
LIGO SCIENTIFIC COLLABORATION AND VIRGO COLLABORATION

(See the end matter for the full list of authors.)

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ABSTRACT

This article provides supplemental information for a Letter reporting the rate of (BBH) coalescences inferred from 16 days of coincident Advanced LIGO observations surrounding the transient (GW) signal GW150914. In that work we reported various rate estimates whose 90% confidence intervals fell in the range 2–600 Gpc⁻³ yr⁻¹. Here we give details on our method and computations, including information about our search pipelines, a derivation of our likelihood function for the analysis, a description of the astrophysical search trigger distribution expected from merging BBHs, details on our computational methods, a description of the effects and our model for calibration uncertainty, and an analytic method for estimating our detector sensitivity, which is calibrated to our measurements.

Key words: gravitational waves – stars: black holes

Supporting material: data behind figures

1. SEARCH PIPELINES

Both the pycbc and gstlal pipelines are based on matched filtering against a bank of template waveforms. See Abbott et al. (2016d) for a detailed description of the pipelines in operation around the time of GW150914; here we provide an abbreviated description.

In the pycbc pipeline, the single-detector signal-to-noise ratio (S/N) is re-weighted by a chi-squared factor (Allen 2005) to account for template-data mismatch (Babak et al. 2013); the re-weighted single-detector S/Ns are combined in quadrature to produce a detection statistic for search triggers.

The gstlal pipeline’s detection statistic, however, is based on a likelihood ratio (Cannon et al. 2013, 2015) constructed from the single-detector S/Ns and a signal-consistency statistic. An analytic estimate of the distribution of astrophysical signals in multiple-detector S/N and signal-consistency statistic space is compared to a measured distribution of single-detector triggers without a coincident counterpart in the other detector to form a multiple-detector likelihood ratio.

Both pipelines rely on an empirical estimate of the search background, making the assumption that triggers of terrestrial origin occur independently in the two detectors. The background estimate is built from observations of single-detector triggers over a long time (gstlal) or through searching over a data stream with one detector’s output shifted in time relative to the other’s by an interval that is longer than the light travel time between detectors, ensuring that no coincident astrophysical signals remain in the data (pycbc). For both pipelines it is not possible to produce an instantaneous background estimate at a particular time; this drives our choice of likelihood function, as described in Section 2.

The gstlal pipeline natively determines the functions \( p_0(x) \) and \( p_1(x) \) for its detection statistic \( x \). For this analysis a threshold of \( x_{\text{min}} = 5 \) was applied, which is sufficiently low that the trigger density is dominated by terrestrial triggers near a threshold. There were \( M = 15,848 \) triggers observed above this threshold in the 17 days of observation time analyzed by gstlal.

For pycbc, the quantity \( x' \) is the re-weighted S/N detection statistic. \(^{137}\) We set a threshold \( x'_{\text{min}} = 8 \), above which \( M' = 270 \) triggers remain in the search. We use a histogram of triggers collected from time-shifted data to estimate the terrestrial trigger density, \( p_0(x') \), and a histogram of the recovered triggers from the injection sets described in Section 2.2 of the Letter to estimate the astrophysical trigger density, \( p_1(x') \). These estimates are shown in Figure 1. The uncertainty in the distribution of triggers from this estimation procedure is much smaller than the uncertainty in the overall rate from the finite number statistics (see, for example, Figure 5). The empirical estimate is necessary to properly account for the interaction of the various single- and double-interferometer thresholds in the pycbc search (Abbott et al. 2016c). At high S/N, where these thresholds are irrelevant, the astrophysical triggers follow an approximately flat-space volumetric density (see Section 3) of

\[
p_1(x') \simeq \frac{3x'^{3/2}}{x'^3},
\]

but they deviate from this at smaller S/N due to threshold effects in the search.

For the pycbc pipeline, a detection statistic \( x' \geq 10.1 \) corresponds to an estimated search false alarm rate (FAR) of one per century.

