GW170814: A Three-Detector Observation of Gravitational Waves from a Binary Black Hole Coalescence

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On August 14, 2017, at 10:30:43 UTC, the Advanced Virgo detector and the two Advanced LIGO detectors coherently observed a transient gravitational-wave signal produced by the coalescence of two stellar mass black holes, with a false-alarm rate of $<1$ in 27 000 years. The signal was observed with a three-detector network matched-filter signal-to-noise ratio of 18. The inferred masses of the initial black holes are $30.5_{-5.0}^{+5.7}M_\odot$ and $25.3_{-4.2}^{+4.8}M_\odot$ (at the 90% credible level). The luminosity distance of the source is $540_{-130}^{+130}$ Mpc, corresponding to a redshift of $z = 0.11_{-0.01}^{+0.03}$. A network of three detectors improves the sky localization of the source, reducing the area of the 90% credible region from 1160 deg$^2$ using only the two LIGO detectors to 60 deg$^2$ using all three detectors. For the first time, we can test the nature of gravitational-wave polarizations from the antenna response of the LIGO-Virgo network, thus enabling a new class of phenomenological tests of gravity.

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I. INTRODUCTION

The era of gravitational-wave (GW) astronomy began with the detection of binary black hole (BBH) mergers, by the Advanced Laser Interferometer Gravitational-Wave Observatory (LIGO) detectors [1], during the first of the Advanced Detector Observation Runs. Three detections, GW150914 [2], GW151226 [3], and GW170104 [4], and a lower significance candidate, LVT151012 [5], have been announced so far. The Advanced Virgo detector [6] joined the second observation run on August 1, 2017.

On August 14, 2017, GWs from the coalescence of two black holes at a luminosity distance of $540_{-130}^{+130}$ Mpc, with masses of $30.5_{-5.0}^{+5.7}M_\odot$ and $25.3_{-4.2}^{+4.8}M_\odot$, were observed in all three detectors. The signal was first observed at the LIGO Livingston detector at 10:30:43 UTC, and at the LIGO Hanford and Virgo detectors with a delay of ~8 ms and ~14 ms, respectively. The signal-to-noise ratio (SNR) time series, the time-frequency representation of the strain data, and the time series data of the three detectors together with the inferred GW waveform, are shown in Fig. 1. The different sensitivities and responses of the three detectors result in the GW producing different values of matched-filter SNR in each detector.

Three methods were used to assess the impact of the Virgo instrument on this detection. (a) Using the best fit waveform obtained from analysis of the LIGO detectors’ data alone, we find that the probability, in 5000 s of data around the event, of a peak in SNR from Virgo data due to noise and as large as the one observed, within a time window determined by the maximum possible time of flight, is 0.3%. (b) A search for unmodeled GW transients demonstrates that adding Advanced Virgo improves the false-alarm rate by an order of magnitude over the two-detector network. (c) We compare the matched-filter marginal likelihood for a model with a coherent BBH signal in all three detectors to that for a model assuming pure Gaussian noise in Virgo and a BBH signal only in the LIGO detectors: the three detector BBH signal model is preferred with a Bayes factor of more than 1600.

Until Advanced Virgo became operational, typical GW position estimates were highly uncertain compared to the fields of view of most telescopes. The baseline formed by the two LIGO detectors allowed us to localize most mergers to roughly annular regions spanning hundreds to about a thousand square degrees at the 90% credible level [7–9]. Virgo adds additional independent baselines, which in cases such as GW170814 can reduce the positional uncertainty by an order of magnitude or more [8].

Tests of general relativity (GR) in the strong field regime have been performed with the signals from the BBH mergers detected by the LIGO interferometers [2–5,10]. In GR, GWs are characterized by two tensor (spin-2) polarizations only, whereas generic metric theories may allow up to six polarizations [11,12]. As the two LIGO instruments have similar orientations, little information about polarizations can be obtained using the LIGO detectors alone. With the addition of Advanced Virgo we can probe, for the first time, gravitational-wave polarizations geometrically by projecting the wave’s amplitude...
onto the three detectors. As an illustration, we perform a test comparing the tensor-only mode with scalar-only and vector-only modes. We find that purely tensor polarization is strongly favored over purely scalar or vector polarizations. With this, and additional tests, we find that GW170814 is consistent with GR.

