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Dynamics of fermions in an amplitude modulated lattice

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We study dynamics of fermions loaded in an optical lattice with a superimposed parabolic trap potential. In the recent Hamburg experiments [J. Heinze et al., Phys. Rev. Lett. 110, 085302 (2013)] on quantum simulation of photoconductivity, a modulation pulse on the optical lattice transferred part of the population of the lowest band to an excited band, leaving a hole in the particle distribution of the lowest band. Subsequent intricate dynamics of both excited particles and holes can be explained by a semiclassical approach based on the evolution of Wigner function. Here we provide a more detailed analysis of the dynamics taking into account the dimensionality of the system and finite temperature effects, aiming at reproducing experimental results on longer timescales. A semiclassical wave packet is constructed more accurately than in the previous theory. As a result, semiclassical dynamics indeed reproduces experimental data and full quantum numerical calculations with much better accuracy. In particular, fascinating phenomenon of collapse and revival of holes is investigated in a more detail. We presume the experimental setup can be used for deeper exploration of nonlinear waves in fermionic gases.

I. INTRODUCTION

Ultracold atoms and molecules in optical lattices may provide a path to construction of quantum simulators (well-controllable systems whose dynamics allow to understand essential features of more complicated condensed matter systems\textsuperscript{1,2}). Recent examples include numerous simulations of Hubbard-type models in optical lattices\textsuperscript{3–8}, in particular the simulation of photoconductivity in\textsuperscript{9}. At the same time, any such simulator, once created, possesses its own intricate dynamics (in some sense, trying to live its own life, and not to be a simplistic version of another system). Such type of behavior indeed showed up in the Hamburg experiment on quantum simulation of photoconductivity\textsuperscript{9} recently, giving rise to unexpectedly rich dynamics even in the case of single-component (non-interacting) fermions. We note that dynamics of many-particle fermionic systems is very interesting even in the absence of interactions, and one can indeed study a variety of effects including quantum-classical correspondence, nonlinear waves, etc. For example, in\textsuperscript{10} dynamics of a density pulse induced by a local quench in a one-dimensional electron system was studied. The spectral curvature led to an “overturn” (population inversion) of the wave, after which the density profile developed strong oscillations. Straightforward realization of such a setup in an optical lattice would require a hugely long optical lattice with thousands of sites. However, one may try to realize this phenomenon in a more compact parabolic optical lattice, in which case the effects found in\textsuperscript{9} and studied in a more detail here are relevant. Another interesting topic is the control of matter waves by high-frequency driving. Not only the band dispersion can be modified by driving\textsuperscript{11–13}, but it is also possible to alter e.g. effective interactions in a system of interacting atoms (like exchange coupling between effective spins, etc.\textsuperscript{13–14}). In the context of the photoconductivity simulation experiment\textsuperscript{9}, one may engineer the excited matter wave packet by varying the length and strength of the modulation pulse.

In this paper, we focus on the dynamics of non-interacting fermions in the amplitude modulated lattices. Though examined experimentally by the Hamburg group\textsuperscript{9}, it is not theoretically investigated in sufficient depth. Here we provide some exact numerical results and a semiclassical description. Especially, we reexamine the hole dynamics that prevails after the holes are created by the amplitude modulation. The temperature effect and the anharmonicity of the trap are also taken into account in our numerical simulation. We find that the hole has intrinsically long coherence time.

In the next Section, we give some additional background and describe our system and methods. Section III contains comparison of experimental dynamics with 1D quantum and semiclassical simulations. Section IV studies 3D dynamics, including effects of the temperature and anharmonicity of the trap. Section V contains conclusions.

II. THE SYSTEM

We consider an optical lattice in a parabolic potential filled by spinless or spin-polarized fermions, as in most experiments such as\textsuperscript{9}. Some techniques and notations used in this paper are also available in the experimental work\textsuperscript{9}, and in a recent numerical study\textsuperscript{17} on single-particle dynamics. We use recoil energy \(E_r = \hbar^2 k_r^2 / 2m\) for the unit of energy, recoil momentum \(k_r = 2\pi / \lambda\) for the unit of (quasi-)momentum, lattice constant \(a = 2 / \lambda\) for the unit of length and rescaled time \(t = E_r t' / \hbar\) for the unit of time. Here \(\hbar, \lambda\), and \(m\) correspond to the Planck
constant, wave length of the optical lattice, and mass of the particle, respectively. To reproduce the experiment numerically, we employ a more accurate numerical setup than in [9], extending it to 3D geometry, and partly taking into account temperature effects by preparing initial conditions correspondingly. In accordance with the experimental procedure, we apply a modulation pulse to the lattice which excites part of the atomic population from the lowest to excited bands. After the pulse, the system is left to evolve for a certain time, and then the band mapping procedure is fulfilled by ramping down the lattice. The resulting particle momentum distribution gives insight into the dynamics of fermions after the excitation. Importantly, we can also evaluate the corresponding semiclassical distribution more accurately than previously [9].