2. DERIVATION OF POISSON MIXTURE MODEL LIKELIHOOD

In this section we derive the likelihood function in Equation (3) of the Letter. Consider a search of the type described in

\(^{137}\) When quoting pipeline-specific values we distinguish pycbc quantities with a prime.
Section 1 over $N_T$ intervals of time, of width $\delta_i$, $i = 1, \ldots, N_T$. Triggers above some fixed threshold occur with an instantaneous rate in time and detection statistic $x$ given by the sum of the terrestrial and astrophysical rates:

$$\frac{dN}{dt} dx (t, x) = R_0(t)p_0(x; t) + R_1(t)V(t)p_1(x; t),$$

where $R_0(t)$ is the instantaneous rate (number per unit time) of terrestrial triggers, $R_1(t)$ is the instantaneous rate density (number per unit time per unit comoving volume) of astrophysical triggers, $p_0$ is the instantaneous density in the detection statistic of terrestrial triggers, $p_1$ is the instantaneous density in the detection statistic of astrophysical triggers, and $V(t)$ is the instantaneous sensitive comoving redshifted volume (Abbott et al. 2016a; see also Equation (15) of the Letter) of the detectors to the assumed source population. The astrophysical rate $R_1$ is to any reasonable approximation constant over our observations so we will drop the time dependence of this term from here on.\footnote{The astrophysical rate can, in principle, also depend on redshift, but in this paper we assume that the BBH coalescence rate is constant in the comoving frame.} Note that $R_0$ and $R_1$ have different units in this expression: the former is a rate (per time), while the latter is a rate density (per time-volume). The density $p_1$ is independent of source parameters as described in Section 3. Let

$$\int dx \frac{dN}{dt} dx = R_0(t) + R_1V(t).$$

If the search intervals $\delta_i$ are sufficiently short, they will contain at most one trigger and the time-dependent terms in Equation (2) will be approximately constant. Then the likelihood for a set of times and detection statistics of triggers, $\{(t_j, x_j)\}_{j=1}^M$, is a product over intervals containing a trigger (indexed by $j$) and intervals that do not contain a trigger (indexed by $k$) of the corresponding Poisson likelihoods

$$\mathcal{L} = \prod_{j=1}^M \left\{ \exp \left[ -\delta_j \frac{dN}{dt} (t_j) \right] \right\} \times \left\{ \prod_{k=1}^{N_T-M} \exp \left[ -\delta_k \frac{dN}{dt} (t_k) \right] \right\}$$

(see Farr et al. 2015, Equation (21), or Loredo & Wasserman 1995 Equation (2.8)).\footnote{There is a typo in Equation (2.8) of Lorodo & Wasserman (1995). The second term in the final bracket is missing a factor of $\delta_t$.} Now let the width of the observation intervals $\delta_i$ go to zero uniformly as the number of intervals goes to infinity. Then the products of exponentials in Equation (4) become an exponential of an integral, and we have

$$\mathcal{L} = \prod_{j=1}^M \left[ \frac{dN}{dt} (t_j, x_j) \right] \exp \left[ -N \right],$$

where

$$N = \int dt \frac{dN}{dt}$$

is the expected number of triggers of both types in the total observation time $T$. As discussed in Section 1, in our search we observe that $R_0$ remains approximately constant and that $p_0$ retains its shape over the observation time discussed here; this assumption is used in our search background estimation procedure (Abbott et al. 2016c). The astrophysical distribution of triggers is universal (Section 3) and also time-independent. Finally, the detector sensitivity is observed to be stable over our 16 days of coincident observations, so $V(t) \approx \text{const}$ (Abbott et al. 2016b). We therefore choose to simply ignore the time dimension in our trigger set. This generates an estimate of the rate that is sub-optimal (i.e., has larger uncertainty) but consistent with using the full data set to the extent that the detector sensitivity varies in time; since this variation is small, the loss of information about the rate will be correspondingly small. We do capture any variation in the sensitivity with time in our Monte Carlo procedure for estimating $\langle VT \rangle$, which is described in Section 2.2 of the Letter.

