n;1}^r \) show only a weak dependence on \( \eta_{\text{ref}} \) for \( \eta_{\text{ref}} > 3 \) or \( \eta_{\text{ref}} < -3 \) at the LHC. Measuring \( r_{n|n;k} \) for \( k > 1 \) provides new information on how the \( v_n \) asymmetry and event-plane twist fluctuate event by event.

If the amount of decorrelation for the \( k \)-th moment of the flow vector is proportional to \( k \), it can be shown that [20]:

\[
r_{n|n;k}(\eta) \approx r_{n|n;1}^k, \quad F_{n;k}^r \approx k F_{n;1}^r.
\]

Deviations from Eq. (4) are sensitive to the detailed EbyE structure of the flow fluctuations in the longitudinal direction.

To estimate the separate contributions of the asymmetry and twist effects, a new observable involving correlations of flow vectors in four \( \eta \) intervals is used [20]:

\[
R_{n|n;2}(\eta) = \frac{\langle q_n^*(\eta) q_n(\eta) q_n(\eta) q_n^*(\eta) \rangle}{\langle q_n^*(\eta) q_n(\eta) q_n(\eta) q_n^*(\eta) \rangle} = \frac{\langle v_n(\eta) v_n(\eta) v_n(\eta) v_n(\eta) \rangle}{\langle v_n(\eta) v_n(\eta) v_n(\eta) v_n(\eta) \rangle} \cos n \left[ \phi_n(\eta_{\text{ref}}) - \phi_n(\eta) \right] \cos n \left[ \phi_n(\eta_{\text{ref}}) - \phi_n(\eta) \right],
\]

where the notation “2” in the subscript indicates that there are two \( q_n \) and two \( q_n^* \) in the numerator and denominator. A schematic illustration of the relations between different flow vectors is shown in the right panel of Fig. 1. Since the effect of an asymmetry is the same in both the numerator and the denominator, this correlator is mainly sensitive to the event-plane twist effects:

\[
R_{n|n;2}(\eta) \approx 1 - 2 F_{n;2}^r \eta, \quad F_{n;2}^r = F_{n;2}^{\text{twi}}.
\]

Therefore, the asymmetry and twist contributions can be estimated by combining Eqs. (3) and (6).

Measurements of longitudinal flow fluctuations can also be extended to correlations between harmonics of different order:

\[
r_{2,3;2,3}(\eta) = \frac{\langle q_2^*(\eta) q_3(\eta) q_3(\eta) q_2^*(\eta) \rangle}{\langle q_2^*(\eta) q_3(\eta) q_3(\eta) q_2^*(\eta) \rangle},
\]

\[
r_{2,2;4}(\eta) = \frac{\langle q_2^*(\eta) q_2(\eta) q_2(\eta) q_2^*(\eta) \rangle}{\langle q_2^*(\eta) q_2(\eta) q_2(\eta) q_2^*(\eta) \rangle},
\]

\[
r_{2,3;5}(\eta) = \frac{\langle q_2^*(\eta) q_3(\eta) q_5(\eta) q_5(\eta) \rangle}{\langle q_2^*(\eta) q_3(\eta) q_5(\eta) q_5(\eta) \rangle}.
\]

where the comma in the subscripts denotes the combination of \( q_n \) of different order. If the longitudinal fluctuations for \( V_2 \) and \( V_3 \) are independent of each other, one would expect \( r_{2,3;2,3} = r_{2;2;1} r_{3;3;1} \) [20]. On the other hand, \( r_{2,2;4} \) and \( r_{2,3;5} \) are sensitive to the \( \eta \) dependence of the correlations between \( v_n \) and event planes of different order, for example \( q_4(\eta) q_4(\eta) q_4(\eta) \) II. Correlations between different orders have been measured previously at the LHC [8,9,23,29].

It is well established that the \( V_4 \) and \( V_5 \) in Pb+Pb collisions contain a linear contribution associated with initial geometry and mode-mixing contributions from lower-order harmonics due to nonlinear hydrodynamic response [8,9,14,21,22]:

\[
V_4 = V_{4L} + \chi_4 V_2^2, \quad V_5 = V_{5L} + \chi_5 V_2 V_3.
\]

where the linear component \( V_{nL} \) is driven by the corresponding eccentricity in the initial geometry [11]. If the linear component of \( v_4 \) and \( v_5 \) is uncorrelated with lower-order harmonics, i.e. \( V_2^2 V_4 \sim 0 \) and \( V_2 V_3 V_5 \sim 0 \), one expects [20]:

\[
r_{2,2;4} \approx r_{2;2;1}, \quad r_{2,3;5} \approx r_{2,3;2,3}.
\]
3 ATLAS detector and trigger

The ATLAS detector [30] provides nearly full solid-angle coverage of the collision point with tracking detectors, calorimeters, and muon chambers, and is well suited for measurements of multi-particle correlations over a large pseudorapidity range. The measurements were performed using the inner detector (ID), minimum-bias trigger scintillators (MBTS), the forward calorimeters (FCal), and the zero-degree calorimeters (ZDC). The ID detects charged particles within $|\eta| < 2.5$ using a combination of silicon pixel detectors, silicon microstrip detectors (SCT), and a strawtube transition-radiation tracker (TRT), all immersed in a 2 T axial magnetic field [31]. An additional pixel layer, the “insertable B-layer” (IBL) [32] installed during the 2013-2015 shutdown between Run 1 and Run 2, is used in the 5.02 TeV measurements. The MBTS system detects charged particles over $2.1 \leq |\eta| \leq 3.9$ using two hodoscopes of hodoscopes of counters positioned at $z = \pm 3.6$ m. The FCal consists of three sampling layers, longitudinal in shower depth, and covers $3.2 < |\eta| < 4.9$. The ZDC are positioned at $\pm 140$ m from the IP, detecting neutrons and photons with $|\eta| > 8.3$.

This analysis uses approximately 7 and 470 $\mu$b$^{-1}$ of Pb+Pb data at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV, respectively, recorded by the ATLAS experiment at the LHC. The 2.76 TeV data were collected in 2010, while the 5.02 TeV data were collected in 2015.

The ATLAS trigger system [33] consists of a level-1 (L1) trigger implemented using a combination of dedicated electronics and programmable logic, and a high-level trigger (HLT) implemented in general-purpose processors. The trigger requires signals in both ZDC or either of the two MBTS counters. The ZDC trigger thresholds on each side are set below the maximum corresponding to a single neutron. A timing requirement based on signals from each side of the MBTS was imposed to remove beam backgrounds. This trigger selected 7 and 22 $\mu$b$^{-1}$ of minimum-bias Pb+Pb data at $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.02$ TeV, respectively. To increase the number of recorded events from very central Pb+Pb collisions, a dedicated L1 trigger was used in 2015 to select events requiring the total transverse energy ($\Sigma E_T$) in the FCal to be more than 4.54 TeV. This ultra-central trigger sampled 470 $\mu$b$^{-1}$ of Pb+Pb collisions at 5.02 TeV and was fully efficient for collisions with centrality 0–0.1% (see Sect. 4).

4 Event and track selection

The offline event selection requires a reconstructed vertex with its $z$ position satisfying $|z_{\text{vertex}}| < 100$ mm. For the $\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb data, the selection also requires a time difference $|\Delta t| < 3$ ns between signals in the MBTS trigger counters on either side of the nominal centre of ATLAS to suppress non-collision backgrounds. A coincidence between the ZDC signals at forward and backward pseudorapidity is required to reject a variety of background processes such as elastic collisions and non-collision backgrounds, while maintaining high efficiency for inelastic processes. The fraction of events containing more than one inelastic interaction (pile-up) is estimated to be less than 0.1% at both collision energies. The pile-up contribution is studied by exploiting the correlation between the transverse energy $\Sigma E_T$ measured in the FCal or the number of neutrons $N_\text{n}$ in the ZDC and the number of tracks associated with a primary vertex $N_{\text{ch}}$. Since the distribution of $\Sigma E_T$ or $N_\text{n}$ in events with pile-up is broader than that for the events without pile-up, pile-up events are suppressed by rejecting events with an abnormally large $\Sigma E_T$ or $N_\text{n}$ as a function of $N_{\text{ch}}$.

The event centrality [34] is characterised by the $\Sigma E_T$ deposited in the FCal over the pseudorapidity range $3.2 < |\eta| < 4.9$ using a calibration employing the electromagnetic calorimeters to set the energy scale [35]. The FCal $\Sigma E_T$ distribution is divided into a set of centrality intervals. A centrality interval refers to a percentile range, starting at 0% relative to the most central collisions. Thus the 0–5% centrality interval, for example, corresponds to the most central 5% of the events. The ultra-central trigger mentioned in Sect. 3 selects events in the 0–0.1% centrality interval with full efficiency. A Monte Carlo Glauber analysis [34,36] is used to estimate the average number of participating nucleons, $N_{\text{part}}$, for each centrality interval. The systematic uncertainty in $N_{\text{part}}$ is less than 1% for centrality intervals in the range 0–20% and increases to 6% for centrality intervals in the range 70–80%. The Glauber model also provides a correspondence between the $\Sigma E_T$ distribution and sampling fraction of the total inelastic Pb+Pb cross section, allowing centrality percentiles to be set. For this analysis, a selection of collisions corresponding to 0–70% centrality is used to avoid diffraction or other processes that contribute to very peripheral collisions. Following the convention used in heavy-ion analyses, the centrality dependence of the results in this paper is presented as a function of $N_{\text{part}}$.

Charged-particle tracks and primary vertices [37] are reconstructed from hits in the ID. Tracks are required to have $p_T > 0.5$ GeV and $|\eta| < 2.4$. For the 2.76 TeV data, tracks are required to have at least nine hits in the silicon detectors with no missing pixel hits and not more than one missing SCT hit, taking into account the presence of known dead modules. For the 5.02 TeV data, tracks are required to have...
at least two pixel hits, with the additional requirement of a hit in the first pixel layer when one is expected, at least eight SCT hits, and at most one missing hit in the SCT. In addition, for both datasets, the point of closest approach of the track is required to be within 1 mm of the primary vertex in both the transverse and longitudinal directions [38].

The efficiency, \( \epsilon(p_T, \eta) \), of the track reconstruction and track selection criteria is evaluated using Pb+Pb Monte Carlo events produced with the HIJING event generator [39]. The generated particles in each event were rotated in azimuthal angle according to the procedure described in Ref. [40] to produce harmonic flow consistent with previous ATLAS measurements [5, 41]. The response of the detector was simulated using GEANT 4 [42, 43] and the resulting events are reconstructed with the same algorithms applied to the data. For the 5.02 TeV Pb+Pb data, the efficiency ranges from 75% to 90% for \( |\eta| > 2 \) for charged particles with \( p_T > 0.8 \) GeV, falling by about 5% as \( p_T \) is reduced to 0.5 GeV. The efficiency varies more strongly with \( \eta \) and event multiplicity. For \( p_T > 0.8 \) GeV, it ranges from 75% at \( \eta \approx 0 \) to about 50% for \( |\eta| > 2 \) in peripheral collisions, while it ranges from 71% at \( \eta \approx 0 \) to about 40% for \( |\eta| > 2 \) in central collisions. The tracking efficiency for the 2.76 TeV data has a similar dependence on \( p_T \) and \( \eta \). The efficiency is used in the particle weight, as described in Sect. 5. However, because the observables studied are ratios (see Sect. 2), uncertainties in detector and reconstruction efficiencies largely cancel. The rate of falsely reconstructed tracks (“fakes”) is also estimated and found to be significant only at \( p_T < 1 \) GeV in central collisions, where its percentage per-track ranges from 2% at \( |\eta| < 1 \) to 8% at the larger \( |\eta| \). The fake rate drops rapidly for higher \( p_T \) and towards more peripheral collisions. The fake rate is accounted for in the tracking efficiency correction following the procedure in Ref. [44].

5 Data analysis

Measurement of the longitudinal flow dynamics requires the calculation of the flow vector \( q_n \) via Eq. (1) in the ID and the FCal. The flow vector from the FCal serves as the reference \( q_n(\Delta \eta) \), while the ID provides the flow vector as a function of pseudorapidity \( q_n(\eta) \).

In order to account for detector inefficiencies and non-uniformity, a particle weight for the \( i \)th-particle in the ID for the flow vector from Eq. (1) is defined as:

\[
w_i^{\text{ID}}(\eta, \phi, p_T) = d_{\text{ID}}(\eta, \phi) / \epsilon(\eta, p_T),
\]

similar to the procedure in Ref. [44]. The determination of track efficiency \( \epsilon(\eta, p_T) \) is described in Sect. 4. The additional weight factor \( d_{\text{ID}}(\eta, \phi) \) corrects for variation of tracking efficiency or non-uniformity of detector acceptance as a function of \( \eta \) and \( \phi \). For a given \( \eta \) interval of 0.1, the distribution in azimuthal bins, \( N(\phi, \eta) \), is built up from reconstructed charged particles summed over all events. The weight factor is then obtained as \( d_{\text{ID}}(\eta, \phi) \equiv \langle N(\eta) \rangle / N(\phi, \eta) \), where \( \langle N(\eta) \rangle \) is the average of \( N(\phi, \eta) \). This “flattening” procedure removes most \( \phi \)-dependent non-uniformity from track reconstruction, which is important for any azimuthal correlation analysis. Similarly, the weight in the FCal for the flow vector from Eq. (1) is defined as:

\[
w_i^{\text{FCal}}(\eta, \phi) = d_{\text{FCal}}(\eta, \phi) E_{T,i},
\]

where \( E_{T,i} \) is the transverse energy measured in the ith tower in the FCal at \( \eta \) and \( \phi \). The azimuthal weight \( d_{\text{FCal}}(\eta, \phi) \) is calculated in narrow \( \eta \) intervals in a similar way to what is done for the ID. It ensures that the ET-weighted distribution, averaged over all events in a given centrality interval, is uniform in \( \phi \). The flow vectors \( q_n(\eta) \) and \( q_n(\Delta \eta) \) are further corrected by an event-averaged offset: \( q_n - \langle q_n \rangle_{\text{evts}} \).

The flow vectors obtained after these reweighting and offset procedures are used in the correlation analysis. The correlation quantities used in \( r_{n|n;k} \) are calculated as:

\[
\langle q_n(\eta) q_{n,k}(\Delta \eta) \rangle = \langle q_n(\eta) q_{n,k}(\Delta \eta) \rangle_{\text{signal}} - \langle q_n(\eta) \rangle_{\text{background}} q_{n,k}(\Delta \eta),
\]

where subscripts “s” and “b” represent the correlator constructed from the same event (“signal”) and from the mixed-event (“background”), respectively. The mixed-event quantity is constructed by combining \( q_n(\eta) \) from each event with \( q_{n,k}(\Delta \eta) \) obtained in other events with similar centrality (within 1%) and similar \( Z_{\text{vtx}} \). The \( \langle q_n(\eta) q_{n,k}(\Delta \eta) \rangle_{\text{b}} \), which is typically more than two orders of magnitude smaller than the corresponding signal term, is subtracted to account for any residual detector non-uniformity effects that result from a correlation between different \( \eta \) ranges.

For correlators involving flow vectors in two different \( \eta \) ranges, mixed events are constructed from two different events. For example, the correlation for \( r_{2,3;5} \) is calculated as:

\[
\langle q_2(\eta) q_3(\eta) q_5^*(\Delta \eta) \rangle = \langle q_2(\eta) q_3(\eta) q_5^*(\Delta \eta) \rangle_{\text{signal}} - \langle q_2(\eta) q_3(\eta) \rangle_{\text{background}} q_5^*(\Delta \eta),
\]

The mixed-event correlator is constructed by combining \( q_2(\eta) q_3(\eta) \) from one event with \( q_5^*(\Delta \eta) \) obtained in another event with similar centrality (within 1%) and similar \( Z_{\text{vtx}} \). On the other hand, for correlators involving more than two different \( \eta \) ranges, mixed events are constructed from two different events, one for each unique \( \eta \) range. One such example is \( R_{n|n;2} \), for which each mixed event is constructed from four different events with similar centrality and \( Z_{\text{vtx}} \).
Most correlators can be symmetrised. For example, in a symmetric system such as Pb+Pb collisions, the condition \(q_n^k(-\eta)q_n^k(\eta) = q_n^k(\eta)q_n^k(-\eta)\) holds. So instead of Eq. (2), the actual measured observable is:

\[
r_{nkle}(\eta) = \frac{q_n^k(-\eta)q_n^k(\eta)q_{\text{ref}}^k(-\eta) + q_n^k(\eta)q_n^k(\eta)q_{\text{ref}}^k(-\eta)}{q_n^k(\eta)q_n^k(\eta)q_{\text{ref}}^k(\eta)},
\]

(18)

The symmetrisation procedure also allows further cancellation of possible differences between \(\eta\) and \(-\eta\) in the tracking efficiency or detector acceptance.

Table 1 gives a summary of the set of correlators measured in this analysis. The analysis is performed in intervals of centrality and the results are presented as a function of \(\eta\) for \(|\eta| < 2.4\). The main results are obtained using 5.02 TeV Pb+Pb data. The 2010 2.76 TeV Pb+Pb data are statistically limited, and are used only to obtain \(n_{n|1}\) and \(n_{n|2}\) to compare with results obtained from the 5.02 TeV data and study the dependence on collision energy.

Figures 2, 3 show the sensitivity of \(r_{2|2:1}\) and \(r_{3|3:1}\), respectively, to the choice of the range of \(\eta_{\text{ref}}\). A smaller \(\eta_{\text{ref}}\) value implies a smaller pseudorapidity gap between \(\eta\) and \(\eta_{\text{ref}}\).

The values of \(n_{n|1}\) generally decrease with decreasing \(\eta_{\text{ref}}\), possibly reflecting the contributions from the dijet correlations [5]. However, such contributions should be reduced in the most central collisions due to large charged-particle multiplicity and jet-quenching [45] effects. Therefore, the decrease of \(n_{n|1}\) in the most central collisions may also reflect the \(\eta_{\text{ref}}\) dependence of \(F_{n|1}\), as defined in Eq. (3).

In this analysis, the reference flow vector is calculated from 4.0 < \(\eta_{\text{ref}}\) < 4.9, which reduces the effect of dijets and provides good statistical precision. For this choice of \(\eta_{\text{ref}}\) range, \(r_{2|2:1}\) and \(r_{3|3:1}\) show a linear decrease as a function of \(\eta\) in most centrality intervals, indicating a significant breakdown of factorisation. A similar comparison for \(r_{4|4:1}\) can be found in the “Appendix”.

Figures 4, 5 show \(r_{2|2:1}\) and \(r_{3|3:1}\) calculated for several \(p_T\) ranges of the charged particles in the ID. A similar comparison for \(r_{4|4:1}\) can be found in the “Appendix”. If the longitudinal-flow asymmetry and twist reflect global properties of the event, the values of \(n_{n|1}\) should not depend strongly on \(p_T\). Indeed no dependence is observed, except for \(r_{2|2:1}\) in the most central collisions and very peripheral collisions. The behaviour in central collisions may be related to the factorisation breaking of the \(v_2\) as a function of \(p_T\) and

![Figure 2](https://example.com/figure2.png) The \(r_{2|2:1}(\eta)\) measured for several \(\eta_{\text{ref}}\) ranges. Each panel shows the results for one centrality range. The error bars are statistical only.
Fig. 3 The $r_{3|3;1}(\eta)$ measured for several $\eta_{\text{ref}}$ ranges. Each panel shows the results for one centrality range. The error bars are statistical only.

Fig. 4 The $r_{2|2;1}(\eta)$ measured in several $p_T$ ranges. Each panel shows the results for one centrality range. The error bars are statistical only.
The behaviour in peripheral collisions is presumably due to increasing relative contributions from jets and dijets at higher $p_T$ and for peripheral collisions. Based on this, the measurements are performed using charged particles with $0.5 < p_T < 3$ GeV.

6 Systematic uncertainties

Since all observables are found to follow an approximately linear decrease with $\eta$, i.e. $D(\eta) \approx 1 - c \eta$ for a given observable $D(\eta)$ where $c$ is a constant, the systematic uncertainty is presented as the relative uncertainty for $1 - D(\eta)$ at $\eta = 1.2$, the mid-point of the $\eta$ range. The systematic uncertainties in this analysis arise from event mixing, track selection, and reconstruction efficiency. Most of the systematic uncertainties enter the analysis through the particle weights in Eqs. (14) and (15). In general, the uncertainties for $r_{n|k}; k$ increase with $n$ and $k$, the uncertainties for $R_{n|2}; n$ increase with $n$, and all uncertainties are larger in the most central and more peripheral collisions. For $r_{2,3|2}; r_{2,2|4}$ and $r_{2,3|5}$, the uncertainties are significantly larger than for the other correlators. Each source is discussed separately below.

The effect of detector azimuthal non-uniformity is accounted for by the weight factor $d(\eta, \phi)$ in Eqs. (14) and (15). The effect of reweighting is studied by setting the weight to unity and repeating the analysis. The results are consistent with the default (weighted) results within statistical uncertainties, so no additional systematic uncertainty is included. Possible residual detector effects for each observable are further removed by subtracting those obtained from mixed events as described in Sect. 5. Uncertainties due to the event-mixing procedure are estimated by varying the criteria for matching events in centrality and $z_{vtx}$. The resulting uncertainty is in general found to be smaller than the statistical uncertainties. The event-mixing uncertainty for $r_{2|2,k}$ and $r_{3|k}$ is less than 1% for $k = 1$ and changes to about 0.4–8% for $k = 2$ and 0.6–10% for $k = 3$, while the uncertainty for $r_{4|4;1}$ and $r_{5|5;1}$ is in the range 1.5–3% and 5–13%, respectively. The uncertainty for $R_{n|2}; n$ is 1.5–6% for $n = 2$ and 3–14% for $n = 3$. The uncertainties for $r_{2,3|2,3}, r_{2,2|4}$ and $r_{2,3|5}$ are typically larger: 1–4%, 1.5–16% and 3–15%.

The systematic uncertainty associated with the track quality selections is estimated by tightening or loosening the requirements on transverse impact parameter $|d_{0}|$ and longitudinal impact parameter $|z_{0}\sin\theta|$ used to select tracks. In each case, the tracking efficiency is re-evaluated and the analysis is repeated. The difference is observed to be larger in the most central collisions where the flow signal is smaller and the influence of falsely reconstructed tracks is higher. The difference is observed to be in the range 0.2–12% for $r_{2|2,k}$ and $r_{3|3,k}$, 1.1–2% for $r_{4|4;1}$, 3–6% for $r_{5|5;1}$, 0.5–13% for $R_{n|2}; n$, and 1–14% for $r_{2,3|2,3}, r_{2,2|4}$ and $r_{2,3|5}$.

From previous measurements [5,6,46], the $y_{\phi}$ signal has been shown to have a strong dependence on $p_T$ but relatively weak dependence on $\eta$. Therefore, a $p_T$-dependent uncer-
tainty in the track reconstruction efficiency $\epsilon(\eta, p_T)$ could affect the measured longitudinal flow correlation, through the particle weights. The uncertainty in the track reconstruction efficiency is due to differences in the detector conditions and known differences in the material between data and simulations. The uncertainty in the efficiency varies between 1% and 4%, depending on $\eta$ and $p_T$ [44]. The systematic uncertainty for each observable in Table 1 is evaluated by repeating the analysis with the tracking efficiency varied up and down by its corresponding uncertainty. 

For $r_{\eta|n,k}$ the uncertainties are in the range 0.1–2%, depending on $n$ and $k$. For $R_{\eta|n,2}$ the uncertainties are in the range 0.1–1%. For $r_{2,3|2,3}$, $r_{2,2|4}$ and $r_{2,3|3}$, the uncertainties are in the range 0.1–2%.

Due to the finite energy resolution and energy scale uncertainty of the FCal, the $q_n(\eta_{\text{ref}})$ calculated from the azimuthal distribution of the $E_T$ via Eqs. (1) and (15) differs from the true azimuthal distribution. However, since $q_n(\eta_{\text{ref}})$ appears in both the numerator and the denominator of the correlators studied in this paper, most of the effects associated with the FCal $E_T$ response are expected to cancel out. Two cross-checks are also performed to study the influence of the FCal response. In the first cross-check, only the FCal towers with $E_T$ above the 50th percentile are used to calculate the $q_n(\eta_{\text{ref}})$. The $|q_n(\eta_{\text{ref}})|$ value is different from the default analysis, but the values of the correlators are found to be consistent. In the second cross-check, HIJING events with imposed flow (see Sect. 4) are used to study the FCal response. The $q_n(\eta_{\text{ref}})$ is calculated using both the generated $E_T$ and the reconstructed $E_T$, and the resulting correlators are compared with each other. The results are found to be consistent. Accordingly, no additional systematic uncertainty is added for the FCal response.

The systematic uncertainties from the different sources described above are added in quadrature to give the total systematic uncertainty for each observable. They are listed in Tables 2, 3 and 4.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Systematic uncertainties in percent for $1-r_{2,2</th>
<th>4}$ and $1-r_{3,3</th>
<th>4}$ at $\eta = 1.2$ in selected centrality intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0–5%</td>
<td>20–30%</td>
<td>40–50%</td>
</tr>
<tr>
<td>Event mixing (%)</td>
<td>0.8</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Track selections (%)</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Reco. efficiency (%)</td>
<td>0.3</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Total (%)</td>
<td>1.0</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>

| Table 3 | Systematic uncertainties in percent for $1-R_{2,2|2}$, $1-R_{3,3|2}$, $1-r_{3,3|4}$, and $1-r_{3,3|5}$ at $\eta = 1.2$ in selected centrality intervals |
|---------|-----------------|-----------------|-----------------|
|         | 0–5% | 20–30% | 40–50% |
| Event mixing (%) | 6.1 | 1.5 | 1.5 |
| Track selections (%) | 3.5 | 0.4 | 0.7 |
| Reco. efficiency (%) | 0.2 | 0.1 | 0.1 |
| Total (%) | 7.1 | 1.6 | 1.7 |

|         | 0–5% | 20–30% | 40–50% |
| Event mixing (%) | 1.8 | 1.5 | 2.7 |
| Track selections (%) | 1.5 | 1.1 | 2.0 |
| Reco. efficiency (%) | 0.3 | 0.3 | 0.6 |
| Total (%) | 2.4 | 1.9 | 3.5 |
Table 4 Systematic uncertainties in percent for $1 - r_{2,3,2,3}$, $1 - r_{2,2,4}$ and $1 - r_{2,3,5}$ at $\eta = 1.2$ in selected centrality intervals

<table>
<thead>
<tr>
<th>Centrality Interval</th>
<th>$1 - r_{2,3,2,3}$ %</th>
<th>$1 - r_{2,2,4}$ %</th>
<th>$1 - r_{2,3,5}$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–5%</td>
<td>4.1</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>20–30%</td>
<td>1.7</td>
<td>1.5</td>
<td>3.4</td>
</tr>
<tr>
<td>40–50%</td>
<td>3.2</td>
<td>2.4</td>
<td>7.8</td>
</tr>
</tbody>
</table>

7 Results

The presentation of the results is structured as follows. Section 7.1 presents the results for $r_{n|n;1}$ and $R_{n|n;2}$ and the comparison between the two collision energies. Section 7.2 shows the results for $r_{n|n;k}$ for $k > 1$. The scaling relation from Eq. (4) is tested and the contributions from $v_n$ FB asymmetry and event-plane twist are estimated. Results for the mixed-harmonic correlators, Eqs. (7)–(9), are presented in Sect. 7.3 and checked for compatibility with the hydrodynamical picture. The measurements are performed using charged particles with $0.5 < p_T < 3$ GeV, and the reference flow vector is calculated with $4.0 < |\eta_{\text{ref}}| < 4.9$. Most results are shown for the $\sqrt{s_{\text{NN}}} = 2.76$ TeV Pb+Pb dataset, which has better statistical precision. The results for the $\sqrt{s_{\text{NN}}} = 5.02$ TeV Pb+Pb dataset are shown only for $r_{n|n;1}$ and $R_{n|n;2}$.

7.1 $r_{n|n;1}$ and $R_{n|n;2}$ at two collision energies

Figure 6 shows $r_{2|2;1}$ in various centrality intervals at the two collision energies. The correlator shows a linear decrease with $\eta$, except in the most central collisions. The decreasing trend is weakest around the 20–30% centrality range, and is more pronounced in both more central and more peripheral collisions. This centrality dependence is the result of a strong centrality dependence of the $v_2$ associated with the average elliptic geometry [47]. The decreasing trend at $\sqrt{s_{\text{NN}}} = 2.76$ TeV is slightly stronger than that at $\sqrt{s_{\text{NN}}} = 5.02$ TeV, which is expected as the collision system becomes less boost-invariant at lower collision energy [24].

Figures 7 and 8 show the results for $r_{3|3;1}$ and $r_{4|4;1}$, respectively, at the two collision energies. A linear decrease as a function of $\eta$ is observed for both correlators, and the rate of the decrease is approximately independent of centrality. This centrality independence could be due to the fact that $v_3$ and $v_4$ are driven mainly by fluctuations in the initial
state. The rate of the decrease is also observed to be slightly stronger at lower collision energy.

The decreasing trend of $r_{\eta_{\eta}}$ for $n = 2–4$ in Figs. 6, 7 and 8 indicates significant breakdown of the factorisation of two-particle flow harmonics into those between different $\eta$ ranges. However, the size of the factorisation breakdown depends on the harmonic order $n$, collision centrality, and collision energy. The results have also been compared with those from the CMS Collaboration [19], with the $\eta_{\text{ref}}$ chosen to be $4.4 < |\eta_{\text{ref}}| < 4.9$ to match the CMS choice of $\eta_{\text{ref}}$. The two results agree very well with each other, and details are shown in the “Appendix”.

Figures 9 and 10 show $R_{2|2,1}$ and $R_{3|3,1}$ in several centrality intervals. Both observables follow a linear decrease with
The measured \( r_{n|n;k} \) and \( R_{n|n;2} \) are parameterised with linear functions,

\[
\begin{align*}
\frac{1}{2} F_{r_{n|n;k}} &= 1 - 2 F_{r_{n|n;k}}(\eta_i)\eta_i \\
\frac{1}{2} F_{R_{n|n;2}} &= 1 - 2 F_{R_{n|n;2}}(\eta_i)\eta_i,
\end{align*}
\]

where the slope parameters are calculated as linear-regression coefficients,

\[
\begin{align*}
F_{r_{n|n;k}} &= \sum_i (1 - r_{n|n;k}(\eta_i))\eta_i \\
F_{R_{n|n;2}} &= \sum_i (1 - R_{n|n;2}(\eta_i))\eta_i,
\end{align*}
\]

which characterise the average \( \eta \)-weighted deviation of \( r_{n|n;1}(\eta) \) and \( R_{n|n;2}(\eta) \) from unity. The sum runs over all data points. If \( r_{n|n;k} \) and \( R_{n|n;2} \) are a linear function in \( \eta \), the linear-regression coefficients are equivalent to a fit to Eq. (19). However, these coefficients are well defined even if the observables have significant nonlinear behaviour, which is the case for \( r_{2|2;k} \) and \( R_{2|2;2} \) in the 0–20% centrality range.

The extracted slope parameters \( F_{r_{n|n;1}} \) and \( F_{R_{n|n;2}} \) are plotted as a function of centrality in terms of \( N_{\text{part}} \), in Figs. 11 and 12, respectively. The values of \( F_{r_{3;1}} \) and \( F_{R_{3;2}} \) first decrease and then increase as a function of increasing \( N_{\text{part}} \). The larger values in central and peripheral collisions are related to the fact that \( v_2 \) is more dominated by the initial geometry fluctuations. The slopes for higher-order harmonics are significantly larger. As a function of \( N_{\text{part}} \), a slight decrease in \( F_{r_{3;1}} \) and \( F_{R_{3;2}} \) is observed for \( N_{\text{part}} > 200 \), as well as an increase in \( F_{r_{4;1}} \) for \( N_{\text{part}} < 100 \). The values of \( F_{r_{n;1}} \) and \( F_{R_{n;2}} \) are larger
with decreasing $\sqrt{S_{NN}}$ as the rapidity profile of the initial state is more compressed due to smaller beam rapidity $y_{beam}$ at lower $\sqrt{S_{NN}}$. This energy dependence has been predicted for $F_{n;1}$ in hydrodynamic model calculations [24], and it is quantified in Fig. 13 via the ratio of $F_{2;1}$ values and of $F_{2;2}$ values at the two energies. The weighted averages of the ratios calculated in the range $30 < N_{\text{part}} < 400$ are given in Table 5. Compared to $\sqrt{S_{NN}} = 5.02$ TeV, the values of $F_{2;1}$ and $F_{2;2}$ at $\sqrt{S_{NN}} = 2.76$ TeV are about 10% higher, and the values of $F_{3;1}$ and $F_{4;1}$ are about 16% higher.

If the change of correlators with $\sqrt{S_{NN}}$ were entirely due to the change of $y_{beam}$ then the correlators would be expected to follow a universal curve when they are rescaled by $y_{beam}$, i.e. $r_n|\nu;k(\eta/y_{beam})$ and $R_n|\nu;l(\eta/y_{beam})$ should not depend on $\sqrt{S_{NN}}$. In this case, the slopes parameters multiplied by the beam rapidity, $\hat{F}_{n;1} \equiv F_{n;1}y_{beam}$ and $\hat{F}_{n;2} \equiv F_{n;2}y_{beam}$, should not depend on $\sqrt{S_{NN}}$. The beam rapidity is $y_{beam} = 7.92$ and 8.52 for $\sqrt{S_{NN}} = 2.76$ and 5.02 TeV, respectively, which leads to a 7.5% reduction in the ratio. Figure 14 shows the ratio of $\hat{F}_{2;1}$ values and of $\hat{F}_{2;2}$ values at the two energies, and the weighted averages of the ratios calculated in the range $30 < N_{\text{part}} < 400$ are given in Table 5. The $y_{beam}$-scaling accounts for a large part of the $\sqrt{S_{NN}}$ dependence. Compared to $\sqrt{S_{NN}} = 5.02$ TeV, the values of $\hat{F}_{2;1}$ and $\hat{F}_{2;2}$ at $\sqrt{S_{NN}} = 2.76$ TeV are about 3% higher, and the values of $\hat{F}_{3;1}$ and $\hat{F}_{4;1}$ are about 8% higher, so this level of difference remains after accounting for the change in the beam rapidity.

7.2 Higher-order moments

The longitudinal correlations of higher-order moments of harmonic flow carry information about the EbyE flow fluctuations in pseudorapidity. In the simple model described in Ref. [20], the decrease in $r_{n|\nu;k}$ is expected to scale with $k$ as given by Eq. (4).

Figure 15 compares the results for $r_{2|2;k}$ for $k = 1–3$ (solid symbols) with $\hat{r}_{2|2;k}$ for $k = 2–3$ (open symbols). The data follow the scaling relation from Eq. (4) in the most
approximately the scaling relation Eq. (4) in all centrality intervals.

Fig. 13 Centrality dependence of ratio of $F_n^{r,v}$ values (left panel) and $F_n^{R,v}$ values (right panel) at 2.76 and 5.02 TeV. The lines indicate the average values in the range $30 < N_{\text{part}} < 400$, with the results and fit uncertainties given by Table 5. The error bars and shaded boxes are statistical and systematic uncertainties, respectively.

Fig. 14 Centrality dependence of ratio of $F_n^{r,v}$ and $F_n^{R,v}$ values (left panel) and $F_n^{R,v}$ values (right panel) at 2.76 and 5.02 TeV. The lines indicate the average values in the range $30 < N_{\text{part}} < 400$, with the results and fit uncertainties given by Table 5. The error bars and shaded boxes are statistical and systematic uncertainties, respectively.

Table 5 Results of the fits to the ratio of $F_n^{r,v}$, $F_n^{R,v}$, $F_n^{r,v}$, $F_n^{R,v}$ values at the two energies in the range $30 < N_{\text{part}} < 400$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$F_n^{r,1}(2.76 \text{ TeV})/F_n^{r,1}(5.02 \text{ TeV})$</th>
<th>$F_n^{R,1}(2.76 \text{ TeV})/F_n^{R,1}(5.02 \text{ TeV})$</th>
<th>$F_n^{r,2}(2.76 \text{ TeV})/F_n^{r,2}(5.02 \text{ TeV})$</th>
<th>$F_n^{R,2}(2.76 \text{ TeV})/F_n^{R,2}(5.02 \text{ TeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$1.100 \pm 0.010$</td>
<td>$1.103 \pm 0.026$</td>
<td>$1.023 \pm 0.009$</td>
<td>$1.025 \pm 0.024$</td>
</tr>
<tr>
<td>3</td>
<td>$1.152 \pm 0.011$</td>
<td>$1.18 \pm 0.08$</td>
<td>$1.071 \pm 0.010$</td>
<td>$1.10 \pm 0.07$</td>
</tr>
<tr>
<td>4</td>
<td>$1.17 \pm 0.036$</td>
<td>$-$</td>
<td>$1.088 \pm 0.033$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

central collisions (0–5% centrality) where $v_2$ is driven by the initial-state fluctuations. In other centrality intervals, where the average geometry is more important for $v_2$, the $r_{2;1}$ slopes ($r_{2;1}^{k}$) and $r_{2;2}^{k}$ slopes ($r_{2;2}^{k}$) of $r_{n;1}$ and $r_{n;2}$, respectively, show stronger decreases with $\eta$ than $r_{2;1}^{k}$.

A similar study is performed for third-order harmonics, and the results are shown in Fig. 16. The data follow approximately the scaling relation Eq. (4) in all centrality intervals.

To quantify the difference between $r_{n;1;1}$ and $r_{n;1;2}$, the slopes ($r_{n;1;1}^{k}$) of $r_{n;1;1}$ and $r_{n;2;2}$ are calculated via Eqs. (19) and (20). The scaled quantities, $F_n^{r,k}/k$, are then compared with each other as a function of centrality in Fig. 17. For second-order harmonics, the data show clearly that over most of the centrality range $F_{2;1}^{r}/3 > F_{2;2}^{r}/2 > F_{2;1}^{r}$, implying $F_{2;1}^{r} > k F_{2;1}^{r}$. However, for the most central and most peripheral collisions, the quantities approach each other on the
Fig. 15 The $r_{2|2;k}$ for $k = 1$–3 compared with $r_{2|1;k}$ for $k = 2$–3 in various centrality intervals for Pb+Pb collisions at 5.02 TeV. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The data points for $k = 2$ or 3 in some centrality intervals are rebinned to reduce the uncertainty.

Fig. 16 The $r_{3|3;k}$ for $k = 1$–3 compared with $r_{3|1;k}$ for $k = 2$–3 in various centrality intervals for Pb+Pb collisions at 5.02 TeV. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The data points for $k = 2$ or 3 in some centrality intervals are rebinned to reduce the uncertainty.
The values of $F_{n,k}^{r} / k$ for $k = 1, 2$ and 3 for $n = 2$ (left panel) and $n = 3$ (right panel), respectively. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The widths of the centrality intervals are not fixed but are optimised to reduce the uncertainty.

The results are shown in Fig. 20. The contributions from the two components are similar to each other for $n = 2$, for which the harmonic flow arises primarily from the average collision shape, as well as for $n = 3$, for which the harmonic flow is driven mainly by fluctuations in the initial geometry.

7.3 Mixed-harmonic correlation

Figure 21 compares the $r_{2,3}^{2,3}$ with the product of $r_{2}^{2,1}$ and $r_{3}^{3,1}$. The data show that they are consistent with each other, suggesting the previously observed anticorrelation between $v_2$ and $v_3$ is a property of the entire event [9,48], and that longitudinal fluctuations of $v_2$ and $v_3$ are uncorrelated. Figure 22 compares $r_{2,2}^{2,2}$ with the mixed-harmonic correlator $r_{2,2}^{2,2}$, as well as $r_{4,4}^{4,4}$. As discussed in Sect. 2 in the context of the first relation in Eq. (10), if the linear and non-linear components of $v_4$ in Eq. (10) are uncorrelated, then $r_{2,2}^{2,2}$ would be expected to be similar to $r_{2,2}^{2,2}$. This is indeed confirmed by the comparisons of the $\eta$ and centrality depen-
The correlation $r_{3;2}(\eta)$ and $R_{3;2}(\eta)$ in various centrality intervals for Pb+Pb collisions at 5.02 TeV. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The data points in 40–50% centrality interval are rebinned to reduce the uncertainty.

The estimated event-plane twist component $F_{n;2}$ and FB asymmetry component $F_{n;2}$ as a function of $N_{\text{part}}$ for $n = 2$ and 3 for Pb+Pb collisions at 5.02 TeV. The error bars and shaded boxes are statistical and systematic uncertainties, respectively.

The decorrelation effects are stronger for the linear component of $v_4$ than for the nonlinear component. Figure 22 also shows that the $\eta$ dependence for $r_{4;1}$ is stronger than for $r_{2;2}$ in all centrality intervals, suggesting that the decorrelation effects are stronger for the linear component of $v_4$ than for the nonlinear component (see Eq. (12)).

A similar study of the influence of the linear and nonlinear effects for $v_5$ was also performed, and results are shown in Fig. 23. The three observables $r_{2;2}, r_{2;3}, r_{3;5}$, and $r_{5;1}$ show similar values in all centrality intervals, albeit with large statistical uncertainties.

The decorrelations shown in Figs. 21, 22, and 23 can be quantified by calculating the slopes of the distributions in each centrality interval and presenting the results as a function of centrality. Following the example for $r_{n;1;2}$, the slopes for the mixed-harmonic correlators are obtained via the linear regression procedure of Eqs. (19) and (20):

$$r_{2;2}(\eta) = 1 - 2F_{r_{2;2}}(\eta), \quad r_{2;3}(\eta) = 1 - 2F_{r_{2;3}}(\eta), \quad r_{3;5}(\eta) = 1 - 2F_{r_{3;5}}(\eta).$$

The results are summarised in Fig. 24, with each panel corresponding to the slopes of distributions in Figs. 21, 22, and 23, respectively. The only significant difference is seen between $F_{4;1}$ and $F_{2;2}$ or $F_{2;4}$.

8 Summary

Measurements of longitudinal flow correlations for charged particles are presented in the pseudorapidity range $|\eta| < 2.4$ using 7 and 470 $\mu$b$^{-1}$ of Pb+Pb data at $\sqrt{s_{NN}} = 2.76$ and 5.02 TeV, respectively, recorded by the ATLAS detector at the LHC. The factorisation of two-particle azimuthal correlations into single-particle flow harmonics $v_n$ is found to be broken, and the amount of factorisation breakdown increases approximately linearly as a function of the $\eta$ separation between the two particles. The slope of this dependence is nearly independent of centrality and $p_T$ for $n > 2$. However, for $n = 2$ the effect is smallest in mid-central collisions and increases toward more central or more peripheral collisions, and in central collisions the effect also depends strongly on $p_T$. Furthermore, the effect is found to be larger.
Fig. 21 The $r_{2,3|2,3}$ (circles) and $r_{2|2;2,3|3,1}$ (squares) as a function of $\eta$ for several centrality intervals. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The $r_{2,3|2,3}$ data in the 50–60% centrality interval are rebinned to reduce the uncertainty.

Fig. 22 Comparison of $r_{2|2;2}$, $r_{2|2;4}$ and $r_{4|4;1}$ for several centrality intervals. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The data points in some centrality intervals are rebinned to reduce the uncertainty.
Fig. 23 Comparison of \( r_{2,3;2,3}, r_{2,3;5} \) and \( r_{5;5,1} \) for several centrality intervals. The error bars and shaded boxes are statistical and systematic uncertainties, respectively. The \( r_{5;5,1} \) data in some centrality intervals are rebinned to reduce the uncertainty.

Fig. 24 Comparison of the slopes of the correlators as a function of \( N_{\text{part}} \) for three groups of correlators: \( r_{2,3;2,3} \) and \( r_{2,2;1,1} \) (for which the slope is \( F_{2;1} + F_{3;1} \)) in Fig. 21 (left panel), \( r_{2,2;1,1} \) in Fig. 22 (middle panel), and \( r_{2,3;2,3}, r_{2,3;5} \) and \( r_{5;5,1} \) in Fig. 23 (right panel). The error bars and shaded boxes are statistical and systematic uncertainties, respectively.

at 2.76 than 5.02 TeV for all harmonics, which cannot be explained entirely by the change in the beam rapidity.

The higher moments of the \( \eta \)-dependent flow correlations are also measured and the corresponding linear coefficients of the \( \eta \) dependence are extracted. The coefficient for the \( k \)th-moment of \( v_n \) scales with \( k \) for \( n > 2 \), but scales faster than \( k \) for \( n = 2 \). The factorisation breakdown is separated into contributions from forward-backward asymmetry of the flow magnitude and event-plane twist, which are found to be comparable to each other.

The longitudinal flow correlations are also measured between harmonic flows of different order. The correlation of \( v_2 v_3 \) between two \( \eta \) ranges is found to factorise into the product of the correlation for \( v_2 \) and the correlation for \( v_3 \), suggesting that the longitudinal fluctuations of \( v_2 \) and \( v_3 \) are independent of each other. The correlations between \( v_4 \) and \( v_2^2 \) suggest that the longitudinal fluctuations of \( v_4 \) have a significant nonlinear contribution from \( v_2 \), i.e. \( v_4 \propto v_2^2 \). Similarly, the correlations between \( v_5 \) and \( v_2 v_3 \) suggest that the longitudinal fluctuations of \( v_5 \) are driven by the nonlinear contribution from \( v_2 v_3 \), i.e. \( v_5 \propto v_2 v_3 \). The results presented in this paper provide new insights into the fluctuations and correlations of harmonic flow in the longitudinal direction, which can be used to improve full three-dimensional viscous hydrodynamic models.
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Appendix

Figures 25 and 26 show a comparison of $r_{2|2;1}$ and $r_{3|3;1}$ between ATLAS and CMS for Pb+Pb collisions at 2.76 TeV, where the ATLAS $\eta_{\text{ref}}$ is chosen to be $4.4 < |\eta_{\text{ref}}| < 4.9$ to match that of the CMS Collaboration. Excellent agreement is observed. Figures 27 and 28 show the detailed $p_T$ and $\eta_{\text{ref}}$ dependence of $r_{4|4;1}$; these figures complement Figs. 2, 3, 4 and 5. Figure 29 compiles the results of $r_{n|n;1}$ and $R_{n|n;2}$ for 0–0.1% ultra-central collisions.