The 1D version of the system is described by the time-dependent Hamiltonian

\[
H = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V_0 \sin^2(k_r x')[1 + \epsilon_0(t')] \cos(\omega't') + \frac{1}{2} m_0 \bar{\nu} x^2
\]

where \(V_0\) is the height of the optical lattice, \(\omega'\) is the frequency of the amplitude modulation, \(\epsilon_0(t')\) is the amplitude of the time modulation which, in the present paper, has a square shape in time, and \(\omega_0\) determines the curvature of the trap potential. Rescaling the Hamiltonian, we get

\[
H = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + s \sin^2(x)[1 + \epsilon_0 \cos(E_\omega t)] + \nu x^2
\]

\[
= H_0 + s \sin^2(x) \epsilon_0 \cos(E_\omega t), \tag{1}
\]

where \(x, s, E_\omega\) and \(\nu\) denote \(x = k_r x', s = V_0/E_r, E_\omega = h\omega'/E_r, \nu = m_0 \bar{\nu}^2/2E_r k_r^2\), and \(H_0 = -\frac{\hbar^2}{2m} + s \sin^2(x) + \nu x^2\), respectively. The parameter \(s\) gives the depth of the optical lattice in units of the recoil energy. As is usual in ultracold atom systems, this parameter can be easily controlled, and its typical value in the Hamburger experiment varied in the range of \(s = 2 - 20\).

Since the trap potential varies slowly as a function of \(x\), one can use semiclassical approach based on dispersion of the unperturbed periodic system without a trap [9]. Let us recall the concept of quasimomentum in the uniform lattice system governed by \(H_B = -\frac{\hbar^2}{2m} + s \sin^2(x)\). The eigenstate \(\phi_n^q\) of \(H_B\) corresponding to energy \(E_n^q\) is the Bloch state represented by

\[
\phi_n^q(x) = e^{iqx} \sum_K C_n^0(K, q) e^{2iKx}, \quad (n = 0, 1, 2, ... \tag{2}
\]

with suitable coefficients \(C_n^0(K, q)\) where \(n, q\) and \(K \in \mathbb{Z}\) represent the band index, quasimomentum, and the corresponding reciprocal vector, respectively. The set \(\{\phi_n^q\}\) serves to label the eigenenergy states of \(H\) in the static limit where \(\epsilon_0(t') = 0\). Namely, diagonalizing \(H\) in the basis of the Bloch states, Eq. (2) yields banded eigenenergies and coefficients labeled by quasimomentum \(q\). We call the lowest energy band with the index \(n = 0\) the ground band hereafter.

The experiments of [9] were done both with interacting fermions composed of two-species and with non-interacting fermionic atoms of a single component. Here we consider the latter case, non-interacting fermions. The initial wave function is obtained from the Slater determinant of single-particle wave functions which retains its form under the influence of time-dependent multi-particle Hamiltonian (being composed of time-dependent single particle wave functions). This is because the Hamiltonian involves only single-particle operators. The expectation value of the number operator

\[
\hat{n} = \sum_{j=1}^N \delta(x - x_j)
\]

with respect to the Slater determinant

\[
\Psi(x_1, x_2, \ldots, x_N; t) = \frac{1}{\sqrt{N!}} \prod_{j=1}^N \psi_j(x_j; t)
\]

yields the same result as that of the Hartree product

\[
\Psi_{\text{Hartree}}(x_1, x_2, \ldots, x_N; t) = \prod_{j=1}^N \psi_j(x_j; t)
\]

at all times, that is

\[
\langle \Psi|\hat{n}|\Psi\rangle = \sum_{j=1}^N |\psi_j(x; t)|^2
\]

so that

\[
\int \langle \Psi|\hat{n}|\Psi\rangle \, dx = N,
\]

each single-particle state being normalized to unity. This feature makes it convenient to study many-particle dynamics by preparing \(N\) eigenstates of a single-particle system and propagating them independently in a time-dependent potential, extracting overall density by mere summation of the single-particle densities.