If we ignore the trigger time, then the appropriate likelihood to use is a marginalization of Equation (5) over the $t_j$. Let

$$\tilde{\mathcal{L}} = \int [\prod_j dt_j] \mathcal{L}$$

$$= \prod_j \left[ \Lambda_0 p_0(x_j) + \Lambda_1 p_1(x_j) \right] \exp \left[ -\Lambda_0 - \Lambda_1 \right],$$

where

$$\Lambda_0 p_0(x) = \int dt R_0(t)p_0(x; t),$$

and

$$\Lambda_1 p_1(x) = \int dt R_1V(t)p_1(x; t),$$

with

$$\int dx p_0(x) = \int dx p_1(x) = 1.$$
Poisson uncertainty in the terrestrial count is uncorrelated in the posterior. One force the terrestrial count to adjust in a meaningful way and the variables are larger than the astrophysical count, changes in the astrophysical count do not near the Poisson uncertainty in the total count. Because this uncertainty is much larger than the astrophysical count, changes in the astrophysical count do not substantially change the expected counts. If we assume that the distribution of source parameters is not substantially change the λ₀ estimate due to the over-whelming number of background triggers in the data set. However, because there are so many more triggers in searches of terrestrial origin than there are in those of astrophysical origin there is little correlation between λ₀ and λ₁, so there is little difference between the posterior we obtain on λ₁ and the posterior we would have obtained had we implemented the tight prior on λ₀. Figure 2 shows the two-dimensional posterior we obtain from Equation (5) of the Letter on λ₀ and λ₁.

We have checked that using a δ-function prior

$$ p(λ₀) = δ(λ₀ - 270) $$

(11)

in the pycbc analysis that is the result of the pipeline λ₀ estimate from timeslides140 (Abbott et al. 2016c) and using a looser prior that is the result of a glslal estimate on a single set of time-slid data produce no meaningful change in our results. Figure 3 shows our canonical rate posterior inferred with the pycbc λ₀ prior in Equation (11) and our canonical broad prior.

3. UNIVERSAL ASTROPHYSICAL TRIGGER DISTRIBUTION

Both the pycbc and glslal pipelines rely on the S/N as part of their detection statistic, x. The S/N of an astrophysical trigger is a function of the detector noise at the time of detection and the parameters of the trigger. Schutz (2011) and Chen & Holz (2014) demonstrate that the distribution of the expected S/N (ρ) in a simple model of a detection pipeline that simply thresholds on S/N, ρ ≥ ρ₀, with sources in the local universe is universal, that is, independent of the source properties. It follows

$$ p(ρ) = \frac{3ρ₀^3}{(ρ)³} $$

(12)

This result follows from the fact that the expected value of the S/N in a matched-filter search for compact binary coalescence (CBC) signals scales inversely with transverse comoving distance (Hogg 1999):

$$ ⟨ρ⟩ = \frac{A(m₁, m₂, a₁, a₂, S(f), z)B(\text{angles})}{D_M} $$

(13)

where A is an amplitude factor that depends on the intrinsic properties (source-frame masses and spins) of the source, the detector sensitivity expressed as a noise power spectral density S(f) as a function of observer frequency and redshift z, and B is an angular factor depending on the location of the source in the sky and the relative orientations of the binary orbit and detector. The redshift enters A only through shifting the source waveform to lower frequency at higher redshift, changing A because the sensitivity varies with observer frequency f. For the redshifts to which we are sensitive to BBH in this observation period this effect on A is small.

If we assume that the distribution of source parameters is constant over the range of distances to which we are sensitive, and ignore the small redshift-dependent sensitivity correction mentioned above, then the distribution of S/N will be governed entirely by the distribution of distances of the sources, which, in the local universe, is approximately

$$ p(D_M) \propto D_M^{5/2} $$

(14)

yielding the distribution of S/N given in Equation (12).

140 While the statistical uncertainty on the pipeline λ₀ estimate is not precisely zero, σ_λ₀/λ₀ ≤ 10⁻³, it is so small that a δ-function prior is appropriate.
suitable for our purposes, independent of the source parameters. Figure 4 shows the distribution of recovered detection statistics for the various injection campaigns with varying source distribution used to estimate sensitive time-volumes in the pycbc pipeline. In each injection campaign $\mathcal{O}(1000)$ signals were recovered. For loud signals, the detection statistic is proportional to $S/N$ in this pipeline, and the distribution is not sensitive to the complicated thresholding in the pipeline, so we recover Equation (12); for quiet signals the interaction of various single-detector thresholds in the pipeline causes the distribution to deviate from this analytic approximation, but it remains independent of the distribution of sources. Note that the empirical distribution of detection statistics, not the analytic one, forms the basis for $p_t$, the foreground distribution used in this rate estimation work.

