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The structure of the Galactic magnetic field from dust polarization maps of the southern Galactic cap


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ABSTRACT

Using data from the Planck satellite, we study the statistical properties of interstellar dust polarization at high Galactic latitudes. Our aim is to advance the understanding of the magnetized interstellar medium (ISM), and to provide a modelling framework of the polarized dust foreground for use in cosmic microwave background (CMB) component-separation procedures. Focusing on the southern Galactic cap ($b < -60^\circ$), we examine the Stokes $I$, $Q$, and $U$ maps at 353 GHz, and particularly the statistical distribution of the polarization fraction ($p$) and angle ($\psi$), in order to characterize the ordered and turbulent components of the Galactic magnetic field (GMF) in the solar neighbourhood. The $Q$ and $U$ maps show patterns at large angular scales, which we relate to the mean orientation of the GMF towards Galactic coordinates ($h_b$, $h_l$) = (70 $\pm$ 5, 24 $\pm$ 5). The histogram of the observed $p$ values shows a wide dispersion up to 25 %. The histogram of $\psi$ has a standard deviation of 12$^\circ$ about the regular pattern expected from the ordered GMF. We build a phenomenological model that connects the distributions of $p$ and $\psi$ to a statistical description of the turbulent component of the GMF, assuming a uniform effective polarization fraction ($p_{\text{eff}}$) of dust emission. To compute the Stokes parameters, we approximate the integration along the line of sight (LOS) as a sum over a set of $N$ independent polarization layers, in each of which the turbulent component of the GMF is obtained from Gaussian realizations of a power-law power spectrum. We are able to reproduce the observed $p$ and $\psi$ distributions using: a $p_{\text{eff}}$ value of 26 %; a ratio of 0.9 between the strengths of the turbulent and mean components of the GMF; and a small value of $N$. The mean value of $p$ (inferred from the fit of the large-scale patterns in the Stokes maps) is 12 $\pm$ 1 %. We relate the polarization layers to the density structure and to the correlation length of the GMF along the LOS. We stress the simplicity of our model (which only involves a few parameters), which can be easily computed on the celestial sphere to produce simulated maps of dust polarization, and thereby to assess component-separation approaches in CMB experiments.

Key words. Interstellar medium: dust – Polarization – Magnetohydrodynamics – Cosmic background radiation – Methods: data analysis

1. Introduction

Interstellar magnetic fields are tied to the interstellar gas. Together with cosmic rays they form a dynamical system that is an important (but debated) facet of the physics of galaxies. Magnetic fields play a pivotal role, because they control the density and distribution of cosmic rays, and they act on the dynamics if the gas. Much of the physics involved in this interplay is encoded in the structure of interstellar magnetic fields. Observations of synchrotron emission and its polarization, as well as Faraday rotation and dust polarization, provide the means to characterize the structure of magnetic fields within galaxies (Haverkorn2013, Lazarian & Pogosyan2016, Beck2016).
Since dust grains are mixed with interstellar gas, dust polarization data are well suited to investigate the physical coupling between the gas dynamics and the magnetic field structure, in other words to characterize magnetohydrodynamical (MHD) turbulence in the interstellar medium (ISM; Brandenburg & Lazarian 2013; Falceta-Gonzalves et al. 2014). Anisotropic dust grains tend to align with their longer axes perpendicular to the local magnetic field, and thus their emission is polarized perpendicular to the magnetic field projection on the plane of the sky (POS). The polarization fraction, $p$, the ratio between the polarized and total intensities of dust thermal emission, depends on the dust polarization properties and the grain alignment efficiency, but also on the structure of the magnetic field (Lazarian 2007). Thus, information on the magnetic field structure is encoded in the Stokes $Q$ and $U$ maps, as well as in the polarization angle $\psi$ and fraction $p$.

For a long time, observations of dust polarization from the diffuse ISM were limited to stellar polarization data available for a discrete set of lines of sight (LOS; Heiles 2000). The Planck data opened a new perspective on this topic. For the first time, we have maps of the dust polarization in emission over the full sky (Planck Collaboration 2016). The Planck maps greatly surpass, in sensitivity and statistical power, the data available from earlier ground-based and balloon-borne observations (e.g., Benoit et al. 2004; Pontieu et al. 2005; Ward-Thompson et al. 2009; Koch et al. 2010; Poolevin et al. 2014; Matthews et al. 2014).

Several studies have already used the Planck data to investigate the link between the dust polarization maps and the structure of the Galactic magnetic field (GMF). Planck Collaboration Int. XIX (2015) presented the first analysis of the polarized sky as seen at 353 GHz (the most sensitive Planck channel for polarized thermal dust emission), focusing on the statistics of $p$ and $\psi$. The comparison with synthetic polarized emission maps, computed from simulations of anisotropic MHD turbulence, shows that the turbulent structure of the GMF is able to reproduce the main statistical properties of $p$ and $\psi$ in nearby molecular clouds (Planck Collaboration Int. XX 2015). This comparison shows that the mean orientation of the GMF with respect to the LOS plays a major role in the quantitative analysis of these statistical properties. An important result is that in the diffuse ISM, the filamentary structure of matter is observed to be statistically aligned with the GMF (McClure-Griffiths et al. 2006; Clark et al. 2014; Planck Collaboration Int. XXXII 2016; Kaliberla et al. 2016).

The spatial structure of the polarization angle has been characterized in Planck Collaboration Int. XIX (2015) using the angle dispersion function $S$. The map of $S$ highlights long, narrow structures of high $S$ that trace abrupt changes of $\psi$ at the interfaces between extended areas within which the polarization angle is ordered. Falgarone et al. (2015) found a correlation between the structures in $S$ and large velocity shears in incompressible magnetized turbulence. The structures seen in the Planck data bear a morphological resemblance to features associated with Faraday rotation in gradient maps of polarized synchrotron emission (Gaensler et al. 2011; Jacobelli et al. 2014), which have been related to fluctuations in the GMF and in the ionized gas density in MHD turbulence (Burkhart et al. 2012). filamentary structures in rotation measure synthesis maps from LOFAR (the Low-Frequency Array) data (Jelic et al. 2015) have been shown to be correlated with the GMF orientation inferred from the Planck dust polarization (Zaroubi et al. 2015). At microwave frequencies, the dust polarization has been demonstrated to be correlated with synchrotron polarization, free from Faraday rotation (Planck Collaboration Int. XXII 2015; Choi & Page 2015). Both emission processes trace the same GMF, but the correlation is not one-to-one due to the difference in the distribution of dust and relativistic electrons in the Galaxy. Jaffe et al. (2013) and Planck Collaboration Int. XII (2016) described the difficulties faced when trying to reproduce the Planck dust polarization data with existing models of the large-scale GMF (Jaffe et al. 2010; Sun & Reich 2010; Jansson & Farrar 2012), which are mainly constrained by synchrotron emission and Faraday rotation measures.

