A GRAVITATIONAL-WAVE STANDARD SIREN MEASUREMENT OF THE HUBBLE CONSTANT

The detection of GW170817 (Abbott et al. 2017a) in both gravitational waves and electromagnetic waves heralds the age of gravitational-wave multi-messenger astronomy. On 17 August 2017 the Advanced Laser Interferometer Gravitational-wave Observatory (LIGO) (LIGO Scientific Collaboration et al. 2015) and Virgo (Acernese et al. 2015) detectors observed GW170817, a strong signal from the merger of a binary neutron-star system. Less than 2 seconds after the merger, a gamma-ray burst (GRB 170817A) was detected within a region of the sky consistent with the LIGO-Virgo-derived location of the gravitational-wave source (Abbott et al. 2017b; Goldstein et al. 2017; Savchenko et al. 2017). This sky region was subsequently observed by optical astronomy facilities (Abbott et al. 2017c), resulting in the identification of an optical transient signal within $\sim 10$ arcsec of the galaxy NGC 4993 (Coulter et al. 2017; Soares-Santos et al. 2017; Valenti et al. 2017; Arcavi et al. 2017; Tanvir et al. 2017; Lipunov et al. 2017). These multi-messenger observations allow us to use GW170817 as a standard siren (Schutz 1986; Holz & Hughes 2005; Dalal et al. 2006; Nissanke et al. 2010, 2013), the gravitational-wave analog of an astronomical standard candle, to measure the Hubble constant. This quantity, which represents the local expansion rate of the Universe, sets the overall scale of the Universe and is of fundamental importance to cosmology. Our measurement combines the distance to the source inferred purely from the gravitational-wave signal with the recession velocity inferred from measurements of the redshift using electromagnetic data. This approach does not require any form of cosmic “distance ladder” (Freedman et al. 2001); the gravitational-wave (GW) analysis can be used to estimate the luminosity distance out to cosmological scales directly, without the use of intermediate astronomical distance measurements. We determine the Hubble constant to be $70.0^{+12.0}_{-8.0}$ km s$^{-1}$ Mpc$^{-1}$ (maximum a posteriori and 68% credible interval). This is consistent with existing measurements (Planck Collaboration et al. 2016; Riess et al. 2016), while being completely independent of them. Additional standard-siren measurements from future gravitational-wave sources will provide precision constraints of this important cosmological parameter.
The Hubble constant \( H_0 \) measures the mean expansion rate of the Universe. At nearby distances \((d \lesssim 50 \text{ Mpc})\) it is well approximated by the expression

\[
v_H = H_0 d, \tag{1}\]

where \( v_H \) is the local “Hubble flow” velocity of a source, and \( d \) is the distance to the source. At such distances all cosmological distance measures (such as luminosity distance and comoving distance) differ at the order of \( v_H/c \) where \( c \) is the speed of light. As \( v_H/c \sim 1\% \) for GW170817 we do not distinguish between them. We are similarly insensitive to the values of other cosmological parameters, such as \( \Omega_m \) and \( \Omega_\Lambda \).

To obtain the Hubble flow velocity at the position of GW170817, we use the optical identification of the host galaxy NGC 4993 (Abbott et al. 2017c). This identification is based solely on the 2-dimensional projected offset and is independent of any assumed value of \( H_0 \). The position and redshift of this galaxy allow us to estimate the appropriate value of the Hubble flow velocity. Because the source is relatively nearby the random relative motions of galaxies, known as peculiar velocities, need to be taken into account. The peculiar velocity is \( \sim 10\% \) of the measured recessional velocity (see Methods).

The original standard siren proposal (Schutz 1986) did not rely on the unique identification of a host galaxy. By combining information from \( \sim 100 \) independent GW detections, each with a set of potential host galaxies, a \( \sim 5\% \) estimate of \( H_0 \) can be obtained even without the detection of any transient optical counterparts (Del Pozzo 2012). This is particularly relevant, as gravitational-wave networks will detect many binary black hole mergers over the coming years (Abbott et al. 2016a), and these are not expected to be accompanied by electromagnetic counterparts. Alternatively, if an EM counterpart has been identified but the host galaxy is unknown, the same statistical method can be applied but using only those galaxies in a narrow beam around the location of the optical counterpart. However, such statistical analyses are sensitive to a number of complicating effects, including the incompleteness of current galaxy catalogs or the need for dedicated follow-up surveys, as well as a range of selection effects (Messenger & Veitch 2013). In what follows we exploit the identification of NGC 4993 as the host galaxy of GW170817 to perform a standard siren measurement of the Hubble constant (Holz & Hughes 2005; Dalal et al. 2006; Nissanke et al. 2010, 2013).

Analysis of the GW data associated with GW170817 produces estimates for the parameters of the source, under the assumption that general relativity is the correct model of gravity (Abbott et al. 2017a). We are most interested in the joint posterior distribution on the luminosity distance and binary orbital inclination angle. For the analysis in this paper we fix the location of the GW source on the sky to the identified location of the counterpart (Coulter et al. 2017). See the Methods section for details.