II. DETECTORS

LIGO operates two 4 km long detectors in the U.S., one in Livingston, LA and one in Hanford, WA [14], while Virgo consists of a single 3 km long detector near Pisa, Italy [15]. Together with GEO600 located near Hanover, Germany [16], several science runs of the initial-era gravitational-wave network were conducted through 2011. LIGO stopped observing in 2010 for the Advanced LIGO upgrade [1]. The Advanced LIGO detectors have been operational since 2015 [17]. They underwent a series of upgrades between the first and second observation runs [4], and began observing again in November 2016.

Virgo stopped observing in 2011 for the Advanced Virgo upgrade, during which many parts of the detector were replaced or improved [6]. Among the main changes are an increase of the finesse of the arm cavities, the use of heavier test mass mirrors that have lower absorption and better surface quality [18]. To reduce the impact of the coating thermal noise [19], the size of the beam in the central part of the detector was doubled, which required modifications of the vacuum system and the input-output optics [20,21]. The recycling cavities are kept marginally stable as in the initial Virgo configuration. The optical benches supporting the main readout photodiodes have been suspended and put under vacuum to reduce the impact of scattered light and acoustic noise. Cryogenic traps have been installed to improve the vacuum level. The vibration isolation and suspension system, already compliant with the Advanced Virgo requirement [22,23], has been further improved to allow for a more robust control of the last-stage pendulum and the accommodation of baffles to mitigate the effect of scattered light. The test mass
mirrors are currently suspended with metallic wires. Following one year of commissioning, Advanced Virgo joined LIGO in August 2017 for the last month of the second observation run.

For Virgo, the noises that are currently limiting the sensitivity at low frequencies are thermal noise of the test mass suspension wires, control noise, the 50 Hz mains line and harmonics, and scattered light driven by seismic noise. At high frequencies, the largest contribution comes from shot noise of the main interferometer beam, with smaller contributions coming from scattered light, and shot noise of a secondary beam used to control the laser frequency. The noise sources that limit LIGO’s sensitivity are described in \([24]\) and \([25]\). For both LIGO and Virgo, commissioning will continue to reach their ultimate designed sensitivities \([26]\).

Several noise sources that are linearly coupled to the GW data channel can be subtracted in postprocessing, using auxiliary sensors (e.g., photodiodes monitoring beam motion) and coupling transfer functions calculated via optimal Wiener filters. This technique was used in the initial detector era \([27–29]\). For LIGO, we remove calibration lines, power mains and harmonics, the effect of some length and angular controls, and the effect of laser beam motion. This noise removal can improve the sensitivity of the LIGO detectors by approximately 20\% \([30]\). For Virgo, we remove the effect of some length controls, and the laser frequency stabilization control. The search pipelines described in Sec. III use the calibrated strain data which were produced in low latency and which have not undergone postprocessing noise subtraction. They also use data quality flags which were produced offline. The source properties, however, described in Sec. V, are inferred using the postprocessing noise-subtracted data. Figure 2 shows the sensitivity of the Advanced LIGO–Advanced Virgo network around the time of GW170814, after the postprocessing removal of several noise sources.

Detection validation procedures at LIGO \([2,31]\), and checks performed at Virgo found no evidence that instrumental or environmental disturbances could account for GW170814. Tests quantifying the detectors’ susceptibility to external environmental disturbances, such as electromagnetic fields \([32]\), indicated that any disturbance strong enough to account for the signal would be clearly detected by the array of environmental sensors. None of the environmental sensors recorded any disturbances consistent with a signal that evolved in time and frequency like GW170814. A noise transient with a central frequency around 50 Hz falls outside the window expected due to the light travel time between the detectors, and has, therefore, no effect on the interpretation of the GW signal.