The GMF structure is also relevant for the modelling of polarized Galactic foregrounds in analyses of the CMB. Thermal emission from Galactic dust is the main polarized foreground at frequencies above 100 GHz (Planck Collaboration X 2016; Planck Collaboration Int. XXX 2016) that have been made publicly available by the Planck consortium are now suitable for such an analysis. Second, we introduce a modelling framework that relates the dust polarization to the GMF structure, its mean orientation, and a statistical description of its random (turbulent) component. This framework is also a step towards a modelling tool for the dust polarization, which may be used to assess component-separation methods in the analysis of CMB polarization (e.g., Planck Collaboration IX 2016).

Our data analysis procedure focuses on the southern Galactic cap, the clearest part of the sky that is directly relevant to CMB observations, in particular those carried out with ground-based telescopes from Antarctica and Chile. This is also the part of the sky where the LOS through the Galaxy is the shortest, and hence is the region best suited to characterize the turbulent component of the GMF.

The paper is organized as follows. We present the Planck data in Sect. 2. Section 3 introduces our model of the GMF structure in the solar neighbourhood and in Sect. 4 we estimate the mean orientation of the GMF in the solar neighbourhood. In Sect. 5 we characterize the turbulent component of the GMF. The data analysis is based on a phenomenological model that we discuss in Sect. 6 and also contains our future perspectives. The paper’s results are summarized in Sect. 7. The approximation

\[\text{http://pla.esac.esa.int} \]

\[\text{http://lambda.gsfc.nasa.gov/product/expt/} \]
tions made to compute the Stokes parameters are presented in Appendix A.

2. Data and conventions

We first introduce the data that we will use, discussing the conventions assumed in the analysis of polarization, and presenting the polarization parameters determined around the south Galactic pole.

2.1. Description of the data

The Planck satellite observed the polarized sky in seven frequency bands from 30 to 353 GHz (Planck Collaboration I 2014). In this paper, we only use the data from the High Frequency Instrument (HFI, Lamarre et al. 2010) at the highest frequency, 353 GHz, where the dust emission is the brightest.

We use the publicly available 353 GHz Stokes $Q$ and $U$ (hereafter, $Q_{353}$ and $U_{353}$) maps (central and right panels in Fig. 1) and the associated noise maps made with the five independent consecutive sky surveys of the Planck cryogenic mission. We refer to publications by the Planck Collaboration for details of the processing of HFI data, including mapmaking, photometric calibration, and photometric uncertainties (Planck Collaboration I 2016; Planck Collaboration VII 2016; Planck Collaboration VIII 2016). The $Q_{353}$ and $U_{353}$ maps are corrected for spectral leakage as described in Planck Collaboration VIII (2016). For the dust total intensity at 353 GHz we use the model map, $D_{353}$, derived from a modified blackbody fit to the Planck data at $\nu \geq 353$ GHz, and IRAS at $\lambda = 100\mu m$ (Planck Collaboration XI 2014). The data used in this fit are corrected for zodiacal emission and CMB anisotropies. $D_{353}$ has a also a lower noise than the corresponding 353 GHz Stokes $I$ Planck map. The $Q_{353}$ and $U_{353}$ maps are initially constructed with an effective beamsize of 4.8', and $D_{353}$ at 5'. The three maps are in HEALPix format[1] with a pixelization $N_{\text{side}} = 2048$. To increase the signal-to-noise ratio at high Galactic latitudes, we smooth the three maps to 1° resolution using a Gaussian approximation to the Planck beam. We reduce the HEALPix resolution to $N_{\text{side}} = 128$ (30.1 pixels) after smoothing. For the polarization maps, we apply the "isomoothing" routine of HEALPix, which decomposes the $Q$ and $U$ maps into $E$ and $B$ maps, applies Gaussian smoothing in harmonic space, and transforms the smoothed $E$ and $B$ back into $Q$ and $U$ maps at $N_{\text{side}} = 128$ resolution.

2.2. Applied conventions in polarization

In terms of $Q_{353}$, $U_{353}$, and $D_{353}$, the quantities $p$ and $\psi$, are defined as

$$p = \frac{\sqrt{Q_{353}^2 + U_{353}^2}}{D_{353}},$$

$$\psi = \frac{1}{2} \tan^{-1}(-U_{353}/Q_{353}),$$

where the minus sign in $\psi$ is needed to change the HEALPix-format maps (or “COSMO convention” for the FITS keyword POLCONV) into the International Astronomical Union (IAU) convention for $\psi$, measured from the local direction to the north Galactic pole with increasing positive values towards the east. Moreover, in this paper we use the version of the inverse tangent function with two signed arguments to resolve the $\pi$ ambiguity ($\psi$ corresponds to orientations not to directions).

When considering dust polarization, the Stokes parameters for linear polarization are integral quantities of the optical depth (see Appendix A and Planck Collaboration Int. XX 2015). An empirical expression for $p$ is

$$p = p_0 F \cos^2 \gamma,$$

where $\gamma$ is the angle between the mean orientation of the GMF and the POS. Therefore, the projection factor, $\cos^2 \gamma$, carries information on the orientation of the GMF with respect to the

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In particular, dust polarization vanishes where the GMF points directly towards or away from the observer. Hereafter, \( p_0 \) is the effective dust polarization fraction, which combines the intrinsic polarization fraction of dust grains \( p_{\text{dust}} \) (the ratio between the polarization and average cross-sections of dust, as defined in Planck Collaboration Int. XX [2015] and \( R \), the Rayleigh reduction factor (related to the degree of dust grain alignment with the GMF). Greenberg [1968], Lee & Draine [1985]. The factor \( F \) accounts for the depolarization due to variations of the GMF orientation along the LOS and within the beam.

\[ F = \frac{1}{2} (1 - \cos^2 \omega) \]

The high degree of order in the polarization properties, including alignment, are homogeneous, the structure of the dust polarization sky reflects the structure of the GMF combined with that of matter. Throughout the paper, we assume that this hypothesis applies to the different Galactic latitudes and the observed dispersion in the distribution? As we will show, the GMF structure in the solar neighbourhood is essential to consider when answering this question.