An analysis of the GW data alone finds that GW170817 occurred at a distance \( d = 43.8^{+2.9}_{-6.9} \text{ Mpc} \) (all values are quoted as the maximum posterior value with the minimal width 68.3% credible interval). We note that the distance quoted here differs from that in other studies (Abbott et al. 2017a), since here we assume that the optical counterpart represents the true sky location of the GW source instead of marginalizing over a range of potential sky locations. The \( \sim 15\% \) uncertainty is due to a combination of statistical measurement error from the noise in the detectors, instrumental calibration uncertainties (Abbott et al. 2017a), and a geometrical factor dependent upon the correlation of distance with inclination angle. The GW measurement is consistent with the distance to NGC 4993 measured using the Tully-Fisher relation, \( d_{TF} = 41.1 \pm 5.8 \text{ Mpc} \) (Sakai et al. 2000; Freedman et al. 2001).
The measurement of the GW polarization is crucial for inferring the binary inclination. This inclination, $\iota$, is defined as the angle between the line of sight vector from the source to the detector and the orbital angular momentum vector of the binary system. For electromagnetic (EM) phenomena it is typically not possible to tell whether a system is orbiting clockwise or counter-clockwise (or, equivalently, face-on or face-off), and sources are therefore usually characterized by a viewing angle: \( \min (\iota, 180^\circ - \iota) \). By contrast, GW measurements can identify the sense of the rotation, and thus $\iota$ ranges from 0 (counter-clockwise) to 180 deg (clockwise).

Previous GW detections by LIGO had large uncertainties in luminosity distance and inclination (Abbott et al. 2016a) because the two LIGO detectors that were involved are nearly co-aligned, preventing a precise polarization measurement. In the present case, thanks to Virgo as an additional detector, the cosine of the inclination can be constrained at 68.3\% (1\sigma) confidence to the range \([-1.00, -0.81]\) corresponding to inclination angles between \([144, 180]\) deg. This implies that the plane of the binary orbit is almost, but not quite, perpendicular to our line of sight to the source ($\iota \approx 180$ deg), which is consistent with the observation of a coincident GRB (LVC, GBM, & INTEGRAL 2017 in prep.; Goldstein et al. 2017, ApJL, submitted; Savchenko et al. 2017, ApJL, submitted). We report inferences on $\cos \iota$ because our prior for it is flat, so the posterior is proportional to the marginal likelihood for it from the GW observations.

EM follow-up of the GW sky localization region (Abbott et al. 2017c) discovered an optical transient (Coulter et al. 2017; Soares-Santos et al. 2017; Valenti et al. 2017; Arcavi et al. 2017; Tanvir et al. 2017; Lipunov et al. 2017) in close proximity to the galaxy NGC 4993. The location of the transient was previously observed by the Distance Less Than 40 Mpc (DLT40) survey on 2017 July 27.99 UT and no sources were found (Valenti et al. 2017). We estimate the probability of a random chance association between the optical counterpart and NGC 4993 to be 0.004\% (see the Methods section for details). In what follows we assume that the optical counterpart is associated with GW170817, and that this source resides in NGC 4993.

To compute $H_0$ we need to estimate the background Hubble flow velocity at the position of NGC 4993. In the traditional electromagnetic calibration of the cosmic “distance ladder” (Freedman et al. 2001), this step is commonly carried out using secondary distance indicator information, such as the Tully-Fisher relation (Sakai et al. 2000), which allows one to infer the background Hubble flow velocity in the local Universe scaled back from more distant secondary indicators calibrated in quiet Hubble flow. We do not adopt this approach here, however, in order to preserve more fully the independence of our results from the electromagnetic distance ladder. Instead we estimate the Hubble flow velocity at the position...
Figure 2. Inference on $H_0$ and inclination. Posterior density of $H_0$ and $\cos \iota$ from the joint GW-EM analysis (blue contours). Shading levels are drawn at every 5% credible level, with the 68.3% (1σ, solid) and 95.4% (2σ, dashed) contours in black. Values of $H_0$ and 1- and 2σ error bands are also displayed from Planck (Planck Collaboration et al. 2016) and SHoES (Riess et al. 2016). As noted in the text, inclination angles near 180 deg ($\cos \iota = -1$) indicate that the orbital angular momentum is anti-parallel with the direction from the source to the detector.

Once the distance and Hubble velocity distributions have been determined from the GW and EM data, respectively, we can constrain the value of the Hubble constant. The measurement of the distance is strongly correlated with the measurement of the inclination of the orbital plane of the binary. The analysis of the GW data also depends on other parameters describing the source, such as the masses of the components (Abbott et al. 2016a). Here we treat the uncertainty in these other variables by marginalizing over the posterior distribution on system parameters (Abbott et al. 2017a), with the exception of the position of the system on the sky which is taken to be fixed at the location of the optical counterpart.