To quantify the deviations from universality, we have preformed two-sample Kolmogorov-Smirnov (KS) tests between all six pairings of the sets of detections statistics recovered in the injection campaigns described in Sections 2 and 3 of the Letter and featured in Figure 4. The most extreme KS $p$-value occurred with the comparison between the injection set with BBH masses drawn flat in log $m$ and the one with masses drawn from a power law (both described in Section 3 of the Letter); this test gave a $p$-value of 0.013. Given that we have performed six identical comparisons we cannot reject the null hypothesis that the empirical distributions used for rate estimation from the pycbc pipeline are identical even at the relatively weak significance $\alpha = 0.05$. Certainly any differences in detection statistic distribution attributable to the BBH population are far too small to matter with the few astrophysical signals in our data set (compared with $\mathcal{O}(1000)$ recovered injections in each campaign).

Because the distribution of detection statistics is, to a very good approximation, universal, we cannot learn anything about the source population from the detection statistic alone; we must instead resort to parameter estimation (PE) follow-up (Veitch et al. 2015; Abbott et al. 2016e) of triggers to determine their parameters. The parameters of the waveform template that produced the trigger can be used to guess the parameters of the source that generated that trigger, but the bias and uncertainty in this estimate are very large compared to the PE estimate. We therefore ignore the parameters of the waveform template that generated the trigger in the assignment of triggers to BBH classes.

### 4. COUNT POSTERIOR

We impose a prior on the $\Lambda$ parameters of:

$$p(\Lambda_1, \Lambda_0) \propto \frac{1}{\sqrt{\Lambda_1}} \frac{1}{\sqrt{\Lambda_0}}. \tag{15}$$

The posterior on expected counts is proportional to the product of the likelihood from Equation (3) of the Letter and the prior from Equation (15):

$$p(\Lambda_1, \Lambda_0 | \{x_j | j = 1, \ldots, M\}) \propto \prod_{j=1}^{M} \left[ \Lambda_1 p_1(x_j) + \Lambda_0 p_0(x_j) \right] \times \exp \left[ -\Lambda_1 - \Lambda_0 \right] \frac{1}{\sqrt{\Lambda_1 \Lambda_0}}. \tag{16}$$

For estimation of the Poisson rate parameter in a simple Poisson model, the Jeffreys prior is $1/\sqrt{\Lambda}$. With this prior, the
posterior mean on $\Lambda$ is $N + 1/2$ for $N$ observed counts. With a prior proportional to $1/\Lambda$ the mean is $N$ for $N > 0$, but the posterior is improper when $N = 0$. For a flat prior, the mean is $N + 1$. Though the behavior of the mean is not identical with our mixture model posterior, it is similar; because we find $\langle \Lambda \rangle \gg 1/2$, the choice of prior among these three reasonable options has little influence on our results here.

For the pycbc data set we find the posterior median and 90% credible range $\Lambda_1 = 3.2^{+4.9}_{-2.4}$ above our threshold. For the gstream set we find the posterior median and 90% credible range $\Lambda_1 = 4.8^{+7.9}_{-3.8}$. Though we have only one event (GW150914) at exceptionally high significance, and one other at marginal significance (LVT151012), the counting analysis shows these to be consistent with the possible presence of several more events of astrophysical origin at a lower detection statistic in both pipelines.

The thresholds applied to the pycbc and gstream triggers for this analysis are not equivalent to each other in terms of either S/N or FAR; instead, both thresholds have been chosen so that the rate of triggers of terrestrial origin ($\Lambda_0 \phi_0$) dominates near the threshold. Since the threshold is set at different values for each pipeline, we do not expect the counts to be the same between pipelines.

The estimated astrophysical and terrestrial trigger rate densities (Equation (1) of the Letter) for pycbc are plotted in Figure 5. We select triggers from a subset of the search parameter space (i.e., our bank of template waveforms) that contains GW150914 as well as the mass range considered for possible alternative populations of BBH binaries in Section 3 of the Letter. There are $M' = 270$ two-detector coincident triggers in this range in the pycbc search (Abbott et al. 2016c). Figure 5 also shows an estimate of the density of triggers that comprise our data set, which agrees well with our inference of the trigger rate.

