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the Municipal Council of the town of Orléans, the Unité d'Enseignement et de Recherche des Sciences Exactes et Naturelles of the Université d'Orléans and the Department of Mathematics of the Université d'Orléans.

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PAUL D. BACSICH, *Primality and model-completions.*

THEOREM 1. *Let T and T' be equivalent equational theories (i.e., the categories of T -algebras and T' -algebras are isomorphic). If T has a model-completion, so does T' .*

An algebra S will be called *crypto-primal* if S is finite, nontrivial, core, simple and generates a congruence-distributive variety \mathcal{K} . Any primal algebra is crypto-primal. More generally, the 2-element distributive $(0, 1)$ -lattice is crypto-primal. Let \mathcal{B} be the variety of boolean algebras, and $S^*: \mathcal{B} \rightarrow \mathcal{K}$ the boolean-power functor.

THEOREM 2. (1) \mathcal{K} has a model-completion. (2) In \mathcal{K} absolutely closed = absolutely pure = boolean power of S . (3) In \mathcal{K} existentially closed = atomless boolean power of S .

The proofs require results of A. Day (*Canadian Journal Mathematics*, vol. 24 (1972), pp. 209–220). By using a characterisation of varieties with epimorphisms surjective we obtain a new proof of

THEOREM 3. *If S is primal then S^* is an isomorphism.*

HENK BARENDREGT, *Solvability in λ -calculus.*

It is proved that the unsolvable terms are in a certain sense *generic*: $FM = I$ and M is unsolvable $\Rightarrow Fx = I$ for all x . From this it follows easily that the set of equations $\{M = M' : M, M' \text{ unsolvable}\}$ can be added consistently to the λ -calculus. Moreover this set has a unique maximal consistent extension \mathcal{H} (see 5).

A closed term M of the λ -calculus is *solvable* iff $\exists N_1 \cdots N_p MN_1 \cdots N_p = I$ for some p .

The main part of the paper consists in proving an arbitrary term M is solvable iff its closure $\lambda x. M$ is solvable.

The following motivation is given to introduce in the λ -calculus and related systems the notion of solvability and in particular to replace the notion ' M has a normal form' by ' M is solvable.'

1. Solvability is an extension of the concept 'having a normal form,' coinciding with it on the λI -terms.

2. 'Having a normal form' is a syntactical notion, whereas solvability is semantical, i.e., it makes sense in models.

3. Solvability can be defined analogously for the theory of combinators. It is invariant under the standard translation(s) of the λ -calculus into combinatory logic, whereas having a normal form is not.

4. In the representation of the partial recursive functions in the λ -calculus the undefined function value can be represented by unsolvable terms. This has the advantage that one can find for the partial recursive functions an intensional (w.r.t. their definitions) representation, which is preserved under the standard translation into the combinators.

5. (Un)solvable terms have a clear meaning in Scott's lattice theoretic models:

$$M \text{ is unsolvable} \Leftrightarrow (M)^{D_\infty} = \perp (= \text{least element of } D_\infty).$$

Moreover for M, N closed

$$D_\infty \vDash M = N \Leftrightarrow \forall F [FM \text{ solvable} \Leftrightarrow FN \text{ solvable}] \Leftrightarrow M = N \in \mathcal{H}^*.$$

6. Solvability considerations show that the intensional version of Gödel's theory of functionals of finite type does not have a pairing function.

HENK BARENDREGT, *A global representation of the recursive functions in the λ -calculus.*

This is an introductory paper to the λ -calculus. A short method to represent the recursive functions is given. This representation has as novelty that it is a *global* representation by terms in *normal form* (global refers to the fact that the recursion equations hold for free variables, not just numerals).