We carry out a Bayesian analysis to infer a posterior distribution on $H_0$ and inclination, marginalized over uncertainties in the recessional and peculiar velocities; see the Methods section for details. Figure 1 shows the marginal posterior for $H_0$. The maximum a posteriori value with the minimal 68.3% credible interval is $H_0 = 70.0^{+12.0}_{-8.0}$ km s$^{-1}$ Mpc$^{-1}$. Our estimate agrees well with state-of-the-art determinations of this quantity, including CMB measurements from Planck (Planck Collaboration et al. 2016) (67.74 ± 0.46 km s$^{-1}$ Mpc$^{-1}$, “TT,TE,EE+lowP+lensing+ext”) and Type Ia supernova measurements from SHoES (Riess et al. 2016) (73.24 ± 1.74 km s$^{-1}$ Mpc$^{-1}$), as well as baryon acoustic oscillations measurements from SDSS (Aubourg et al. 2015), strong lensing measurements from H0LiCOW (Bonvin et al. 2017), high-$l$ CMB measurements from SPT (Henning et al. 2017), and Cepheid measurements from the HST key project (Freedman et al. 2001). Our measurement is a new and independent determination of this quantity. The close agreement indicates that, although each method may be affected by different systematic uncertainties, we see no evidence at present for a systematic difference between GW and established EM-based estimates. As has been much remarked upon, the Planck and SHoES re-
sults are inconsistent at $\gtrsim 3\sigma$ level. Our measurement does not resolve this tension, and is broadly consistent with both.

One of the main sources of uncertainty in our measurement of $H_0$ is due to the degeneracy between distance and inclination in the GW measurements. A face-on or face-off binary far away has a similar gravitational-wave amplitude to an edge-on binary closer in. This relationship is captured in Figure 2, which shows posterior contours in the $H_0$–$\cos \iota$ parameter space.

The posterior in Figure 1 results from the vertical projection of Figure 2, marginalizing out uncertainties in the cosine of inclination to derive constraints on the Hubble constant. Alternatively, it is possible to project horizontally, and thereby marginalize out the Hubble constant to derive constraints on the cosine of inclination. If instead of deriving $H_0$ independently we take the existing constraints on $H_0$ (Planck Collaboration et al. 2016; Riess et al. 2016) as priors, we are able to significantly improve our constraints on $\cos \iota$ as shown in Figure 3. Assuming the Planck value for $H_0$, the minimal 68.3% credible interval for the cosine of inclination is $[-1.00, -0.92]$ (corresponding to an inclination angle range $[157, 177] \degree$). For the SHoES value of $H_0$, it is $[-0.97, -0.85]$ (corresponding to an inclination angle range $[148, 166] \degree$). For this latter SHoES result we note that the face-off $\iota = 180 \degree$ orientation is just outside the 90% confidence range. It will be particularly interesting to compare these constraints to those from modeling of the short GRB, afterglow, and optical counterpart associated with GW170817 (Abbott et al. 2017c).

We have presented a standard siren determination of the Hubble constant, using a combination of a GW distance and an EM Hubble velocity estimate. Our measurement does not use a “distance ladder”, and makes no prior assumptions about $H_0$. We find $H_0 = 70.0^{+12.0}_{-8.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is consistent with existing measurements (Riess et al. 2016; Planck Collaboration et al. 2016). This first GW–EM multi-messenger event demonstrates the potential for cosmological inference from GW standard sirens. We expect that additional multi-messenger binary neutron-star events will be detected in the coming years, and combining subsequent independent measurements of $H_0$ from these future standard sirens will lead to an era of precision gravitational-wave cosmology.
METHODS

PROBABILITY OF OPTICAL COUNTERPART ASSOCIATION WITH NGC 4993

We calculate the probability that an NGC 4993-like galaxy (or brighter) is misidentified as the host by asking how often the centre of one or more such galaxies falls by random chance within a given angular radius \( \theta \) of the counterpart. Assuming Poisson counting statistics this probability is given by

\[
P = 1 - \exp \left[ -\pi \theta^2 S(<m) \right]
\]

where \( S(<m) \) is the surface density of galaxies with apparent magnitude equal to or brighter than \( m \). From the local galaxy sample distribution in the infrared (K-band) apparent magnitude (Huang et al. 1998) we obtain \( S(<K) = 0.68 \times 10^{0.64(K-10.0)-0.7} \text{ deg}^{-2} \). As suggested by (Bloom et al. 2002), we set \( \theta \) equal to twice the half-light radius of the galaxy, for which we use NGC 4993’s diameter of \( \sim 1.1 \) arcmin, as measured in the near infrared band (the predominant emission band for early-type galaxies). Using \( K = 9.2 \) mag taken from the 2MASS survey (Skrutskie et al. 2006) for NGC 4993, we find the probability of random chance association is \( P = 0.004\% \).

FINDING THE HUBBLE VELOCITY OF NGC 4993

In previous EM determinations of the cosmic “distance ladder”, the Hubble flow velocity of the local calibrating galaxies has generally been estimated using redshift-independent secondary galaxy distance indicators, such as the Tully-Fisher relation or type Ia supernovae, calibrated with more distant samples that can be assumed to sit in quiet Hubble flow (Freedman et al. 2001). We do not adopt this approach for NGC 4993, however, in order that our inference of the Hubble constant is fully independent of the electromagnetic distance scale. Instead we estimate the Hubble flow velocity at the position of NGC 4993 by correcting its measured recessional velocity for local peculiar motions.