\section{Model framework}

The polarization of thermal dust emission results from the alignment of elongated grains with respect to the GMF (Stein 1966, Hildebrand 1988). Within the hypothesis that grain polarization properties, including alignment, are homogeneous, the structure of the dust polarization sky reflects the structure of the GMF combined with that of matter. Throughout the paper, we assume that this hypothesis applies to the diffuse ISM, where radiative torques provide a mechanism to efficiently align grains (Dolginov & Mitrofanov 1976, Hoang & Lazarian 2014, Andersson et al. 2015). Our data modelling focusses on the structure of the GMF. This section describes the model framework (Sect. 3.1) and how we proceed to fit it to the data (Sect. 3.2).

\subsection{Magnetic field modelling}

We now introduce the framework we use to model the GMF structure within the solar neighbourhood. The integral equations of the Stokes \( I, Q, \) and \( U \) parameters are recalled in Appendix A.

We follow earlier works (e.g., Chandrasekhar & Fermi 1953, Hildebrand et al. 2009), expressing the GMF (\( B \)) as the sum of its mean (\( B_0 \)) and turbulent (\( B_t \)) components:

\[ B = B_0 + B_t \]

\[ B = B_0 + B_t \]
We introduce and discuss the assumptions we make about each of these two components.

Our model aims at describing dust polarization towards the southern Galactic cap at Galactic latitudes \( b \leq -60^\circ \). We focus on the solar neighbourhood and thereby ignore the structure of the GMF on Galaxy-wide scales. We also ignore the change of its orientation from the disk to the halo (Haverkorn 2015), because dust emission arises mainly from a thin disk. The dust scale height is not measured in the solar neighbourhood, but modelling of the dust emission from the Milky Way indicates that the dust scale height at the solar distance to the Galactic centre is approximately 200 pc (Drimmel & Spergel 2001). Observations of the edge-on spiral galaxy NGC 891, a galaxy analogous to the Milky Way, give a comparable scale height of around 150 pc (Bocchio et al. 2016). These estimates are in agreement with the scale height of the neutral atomic gas in the Milky Way, inferred from HI observations (Dickey & Lockman 1990; Kalberla et al. 2007). Hence, we assume that the vector \( B_0 \) has a fixed orientation, which represents the mean orientation of the GMF in the solar neighbourhood.

Radio observations of synchrotron emission and polarization reveal a wealth of structures down to pc and sub-pc scales (e.g., Reich et al. 2004; Gaensler et al. 2011; Jacobelli et al. 2013, 2014), such as filaments, canals, lenses, and rings, which carry valuable information about \( B \). (Fletcher & Shukurov 2006). Heiles (1995) and Haverkorn (2015) reviewed observations that characterize this random component, concluding that it has a strength of about 5 \( \mu \)G, comparable to that of \( B_0 \). Jones et al. (1992) reached a similar conclusion from stellar polarization data.

Fig. 4. Mollweide (top) and orthographic (bottom) projections of the model Stokes parameters, \( q_A \) (left) and \( u_A \) (right), for a uniform direction of the GMF towards \((l_0, b_0) = (80^\circ, 0^\circ)\); these are roughly the values inferred from starlight polarization (Heiles 1996). The orthographic projections are centred on the Galactic poles. Galactic coordinates in degrees are shown on all plots.

The turbulent component of the GMF is significant. To take it into account, we follow earlier works (e.g., Waelkens et al. 2009; Fauvet et al. 2011), modelling each component of the \( B_t \) vector with Gaussian realizations. To model dust polarization over the celestial sphere, earlier studies (e.g., Miville-Deschênes et al. 2008; Fauvet et al. 2011; O’Dea et al. 2012) computed independent realizations of the components of \( B_t \), for each LOS. This approach ignores the angular coherence of \( B_t \) over the sky, which, however, is essential to match the correlated patterns seen in the Planck maps of the dust \( p \) and \( \psi \) (Planck Collaboration Int. XIX 2015). Because of this, we use a different method. We model \( B_t \) with Gaussian realizations on the celestial sphere, computed for an angular power spectrum \( C_\ell \) scaling as a power-law \( \ell^{\alpha} \) for \( \ell \geq 2 \). The amplitude of the spectrum is parametrized by the ratio \( f_M \) between the standard deviation of \(|B_t|\) and \(|B_0|\).

Our spectrum does not have a low \( \ell \) cut-off, which would represent the scale of energy injection of the turbulent energy cascade. Here, since we compare the model and the data over a field with an angular extent of 60° (about 1 radian), we implicitly assume that the injection scale is larger than, or comparable to, the scale height of the dust emission (approximately 200 pc, Drimmel & Spergel 2001). The scale of the warm ionized medium (WIM) is larger (about 1–1.5 kpc, Gaensler et al. 2008), but the WIM is not a major component of the dust emission from the diffuse ISM (Planck Collaboration Int. XVII 2014). The range of distances involved in the modelling of dust polarization at high Galactic latitudes is small, because there is little interstellar matter within the local bubble, i.e., within 50–100 pc of the Sun (Lallement et al. 2014). The local bubble may extend to larger distances towards the Galactic poles, but this
possibility is not well constrained by existing data. In any case, it is reasonable to assume that most of the dust emission at high Galactic latitudes arises from a limited range of distances, which sets a rough correspondence between angles and physical scales in our model.

To compute the Stokes parameters, we approximate the integration along the line of sight (LOS) with a sum over a set of \( N \) polarization layers with independent realizations of \( B_0 \). The layers are a phenomenological means to represent the variation of \( B \) along the LOS. Our modelling of \( B_0 \) is continuous over the celestial sphere, while we use a set of independent orientations along the LOS. At first sight, this may be considered as physically inconsistent. However, in Sect. 6 we relate the polarization layers to the density structure and to the correlation length of \( B_0 \) along the LOS. Our modelling does not take into account explicitly the density structure of matter along the LOS; the source function (presented in Eqs. A.1b and A.1c) is assumed to be constant along the LOS. It also ignores the alignment observed between the filamentary structure of the diffuse ISM and the magnetic field.

### 3.2. Data fitting in three steps: A, B, and C

In the following two sections, we present three steps in our data-fitting, labelled steps A, B, and C. Step A only takes into account the mean field \( B_0 \). In Sect. 3.2, we determine the orientation of \( B_0 \) by fitting the regular patterns seen in the Planck \( Q_{353} \) and \( U_{353} \) maps shown in Fig. 1. The other two models involve both \( B_0 \) and \( B_1 \), as required to reproduce the 1-point statistics of \( \psi \) and \( p \). In step B (Sect. 3.3), \( B_1 \) is computed from random realizations on the sphere. In this model, the depolarization due to changes in the orientation of \( B_1 \) along the LOS is accounted for with an \( F \) factor in Eq. (2) that is uniform over the sky. This simplifying assumption is often made in analysing polarization data. Step C in Sect. 3.3 is an extension of step B, where we introduce variations of the \( F \) factor over the sky by summing Stokes parameters over \( N \) polarization layers along the LOS.