LIGO is calibrated by inducing test-mass motion using photon pressure from modulated auxiliary lasers \([33,34]\), and Virgo is calibrated using electromagnetic actuators \([35,36]\). Frequency-dependent calibration uncertainties are determined for both LIGO detectors for GW170814 using the method in \([37]\), and used for estimation of the properties of this event; the maximum 1-\(\sigma\) uncertainty for the strain data in the frequency range 20–1024 Hz is 7\% in amplitude and 4° in phase. The maximum 1-\(\sigma\) uncertainties for Virgo are 8\% in amplitude and 3° in phase over the same frequency range. The estimation of properties of GW170814 use these maximum values for the Virgo uncertainty over the whole frequency range. Uncertainties in the time stamping of the data are 10 \(\mu\)s for LIGO and 20 \(\mu\)s for Virgo, which does not limit the sky localization.

### III. SEARCHES

GW170814 was first identified with high confidence \(~30\) s after its arrival by two independent low-latency matched-filter pipelines \([38–44]\) that filter the data against a collection of approximate gravitational-wave templates \([45–53]\), triggering an alert that was shared with partners for electromagnetic follow-up \([54]\).

The significance estimates for this event were found by the two matched-filter pipelines, and a fully coherent unmodeled search pipeline \([55]\), analyzing 5.9 days of coincident strain data from the Advanced LIGO detectors spanning August 13, 2017 to August 21, 2017. The matched-filter pipelines do not currently use data from Virgo for significance estimates. Coherent searches, however, use the Virgo data to improve significance estimates.

The analysis was performed over the same source parameter space as the GW170104 matched-filter analysis \([4]\) and with additional data quality information unavailable in low latency \([5,31]\), although the noise-subtracted data described in Sec. II were not used. Both pipelines identified GW170814 with a Hanford-Livingston network SNR of 15, with ranking statistic values from the two pipelines corresponding to a

![Figure 2](image-url)
false-alarm rate of 1 in 140 000 years in one search [38,39] and 1 in 27 000 years in the other search [40–44,56], clearly identifying GW170814 as a GW signal. The difference in significance is due to the different techniques used to rank candidate events and measure the noise background in these searches; however, both report a highly significant event.

The significance of GW170814 was confirmed on the full network of three detectors by an independent coherent analysis that targets generic gravitational-wave transients with increasing frequency over time [55]. This more generic search reports a false-alarm rate $< 1$ in 5900 years. By comparison, when we limit this analysis to the two LIGO detectors only, the false-alarm rate is approximately 1 in 300 years; the use of the data from Virgo improves significance by more than an order of magnitude. Moreover, this independent approach recovers waveforms and SNRs at the three detectors which are compatible with respect to the coherent analyses used to infer source properties (see Sec. V).

IV. LOCALIZATION

Some compact object mergers are thought to produce not just GWs but also broadband electromagnetic emission. LIGO and Virgo have been distributing low-latency alerts and localizations of GW events to a consortium now consisting of ground- and space-based facilities who are searching for gamma-ray, x-ray, optical, near-infrared, radio, and neutrino counterparts [57–59].

For the purpose of position reconstruction, the LIGO-Virgo GW detector network can be thought of as a phased array of antennas. Any single detector provides only minimal position information, its slowly varying antenna pattern favoring two broad regions perpendicular to the plane of the detectors’ arms [60,61]. However, with a network of detectors, sky position can be inferred by triangulation employing the time differences [62,63], phase differences, and amplitude ratios on arrival at the sites [64].

An initial rapid localization was performed by coherent triangulation of the matched-filter estimates of the times, amplitudes, and phases on arrival [65]. The localization was then progressively refined by full coherent Bayesian parameter estimation [66], using more sophisticated waveform models and treatment of calibration systematics, as described in the next section.