NGC 4993 resides in a group of galaxies whose center-of-mass recession velocity relative to the Cosmic Microwave Background (CMB) frame (Hinshaw et al. 2009) is (Crook et al. 2007, 2008) \( 3327 \pm 72 \text{ km s}^{-1} \). We assume that all of the galaxies in the group are at the same distance and therefore have the same Hubble flow velocity, which we assign to be the Hubble velocity of GW170817. This assumption is accurate to within 1% given that the radius of the group is \( \sim 0.4 \) Mpc. To calculate the Hubble flow velocity of the group, we correct its measured recessional velocity by the peculiar velocity caused by the local gravitational field. This is a significant correction (Springob et al. 2014; Carrick et al. 2015); typical peculiar velocities are 300 km s\(^{-1}\), equivalent to \( \sim 10\% \) of the total recessional velocity at a distance of 40 Mpc.

We employ the 6dF galaxy redshift survey peculiar velocity map (Springob et al. 2014; Jones et al. 2009), which used more than 8,000 Fundamental Plane galaxies to map the peculiar velocity field in the Southern hemisphere out to redshift \( z \approx 0.055 \). We weight the peculiar velocity corrections from this catalog with a Gaussian centered on NGC 4993’s sky position and with a width of \( 8h^{-1} \) Mpc; the kernel width is independent of \( H_0 \) and is equivalent to a width of 800 km s\(^{-1}\) in velocity space, typical of the widths used in the catalog itself. There are 10 galaxies in the 6dF peculiar velocity catalog within one kernel width of NGC 4993. In the CMB frame (Hinshaw et al. 2009), the weighted radial component of the peculiar velocity and associated uncertainty is \( \langle v_p \rangle = 310 \pm 69 \text{ km s}^{-1} \).

We verified the robustness of this peculiar velocity correction by comparing it with the velocity field reconstructed from the 2MASS redshift survey (Carrick et al. 2015; Huchra et al. 2012). This exploits the linear relationship between the peculiar velocity and mass density fields smoothed on scales larger than about \( 8h^{-1} \) Mpc, and the constant of proportionality can be determined by com-
comparison with radial peculiar velocities of individual galaxies estimated from e.g. Tully-Fisher and Type Ia supernovae distances. Using these reconstructed peculiar velocities, which have a larger associated uncertainty (Carrick et al. 2015) of 150 km s$^{-1}$, at the position of NGC 4993 we find a Hubble velocity in the CMB frame of $v_H = 3047$ km s$^{-1}$ – in excellent agreement with the result derived using 6dF. We adopt this larger uncertainty on the peculiar velocity correction in recognition that the peculiar velocity estimated from the 6dF data may represent an imperfect model of the true bulk flow at the location of NGC 4993. For our inference of the Hubble constant we therefore use a Hubble velocity $v_H = 3017 \pm 166$ km s$^{-1}$ with 68.3% uncertainty.

Finally, while we emphasise again the independence of our Hubble constant inference from the electromagnetic distance scale, we note the consistency of our GW distance estimate to NGC 4993 with the Tully-Fisher distance estimate derived by scaling back the Tully-Fisher relation calibrated with more distant galaxies in quiet Hubble flow (Sakai et al. 2000). This also strongly supports the robustness of our estimate for the Hubble velocity of NGC 4993.

**SUMMARY OF THE MODEL**

Given observed data from a set of GW detectors, $x_{GW}$, parameter estimation is used to generate a posterior on the parameters that determine the waveform of the GW signal. Parameters are inferred within a Bayesian framework (Veitch et al. 2015) by comparing strain measurements (Abbott et al. 2017a) in the two LIGO detectors and the Virgo detector with the gravitational waveforms expected from the inspiral of two point masses (Hannam et al. 2014) under general relativity. We use algorithms for removing short-lived detector noise artifacts (Abbott et al. 2017a; Cornish & Littenberg 2015) and we employ approximate point-particle waveform models (Buonanno & Damour 1999; Blanchet 2014; Hannam et al. 2014). We have verified that the systematic changes in the results presented here from incorporating non-point-mass (tidal) effects (Hinderer & Flanagan 2008; Vines et al. 2011) and from different data processing methods are much smaller than the statistical uncertainties in the measurement of $H_0$ and the binary orbital inclination angle.

From this analysis we can obtain the parameter estimation likelihood of the observed GW data, marginalized over all parameters characterizing the GW signal except $d$ and $\cos \iota$,

$$p(x_{GW} \mid d, \cos \iota) = \int p(x_{GW} \mid d, \cos \iota, \lambda) p(\lambda) d\lambda.$$  

(2)

The other waveform parameters are denoted by $\lambda$, with $p(\lambda)$ denoting the corresponding prior.

Given perfect knowledge of the Hubble flow velocity of the GW source, $v_H$, this posterior distribution can be readily converted into a posterior on $\cos \iota$ and $H_0 = v_H/d$,

$$p(H_0, \cos \iota \mid x_{GW}) \propto (v_H/H_0^2) p(x_{GW} \mid d = v_H/H_0, \cos \iota) \times p_d(v_H/H_0) p_\iota(\cos \iota),$$  

(3)

where $p_d(d)$ and $p_\iota(\cos \iota)$ are the prior distributions on distance and inclination. For the Hubble velocity $v_H = 3017$ km s$^{-1}$, the maximum a posteriori distance from the GW measurement of 43.8 Mpc corresponds to $H_0 = 68.9$ km s$^{-1}$ Mpc$^{-1}$, so this procedure would be expected to generate a posterior on $H_0$ that peaks close to that value.