Our model has six parameters: the two coordinates defining the orientation of \( B_0 \), \( f_M \) quantifying the dispersion of \( B \) around \( B_0 \); the number of layers, \( N \); the index \( \alpha_M \); and the effective polarization fraction of dust emission, \( p_0 \). The parameters are not all fit simultaneously because they are connected to the data in different ways. The coordinates of \( B_0 \) relate to the large-scale patterns in the \( Q_{353} \) and \( U_{353} \) maps and do not depend on the other parameters. The triad of parameters \( f_M, N, \) and \( \alpha_M \) describe statistical properties of the polarization maps. We determine \( f_M, N, \) and \( p_0 \) simultaneously by fitting the 1-point statistics of both \( \psi \) and \( p \). To constrain \( \alpha_M \) it is necessary to use 2-point statistics (i.e., power spectra); this is not done in this paper, but will be the specific topic of a future paper.

### 4. The mean orientation of the magnetic field

In this step A of our data modelling, we determine the orientation of the mean field \( B_0 \), ignoring \( B_1 \).

#### 4.1. Description of step A

We show that the ordered magnetic field produces well-defined polarization patterns in the \( Q_{353} \) and \( U_{353} \) maps, resulting from the variation across the observed region of the angle between the LOS and the ordered field.

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*Fig. 5.* Same as in the left panel of Fig. 1 but now highlighting the \( b < -60^\circ \) region, excluding the brightest clouds (in grey on the image) that has been used to fit step A to the data.

Given a Cartesian reference frame \( xyz \), each point on the sphere can be identified by a pair of angular coordinates, hereafter the Galactic longitude and latitude, \( l \) and \( b \). The reference frame is chosen to be centred at the observer with \( \hat{z} = (0,0,1) \) pointing towards the north Galactic pole, \( \hat{x} = (1,0,0) \) towards the Galactic centre, and \( \hat{y} = (0,1,0) \) towards positive Galactic longitude.

We define the uniform direction of \( B_0 \) through the unit vector \( \hat{B}_0 \), which depends on the coordinates \( (l_0,b_0) \) as

\[
\hat{B}_0 = \cos l_0 \cos b_0 \hat{x} + \sin l_0 \cos b_0 \hat{y} + \sin b_0 \hat{z}
\]

We define the generic LOS unit vector \( \hat{r} \) as \( \cos l \hat{x} + \sin l \cos b \hat{y} + \sin b \hat{z} \) on a full-sky HEALPix grid.

Combining \( \hat{r} \) and \( \hat{B}_0 \), we can derive the POS component of \( \hat{B}_0, \hat{B}_{0\perp} \), as

\[
\hat{B}_{0\perp} = \hat{B}_0 - \hat{B}_{0||} = \hat{B}_0 - (\hat{B}_0 \cdot \hat{r})\hat{r},
\]

where \( \hat{B}_{0||} \) is the component of \( \hat{B}_0 \) along \( \hat{r} \). In order to define the \( \psi \) and \( \gamma \) angles for a given \( \hat{r} \), we need to derive the north and east directions, tangential to the sphere, which correspond to

\[
\hat{n} = \frac{(\hat{r} \times \hat{z}) \times \hat{r}}{|(\hat{r} \times \hat{z}) \times \hat{r}|},
\]

\[
\hat{e} = \frac{\hat{r} \times \hat{n}}{|\hat{r} \times \hat{n}|},
\]

respectively. The polarization angle is the complement of that between \( \hat{B}_{0\perp} \) and \( \hat{n} \), and \( \gamma \) the angle between \( \hat{B}_0 \) and \( \hat{B}_{0\perp} \). From
Appendix A, this is a small e

(Planck Collaboration Int. XX 2015). However, as detailed in

university of dust emission also depends on the GMF geometry

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for (best fits the data, we explore the space of Galactic coordinates

At first glance, the “butterfly” patterns in the

4.2. Fitting step A to the Planck data

Eqs. (4) and (5), we derive

\[
\psi_A = 90^\circ - \arccos \left( \frac{\hat{B}_{0, A} \cdot \hat{n}}{|\hat{B}_{0, A}|} \right),
\]

\[
\cos^2 \gamma_A = 1 - |\hat{B}_0 \cdot \hat{n}|^2,
\]

where the subscript “A” stands for step A, and the sign of arccos is imposed by the sign of $\hat{B}_{0, A} \cdot \hat{e}$.

Using Eqs. (1) and (2), we can produce an analytical expressions for the modelled Stokes parameters normalized to the total intensity times $p_0 F$, $q_A$ and $u_A$, as follows:

\[
q_A = \cos^2 \gamma_A \cos 2\psi_A;
\]

\[
u_A = -\cos^2 \gamma_A \sin 2\psi_A.
\]

We stress that $q_A$ and $u_A$ only show patterns generated by projection effects. For illustration, in Fig. 4 we present maps of $q_A$ and $u_A$ for a uniform direction of the GMF towards ($l_0, b_0$) = (80°, 0°), roughly the direction inferred from starlight polarization data (Heiles 1996). We note that the total intensity of dust emission also depends on the GMF geometry (Planck Collaboration Int. XX 2015). However, as detailed in Appendix A, this is a small effect that does not alter our results.

**4.2. Fitting step A to the Planck data**

At first glance, the “butterfly” patterns in the $Q_{353}$ and $U_{353}$ maps around the south Galactic pole in Fig. 4 resemble those produced with step A in Fig. 4. In order to find the orientation of $\hat{B}_0$ that best fits the data, we explore the space of Galactic coordinates for ($l_0, b_0$), spanning Galactic longitudes between 0° and 180°, and latitudes between −90° and 90°. From Eqs. (1, 2, and 7), we simultaneously fit step A to $Q_{353}$ and $U_{353}$ with the corresponding errors, as

\[
Q_{353} = p_{0, A} q_A D_{353},
\]

\[
U_{353} = p_{0, A} u_A D_{353},
\]

where the factor $p_{0, A}$ represents an average of the product $p_0 F$ in Eq. (2) over the region where we perform the fit. For each ($l_0, b_0$) pair we perform a linear fit to determine $p_{0, A}$. The fit is carried out for the southern polar cap at $b < -60^\circ$, after masking the most intense localized structures around the south Galactic pole, as shown in Fig. 5. To remove these regions from the analysis, we fit a Gaussian profile to the histogram of pixel values of $D_{353}$ below $b = -60^\circ$. We then mask all pixels with $D_{353} > \bar{D}_{353} + 4 \sigma_{353}$, where $\bar{D}_{353}$ and $\sigma_{353}$ are the mean and the standard deviation of the Gaussian fit.

The fit is done over an area of 2652 deg$^2$, corresponding to 2652 independent data beams. Since the number of parameters is 3, the number of degrees of freedom, $N_{\text{dof}}$, is large. We find a best-fit direction of the mean GMF towards Galactic coordinates $l_0 = 70^\circ \pm 5^\circ$ and $b_0 = 24^\circ \pm 5^\circ$. The value of $p_{0, A}$ corresponding to this direction is $(12 \pm 1)\%$. The statistical errors are small but there are significant uncertainties on the three parameters from residual, uncorrected, systematic effects in the data. We quote these uncertainties, which we estimated repeating the fit on maps produced with ten different subsets of the data (Planck Collaboration ES 2015). We notice that, because of the 180° ambiguity in the definition of $\psi$, the opposite direction ($l_0 + \pi, -b_0$) is an equivalent solution of our fit. However, the chosen solution is the closest to the mean GMF direction derived from observations of pulsars in the solar neighbourhood.
Planck Collaboration: The local structure of the Galactic magnetic field

Fig. 7. Orthographic projections centred on the south Galactic pole of $Q_{353}^R$ (left) and $U_{353}^R$ (right), the Stokes parameters in a reference frame rotated with respect to the best-fit direction of the uniform component of the GMF towards $(l_0, b_0) = (70^\circ, 24^\circ)$. The sky for the masked $b > -60^\circ$ region appears in grey.

Rand & Kulkarni 1989, Ferrière 2015), which, unlike dust polarization are sensitive to the sign of the GMF. Our determination of $l_0$ is in agreement with earlier values derived from starlight polarization (e.g. Heiles 1996). The positive value of $b_0$ is consistent with the positive sign of the median value of rotation measures derived from observations of extragalactic radio sources in the direction of the southern Galactic cap (Taylor et al. 2009, Mao et al. 2010). For illustration, we show the best-fit model maps of $q_A$ and $u_A$ around the south pole in Fig. 6.

We note that the obtained value of $p_{0A}$ is a substantial fraction of the maximum $p (> 18\%)$ reported in Planck Collaboration Int. XIX (2015) at intermediate Galactic latitudes. This result confirms that dust polarization is important at high Galactic latitudes. We also stress that this value of $p_{0A}$ is only a lower limit to the effective dust polarization fraction, because step A does not take into account any depolarizing effects along the LOS, associated with variations of the GMF orientation.

5. The turbulent component of the magnetic field

The Planck maps show structures in polarization on a wide range of scales (Fig 1), not accounted for by the single field orientation of step A, which we associate with the turbulent component of the magnetic field $B_t$. In Sects. 5.1 and 5.2, $B_t$ is assumed to vary only across the sky (step B), while in Sect. 5.3 we take into account its variations both across the sky and along the LOS (step C).

5.1. Step B: dispersion of the polarization angle

In Sect. 4.2 we found that the best-fit orientation of $B_0$ in step A is given by $(l_0, b_0) = (70^\circ, 24^\circ)$. We can now obtain maps of the corresponding normalized Stokes parameters, $u_{0A}$ and $q_{0A}$, as well as a map of the associated polarization angle

$$\psi_{0A} = \frac{1}{2} \tan^{-1}(-u_{0A}, q_{0A}).$$  \hspace{1cm} (9)

The angle $\psi_{0A}$ allows us to rotate, at each point on the sky, the reference direction used to compute the Stokes parameters $(Q_{353}, U_{353})$. With this new reference, the $q_A$ map in Fig. 6 would be that of $\cos^2\psi_A$ and $u_A$ would be null (see Eq. 7). To obtain the rotated values $Q_{353}^R$ and $U_{353}^R$, we apply to the data the following rotation matrix (e.g.,Delabrouille et al. 2009):

$$\begin{pmatrix} Q_{353}^R \\ U_{353}^R \end{pmatrix} = \begin{pmatrix} \cos 2\psi_{0A} & \sin 2\psi_{0A} \\ -\sin 2\psi_{0A} & \cos 2\psi_{0A} \end{pmatrix} \begin{pmatrix} Q_{353} \\ U_{353} \end{pmatrix}.$$ \hspace{1cm} (10)

The maps of $Q_{353}^R$ and $U_{353}^R$ are shown in Fig. 7 where the butterfly patterns, caused by the uniform component of the GMF, are now removed by the change of reference. The polarization angle that can be derived from $Q_{353}^R$ and $U_{353}^R$ as

$$\psi_R = \frac{1}{2} \tan^{-1}(-U_{353}^R, Q_{353}^R),$$  \hspace{1cm} (11)

represents the dispersion of $B_t$ around $B_{0A}$. The histogram of $\psi_R$ for $b < -60^\circ$, shown in the top panel of Fig. 8 (black dots with Poisson noise as error bars), has a 1 $\sigma$ dispersion of 12$^\circ$.

To characterize $B_t$, it is necessary to account for projection effects (Falceta-Gonçalves et al.)
model reproduces the histogram of $\psi_R$ and mean components of the GMF, which represents the ratio between the strengths of the turbulent B dispersion of metric model, which we use in this paper to characterize the Planck Collaboration Int. XXXII (2016) describes a geometric model, which we use in this paper to characterize the 3D dispersion of $B$ with respect to $B_0$, given the histogram of $\psi_R$. Each component of $B_i$ is obtained with an independent realization of a Gaussian field with an angular power spectrum equal to a power law of index $\alpha_M$, for multipoles $\ell \geq 2$. The degree of alignment between $B$ and $B_0$ is parameterized by $f_M$, which represents the ratio between the strengths of the turbulent and mean components of the GMF.

In the top panel of Fig. 8 we show that for $f_M = 0.4$ the model reproduces the histogram of $\psi_R$ fairly well. We computed 20 different Gaussian realizations to take into account the statistical variance of the model. The green line represents the average of the 20 realizations, whereas the green shaded regions are the $\pm 1 \sigma$ (light) and $\pm 2 \sigma$ (dark) variations of the model. In these calculations, as in Planck Collaboration Int. XXXII (2016), the spectral index $\alpha_M$ has a value of $-1.5$. This specific choice does not impact the distribution of $\psi_R$, or that of $p$. However, we note that the variance of the histogram, i.e., the dispersion of histogram values between independent realizations, increases for decreasing values of $\alpha_M$.

5.2. Step B: histogram of the polarization fraction

We showed that the structure of the GMF on the sphere allows us to reproduce $\psi_R$ over the southern Galactic cap. Here, we characterize the distribution of $p$ at $b < -60^\circ$ and we show that step B is not sufficient to describe the data.

As already discussed above, the noise bias on $p$ represents an intrinsic problem. To circumvent it, we compute unbiased values of $p^2$ by multiplying Stokes parameters from subsets of the data. Doing this, instead of using $\rho_{\text{NIS}}$ as in Sect. 2.3, gives us control over the level of noise in the data, as we now demonstrate. We use the year-maps (denoted by the indices “Y1” and “Y2”), which have uncorrelated instrumental noise, and compute $p^2$ as

$$p^2 = \frac{Q_{353}^Y Q_{353}^{Y2} + U_{353}^Y U_{353}^{Y2}}{(D_{353})^2}.$$ 

We also estimate $p^2$ from the so-called “DetSet” maps (made from different subsets of detectors, see Planck Collaboration ES 2015), and we find good agreement between the two estimates using distinct subsets of the data. In order to model $p^2$, we make use of the results obtained from fitting steps A and B to the data. Given $B_{\ell_0}$ pointing towards $(\ell_0, b_0) = (70^\circ, 24^\circ)$, we add $B_i$ to it with normalization parameter $f_M = 0.4$. In doing so, we now produce the two variables $q_\alpha$ and $u_\alpha$, as $q_\alpha$ and $u_\alpha$ in Eq. (7), where now the angles take into account the turbulent component of the GMF. We then make realizations of the Planck statistical noise ($n_{\delta q}$ and $n_{\delta u}$, with $i = 1, 2$), and, as in Eq. (8), we produce two pairs of independent samples of modelled Stokes $Q$ and $U$. 

Fig. 8. Results of step B. Top: Histogram of $\psi_R$, the polarization angle inferred from the Stokes parameters rotated with respect to the best-fit uniform direction of the GMF ($Q_{353}^Y$ and $U_{353}^Y$), over the southern Galactic cap (black dots). The error bars represent the Poisson noise within each bin of the histogram. The green line represents the mean of the step B results for $f_M = 0.4$ over 20 different realizations. The green shaded regions correspond to $\pm 1 \sigma$ (light green) and $\pm 2 \sigma$ (dark green) variations of the model. Bottom: Histogram of $p^2$ obtained when combining the Year 1 and Year 2 maps (black dots). The error bars here represent the Poisson noise within each bin of the histogram. Step B is now shown in blue. The dashed vertical line corresponds to a value of the polarization fraction of 12%.

Fig. 9. Cartoon illustrating, for step C, the integration of $q_C$ along the LOS, with four distinct polarization layers for the same value of $f_M$ and the same mean orientation of the GMF. Each map in this cartoon is a realization of the model.
as

\[
\begin{align*}
Q_{M_i} &= p_0 q_i B_{SS3} + n_{Q_i}, \\
U_{M_i} &= p_0 u_i B_{SS3} + n_{U_i},
\end{align*}
\]

(13)
in which \(i = 1, 2\) and \(p_0 = 12\%\). Thus, the modelled \(p^2\) results from

\[
p_{M}^2 = \frac{Q_{M1}Q_{M2} + U_{M1}U_{M2}}{(D_{SS3})^2},
\]

(14)

In the bottom panel of Fig. 9, we show the comparison between the histograms of \(p^2\) for the data (black dots) and for the model. In particular, we present the average over 20 realizations of step B (blue line) and the corresponding \(\pm 1\sigma\) (light blue shaded region) and \(\pm 2\sigma\) (dark blue) variations. The dashed vertical line refers to the value of \(p_0 = 12\%\). We notice that our modelling of \(p^2\) seems to appropriately take into account the data noise, since it nicely fits the negative \(p^2\) values, which result from noise in the combination of the individual year maps.

However, from Fig. 9 it is clear that our description of the GMF structure using step B does not provide a satisfactory characterization of the distribution of \(p^2\). The data show a more prominent peak in the distribution towards very low \(p^2\) values than seen in the model, for which the histogram peaks near the value of \(p_0\). Moreover, the large dispersion in the data, also found by Planck Collaboration Int. XIX (2015) at intermediate Galactic latitudes, produces a long tail in the distribution towards high values of \(p^2\), which is not reproduced by the model.

5.3. Step C: line-of-sight depolarization

Now we consider the effect of depolarization, associated with variations of the GMF orientation along the LOS. This additional step is essential to account for the dispersion of \(p\) and correctly estimate the amplitude of the turbulent component of the GMF with respect to its mean component, because the dispersion of the polarization angle is reduced by averaging along the LOS (Myers & Goodman 1991; Jones et al. 1992; Houde et al. 2009). Figure 10 illustrates step C with a simple cartoon. In order to account for the LOS integration that characterizes the polarization data, we produce \(N\) distinct maps of \(q_B\) and \(u_B\) (with \(i\) from 1 to \(N\)), for a common, but freely varying value of \(f_M\), while fixing \(\alpha_M = -1.5\) (as in step B), and for the best-fit orientation of \(B_i\) obtained with step A. The Gaussian realizations of \(B_i\) are different for each layer. All layers have the same \(B_0\) but an independent \(B_i\) in Eq. (3). Then, we model the LOS depolarization by averaging the Stokes parameters over the \(N\) layers as follows:

\[
\begin{align*}
q_C &= \frac{\sum_{i=1}^{N} q_B(i)}{N}, \\
u_C &= \frac{\sum_{i=1}^{N} u_B(i)}{N}.
\end{align*}
\]

(15)

We follow the same procedure as in Sects. 5.1 and 5.2 with \(q_B\) and \(u_B\) replaced by \(q_C\) and \(u_C\), to obtain model distributions of \(p^2\) and \(\psi_R\).

Given \(\alpha_M\), the modelled distributions of \(p^2\) and \(\psi_R\) depend on three main parameters, namely \(p_0\), \(f_{38}\), and \(N\). We fit the data exploring the parameter spaces of \(p_0\) between 15% and 40% with steps of 1%, of \(f_{38}\) between 0.2 and 1.8 with steps of 0.1, and of \(N\) between 1 and 17 with steps of 1. The distributions of \(p^2\) and \(\psi_R\) have about 200 bins each. For each triad of parameters we compute maps of the reduced \(\chi^2\) for the combined \(p^2\) and \(\psi_R\) fit, using

\[
\chi^2_{\text{tot}} = \chi^2_{p^2} + \chi^2_{\psi_R},
\]

(16)

where in computing the \(\chi^2\) distributions we fit the data with the mean of the 20 realizations, and we add their dispersion in quadrature to the error bar of the observations. Fitting the distribution of \(\psi_R\) between \(-40^\circ\) and \(40^\circ\) (where most of the data lie), we obtain a best fit for a minimum \(\chi^2_{\text{tot}}\) of 2.8, for \(p_0 = 26\%\), \(f_{38} = 0.9\), and \(N = 7\). In Fig. 10 we show three maps of \(\chi^2_{\text{tot}}\); each one corresponds to the parameter space for two parameters given the best-fit value of the third one. The \(\chi^2_{\text{tot}}\) maps reveal some correlation among the three parameters. The variance of each model among the 20 different realizations represents the
dominant uncertainty of the fit, and it is correlated between the bins of the histogram. Repeating the χ²-minimization for each one of the 20 realizations, the fit constrains the range of values for the main parameters to 0.8 < f_M < 1, 5 < N < 9, and 23% < p_0 < 29%. Step C generates a mean value of the depolarization factor F that is about 0.5, and thus leads to an estimate of p_0 twice larger than in step A. The best-fit value of (26 ± 3)% is comparable with the maximum value of the observed reported in Planck Collaboration Int. XIX (2015).

As in Fig. 8 the histograms of p² and ψ_R for the best-fit triad are shown in the bottom and top panels of Fig. 11 respectively. The top panel of Fig. 11 shows that if we consider a few (N = 7) independent polarization layers along the LOS, this provides us with an estimate of f_M that is closer to equality between the turbulent and mean components of the GMF than for step B (for which N = 1, see Sect. 5.1). A value of f_M = 0.9 with N = 1 would generate a much broader distribution of ψ_R than the observed one. The bottom panel of Fig. 11 shows that step C, unlike step B, can reproduce the histogram of p² quite well. The combination of a small number of independent polarization layers along the LOS produces the large dispersion in p² that is observed in the data.

6. Discussion

We have presented a phenomenological model that is able to describe the 1-point statistics of p and ψ for the Planck dust polarization data around the south Galactic pole, using a few parameters to describe the uniform and turbulent components of the GMF. We stress that our model is not entirely physical and certainly not unique. We made several assumptions, including: a single orientation of the mean field B_0; a uniform ratio f_M of the turbulent to mean strengths of the GMF along the LOS; a fixed value for the number of polarization layers, N, independent of the total dust intensity (unlike what was considered by Jones et al. [1992]); and isotropy of the turbulent component, B_t.

These assumptions restrict us from fitting the data over a larger portion of the sky than the southern Galactic cap. For the time being, we limit our study to this sky area. We now discuss the interpretation of our model in relation to the ISM physics and we present future perspectives on the modelling.

6.1. The density structure of the ISM

Our description of the turbulent component of the GMF along the LOS is based on a finite number of independent lay-
ers, rather than on a continuous variation computed from the power spectrum of the GMF, as was included in some earlier models (e.g., Miville-Deschênes et al. 2008; O’Dea et al. 2012). The density structure of the diffuse ISM provides one argument in favour of this approximation.

If we are in practice observing a finite number of localized density structures from the cold neutral medium (CNM) along the LOS, then the discretization of the GMF orientation is appropriate. Such structures appear as extended features on the sky in dust emission maps, with a power-law power spectrum. This statement is exemplified by the images and the power spectrum of the GMF, as was included in some earlier works, rather than on a continuous variation computed from the GMF. Following Eilek (1989), we derive the correlation length of the turbulent component of the GMF, which is the lag of $\sigma_B$ from the 2-point auto-correlation function, $C_B$, of each of the three components of $B$: 

$$\int C_B(s) \, ds = l_s \sigma_B^2,$$  

where $s$ is the lag of $C_B$ along one given direction and $\sigma_B$ is the dispersion of $B$. In this framework, the number of correlation lengths along the LOS is $N_\alpha = L/l_s$, where $L$ is the extent of matter along the LOS. We compute $C_B$ from Gaussian realizations of $B$ for power-law spectra and, from there, $N_\alpha$ integrates Eq. (17) up to the lag where $C_B(s) = 0$. $N_\alpha$ depends on the spectral index $\alpha$ of the power spectrum of the components of $B$. We find values of $N_\alpha$ of 16, 10, 6, and 5 for spectral indices of the power law spectrum $\alpha = -1.5$, $-2$, $-2.5$, and $-3$, respectively.

We can now compute the Stokes parameters for this continuous description of $B$, and the mean orientation, $B_\theta$ (determined in Sect. 3), through the integral equations described in Appendix A for several values of $\alpha$ using a constant source function as in step C. The Gaussian realizations and the integrals are computed over 1024 points along each LOS at $b < -60^\circ$. In this approach, used earlier by Miville-Deschênes et al. (e.g., 2008; O’Dea et al. (e.g., 2012), there is no correlation of $B$ between nearby pixels on the sky, hence, we cannot produce realistic images but we do sample the 1-point distribution of $p^2$.

The histograms of $p^2$ (normalized to unity with $p_0$) are presented in Fig. 13 for several values of $\alpha$, with $f_{H_\alpha} = 0.9$ and no data noise. We use the same binning as in Fig. 12 to allow for a direct comparison between the two sets of histograms. The continuous description of $B$ matches the standard deviation of $\psi_B$ measured in the Planck data for $\alpha = -3$. However, the corresponding histogram of $(p/p_0)^2$ in Fig. 13 is narrower than the one for $N = 7$ in Fig. 12, which fits the data better. We conclude that the number of polarization layers may be interpreted as the number of effective modes contributing to the variations of the orientation of $B$ along the LOS within the WNM and WIM. From this view point, the low value of $\sigma_B$ derived from the data indicates the steepness of the power spectrum of $B$; however, this interpretation does not fully account for the data, because it ignores the density structure of the diffuse ISM (i.e., the CNM).

![Fig. 13. Model histograms of $p^2$ normalized to unity with $p_0$, obtained for a continuously varying GMF orientation along the LOS, with $f_{H_\alpha} = 0.9$ for several values of $\alpha$, between 0 (black curve) and $-3$ (yellow curve). To facilitate the comparison of the histogram of $(p/p_0)^2$ with that in Fig. 12 we have used the same bin width (0.01) to compute both histograms.](image-url)
6.3. Future perspectives

We now briefly outline a few future directions that could be taken to extend our data analysis and modelling.

We have started to investigate the impact of the GMF structure on the statistics of the polarization parameters. In the upcoming paper we will use the model presented in this work to reproduce the dust polarization power spectra measured by Planck (Planck Collaboration Int. XXX (2016)) and constrain the value of $\sigma_3$, the value of which is left open in this paper. Another future project will be to introduce the density structure and its correlation with the orientation of the GMF within each polarization layer. Such a study will enable us to assess the respective contributions of the density and the GMF structure to the statistics of the dust polarization data.

In the present work, we have aimed at providing a phenomenological method to compute realizations of the dust polarization sky for component separation in measurements of the polarization of the GMF. We want to stress the simplicity of our approach, which allowed us to characterize the high latitude polarization sky with very few parameters. This framework might be useful to predict the expected accuracy of component-separation methods in future CMB experiments. Planck Collaboration Int. XXXVIII (2016) and Clark et al. (2015) associated the asymmetry between EE and BB power spectra of dust polarization (i.e., $C^E_E \approx 0.5 C^B_B$) with the correlation between the structure of the GMF and the distribution of interstellar matter. Future models will need to take this correlation into account in order to realistically assess the accuracy to which, for a given experiment, dust and CMB polarization can be separated.

7. Summary

We have analysed the Planck maps of the Stokes parameters at high Galactic latitudes over the sky area $b < -60^\circ$, which is well suited for describing the Galactic magnetic field (GMF) structure in the diffuse interstellar medium (ISM), and is directly relevant for cosmic microwave background (CMB) studies. We characterized the structure of the Stokes parameter maps at 353 GHz, as well as the statistics of the polarization fraction $p$ and angle $\psi$. We presented simple geometrical models, which relate the data to the structure of the GMF in the solar neighbourhood. Combining models of the turbulent and ordered components of the GMF, we have reproduced the patterns of the Stokes $Q$ and $U$ maps at large angular scales, as well as the histograms of $p$ and $\psi$. The main results of the paper are listed below.

- We find that the histogram of $p$ at high Galactic latitudes has a similar dispersion as that measured over the whole sky, although with a smaller depolarization, caused by line-of-sight (LOS) variations of the GMF orientation, on and near the Galactic plane.
- The Stokes $Q$ and $U$ maps show regular patterns at large scales, which we associate with the mean orientation of the GMF in the solar neighbourhood. We build a geometric model and find a mean orientation towards Galactic coordinates $(l_0, b_0) = (70^\circ, 24^\circ)$, compatible with previous estimates. The fit also provides us with the average value of $p$ at $b \leq -60^\circ$, which is $(12 \pm 1) \%$.
- By means of a simple description of the turbulent component of the GMF (Gaussian and isotropic), we manage to account for both the dispersion of $\psi$ and the histogram of $p$. The effect of depolarization caused by the GMF fluctuations along the LOS is introduced through an approximation where the integrals along the LOS are replaced by a discrete sum over only a few independent polarization layers. This approach successfully reproduces the $p$ and $\psi$ distributions using $N \approx 4$–9 layers.
- The integration along the LOS generates a mean depolarization factor that is about 0.5 and thus leads to an estimate of $p_0$ about twice greater than the average value of $p$. The best-fit value of the effective polarization of dust, which combines the intrinsic polarization of dust grains and their degree of alignment with the GMF, is $\langle 26 \pm 3 \rangle \%$.
- Our description of the turbulent component of the GMF corresponds to a rough equality between the turbulent and mean strengths of the GMF. The same conclusion was reached from modelling the dispersion of polarization angles measured for CNM filamentary structures by Planck Collaboration Int. XXXII (2016). We extend this to the diffuse ISM observed in the high latitude sky, which comprises of both WNM and CNM gas.

The present study represents the first step towards the characterization of the magnetized properties of the diffuse ISM by means of the Planck data. We argue that both the density structure and the effective correlation length of the GMF contribute to account for the large dispersion of $p$ observed in the data. This can be further investigated using MHD numerical simulations. The next step in our modelling of dust polarization at high Galactic latitudes will be to fit the $E$ and $B$ power spectra. This will constrain the spectral index of the GMF power spectrum, providing information on the turbulent energy cascade in the diffuse ISM. It is also a required step before using our model to compute simulated maps for assessing component-separation methods in CMB polarization projects.

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References


Appendix A: Approximations for dust polarization

In this Appendix, we detail the approximations made to model the Stokes parameters for linear polarization from dust emission. For the sake of clarity, we recall the integral equations of the Stokes parameters I, Q, and U from Planck Collaboration Int. XX (2015):

\[ I = \int S_{\nu} e^{-\tau_{\nu}} \left[ 1 - p_{0}(\cos^{2}\gamma - \frac{2}{3}) \right] d\tau_{\nu}; \] (A.1a)

\[ Q = \int p_{0} S_{\nu} e^{-\tau_{\nu}} \cos(2\psi) \cos^{2}\gamma d\tau_{\nu}; \] (A.1b)

\[ U = \int p_{0} S_{\nu} e^{-\tau_{\nu}} \sin(2\psi) \cos^{2}\gamma d\tau_{\nu}. \] (A.1c)

Here \( \tau_{\nu} \) is the optical depth and \( S_{\nu} \) is the source function of dust emission, while \( p_{0} \) and the angles (\( \psi, \gamma \)) are the same as in Sect. 2.2. We make two additional points: (1) in order to relate \( p \) as shown in Eq. (3) to the mean orientation of the GMF with respect to the POS (the angle \( \gamma \)), we need to assume that all parameters in Eqs. (A.1a), (A.1b), and (A.1c) are roughly uniform along the LOS; and (2) the total intensity in Eq. (A.1a) also depends on the GMF orientation through the angle \( \gamma \). However, throughout our modelling procedure, we neglect this dependence.

In general the corrections to Stokes I caused by the GMF geometry are small, ranging roughly between \(-7\%\) and \(+13\%\) for \( p_{0} \approx 20\% \) (Planck Collaboration Int. XIX, 2015). In our study, we focus on a region of the sky where the depolarization produced by \( \cos^{2}\gamma \) is small (\( \cos^{2}\gamma \) is mostly close to unity over the southern Galactic cap). Hence, in our study, the correction to Eq. (A.1a) would always be negative and less than \(-10\%\). Thus, in Sect. 5.3, we might estimate a value of \( p_{0} \) slightly greater than the true value that we would have obtained by modelling the GMF correction for Stokes I. In practice, Eq. (8) in Sect. 4.2 would change as follows:

\[ Q_{353} = \frac{p_{0} q_{353}}{1 - p_{0}(\cos^{2}\gamma - \frac{2}{3})} D_{353}; \]

\[ U_{353} = \frac{p_{0} u_{353}}{1 - p_{0}(\cos^{2}\gamma - \frac{2}{3})} D_{353}. \] (A.2)

The fits of steps A, B, and C would then not be linear in \( p_{0} \) any more, substantially complicating the fit. We argue that, considering the overall approximations (analytical and astrophysical) of our models, the GMF geometry in Stokes I is a minor issue.

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