### III. Exact and semiclassical dynamics of fermions following a modulation pulse

Here we reproduce the experimental procedure of [9] numerically, that is, first prepare the ground state of this fermionic system in the combined lattice and parabolic potential at temperature \(T = 0\), excite it with a lattice modulation pulse, let it evolve, and make a band mapping by turning off the lattice potential and trap.

In Fig. (a) we show the energy levels depicted as a function of position for the particular case of \(s = 2\) and \(\nu = 4.49 \times 10^{-5}\). The figure displays the local atomic density \(|\psi_j(x)|^2\) for each energy eigenstate of \(H_0\) pertaining
to eigenenergy $\epsilon_j$, using a darker shade for a higher density. To its right, Fig. 1(b) shows, as a reference the energy bands in the absence of the harmonic confinement as a function of quasimomentum. In order to make a rigorous quantum simulation, we numerically solved the Time-Dependent Schrödinger Equation (TDSE) with the initial Slater determinant, Eq. (3) and the total Hamiltonian $\sum_j H(x_j)$. In this paper, we fix the depth of the optical lattice to $s = 2$, the parabolic trap strength to $\nu = 4.49 \times 10^{-5}$, the modulation amplitude to $\epsilon_0 = 0.4$ and the pulse duration to 0.5ms, respectively, thus following the Hamburg experiment closely.

![FIG. 1: (Color online) (a) Eigenstates of the Hamiltonian $H_0$ displayed at the corresponding energies as functions of position. The bluer, the higher probability density. The lowest eigenenergy is set to 0. The Fermi energy equals 0.97 when $N = 81$ atoms are present. It is thus possible, for instance, to read off that the hole dynamics is constrained to the range of $(-60, 60)$ in position space. (b) Energy band structure of uniform lattice system $H_B$. To create a single hole at $q = 0$ in the ground band, atoms are excited from ground to second band with excitation energy $E_\omega = \Delta E_0^20 = 4.223$.](image)

The parabolic trap varies gradually from site to site, so that we exploit the free band structure Fig. 1(b) for labeling the bands in what follows. On the other hand, the actual numerical calculations are done using the energy eigenstates of $H_0$. The concept of Rabi interband oscillations (oscillations between eigenstates belonging to different bands at the same quasimomentum) proved to be useful in [17], so that we show in Fig. 2 some Rabi frequencies $\Omega$ for the present system. A resonant excitation occurs at such values of $q$, where the band energy difference matches $\hbar \omega'$. The behavior of $\Omega$ over a range of $q$ wider than in the bosonic case is thus relevant for fermions. One observes that the frequency $\Omega$ for the 0th to the second band transition has the largest overall values, and peaks near $q = 0$. The Rabi frequency for the 0th to the first comes next, except that its value plummets exactly to zero at $q = 0$.

In the case of $q = 0$ a single hole is created, while a pair of holes appear in the quasimomentum distribution when the modulation frequency and the inter-band energy difference match at some $q \neq 0$. In the particular case of $s = 2$, a single hole is formed approximately at $\omega' = E_{q=0}/\hbar = 4.223 E_r/\hbar$, so we fix the modulation energy to $E_\omega = 4.223$.

At the moment the amplitude modulation is turned off, a hole of a considerable depth is created in the ground band. The subsequent time evolution of the matter wave packet has rich dynamical features.

The single-particle quasimomentum density distribution is given by the projection of the wave function onto the Bloch states, namely

$$|\Psi_n(q; t)|^2 = \sum_{j=1}^N |\langle \phi_j^n(x) | \psi_j(x; t) \rangle|^2,$$

thus the band population corresponds to $B_n(t) = \sum_q |\Psi_n(q; t)|^2$. The populations of atoms of the first and second bands are shown in Fig. 3 the numbers of atoms are complementary, and the sum remains nearly constant. More precisely, the number of atoms in the ground band fluctuates on the order of $10^{-3}$ due to the coupling between ground and 1st band. This is too small to affect the following discussion.

![FIG. 2: Some Rabi frequencies for ground to first (red solid line), ground to second (green dashed line) and ground to third (blue dashed line) channels as functions of quasimomentum. See Ref. [17] for more detailed discussions.](image)

![FIG. 3: (a) Quasimomentum distributions of the first and second bands. Number of atoms $N = 81$ corresponds roughly to the cubic root of $N_{3d} = 10^7$ atoms in a 3-dimensional trap. Time $t = 0$ corresponds to the beginning of the amplitude modulation; the modulation halts at $t = 0.5 ms$. (b) The band populations of the first and second bands as functions of time.](image)

The hole dynamics in the ground band is almost decoupled from the higher bands and was discussed in [9].
both from semiclassical point of view and via numerical experiments. A somewhat unsatisfactory aspect that remained is that the results of the semiclassical and numerical dynamics deviate from the experimental data on longer timescales. As we show below, semiclassical wave packets can be constructed more accurately, allowing us to observe some new features of the dynamics.