While the above analysis is conceptually straightforward, it makes a number of assumptions. In practice, the Hubble-flow velocity cannot be determined exactly and it must be corrected for uncertain peculiar velocities. The above does not explicitly set a prior on $H_0$, but instead inherits a $1/H_0^4$ prior from the usual $p_d(d) \propto d^2$ prior used in GW parameter estimation. In addition, the logic in this model is that a redshift has been obtained first and the distance is then measured using GWs. As GW detectors cannot be pointed,
we cannot target particular galaxies or redshifts for GW sources. In practice, we wait for a GW event to trigger the analysis and this introduces potential selection effects which we must consider. We will see below that the simple analysis described above does give results that are consistent with a more careful analysis for this first detection. However, the simple analysis cannot be readily extended to include second and subsequent detections, so we now describe a more general framework that does not suffer from these limitations.

We suppose that we have observed a GW event, which generated data \( x_{GW} \) in our detectors, and that we have also measured a recessional velocity for the host, \( v_r \), and the peculiar velocity field, \( \langle v_p \rangle \), in the vicinity of the host. These observations are statistically independent and so the combined likelihood is

\[
\begin{align*}
p(x_{GW}, v_r, \langle v_p \rangle | d, \cos \iota, v_p, H_0) = \\
p(x_{GW} | d, \cos \iota) p(v_r | d, v_p, H_0) p(\langle v_p \rangle | v_p).
\end{align*}
\]  

The quantity \( p(v_r | d, v_p, H_0) \) is the likelihood of the recessional velocity measurement, which we model as

\[
p(v_r | d, v_p, H_0) = N[v_p + H_0 d, \sigma_{v_r}^2](v_r)
\]

where \( N[\mu, \sigma^2](x) \) is the normal (Gaussian) probability density with mean \( \mu \) and standard deviation \( \sigma \) evaluated at \( x \). The measured recessional velocity, \( v_r = 3327 \text{ km s}^{-1} \), with uncertainty \( \sigma_{v_r} = 72 \text{ km s}^{-1} \), is the mean velocity and standard error for the members of the group hosting NGC 4993 taken from the two micron all sky survey (2MASS) (Crook et al. 2007, 2008), corrected to the CMB frame (Hinshaw et al. 2009). We take a similar Gaussian likelihood for the measured peculiar velocity, \( \langle v_p \rangle = 310 \text{ km s}^{-1} \), with uncertainty \( \sigma_{v_p} = 150 \text{ km s}^{-1} \):

\[
p(\langle v_p \rangle | v_p) = N[v_p, \sigma_{v_p}^2](\langle v_p \rangle).
\]

From the likelihood (4) we derive the posterior

\[
p(H_0, d, \cos \iota, v_p | x_{GW}, v_r, \langle v_p \rangle)
\]  

\[
\propto \frac{p(H_0)}{\mathcal{N}_s(H_0)} p(x_{GW} | d, \cos \iota) p(v_r | d, v_p, H_0) \times p(\langle v_p \rangle | v_p) p(d) p(v_p) p(\cos \iota),
\]  

where \( p(H_0) \), \( p(d) \), \( p(v_p) \) and \( p(\cos \iota) \) are the parameter prior probabilities. Our standard analysis assumes a volumetric prior, \( p(d) \propto d^2 \), on the Hubble distance, but we explore sensitivity to this choice below. We take a flat-in-log prior on \( H_0 \), \( p(H_0) \propto 1/H_0 \), impose a flat (i.e. isotropic) prior on \( \cos \iota \), and a flat prior on \( v_p \) for \( v_p \in [-1000, 1000] \text{ km s}^{-1} \). These priors characterise our beliefs about the cosmological population of GW events and their hosts before we make any additional measurements or account for selection biases. The full statistical model is summarized graphically in Extended Data Figure 1. This model with these priors is our canonical analysis.

In Eq. (7), the term \( \mathcal{N}_s(H_0) \) encodes selection effects (Loredo 2004; Mandel et al. 2016; Abbott et al. 2016a). These arise because of the finite sensitivity of our detectors. While all events in the Universe generate a response in the detector, we will only be able to identify, and hence use, signals that generate a response of sufficiently high amplitude. The decision about whether to include an event in the analysis is a property of the data only, in this case \( \{x_{GW}, v_r, \langle v_p \rangle\} \), but the fact that we condition our analysis on a signal being detected, i.e., the data exceeding these thresholds, means that the likelihood must be renormalized to become the likelihood for detected events. This is the role of

\[
\mathcal{N}_s(H_0) = \int_{\text{detectable}} d\tilde{\lambda} dd dv_p d\cos \iota dx_{GW} dv_r d\langle v_p \rangle
\]  

\[
\times \left[p(x_{GW} | d, \cos \iota, \tilde{\lambda}) p(v_r | d, v_p, H_0) \times p(\langle v_p \rangle | v_p) p(\tilde{\lambda}) p(d) p(v_p) p(\cos \iota)\right],
\]

where the integral is over the full prior ranges of the parameters, \( \{d, v_p, \cos \iota, \tilde{\lambda}\} \), and over data sets.
that would be selected for inclusion in the analysis, i.e., exceed the specified thresholds. If the integral was over all data sets it would evaluate to 1, but because the range is restricted there can be a non-trivial dependence on parameters characterizing the population of sources, in this case \( H_0 \).