We use the Stan and emcee Markov Chain Monte Carlo samplers (Foreman-Mackey et al. 2013; Stan Development Team 2015a, 2015b) to draw samples from the posterior in Equation (5) of the Letter for the two pipelines. We have assessed the convergence and mixing of our chains using empirical estimates of the autocorrelation length in each parameter (Sokal 1996), the Gelman-Rubin $R$ convergence statistic (Gelman & Rubin 1992), and through visual inspection of chain plots. By all measures, the chains appear to be well-converged to the posterior distribution.

Table 1 contains the full results on expected counts and associated sensitive time-volumes for both pipelines.

5. CALIBRATION UNCERTAINTY

The LIGO detectors are subject to uncertainty in their calibration, in both the measured amplitude and the phase of the GW strain. Abbott et al. (2016b) discussed the methods used to calibrate the strain output of the detector during the 16 days of coincident observations discussed here. Abbott et al. (2016b) estimated that the reported strain is accurate to within 10% in amplitude and 10 degrees in phase between 20 Hz and 1 kHz throughout the observations.

The S/Ns reported by our searches are quadratically sensitive to calibration errors because they are maximized over arrival time, waveform phase, and a template bank of waveforms (Allen 1996; Brown & LIGO Scientific Collaboration 2004). Abbott et al. (2016c) demonstrated that the other search pipeline outputs are also not affected to a significant degree by the calibration uncertainty present during our observing run. Therefore, we ignore the effects of calibration on the pipeline detection statistics $x$ and $x'$ that we use here to estimate rates from the pycbc and gstream pipelines.

The amplitude calibration uncertainty in the detector results, at leading order, in a corresponding uncertainty between the luminosity distances of sources measured from real detector outputs (Abbott et al. 2016e) and the luminosity distances used to produce injected waveforms used to estimate sensitive time-volumes in this work. A 10% uncertainty in $d_L$ at these redshifts corresponds to an approximately 30% uncertainty in volume. We model this uncertainty by treating $\langle VT \rangle$ as a...
parameter in our analysis, and imposing a log-normal prior:

$$p(\log \langle VT\rangle) \propto N \left( \log \mu, \frac{\sigma}{\mu} \right),$$

(17)

where $\mu$ is the Monte Carlo estimate of sensitive time-volume produced from the injection campaigns described in Section 2.2 of the Letter and

$$\sigma^2 = \sigma^2_{\text{cal}} + \sigma^2_{\text{stat}},$$

(18)

with $\sigma_{\text{cal}} = 0.3\mu$ and $\sigma_{\text{stat}}$ is the estimate of the Monte Carlo uncertainty from the finite number of recovered injections reported above. In all cases $\sigma_{\text{cal}} \gg \sigma_{\text{stat}}$.

Since the likelihood in Equations (3) or (10) of the Letter does not constrain $\langle VT\rangle$ independently of $R$, sampling over $\langle VT\rangle$ at the same time as $\Lambda$ and $R$ has the effect of convolving the log-normal distribution of $\langle VT\rangle$ with the posterior on $\Lambda$ in the inference of $R$.

Table 1

<table>
<thead>
<tr>
<th>Assumption</th>
<th>$\Lambda$ (Gpc yr$^{-1}$)</th>
<th>$\langle VT\rangle$ (Gpc yr$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pycbc</td>
<td>2.1$^{+1.3}_{-1.1}$</td>
<td>0.130$^{+0.004}_{-0.001}$</td>
</tr>
<tr>
<td>gstlal</td>
<td>3.6$^{+1.2}_{-0.9}$</td>
<td>0.21$^{+0.14}_{-0.08}$</td>
</tr>
<tr>
<td>GW150914</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LVT151012</td>
<td>2.0$^{+0.9}_{-1.0}$</td>
<td>0.032$^{+0.002}_{-0.001}$</td>
</tr>
<tr>
<td>Both</td>
<td>4.5$^{+5.5}_{-5.3}$</td>
<td>0.048$^{+0.001}_{-0.001}$</td>
</tr>
<tr>
<td>Flat in log mass</td>
<td>3.2$^{+4.9}_{-2.4}$</td>
<td>0.050$^{+0.002}_{-0.001}$</td>
</tr>
<tr>
<td>Power Law ($-2.35$)</td>
<td>4.8$^{+7.9}_{-3.8}$</td>
<td>0.080$^{+0.001}_{-0.001}$</td>
</tr>
</tbody>
</table>

Note. See Sections 2.1, 2.2, 3, and 4 of the Letter.