The localization of GW170814 is shown in Fig. 3. For the rapid localization from Hanford and Livingston, the 90% credible area on the sky is 1160 deg$^2$ and shrinks to 100 deg$^2$ when including Virgo data. The full parameter estimation further constrains the position to a 90% credible area of 60 deg$^2$ centered at the maximum a posteriori position of right ascension $\text{RA} = 03^\text{h} 11^\text{m}$ and declination $\text{dec} = -44^\circ 57^\prime$ (J2000). The shift between the rapid localization and the full parameter estimation is partly due to the noise removal and final detector calibration, described in the previous section, that was applied for the full parameter estimation but not the rapid localization.

Incorporating Virgo data also reduces the luminosity distance uncertainty from $570^{+300}_{-230}$ Mpc (rapid localization) to $540^{+130}_{-210}$ Mpc (full parameter estimation). As with the previous paragraph, the three-dimensional credible volume and number of possible host galaxies also decreases by an order of magnitude [67–69], from $71 \times 10^6$ Mpc$^3$, to $3.4 \times 10^6$ Mpc$^3$, to $2.1 \times 10^6$ Mpc$^3$.

![Figure 3](image-url)
Follow-up observations of GW170814 were conducted by 25 facilities in neutrinos [70–72], gamma rays [73–81], x rays [82–85], and in optical and near infrared [86–98]. No counterpart has been reported so far.

V. SOURCE PROPERTIES

The parameters of the source are inferred through a coherent Bayesian analysis [66,99] of offline noise-subtracted data for the LIGO and Virgo detectors using two independently developed waveform models.

Both of these waveform models are calibrated to partially overlapping sets of numerical-relativity simulations of binary black hole coalescences, following from the initial breakthroughs reported in [100–102]. One model includes the full two-spin inspiral dynamics in the absence of precession [53,103–109,109], whereas the other model includes an effective treatment of the spin-precession dynamics through a rotation of an originally nonprecessing model [110–113]. Previous studies [114–117] have investigated the effect of systematic waveform modeling in sources we believe to be similar to GW170814, albeit for a different detector network configuration. Based on these analogous investigations, and due to the brevity and amplitude of GW170814, we expect systematic biases to be significantly smaller than the statistical error reported in this work.

In addition to a waveform model, the coherent Bayesian analysis also incorporates the detectors’ noise power spectral densities at the time of the event [13,118] and marginalizes over the calibration uncertainties described in Sec. II, as in [4,5,119]. We note that the likelihood used in our analyses assumes that the noise in the detectors is Gaussian in the 4 s window around the event. While some non-Gaussian and nonstationary features exist in the data, initial investigations suggest that the non-Gaussian features in the data do not significantly impact the reported parameters, but we defer a detailed study of these effects to future work. The coherent Bayesian analysis recovers the maximum matched-filter SNR across the LIGO-Virgo network of 18.3 [66,120], with individual detector matched-filter SNRs of 9.7, 14.8, and 4.8 in LIGO Hanford, LIGO Livingston, and Virgo, respectively.

Table I shows source parameters for GW170814, where we quote the median value and the symmetric 90% credible intervals. The final mass (or equivalently the energy radiated), final spin, and peak luminosity are computed from averages of fits to numerical relativity simulations [121–125]. The reported uncertainties account for both statistical and systematic uncertainties from averaging over the two waveform models used. An independent calculation using direct comparison to numerical relativity gives consistent parameters [114].

The inferred posterior distributions for the two black hole masses $m_1$ and $m_2$ are shown in Fig. 4. GW170814 allows for measurements of comparable accuracy of the total binary mass $M = m_1 + m_2$, which is primarily governed by the merger and ringdown, and the chirp mass $\mathcal{M} = (m_1m_2)^{3/5}/M_*^{1/5}$, determined by the binary inspiral [64,131–137], similarly to both GW150914 [99] and GW170104 [4].