Fig. 25 The values of $r_{2|2;1}$ measured by ATLAS and by CMS [19] for Pb+Pb collisions at 2.76 TeV, for the same reference pseudorapidity $4.4 < |\eta_{\text{ref}}| < 4.9$.
Fig. 26 The values of $r_{3;1}$ measured by ATLAS and by CMS [19] for Pb+Pb collisions at 2.76 TeV, for the same reference pseudorapidity $|\eta_{\text{ref}}| < 4.9$.

Fig. 27 The $r_{4;1}(\eta)$ measured for several $\eta_{\text{ref}}$ ranges for Pb+Pb collisions at 5.02 TeV. Each panel represents one centrality range. The error bars are statistical only. The data points in some centrality intervals are rebinned to reduce the uncertainty.
Fig. 28 The $r_{n;1}(\eta)$ measured in several $p_T$ ranges for Pb+Pb collisions at 5.02 TeV. Each panel shows the results for one centrality range. The error bars are statistical only. The data points in some centrality intervals are rebinned to reduce the uncertainty.

Fig. 29 The $r_{n;1}(\eta)$ (left panel) and $R_{n;2}(\eta)$ (right panel) in ultra-central Pb+Pb collisions at 5.02 TeV. The error bars and shaded boxes are statistical and systematic uncertainties, respectively.

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18 Albert Einstein Center for Fundamental Physics, Laboratory for High Energy Physics, University of Bern, Bern, Switzerland

19 School of Physics and Astronomy, University of Birmingham, Birmingham, UK

20 (a) Department of Physics, Bogazici University, Istanbul, Turkey; (b) Department of Physics Engineering, Gaziantep University, Gaziantep, Turkey; (c) Faculty of Engineering and Natural Sciences, Istanbul Bilgi University, Istanbul, Turkey; (d) Faculty of Engineering and Natural Sciences, Bahcesehir University, Istanbul, Turkey

21 Centro de Investigaciones, Universidad Antonio Narino, Bogotá, Colombia

22 (a) INFN Sezione di Bologna, Bologna, Italy; (b) Dipartimento di Fisica e Astronomia, Università di Bologna, Bologna, Italy

23 Physikalisches Institut, University of Bonn, Bonn, Germany

24 Department of Physics, Boston University, Boston, MA, USA

25 Department of Physics, Brandeis University, Waltham, MA, USA

26 (a) Universidade Federal do Rio De Janeiro COPPE/EE/IF, Rio de Janeiro, Brazil; (b) Electrical Circuits Department, Federal University of Juiz de Fora (UFJF), Juiz de Fora, Brazil; (c) Federal University of Sao Joao del Rei (UFSJ), Sao Joao del Rei, Brazil; (d) Instituto de Fisica, Universidade de Sao Paulo, Sao Paulo, Brazil

27 Physics Department, Brookhaven National Laboratory, Upton, NY, USA

28 (a) Transilvania University of Brasov, Brasov, Romania; (b) Horia Hulubei National Institute of Physics and Nuclear Engineering, Bucharest, Romania; (c) Department of Physics, Alexandru Ioan Cuza University of Iasi, Iasi, Romania; (d) Physics Department, National Institute for Research and Development of Isotopic and Molecular Technologies, Cluj-Napoca, Romania; (e) University Politehnica Bucharest, Bucharest, Romania; (f) West University in Timisoara, Timisoara, Romania

29 Departamento de Física, Universidad de Buenos Aires, Buenos Aires, Argentina

30 Cavendish Laboratory, University of Cambridge, Cambridge, UK

31 Department of Physics, Carleton University, Ottawa, ON, Canada

32 CERN, Geneva, Switzerland

33 Enrico Fermi Institute, University of Chicago, Chicago, IL, USA

34 (a) Departamento de Física, Pontificia Universidad Católica de Chile, Santiago, Chile; (b) Departamento de Física, Universidad Técnica Federico Santa María, Valparaíso, Chile

35 (a) Institute of High Energy Physics, Chinese Academy of Sciences, Beijing, China; (b) Department of Physics, Nanjing University, Nanjing, Jiangsu, China; (c) Physics Department, Tsinghua University, Beijing 100084, China; (d) University of Chinese Academy of Science (UCAS), Beijing, China

36 (a) Department of Modern Physics and State Key Laboratory of Particle Detection and Electronics, University of Science and Technology of China, Hefei, Anhui, China; (b) School of Physics, Shandong University, Jinan, Shandong, China; (c) Department of Physics of Astronomy, Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education, Shanghai Key Laboratory for Particle Physics and Cosmology, Shanghai Jiao Tong University, Shanghai (also at PKU-CHEP), Shanghai, China

37 Université Clermont Auvergne, CNRS/IN2P3, LPC, Clermont-Ferrand, France

38 Nevis Laboratory, Columbia University, Irvington, NY, USA

39 Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark

40 (a) INFN Gruppo Collegato di Cosenza, Laboratori Nazionali di Frascati, Frascati, Italy; (b) Dipartimento di Fisica, Università della Calabria, Rende, Italy

41 (a) Faculty of Physics and Applied Computer Science, AGH University of Science and Technology, Kraków, Poland; (b) Marian Smoluchowski Institute of Physics, Jagiellonian University, Kraków, Poland