In the current analysis, there are in principle selection effects in both the GW data and the EM data. However, around the time of detection of GW170817, the LIGO-Virgo detector network had a detection horizon of \( \sim 190 \text{ Mpc} \) for binary neutron star (BNS) events (Abbott et al. 2017a), within which EM measurements are largely complete. For example, the counterpart associated with GW170817 had brightness \( \sim 17 \text{ mag} \) in the I band at 40 Mpc (Valenti et al. 2017; Arcavi et al. 2017; Tanvir et al. 2017; Lipunov et al. 2017; Coulter et al. 2017); this source would be \( \sim 22 \text{ mag} \) at 400 Mpc, and thus still detectable by survey telescopes such as DECam well beyond the GW horizon. Even the dimmest theoretical lightcurves for kilonovae are expected to peak at \( \sim 22.5 \text{ mag} \) at the LIGO–Virgo horizon (Metzger & Berger 2012). We therefore expect that we are dominated by GW selection effects at the current time and can ignore EM selection effects. The fact that the fraction of BNS events that will have observed kilonova counterparts is presently unknown does not modify these conclusions, since we can restrict our analysis to GW events with kilonova counterparts only.

In the GW data, the decision about whether or not to analyse an event is largely determined by the signal-to-noise ratio (SNR), \( \rho \), of the event. A reasonable model for the selection process is a cut in SNR, i.e., events with \( \rho > \rho_s \) are analysed (Abbott et al. 2016b). In that model, the integral over \( x_{\text{GW}} \) in Eq. (8) can be replaced by an integral over SNR from \( \rho_s \) to \( \infty \), and \( p(x_{\text{GW}} | d, \cos \iota, \vec{\lambda}) \) replaced by \( p(\rho | d, \cos \iota, \vec{\lambda}) \) in the integrand. This distribution depends on the noise properties of the operating detectors, and on the intrinsic strain amplitude of the source. The former are clearly independent of the population parameters, while the latter scales like a function of the source parameters divided by the luminosity distance. The dependence on source parameters is on redshifted parameters, which introduces an explicit redshift dependence. However, within the \( \sim 190 \text{ Mpc} \) horizon, redshift corrections are at most \( \lesssim 5\% \), and the Hubble constant measurement is a weak function of these, meaning the overall impact is even smaller. At present, whether or not a particular event in the population ends up being analysed can therefore be regarded as a function of \( d \) only. When GW selection effects dominate, only the terms in Eq. (8) arising from the GW measurement matter. As these are a function of \( d \) only and we set a prior on \( d \), there is no explicit \( H_0 \) dependence in these terms. Hence, \( \mathcal{N}_0(H_0) \) is a constant and can be ignored. This would not be the case if we set a prior on the redshifts of potential sources instead of their distances, since then changes in \( H_0 \) would modify the range of detectable redshifts. As the LIGO–Virgo detectors improve in sensitivity the redshift dependence in the GW selection effects will become more important, as will EM selection effects. However, at that point we will also have to consider deviations in the cosmological model from the simple Hubble flow described in Eq. (1) of the main article.

Marginalising Eq. (7) over \( d, v_p \) and \( \cos \iota \) then yields

\[
p(H_0 | x_{\text{GW}}, v_r, \langle v_p \rangle) \propto p(H_0) \int dd dv_p d\cos \iota \times p(x_{\text{GW}} | d, \cos \iota) p(v_r | d, v_p, H_0) \times p(\langle v_p \rangle | v_p) p(d|v_p) p(\cos \iota). \tag{9}
\]

The posterior computed in this way was shown in Figure 1 in the main article and has a maximum a posteriori value and minimal 68.3% credible interval of \( 70.0^{+12.0}_{-8.0} \text{ km s}^{-1} \text{ Mpc}^{-1} \), as quoted in the main article. The posterior mean is \( 78 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and the standard deviation is \( 15 \text{ km s}^{-1} \text{ Mpc}^{-1} \). Various other summary statistics are given in Extended Data Table 1.

**ROBUSTNESS TO PRIOR SPECIFICATION**
Extended Data Figure 1. Graphical model illustrating the statistical relationships between the data and parameters. Open circles indicate parameters which require a prior; filled circles described measured data, which are conditioned on in the analysis. Here we assume we have measurements of the GW data, $x_{GW}$, a recessional velocity (i.e. redshift), $v_r$, and the mean peculiar velocity in the neighborhood of NGC 4993, $\langle v_p \rangle$. Arrows flowing into a node indicate that the conditional probability density for the node depends on the source parameters; for example, the conditional distribution for the observed GW data, $p(x_{GW} \mid d, \cos \iota)$, discussed in the text, depends on the distance and inclination of the source (and additional parameters, here marginalized out).