The orbital evolution is dominated by the black hole masses and the components of their spins $\mathbf{S}_{i,2}$ perpendicular to the orbital plane, and other spin components affect the GW signal on a subdominant level. The dominant spin effects are represented through the effective inspiral spin parameter $\chi_{\text{eff}} = (m_1a_1\cos\theta_{LS_1} + m_2a_2\cos\theta_{LS_2})/M$ which is approximately conserved throughout the evolution of the binary orbit [138–141]. Here, $\theta_{LS_i}$ is the angle between the black hole spin $\mathbf{S}_i$ and the Newtonian orbital angular momentum $\mathbf{L}$ for both the primary ($i = 1$) and secondary ($i = 2$) black holes, and $a_i = |c\mathbf{S}_i/Gm^2|$ is the dimensionless spin magnitude of the initial ($i = 1, 2$) and final ($i = f$) black holes. For $a_{1,2}$, this analysis assumed a uniform prior distribution between 0 and 0.99, with no restrictions on the spin orientations. As with GW150914 and GW170104, $\chi_{\text{eff}}$ is consistent with having a arbitrarily small value [4,5]. The spin components orthogonal to $\mathbf{L}$ are interesting, as they lead to a precession of the binary orbit [142,143] and are here quantified by the effective precession spin parameter $\chi_p$ [112,143]. As for previous events [4,5,116,130], the $\chi_p$ posterior distribution is dominated by assumptions about the prior, as shown in Fig. 4. Given these assumptions, as well as statistical and systematic uncertainties, we cannot draw further robust conclusions about the transverse components of the spin. The event, GW170814, is consistent with the population of BBHs, physical parameters, and merger rate reported in previous BBH papers [5,144,145].

The accuracy with which parameters influencing the phase evolution of the observed GW, the black hole masses

| Primary black hole mass $m_1$ | $30.5^{+5.7}_{-3.0}M_\odot$ |
| Secondary black hole mass $m_2$ | $25.3^{+2.8}_{-4.2}M_\odot$ |
| Chirp mass $\mathcal{M}$ | $24.1^{+1.4}_{-1.1}M_\odot$ |
| Total mass $M$ | $55.9^{+3.2}_{-2.7}M_\odot$ |
| Final black hole mass $M_f$ | $53.2^{+3.2}_{-2.5}M_\odot$ |
| Radiated energy $E_{\text{rad}}$ | $2.7^{+0.4}_{-0.3}M_\odot c^2$ |
| Peak luminosity $L_{\text{peak}}$ | $3.7^{+0.5}_{-0.6} \times 10^{56} \text{ erg s}^{-1}$ |
| Effective inspiral spin parameter $\chi_{\text{eff}}$ | $0.06^{+0.12}_{-0.12}$ |
| Final black hole spin $a_f$ | $0.70^{+0.07}_{-0.05}$ |
| Luminosity distance $D_L$ | $540^{+130}_{-210} \text{ Mpc}$ |
| Source redshift $z$ | $0.11^{+0.04}_{-0.03}$ |
and spins, can be measured is determined by the network SNR. For GW170814 this is dominated by the two LIGO detectors. The inclusion of Virgo data into the coherent analysis significantly improves the inference of parameters describing the binary’s position relative to the Earth, as shown in Fig. 3, since those parameters are predominantly determined by the relative amplitudes and arrival times observed in the detector network [67,146,147]. Because of the inferred orientation of the binary, we do not see a significant improvement in parameters such as inclination and polarization angle for GW170814.

VI. TESTS OF GENERAL RELATIVITY

To determine the consistency of the signal with GR, we allowed the post-Newtonian (PN) and additional coefficients describing the waveform to deviate from their nominal values [148–150], as was done for previous detections [2–5,10]. In addition to previously tested coefficients, these analyses were expanded to also explicitly consider phase contributions at effective $-1\text{PN}$ order, i.e., with a frequency dependence of $f^{-7/3}$. Additionally, as in [2–4], we check that the inspiral and merger-ringdown regimes are mutually consistent, and check for possible deviations from GR in the propagation of GWs due to a massive graviton and/or Lorentz invariance violation. Preliminary results of all these tests show no evidence for disagreement with the predictions of GR; detailed investigations are still ongoing, and full results will be presented at a later date.

VII. GRAVITATIONAL-WAVE POLARIZATIONS

One of the key predictions of GR is that metric perturbations possess two tensor degrees of freedom [151,152]. These two are only a subset of the six independent modes allowed by generic metric theories of gravity, which may in principle predict any combination of tensor (spin-2), vector (spin-1), or scalar (spin-0) polarizations [11,12]. While it may be that any generic theory of gravity will be composed of a potential mixture of polarization modes, an investigation of this type is beyond the scope of this Letter. However, a simplified first investigation that serves to illustrate the potential power of this new phenomenological test of gravity is to consider models where the polarization states are pure tensor, pure vector, or pure scalar only.

So far, some evidence that GWs are described by the tensor (spin-2) metric perturbations of GR has been obtained from measurements of the rate of orbital decay of binary pulsars, in the context of specific beyond-GR theories (see, e.g., [153,154] or [155,156] for reviews), and from the rapidly changing GW phase of BBH mergers observed by LIGO, in the framework of parametrized models [2,4,10]. The addition of Advanced Virgo provides us with another, more compelling, way of probing the nature of polarizations by studying GW geometry directly through the projection of the metric perturbation onto our detector network [157–159].

FIG. 4. Posterior probability density for the source-frame masses $m_1$ and $m_2$ (top) and the effective inspiral and precession spin parameters $\chi_{\text{eff}}$ and $\chi_p$ (bottom) measured at a gravitational-wave frequency of 20 Hz, well before the merger. The dashed lines mark the 90% credible interval for the one-dimensional marginalized distributions. The two-dimensional plots show the contours of the 50% and 90% credible regions plotted over a color-coded posterior density function. For GW170814, both $\chi_{\text{eff}}$ and $\chi_p$ are influenced by their respective prior distributions, shown in green. While the GW observation provides additional constraints for the $\chi_{\text{eff}}$ posterior, there is only a marginal information gain for $\chi_p$. (Kullback–Leibler divergence between the prior and posterior distribution of 0.08 nat [4,129,130].)
The coherent Bayesian analysis described in Sec. V is repeated after replacing the standard tensor antenna response functions with those appropriate for scalar or vector polarizations [160]. In our analysis, we are interested in the geometric projection of the GW onto the detector network; therefore, the details of the phase model itself are less relevant as long it is a faithful representation of the fit to the data in Fig 1. Hence, we assume a GR phase model. We find Bayes’ factors of more than 200 and 1000 in favor of the purely tensor polarization against purely vector and purely scalar, respectively. We also find that, as expected, the reconstructed sky location and distance change significantly depending on the polarization content of the source, with nonoverlapping 90% credible regions for tensor, vector, and scalar. The inferred masses and spins are always the same, because that information is encoded in the signal phasing. These are only the simplest possible phenomenological models, but a more intensive study involving mixed-polarization states, using both matched-filter and generic GW transient models, is currently under way. Similar tests were inconclusive for previous events [10] because the two LIGO detectors are very nearly coaligned, and record the same combination of polarizations.

VIII. CONCLUSION

On August 1, 2017, Advanced Virgo joined the two Advanced LIGO detectors in the second Advanced Detector Observation Runs. On August 14, 2017, a GW signal coming from the merger of two stellar mass black holes was observed with the Virgo and LIGO detectors. The three-detector detection of GW170814 allowed for a significant reduction in the search volume for the source. The black hole characteristics of GW170814 are similar to GW150914 and GW170104, and are found to be consistent with the astrophysical population and merger rate determined with previous detections. The addition of Virgo has allowed us to probe the polarization content of the signal for the first time; we find that the data strongly favor pure tensor polarization of gravitational waves, over pure scalar or pure vector polarizations. Data for this event are available at the LIGO Open Science Center [161].

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