Figure 6. Posterior probability that coincident triggers in our analysis come from an astrophysical source (see Equation (7) of the Letter), taking into account the astrophysical and terrestrial expected counts estimated in Section 2.1 of the Letter. Left: the gstlal triggers with $x > 5$; right: pycbc triggers with $x' > 8$. GW150914 is not shown in the plot because its probability of astrophysical origin is effectively 100%. The only two triggers with $P_1 \geq 50\%$ are GW150914 and LVT151012. For GW150914, we find $P_1 = 1$ to very high precision; for LVT151012, the gstlal pipeline finds $P_1 = 0.84$ and the pycbc pipeline finds $P_1 = 0.91$. (The data used to create this figure are available.)

Figure 7. Rate at which sensitive time-volume accumulates with redshift. Curves labeled by component masses in $M_\odot$ are computed using the approximate prescription described in Section 6, assuming sources with fixed mass in the comoving frame and without spin; the GW150914 and LVT151012 curves are determined from the Monte Carlo injection campaign described in Section 2.2 of the Letter. (The data used to create this figure are available.)

In spite of the 30% relative uncertainty in $\langle VT\rangle$ from calibration uncertainty, the counting uncertainty on $R$ from the small number of detected events dominates the width of the posterior on $R$. 

6. ANALYTIC SENSITIVITY ESTIMATE

As a rough check on our $\langle VT \rangle$ estimates and the integrand $d\langle VT \rangle/dz$, we find that the following approximate, analytic procedure also produces a good approximation to the pybce Monte Carlo estimate in Table 1.

1. Generate inspiral–merger–ringdown waveforms in a single detector at various redshifts from the source distribution $s(\theta)$ with random orientations and sky positions.
2. Using the high-sensitivity early Advanced LIGO noise power spectral density from Abbott et al. (2016f), compute the S/N in a single detector.
3. Consider a signal found if the S/N is greater than 8.

Employed with the source distributions described above, this approximate procedure yields $\langle VT \rangle_1 \approx 0.107$ Gpc$^3$ yr and $\langle VT \rangle_2 \approx 0.0225$ Gpc$^3$ yr for the sensitivity to the two classes of merging BBH system. Figure 7 shows the sensitive time-volume integrand,

$$\frac{d\langle VT \rangle}{dz} = T \frac{1}{1 + z} \frac{dV}{dz} \int d\theta s(\theta)f(z, \theta),$$

estimated from this procedure for systems with various parameters superimposed on the Monte Carlo estimates from the injection campaign described above.

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AUTHORS

J. Zweizig

(LIGO Scientific Collaboration and Virgo Collaboration)