42 Institute of Nuclear Physics, Polish Academy of Sciences, Kraków, Poland

43 Physics Department, Southern Methodist University, Dallas, TX, USA

44 Physics Department, University of Texas at Dallas, Richardson, TX, USA

45 DESY, Hamburg and Zeuthen, Germany

46 Lehrstuhl für Experimentelle Physik IV, Technische Universität Dortmund, Dortmund, Germany

47 Institut für Kern- und Teilchenphysik, Technische Universität Dresden, Dresden, Germany

48 Department of Physics, Duke University, Durham, NC, USA

49 SUPA-School of Physics and Astronomy, University of Edinburgh, Edinburgh, UK

50 INFN Laboratori Nazionali di Frascati, Frascati, Italy

51 Fakultät für Mathematik und Physik, Albert-Ludwigs-Universität, Freiburg, Germany
139 Santa Cruz Institute for Particle Physics, University of California Santa Cruz, Santa Cruz, CA, USA
140 Department of Physics, University of Washington, Seattle, WA, USA
141 Department of Physics and Astronomy, University of Sheffield, Sheffield, UK
142 Department of Physics, Shinshu University, Nagano, Japan
143 Department Physik, Universität Siegen, Siegen, Germany
144 Department of Physics, Simon Fraser University, Burnaby, BC, Canada
145 SLAC National Accelerator Laboratory, Stanford, CA, USA
146 (a) Faculty of Mathematics, Physics and Informatics, Comenius University, Bratislava, Slovak Republic; (b) Department of Subnuclear Physics, Institute of Experimental Physics of the Slovak Academy of Sciences, Kosice, Slovak Republic
147 (a) Department of Physics, University of Cape Town, Cape Town, South Africa; (b) Department of Physics, University of Johannesberg, Johannesburg, South Africa; (c) School of Physics, University of the Witwatersrand, Johannesburg, South Africa
148 (a) Department of Physics, Stockholm University, Stockholm, Sweden; (b) The Oskar Klein Centre, Stockholm, Sweden
149 Physics Department, Royal Institute of Technology, Stockholm, Sweden
150 Departments of Physics and Astronomy and Chemistry, Stony Brook University, Stony Brook, NY, USA
151 Department of Physics and Astronomy, University of Sussex, Brighton, UK
152 School of Physics, University of Sydney, Sydney, Australia
153 Institute of Physics, Academia Sinica, Taipei, Taiwan
154 Department of Physics, Technion: Israel Institute of Technology, Haifa, Israel
155 Raymond and Beverly Sackler School of Physics and Astronomy, Tel Aviv University, Tel Aviv, Israel
156 Department of Physics, Aristotle University of Thessaloniki, Thessaloniki, Greece
157 International Center for Elementary Particle Physics and Department of Physics, The University of Tokyo, Tokyo, Japan
158 Graduate School of Science and Technology, Tokyo Metropolitan University, Tokyo, Japan
159 Department of Physics, Tokyo Institute of Technology, Tokyo, Japan
160 Tomsk State University, Tomsk, Russia
161 Department of Physics, University of Toronto, Toronto, ON, Canada
162 (a) INFN-TIFPA, Trento, Italy; (b) University of Trento, Trento, Italy
163 (a) TRIUMF, Vancouver, BC, Canada; (b) Department of Physics and Astronomy, York University, Toronto, ON, Canada
164 Faculty of Pure and Applied Sciences, and Center for Integrated Research in Fundamental Science and Engineering, University of Tsukuba, Tsukuba, Japan
165 Department of Physics and Astronomy, Tufts University, Medford, MA, USA
166 Department of Physics and Astronomy, University of California Irvine, Irvine, CA, USA
167 (a) INFN Gruppo Collegato di Udine, Sezione di Trieste, Udine, Italy; (b) ICTP, Trieste, Italy; (c) Dipartimento di Chimica, Fisica e Ambiente, Università di Udine, Udine, Italy
168 Department of Physics and Astronomy, University of Uppsala, Uppsala, Sweden
169 Department of Physics, University of Illinois, Urbana, IL, USA
170 Instituto de Fisica Corpuscular (IFIC), Centro Mixto Universidad de Valencia - CSIC, Valencia, Spain
171 Department of Physics, University of British Columbia, Vancouver, BC, Canada
172 Department of Physics and Astronomy, University of Victoria, Victoria, BC, Canada
173 Department of Physics, University of Warwick, Coventry, UK
174 Waseda University, Tokyo, Japan
175 Department of Particle Physics, The Weizmann Institute of Science, Rehovot, Israel
176 Department of Physics, University of Wisconsin, Madison, WI, USA
177 Fakultät für Physik und Astronomie, Julius-Maximilians-Universität, Würzburg, Germany
178 Fakultät für Mathematik und Naturwissenschaften, Fachgruppe Physik, Bergische Universität Wuppertal, Wuppertal, Germany
179 Department of Physics, Yale University, New Haven, CT, USA
180 Yerevan Physics Institute, Yerevan, Armenia
181 Centre de Calcul de l’Institut National de Physique Nucléaire et de Physique des Particules (IN2P3), Villeurbanne, France
182 Academia Sinica Grid Computing, Institute of Physics, Academia Sinica, Taipei, Taiwan
a Also at Department of Physics, King’s College London, London, UK
b Also at Institute of Physics, Azerbaijan Academy of Sciences, Baku, Azerbaijan
c Also at Novosibirsk State University, Novosibirsk, Russia
d Also at TRIUMF, Vancouver, BC, Canada
e Also at Department of Physics and Astronomy, University of Louisville, Louisville, KY, USA
f Also at Physics Department, An-Najah National University, Nablus, Palestine
g Also at Department of Physics, California State University, Fresno, CA, USA
h Also at Department of Physics, University of Fribourg, Fribourg, Switzerland
i Also at II Physikalisches Institut, Georg-August-Universität, Göttingen, Germany
j Also at Departament de Física de la Universitat Autònoma de Barcelona, Barcelona, Spain
k Also at Departamento de Física e Astronomia, Faculdade de Ciencias, Universidade do Porto, Porto, Portugal
l Also at Tomsk State University, Tomsk, and and Moscow Institute of Physics and Technology State University, Dolgoprudny, Russia
m Also at The Collaborative Innovation Center of Quantum Matter (CICQM), Beijing, China
n Also at Universita di Napoli Parthenope, Napoli, Italy
o Also at Institute of Particle Physics (IPP), Canada
p Also at Horia Hulubei National Institute of Physics and Nuclear Engineering, Bucharest, Romania
q Also at Department of Physics, St. Petersburg State Polytechnical University, St. Petersburg, Russia
r Also at Borough of Manhattan Community College, City University of New York, New York, USA
s Also at Department of Financial and Management Engineering, University of the Aegean, Chios, Greece
t Also at Centre for High Performance Computing, CSIR Campus, Rosebank, Cape Town, South Africa
u Also at Louisiana Tech University, Ruston, LA, USA
v Also at Institutio Catalana de Recerca i Estudis Avancats, ICREA, Barcelona, Spain
w Also at Department of Physics, The University of Michigan, Ann Arbor MI, United States of America
x Also at Graduate School of Science, Osaka University, Osaka, Japan
y Also at Fakultät für Mathematik und Physik, Albert-Ludwigs-Universität, Freiburg, Germany
z Also at Institute for Mathematics, Astrophysics and Particle Physics, Radboud University Nijmegen/Nikhef, Nijmegen, The Netherlands
aa Also at Department of Physics, The University of Texas at Austin, Austin, TX, USA
ab Also at Institute of Theoretical Physics, Ilia State University, Tbilisi, Georgia
ac Also at CERN, Geneva, Switzerland
ad Also at Georgian Technical University (GTU), Tbilisi, Georgia
ae Also at Ochadai Academic Production, Ochanomizu University, Tokyo, Japan
af Also at Manhattan College, New York, NY, USA
ag Also at The City College of New York, New York NY, USA
ah Also at Departamento de Física Teorica y del Cosmos, Universidad de Granada, Granada, Portugal
ai Also at Department of Physics, California State University, Sacramento, CA, USA
aj Also at Moscow Institute of Physics and Technology State University, Dolgoprudny, Russia
ak Also at Departement de Physique Nucleaire et Corpusculaire, Université de Genève, Geneva, Switzerland
al Also at Institut de Física d’Altes Energies (IAFE), The Barcelona Institute of Science and Technology, Barcelona, Spain
am Also at School of Physics, Sun Yat-sen University, Guangzhou, China
an Also at Institute for Nuclear Research and Nuclear Energy (INRNE) of the Bulgarian Academy of Sciences, Sofia, Bulgaria
ao Also at Faculty of Physics, M.V. Lomonosov Moscow State University, Moscow, Russia
ap Also at National Research Nuclear University MEPhI, Moscow, Russia
aq Also at Department of Physics, Stanford University, Stanford, CA, USA
ar Also at Institute for Particle and Nuclear Physics, Wigner Research Centre for Physics, Budapest, Hungary
as Also at Faculty of Engineering, Giresun University, Giresun, Turkey
at Also at CPPM, Aix-Marseille Université and CNRS/IN2P3, Marseille, France
au Also at Department of Physics, Nanjing University, Jiangsu, China
av Also at Department of Physics, Academia Sinica, Taipei, Taiwan
\textsuperscript{aw} Also at University of Malaya, Department of Physics, Kuala Lumpur, Malaysia
\textsuperscript{ax} Also at LAL, Univ. Paris-Sud, CNRS/IN2P3, Université Paris-Saclay, Orsay, France
*Deceased