FIG. 4: (a) Time evolution of quasimomentum distribution of the ground band. (b) Differential quasimomentum distribution \( h(q; t) \) of the ground band as a function of time. (c) Quasimomentum distributions at \( t = 0 \text{ms} \) (red solid line) and \( t = 0.5 \text{ms} \) (green dashed line) in the ground band. (d) The relative hole depth \( d(t) \) as a function of time according to the full quantum evolution (‘TDSE’), semiclassical TW A evolution (‘Wigner’), experimental data for \( N = 81, s = 2 \) (‘Experiment’) and previous semiclassical approach (‘Gaussian’).

In Figs. 4(a), (b), we show the results of our numerical experiment corresponding to Figure SF1 in [9]. Fig. 4(a) displays time evolution of quasimomentum distribution of the ground band and (b) shows differential distribution \( h(q; t) = |\Psi_0(q; t = 0)|^2 - |\Psi_0(q; t)|^2 \). As we discussed above, the amplitude modulation with \( E_0 = 4.223 \) makes a single hole at the end of the pulse (Fig. 4(c)). After the modulation, the hole density spreads out in the ground band and comes back to \( q = 0 \) periodically as seen in Fig. 4(b). The experimental paper focused on this revival feature and analyzed it with a semiclassical approach. In Fig. 4(d), we show a relative hole depth \( d(t) \) as a function of time and compare it to the experimental result. Our TDSE approach shows good agreement with the experimental result.

To analyze the dynamics of the hole in detail, let us recall the semiclassical approach used in Ref. 9. A classical Hamiltonian for a single particle in the ground band is approximately given by \( H_c = -J \cos(\pi q) + \nu x^2 \)

where \( J \) is the tunneling parameter for the ground band. \( H_c \) has the same form as that of the nonlinear pendulum (Fig. 5(a)), with the role of the coordinate and momentum exchanged [9, 14]. The dynamics can be divided into two regimes, dipole oscillations corresponding to motion inside the separatrix on the phase portrait, and Bloch oscillations corresponding to motion outside the separatrix. The dipole oscillations contribute to the hole revival, however \( H_c \) does not lead to an isochronous motion, so that perfect hole revivals are prevented. The initial distribution of fermions is approximated by an ensemble of classical phase points enclosed by Fermi energy. Quantum observables are obtained by averaging over this classical ensemble. As a result of the amplitude modulation of the lattice, a hole is formed in the ground band. Ref. 9 approximated the hole by the gaussian
two traveling wave packets. Hole density at 9 ms. However, after that the density behaves as if consisted of

density in the quasimomentum space behaves like gaussian up to position space, and (b) in quasimomentum space. Hole den-

tribution is shown in Fig. 5.

Wigner function according to quantum and classical evo-

sion over tens of milliseconds. The dynamics of

that are almost indistinguishable from the full quantum
cal equations of motion. Such approach produces results
bution, which can be time-propagated using the classi-

them up, and obtains quasiclassical ‘probability’ distri-

the Truncated Wigner Approximation (TWA), one sums

wave packet constructed of single-particle quasimomen-
them description of the hole wave packet, that is the

that resides inside the separatrix. Figs. 8 (b) and

by integrating the Wigner distribution over the quasi-

FIG. 6: Hole density represented in position space can be ob-
tained by integrating the Wigner distribution over the quasi-
momentum coordinate. Likewise for quasimomentum repre-
sentation. Hole density shown at \( t = 0.5, 3, 6 \) and 9 ms (a) in

position space, and (b) in quasimomentum space. Hole den-
sity in the quasimomentum space behaves like gaussian up to
9 ms. However, after that the density behaves as if consisted of
two traveling wave packets. Hole density at \( t = 11, 13, 15 \) and

17 ms shown (c) in position space and (d) in quasimomentum

We also show the time evolution of the hole density in the

position and quasimomentum space in Fig. 6. These

figures clearly show that an initial density profile in the

position space is more rectangular than gaussian. And

this makes the hole behave like a pair of symmetrically

propagating wave packets in phase space. As easily seen,
the present numerical implementation reproduces experi-
mental data better than the previous semiclassical ap-