Our canonical analysis uses a uniform volumetric prior on distance, $p(d) \propto d^2$. The distribution of galaxies is not completely uniform due to clustering, so we explore sensitivity to this prior choice. We are free to place priors on any two of the three variables $\{d, H_0, z\}$, where $z = H_0 d/c$ is the Hubble flow redshift of NGC 4993. A choice of prior for two of these variables induces a prior on the third which may or may not correspond to a natural choice for that parameter. A prior on $z$ could be obtained from galaxy catalog observations (Dalya et al. 2016), but must be corrected for incompleteness. When setting a prior on $H_0$ and $z$, the posterior becomes

$$p(H_0, z, \cos \iota, v_p, \nu_v, \langle v_p \rangle)$$

$$\propto p(H_0) \frac{p(x_{GW} \mid d = cz/H_0, \cos \iota)}{N_s(H_0)} p(v_r \mid z, v_p) \times p(\langle v_p \rangle \mid v_p) p(z) p(v_p) p(\cos \iota), \quad (10)$$

but now

$$N_s(H_0) = \int_{\text{detectable}} dz \, dv_p \, d\cos \iota \, dx_{GW} \, dv_r \, d\langle v_p \rangle \times p(x_{GW} \mid d = cz/H_0, \cos \iota) p(v_r \mid z, v_p) \times p(\langle v_p \rangle \mid v_p) p(z) p(v_p) p(\cos \iota). \quad (11)$$

When GW selection effects dominate, the integral is effectively

$$N_s(H_0) = \int dz \, d\cos \iota \, dx_{GW} \times p(x_{GW} \mid d = cz/H_0, \cos \iota) p(z) p(\cos \iota) = \int dd \, d\cos \iota \, dx_{GW} \times p(x_{GW} \mid d, \cos \iota) p(dH_0/c) p(\cos \iota) (H_0/c), \quad (12)$$

which has an $H_0$ dependence, unless $p(z)$ takes a special, $H_0$-dependent form, $p(z) = f(z/H_0)/H_0$. However, if the redshift prior is volumetric, $p(z) \propto z^2$, the selection effect term is $\propto H_0^3$, which cancels a similar correction to the likelihood and gives a posterior on $H_0$ that is identical to the canonical analysis.

For a single event, any choice of prior can be mapped to our canonical analysis with a different prior on $H_0$. For any reasonable prior choices on $d$ or $z$, we would expect to gradually lose sensitivity to the particular prior choice as further observed events are added to the analysis. However, to illustrate the uncertainty that comes from the prior choice for this first event, we compare in Extended Data Figure 2 and Extended Data Table 1 the results from the canonical prior choice $p(d) \propto d^2$ to those from two other choices: using a flat prior
on $z$, and assuming a velocity correction due to the peculiar velocity of NGC 4993 that is a Gaussian with width 250 km s$^{-1}$. (To do the first of these, the posterior samples from GW parameter estimation have to be re-weighted, since they are generated with the $d^2$ prior used in the canonical analysis. We first “undo” the default prior before applying the desired new prior.)

The choice of a flat prior on $z$ is motivated by the simple model described above, in which we imagine first making a redshift measurement for the host and then use that as a prior for analysing the GW data. Setting priors on distance and redshift, the simple analysis gives the same result as the canonical analysis, but now we set a prior on redshift and $H_0$ and obtain a different result. This is to be expected because we are making different assumptions about the underlying population, and it arises for similar reasons as the different biases in peculiar velocity measurements based on redshift-selected or distance-selected samples (Strauss & Willick 1995). As can be seen in Extended Data Table 1, the results change by less than $1\sigma$, as measured by the statistical error of the canonical analysis.

By increasing the uncertainty in the peculiar velocity prior, we test the assumptions in our canonical analysis that (1) NGC 4993 is a member of the nearby group of galaxies, and (2) that this group has a center-of-mass velocity close to the Hubble flow. The results in Extended Data Table 1 summarizes changes in the values of $H_0$ and in the error bars.

We conclude that the impact of a reasonable change to the prior is small relative to the statistical uncertainties for this event.

**INCORPORATING ADDITIONAL CONSTRAINTS ON $H_0$**

By including previous measurements of $H_0$ (Planck Collaboration et al. 2016; Riess et al. 2016) we can constrain the orbital inclination more precisely. We do this by setting the $H_0$ prior in Eq. (7) to $p(H_0 | \mu_{H_0}, \sigma_{H_0}^2) = N[\mu_{H_0}, \sigma_{H_0}^2],$

Extended Data Figure 2. Using different assumptions compared to our canonical analysis. The posterior distribution on $H_0$ discussed in the main text is shown in black, the alternative flat prior on $z$ (discussed in the Methods section) gives the distribution shown in blue, and the increased uncertainty (250 km s$^{-1}$) applied to our peculiar velocity measurement (also discussed in the Methods section) is shown in pink. Minimal 68.3% ($1\sigma$) credible intervals are shown by dashed lines.