1 LIGO, California Institute of Technology, Pasadena, CA 91125, USA
2 Louisiana State University, Baton Rouge, LA 70803, USA
3 Università di Salerno, Fisciano, I-84084 Salerno, Italy
4 INFN, Sezione di Napoli, Complesso Universitario di Monte S. Angelo, I-80126 Napoli, Italy
5 University of Florida, Gainesville, FL 32611, USA
6 LIGO Livingston Observatory, Livingston, LA 70754, USA
7 Laboratoire d’Annecy-le-Vieux de Physique des Particules (LAPP), Université Savoie Mont Blanc, CNRS/IN2P3, F-74931 Annecy-le-Vieux, France
8 Albert-Einstein-Institut, Max-Planck-Institut für Gravitationsphysik, D-30167 Hannover, Germany
9 Nikhef, Science Park, 1098 XG Amsterdam, Netherlands
10 LIGO, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
11 Instituto Nacional de Pesquisas Espaciais, 12227-010 São José dos Campos, São Paulo, Brazil
12 INFN, Gran Sasso Science Institute, I-67100 L’Aquila, Italy
13 INFN, Sezione di Roma Tor Vergata, I-00133 Roma, Italy
14 Inter-University Centre for Astronomy and Astrophysics, Pune 411007, India
15 International Centre for Theoretical Sciences, Tata Institute of Fundamental Research, Bangalore 560012, India
16 University of Wisconsin-Milwaukee, Milwaukee, WI 53201, USA
17 Leibniz Universität Hannover, D-30167 Hannover, Germany
18 Università di Pisa, I-56127 Pisa, Italy
19 INFN, Sezione di Pisa, I-56127 Pisa, Italy
20 Australian National University, Canberra, ACT 0200, Australia
21 The University of Mississippi, University, MS 38677, USA
22 California State University Fullerton, Fullerton, CA 92831, USA
23 LAL, Université Paris-Sud, CNRS/IN2P3, Université Paris-Saclay, 91400 Orsay, France
24 Chennai Mathematical Institute, Chennai 603103, India
25 Università di Roma Tor Vergata, I-00133 Roma, Italy
26 University of Southampton, Southampton SO17 1BJ, UK
27 Universität Hamburg, D-22761 Hamburg, Germany
28 INFN, Sezione di Roma, I-00185 Roma, Italy
29 Albert-Einstein-Institut, Max-Planck-Institut für Gravitationsphysik, D-14476 Potsdam-Golm, Germany
30 APC, AstroParticule et Cosmologie, Université Paris Diderot, CNRS/IN2P3, CEA/Irfu, Observatoire de Paris, Sorbonne Paris Cité, F-75205 Paris Cedex 13, France
31 Montana State University, Bozeman, MT 59717, USA
32 Università di Perugia, I-06123 Perugia, Italy
33 INFN, Sezione di Perugia, I-06123 Perugia, Italy
34 European Gravitational Observatory (EGO), I-56021 Cascina, Pisa, Italy
35 Syracuse University, Syracuse, NY 13244, USA
36 SUPA, University of Glasgow, Glasgow G12 0QX, UK
37 LIGO Hanford Observatory, Richland, WA 99352, USA
38 Wigner RCP, RMKI, H-1121 Budapest, Konkoly Thege Miklós út 29-33, Hungary
39 Columbia University, New York, NY 10027, USA
40 Stanford University, Stanford, CA 94305, USA
41 Università di Padova, Dipartimento di Fisica e Astronomia, I-35131 Padova, Italy
42 INFN, Sezione di Padova, I-35131 Padova, Italy
43 CAMK-PAN, 00-716 Warsaw, Poland
44 University of Birmingham, Birmingham B15 2TT, UK
45 Université degli Studi di Genova, I-16146 Genova, Italy
46 INFN, Sezione di Genova, I-16146 Genova, Italy
47 RRCAT, Indore MP 452013, India
48 Faculty of Physics, Lomonosov Moscow State University, Moscow 119991, Russia
49 SUPA, University of the West of Scotland, Paisley PA1 2BE, UK
50 University of Western Australia, Crawley, WA 6009, Australia
51 Department of Astrophysics/IMAPP, Radboud University Nijmegen, P.O. Box 9010, 6500 GL Nijmegen, Netherlands
52 Artemis, Université Côte d’Azur, CNRS, Observatoire Côte d’Azur, CS F-06232, Nice cedex 4, France
53 MTA Eötvös University, “Lendület” Astroparticle Physics Research Group, Budapest 1117, Hungary
54 Institut de Physique de Rennes, CNRS, Université de Rennes 1, F-35042 Rennes, France
55 Washington State University, Pullman, WA 99164, USA
56 Università degli Studi di Urbino “Carlo Bo,” I-61029 Urbino, Italy
57 INFN, Sezione di Firenze, I-50019 Sesto Fiorentino, Firenze, Italy
58 University of Oregon, Eugene, OR 97403, USA
59 Laboratoire Kastler Brossel, UPMC-Sorbonne Universités, CNRS, ENS-PSL, Université de Paris, F-75005 Paris, France
60 Astronomical Observatory Warsaw University, 00-478 Warsaw, Poland
61 Vrije Universiteit Amsterdam, 1081 HV Amsterdam, Netherlands
62 University of Maryland, College Park, MD 20742, USA
63 Center for Relativistic Astrophysics and School of Physics, Georgia Institute of Technology, Atlanta, GA 30332, USA
64 Institut Lumière Matière, Université de Lyon, Université Claude Bernard Lyon 1, UMR CNRS 5306, F-69622 Villeurbanne, France
65 Laboratoire des Matériaux Avancés (LMA), IN2P3/CNRS, Université de Lyon, F-69622 Villeurbanne, Lyon, France
66 Universidad de las Illes Balears, IAC—IEEC, E-07122 Palma de Mallorca, Spain
67 Università di Napoli “Federico II,” Complesso Universitario di Monte S. Angelo, I-80126 Napoli, Italy
68 NASA/Goddard Space Flight Center, Greenbelt, MD 20771, USA
69 Canadian Institute for Theoretical Astrophysics, University of Toronto, Toronto, ON M5S 3H8, Canada
70 Tsinghua University, Beijing 100084, China
71 Texas Tech University, Lubbock, TX 79409, USA
<table>
<thead>
<tr>
<th>Institution</th>
<th>City, State, Country</th>
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<tr>
<td>The Pennsylvania State University, University Park</td>
<td>PA 16802, USA</td>
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<td>30013 Taiwan</td>
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<td>University of Chicago, Chicago, IL 60637, USA</td>
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<tr>
<td>Caltech CaRT, Pasadena</td>
<td>CA 91215, USA</td>
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<td>Korea Institute of Science and Technology Information, Daejeon 305-806, Korea</td>
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<tr>
<td>Carleton College, Northfield, MN 55057, USA</td>
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<tr>
<td>Università di Roma “La Sapienza,” I-00185 Roma, Italy</td>
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<td>University of Brussels, Brussels B-1050, Belgium</td>
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<td>Sonoma State University, Rohnert Park, CA 94928, USA</td>
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<td>University of Sannio at Benevento, I-82100 Benevento, Italy</td>
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<td>INFN, Sezione di Napoli, I-80100 Napoli, Italy</td>
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<td>Montclair State University, Montclair, NJ 07043, USA</td>
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<td>Università di Trento, Dipartimento di Fisica, I-38123 Povo, Trento, Italy</td>
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<td>INFN, Trento Institute for Fundamental Physics and Applications, I-38123 Povo, Trento, Italy</td>
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<td>Cardiff University, Cardiff CF24 3AA, UK</td>
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<td>National Astronomical Observatory of Japan, 2-21-1 Osaka, Mitaka, Tokyo</td>
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<td>School of Mathematics, University of Edinburgh, Edinburgh EH9 3FD, UK</td>
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<td>Indian Institute of Technology, Gandhinagar Ahmedabad Gujarat 382424, India</td>
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<td>Institute for Plasma Research, Bhat, Gandhinagar 382428, India</td>
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<td>Institute of Applied Physics, Nizhny Novgorod, 603950, Russia</td>
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<td>ESPCI CNRS, F-75005 Paris, France</td>
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<td>Università di Camerino, Dipartimento di Fisica, I-62032 Camerino, Italy</td>
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<td>Southern University and A&amp;M College, Baton Rouge, LA 70813, USA</td>
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<td>College of William and Mary, Williamsburg, VA 23187, USA</td>
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<td>Instituto de Física Teórica, University Estadual Paulista/ICTP South American Institute for Fundamental Research, São Paulo SP 01140-070, Brazil</td>
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<td>University of Cambridge, Cambridge CB2 1TN, UK</td>
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<td>Rutherford Appleton Laboratory, HSIC, Chilton, Didcot, Oxon OX11 0QX, UK</td>
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<td>Whitman College, 345 Boyer Avenue, Walla Walla, WA 99362 USA</td>
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<td>Hobart and William Smith Colleges, Geneva, NY 14456, USA</td>
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<td>Janusz Gil Institute of Astronomy, University of Zielona Góra, 65-265 Zielona Góra, Poland</td>
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<td>Andrews University, Berrien Springs, MI 49104, USA</td>
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<tr>
<td>Abilene Christian University, Abilene, TX 79699, USA</td>
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Abbott et al.