The single-particle density distribution is represented as

\[
|\Psi(x;t)|^2 = \sum_j |\psi_j(x;t)|^2 \frac{1}{e^{\beta(\epsilon_j - \mu)} + 1} \tag{5}
\]

where \( \beta = \frac{E_r}{k_B T} \), \( \mu \) and \( \epsilon_j \) are scaled inverse tem-

perature, scaled chemical potential, and eigenenergies of

the unperturbed i.e. unmodulated system, respectively.
The scaled chemical potential is calculated by solving

\[
N - \sum_j \frac{1}{e^{\beta(\epsilon_j - \mu)} + 1} = 0.
\]

To compare our results with the experimental ones, we employ the Fermi tempera-
ture \( T_F = \frac{\epsilon_F}{k_B} \) as the unit of temperature, where \( \epsilon_F \)
is the Fermi energy. The results shown in Fig. 7 for this

case reveal modest difference from \( T = 0 \).

To distinguish different roles played by states near the
bottom of the ground band \( q \sim 0 \) and those near the
band edge \( q \sim \pm 1 \), we follow the dynamics of the low and

high energy eigenstates separately, retaining only those

states that reside inside the separatrix. Figs. 8 (b) and

(c) show the subtracted quasimomentum density distribu-
tions constructed for limited ranges of index \( j = 1 - 5 \) and

\( j = 40 - 45 \), respectively. The low energy range

\( (j = 1 - 5) \) mainly consists of Bloch states \( \phi_q \) with \( q \sim 0 \),

IV. DISCUSSIONS: DEPHASING OF THE HOLE DUE TO DIMENSIONALITY, FINITE TEMPERATURE, AND IMPERFECT TRAP

A. Finite temperature effects

We include thermal effects by preparing a correspond-
ing initial state mixture. During and after the modula-
pulse the system is assumed to be decoupled from the
environment, i.e. all thermal effects in the present
consideration are only due to the initial distribution.

The single-particle density distribution is represented as

\[
|\Psi(x;t)|^2 = \sum_j |\psi_j(x;t)|^2 \frac{1}{e^{\beta(\epsilon_j - \mu)} + 1} \tag{5}
\]
thus representing the hole state localized near \(q \sim 0\). The hole revival near \(q \sim \pm 1\) seemingly lags behind because of the location close to the separatrix where the trajectories evolve very slowly (see Fig. 8(c)). In this way, the revival time of the trajectories vary over a range depending on whether they evolve near \(q \sim 0\) or near \(q \sim \pm 1\). The time lag between low and high energy trajectories could be clearly seen in Fig. 8(c). This lag makes the revival tail off so that the hole peaks become asymmetric. Indeed, Fig. 8(a) shows the asymmetry less conspicuous. Indeed, Fig. 8(a) shows the 1D Fermi distribution with \(T=0\) and \(0.2T_F\), the latter clearly indicating the reduced contribution from the high energy region due to thermal excitation.

\[
\Psi(x,y,z,t) = \sum_{j_x,j_y,j_z} |\psi_{j_x}(x;t)\psi_{j_y}(y;t)\psi_{j_z}(z;t)|^2 \times \frac{1}{e^{\beta(\epsilon_{j_x}+\epsilon_{j_y}+\epsilon_{j_z})} + 1}
\]

where the total energy is \(\epsilon_{j_x}+\epsilon_{j_y}+\epsilon_{j_z}\), and the 3D scaled chemical potential is calculated by solving \(N_{3d} - \sum_{j_x,j_y,j_z} e^{\beta(\epsilon_{j_x}+\epsilon_{j_y}+\epsilon_{j_z})} = 0\). Here, we assume for simplicity that the trap is an isotropic 3D parabolic lattice and the amplitude modulation is applied only to the \(x\)-direction. Therefore, the density reduced distribution is given by

\[
|\Psi(x,t)|^2 = \sum_{j_x} W(j_x) |\psi_{j_x}(x;t)|^2,
\]

where the weight function \(W(j_x) = \sum_{j_y,j_z} e^{\beta(\epsilon_{j_x}+\epsilon_{j_y}+\epsilon_{j_z})} + 1\). We set the number of atoms \(N_{3d}\) to \(5.3 \times 10^5\). The weight functions with temperature \(T = 0\) and \(0.2T_F\) are shown in Fig. 9(a). The functions resemble the 1D Fermi distributions, therefore the discussions for the 1D case remain applicable. Note that, in the case of \(T = 0\), the component of high energy region below the separatrix is suppressed. Thus we observe much more robust hole-revival than in the 1D case. However, the results shown in Fig. 9(b) with \(T = 0.2T_F\) are not dramatically different from the 1D case. No dramatic change is thus reproduced by any of these effects arising from the density distribution.