where for ShoES (Riess et al. 2016) $\mu_{H_0} = 73.24$ km s$^{-1}$ Mpc$^{-1}$ and $\sigma_{H_0} = 1.74$ km s$^{-1}$ Mpc$^{-1}$, while for Planck (Planck Collaboration et al. 2016) $\mu_{H_0} = 67.74$ km s$^{-1}$ Mpc$^{-1}$ and $\sigma_{H_0} = 0.46$ km s$^{-1}$ Mpc$^{-1}$. The posterior on $\cos \iota$ is then

$$p(\cos \iota \mid x_{GW}, v_r, \langle v_p \rangle, \mu_{H_0}, \sigma_{H_0}^2) \propto \int ddv_p dH_0 \times p(x_{GW} \mid d, \cos \iota) p(v_r \mid d, v_p, H_0) p(\langle v_p \rangle \mid v_p) \times p(H_0 \mid \mu_{H_0}, \sigma_{H_0}^2) p(d \mid v_p).$$

This posterior was shown in Figure 3 of the main article.

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## Extended Data Table 1. Summary of constraints on the Hubble constant, binary inclination, and distance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>68.3% Symm.</th>
<th>68.3% MAP</th>
<th>90% Symm.</th>
<th>90% MAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$ (km s$^{-1}$ Mpc$^{-1}$)</td>
<td>74.0$^{+16.0}_{-8.0}$</td>
<td>70.0$^{+12.0}_{-8.0}$</td>
<td>74.0$^{+33}_{-12}$</td>
<td>70.0$^{+28}_{-11}$</td>
</tr>
<tr>
<td>$H_0$ (km s$^{-1}$ Mpc$^{-1}$) (flat in z prior)</td>
<td>81$^{+27}_{-13}$</td>
<td>71.0$^{+23.0}_{-9.0}$</td>
<td>81$^{+50}_{-17}$</td>
<td>71.0$^{+48}_{-11}$</td>
</tr>
<tr>
<td>$H_0$ (km s$^{-1}$ Mpc$^{-1}$) (250 km s$^{-1}$ $\sigma_{\nu}$)</td>
<td>74.0$^{+16.0}_{-9.0}$</td>
<td>70.0$^{+14.0}_{-9.0}$</td>
<td>74.0$^{+33}_{-14}$</td>
<td>70.0$^{+29}_{-14}$</td>
</tr>
<tr>
<td>$\cos \iota$ (GW only)</td>
<td>$-0.88_{-0.09}^{+0.18}$</td>
<td>$-0.974_{-0.026}^{+0.164}$</td>
<td>$-0.88_{-0.11}^{+0.32}$</td>
<td>$-0.974_{-0.026}^{+0.332}$</td>
</tr>
<tr>
<td>$\cos \iota$ (SHoES)</td>
<td>$-0.901_{-0.057}^{+0.065}$</td>
<td>$-0.912_{-0.059}^{+0.061}$</td>
<td>$-0.901_{-0.083}^{+0.106}$</td>
<td>$-0.912_{-0.086}^{+0.095}$</td>
</tr>
<tr>
<td>$\cos \iota$ (Planck)</td>
<td>$-0.948_{-0.036}^{+0.052}$</td>
<td>$-0.982_{-0.016}^{+0.060}$</td>
<td>$-0.948_{-0.046}^{+0.091}$</td>
<td>$-0.982_{-0.018}^{+0.104}$</td>
</tr>
<tr>
<td>$\iota$/deg (GW only)</td>
<td>$152_{-17}^{+14}$</td>
<td>$167_{-23}^{+13}$</td>
<td>$152_{-27}^{+20}$</td>
<td>$167_{-37}^{+13}$</td>
</tr>
<tr>
<td>$\iota$/deg (SHoES)</td>
<td>$154.0_{-8.0}^{+9.0}$</td>
<td>$156.0_{-7.0}^{+10.0}$</td>
<td>$154.0_{-12}^{+15}$</td>
<td>$156.0_{-11}^{+21}$</td>
</tr>
<tr>
<td>$\iota$/deg (Planck)</td>
<td>$161.0_{-8.0}^{+8.0}$</td>
<td>$169.0_{-12.0}^{+8.0}$</td>
<td>$161.0_{-12}^{+12}$</td>
<td>$169.0_{-18}^{+11}$</td>
</tr>
<tr>
<td>$d$/ (Mpc)</td>
<td>$41.1_{-7.3}^{+4.0}$</td>
<td>$43.8_{-6.9}^{+2.9}$</td>
<td>$41.1_{-12.6}^{+5.6}$</td>
<td>$43.8_{-13.1}^{+5.6}$</td>
</tr>
</tbody>
</table>

**Note:** We give both one-sigma (68.3%) and 90% credible intervals for each quantity. “Symm.” refers to a symmetric interval (e.g. median and 5% to 95% range), while “MAP” refers to maximum a posteriori intervals (e.g. MAP value and smallest range enclosing 90% of the posterior). Values given for $\iota$ are derived from arc-cosine transforming the corresponding values for $\cos \iota$, so the “MAP” values differ from those that would be derived from the posterior on $\iota$.

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