B. Three dimensional effects

Including the 3D effect requires the density of states arising from the other two degrees of freedom, ignored up to now. We extend the 1D finite temperature representation to 3D with eigenfunctions \(\psi_{j_x},\psi_{j_y}\) and \(\psi_{j_z}\), and eigenenergies \(\epsilon_{j_x}^2,\epsilon_{j_y}^2\) and \(\epsilon_{j_z}^2\) for each direction,

\[
|\Psi(x,y,z,t)|^2 = \sum_{j_x,j_y,j_z} |\psi_{j_x}(x;t)\psi_{j_y}(y;t)\psi_{j_z}(z;t)|^2 \times \frac{1}{e^{\beta(\epsilon_{j_x}+\epsilon_{j_y}+\epsilon_{j_z})} + 1}
\]

C. Imperfect trap

We assume the same degree of imperfection may occur in the Hamburg experiment as in the Aarhus experiment [18]. In adding an extra cubic potential term
\( \xi x^3 \) to the Hamiltonian \( H_0 \), we set the cubic parameter \( \xi = 5 \times 10^{-8} \) slightly above the experimental value of \( \xi \sim 2 \times 10^{-8} \) measured in the Aarhus experiment to see how a deformation of the trap potential affects the dynamics of the ground band atoms. Note that, if the cubic potential is used without restriction, it ends up allowing a loss of atoms due to tunneling. However, we diagonalize the Hamiltonian defined in the limited position space \( |x| < \nu/\xi \) so that the potential term is always positive. This insures that the number of atoms is conserved in our numerical simulation. Here we use a 1D system as given by Eq. (5). The results shown in Fig. 10(a) indicate little difference from the perfectly parabolic trap. Although no fundamental difference shows up at \( q = 0 \), we observe some asymmetry induced by the cubic term in the side wings in (b). Here we define \( a(t, q) = |\Psi(t, q)|^2 - |\Psi(t, -q)|^2 \) as a measure of asymmetry. Figs. 10(c) and (d) show the hole depth \( d(t) \) at temperatures \( T = 0 \) and \( 0.2T_F \), respectively. We do not observe any qualitative differences stemming from the cubic parameter because it modifies neither the phase space structure nor the separatrix in the ground band strongly. However, the cubic term \( \xi = 5 \times 10^{-8} \) modifies the phase space structure in the higher bands, and totally breaks symmetric features. Since no such asymmetric features were observed in the Hamburg experiment[9], the cubic parameter would not be bigger than \( \xi = 5 \times 10^{-8} \) in the actual experiment.

V. CONCLUSIONS

In this paper, we reexamined the hole dynamics of a fermionic system in an amplitude modulated parabolic lattice using numerical calculations as well as the refined semiclassical approach. It is remarkable that careful preparation of the semiclassical distribution allows to reproduce the exact quantum dynamics on long timescales using the same classical Hamiltonian as in [9]. Several qualitative features of the dynamics are successfully reproduced. At the same time, decay of the hole wave packet in the experiment happens faster than in our numerical calculations. To clarify what affects the decay of the hole state, we checked three possible causes: finite initial temperature of the system, three-dimensionality of the setup, and anharmonic distortion of the parabolic trap. None of them can explain the remaining discrepancy. We also checked two other causes possibly attributable to the imperfection of the amplitude modulation, namely a mixture of the second harmonics, and a white noise in the excitation frequency. We find that they do not substantially alter the hole dynamics. The discrepancy from the experiment may be caused by loss of atoms or heating by environment during dynamical evolution of the atoms after the pulse. A closer experimental examination might be useful. Numerical results show the hole dynamics has an intrinsically long coherence time in the limit of a perfectly isolated lattice system. It leads to a suggestion that the hole state could be used as a quantum element for an interferometer, as its bosonic counterpart of BEC. Thus the manipulations of the hole state subject to a two-color optical lattice[10] is a possible future extension of this work. Another natural extension is a study of interacting mixture of fermionic atoms[9], and dynamics of nonlinear waves[10] in such systems.

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[2] M.Lewenstein et al., Ultracold Atoms in Optical Lattices:


